15 April, 2020

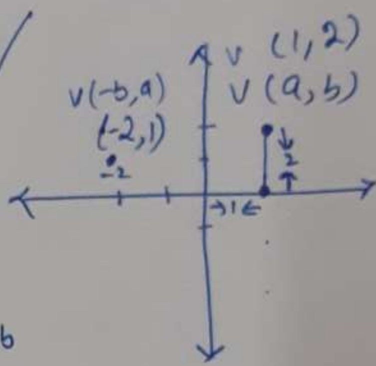
Q[1-40] Exercise 4.3 ①

Q6 ⓐ Show that $v=(a,b)$ & $w=(-b,a)$ are orthogonal vectors

Consider $v\cdot w = (a,b)\cdot(-b,a)$

$= -ab + ba$

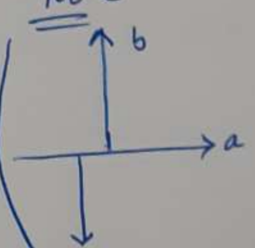
$= 0 \Rightarrow v \perp w$.



ⓑ Use the result in part (a) to find the two vectors that are orthogonal to $v=(2,-3)$

$\underset{a}{2}$, $\underset{b}{-3}$

Note



if $a \perp b$
$\Rightarrow a \perp -b$

using result ⓐ, we have

$$w = (-b, a) = (-(-3), 2) = (3,2)$$

& $-w = (-3,-2)$ Here $v\cdot w = (2,-3)\cdot(3,2)$

$= 6-6 = 0 \Rightarrow v \perp w$

$v\cdot(-w) = (2,-3)(-3,-2)$

$= -6+6$

$= 0 \Rightarrow v \perp -w$

ⓒ Find two unit vectors that are orthogonal to $(-3,4)$

Let ① $\underline{u}=(x,y)$ is orthogonal to $v=(-3,4)$

ⓐ $\underline{u}\cdot\underline{v}=0 \Rightarrow \boxed{-3x+4y=0} \rightarrow ①$

② u is considered as unit $\Rightarrow |u|=1 \Rightarrow \sqrt{x^2+y^2}=1$

$\Rightarrow \boxed{x^2+y^2=1} \rightarrow ②$

From ① $-3x = -4y$ ②

$x = \frac{4}{3}y \xