Exercise 3.1 Total Q (7–15)

Q:- 7–8 (x_1, y_1) (x_2, y_2)

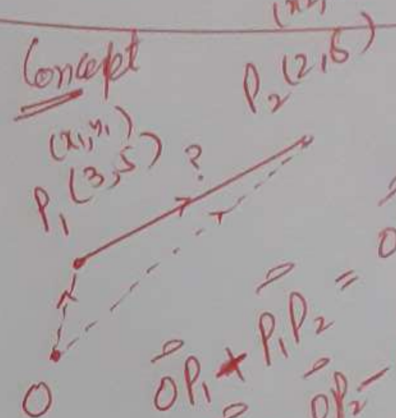
7(a):- $P_1(3,5)$, $P_2(2,8)$

Find the components of the vector $\overrightarrow{P_1P_2}$

$\overrightarrow{P_1P_2} = (x_2 - x_1,\ y_2 - y_1)$

$= (2-3,\ 8-5)$

$= (-1, 3)$



$\overrightarrow{OP_1} + \overrightarrow{P_1P_2} = \overrightarrow{OP_2}$

$\overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1}$

$= (x_2 - x_1, y_2 - y_1)$

Do: Q8

Q(9) (a) $A(1,1)$ $u = (1,2)$ B ?

$x_1\ x_2$, $u_1\ u_2$

$\vec{u} = \overrightarrow{AB}$

$\vec{u} = \overrightarrow{OB} - \overrightarrow{OA}$

$\vec{u} + \overrightarrow{OA} = \overrightarrow{OB}$

$\overrightarrow{OB} = \vec{u} + \overrightarrow{OA}$

$= (u_1 + x_1,\ u_2 + x_2)$

$= (1+1,\ 2+1)$

$\overrightarrow{OB} = (2,3)$

$\Rightarrow B = (2,3)$

OR

Directly

$B = (u_1 + x_1,\ u_2 + x_2)$

$= (1+1, 2+1)$

$= (2,3)$

Do: Q10

(b) A ? $u = (1,1,3)$ $B(-1,-1,2)$

$u_1\ u_2\ u_3$, $y_1\ y_2\ y_3$

$\vec{u} = \overrightarrow{AB}$

$\vec{u} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{OA} = \overrightarrow{OB} - \vec{u}$

$= (y_1 - u_1,\ y_2 - u_2,\ y_3 - u_3)$

$= (-1-1,\ -1-1,\ 2-3)$

$\overrightarrow{OA} = (-2, -2, -1)$

$A = (-2, -2, -1)$

Q(11) (a) Given (i) $\hat{u} = \hat{v}$ (ii) we assume that $|\vec{u}| = |\vec{v}|$ *

Consequently $\vec{u} = \vec{v}$ $\quad$ ($\because \vec{u} = |u|\,\hat{u}$ Definition).

$P(x_1, x_2, x_3) = ?$ $\quad \vec{u}(u_1, u_2, u_3)$ $\quad Q(3, 0, -5)$

$\vec{v} = (4, -2, -1)$

$\vec{u} = \overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$

$\overrightarrow{OP} = \overrightarrow{OQ} - \vec{u} = (3-4,\ 0+2,\ -5+1)$

$\overrightarrow{OP} = (-1, 2, -4) \Rightarrow P = (-1, 2, -4)$ initial point ?

(11) (b) (i) $\vec{u}$ is oppositely directed to v

(2)

$\Rightarrow \quad \hat{u} = -\hat{v}$

(ii) Assume that $|\vec{u}| = |\vec{v}|$

Initial Point = ?

$\Rightarrow \quad \vec{u} = |\vec{u}|\,\hat{u}$

$= |\vec{v}|\,(-\hat{v})$

$\vec{u} = -\vec{v}$

$\Rightarrow \quad \vec{u} = -(4,-2,-1) = (-4,+2,+1)$

P ——— Q(3,0,-5)

u(-4,2,1)

$\vec{U} = \vec{PQ}$

$= \vec{Q} - \vec{P}$

$\vec{P} = \vec{Q} - \vec{u} = (3,0,-5)-(-4,2,1)$

$= (3,0,-5)+(4,-2,-1)$

$= (3+4,\ 0-2,\ -5-1)$

$= (7,-2,-6)$

Do: Q_{12}

Q_{15} $\quad \underline{u} = (-3,2,1,0),\ v=(4,7,-3,2)\ \ w=(5,-2,8,1)$

Compute $(6\underline{v}-\underline{w})-(4\underline{u}+\underline{v})$

$= [6(4,7,-3,2)-(5,-2,8,1)] - [4(-3,2,1,0)+(4,7,-3,2)]$

$= [(24,42,-18,12)-(5,-2,8,1)] - [(-12,8,4,0)+(4,7,-3,2)]$

$= [(24-5,\ 42+2,\ -18-8,\ 12-1] - [-12+4,\ 8+7,\ 4-3,\ 0+2]$

$= (19,44,-26,11)-(-8,15,1,2)$

$= (19+8,\ 44-15,\ -26-1,\ 11-2) = (27,29,-27,9)$

Do: Q_{17}

Exercise 3.2

Q_{1-23} (3)

Q1: (a) $\underline{v} = (4,-3)$

(i) Norm of $\underline{v}$ = $\|\underline{v}\| = \sqrt{(4)^2+(-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$

(ii) Unit Vector that has same direction as of $\underline{v}$

let $\underline{u}$ = required vector

$\underline{u} = |\underline{u}|\,\hat{u}$ (Formula

$= |\underline{u}|\,\hat{v}$ ($\because \hat{u} = \hat{v}$

$= (1)\,\hat{v}$ ($\because$ u is unit $|\underline{u}| = 1$

$= \hat{v}$

$= \dfrac{\underline{v}}{\|\underline{v}\|}$ (Formula)

$= \dfrac{(4,-3)}{\sqrt{16+9}} = \dfrac{(4,-3)}{5} = \left(\dfrac{4}{5}, \dfrac{-3}{5}\right)$

(iii) Unit vector with opposite direction of $\underline{v}$

$\underline{u} = |\underline{u}|\,\hat{u} = |\underline{u}|\,(-\hat{v}) = -(1)\,\hat{v}$

$\underline{u} = -\dfrac{\underline{v}}{\|\underline{v}\|} = -\left(\dfrac{4}{5}, \dfrac{-3}{5}\right) = \left(-\dfrac{4}{5}, \dfrac{3}{5}\right)$

4

Q_3:- $\underline{u} = (2,-2,3)$ $v = (1,-3,4)$ & $w = (3,6,-4)$

(a) $\| \underline{u} + \underline{v} \| = ?$

$\underline{u} + \underline{v} = (2,-2,3) + (1,-3,4) = (2+1, -2-3, 3+4)$

$= (3,-5,7)$

$\| \underline{u} + \underline{v} \| = \sqrt{(3)^2 + (-5)^2 + (7)^2} = \sqrt{9+25+49} = \sqrt{83}$

(b) $\| \underline{u} \| + \| \underline{v} \| = ?$

$= \sqrt{(2)^2 + (-2)^2 + (3)^2} + \sqrt{(1)^2 + (-3)^2 + (4)^2}$

$= \sqrt{4+4+9} + \sqrt{1+9+16}$

$= \sqrt{17} + \sqrt{26}$

(c) $\| -2u + 2v \| = ?$

$-2\underline{u} + 2\underline{v} = -2(2,-2,3) + 2(1,-3,4)$

$= (-4,4,-6) + (2,-6,8)$

$= (-4+2, 4-6, -6+8)$

$= (-2,-2,2)$

$\| -2u + 2v \|$

$= \sqrt{(-2)^2 + (-2)^2 + (2)^2}$

$= \sqrt{4+4+4}$

$= \sqrt{12} = \sqrt{2 \times 3 \times 3}$

$= 3\sqrt{2}$

(d) $\|3u-5v+w\|$

Consider

$$3u-5v+w = 3(2,-2,3) - 5(1,-3,4) + (3,6,-4)$$

$$= (6,-6,9) - (5,-15,20) + (3,6,-4)$$

$$= (6-5+3, -6+15+6, 9-20-4)$$

$$= (4, 15, -15)$$

$$\|3u-5v+w\| = \sqrt{(4)^2+(15)^2+(-15)^2}$$

Do: Q_2, Q_4 ; Q_6

Q_7 Find K

$V = (-2, 3, 0, 6)$

Given $\|KV\| = 5$

$$\|K(-2,3,0,6)\| = 5$$

$$\|(-2K, 3K, 0, 6K)\| = 5$$

$$\sqrt{(-2K)^2+(3K)^2+(0)^2+(6K)^2} = 5$$

$$4K^2+9K^2+36K^2 = (5)^2$$

$$49K^2 = 25$$

$$K^2 = \frac{25}{49}$$

$$K = \sqrt{\frac{25}{49}}$$

$$K = \pm\frac{5}{7}$$ ✓

Do: Q_8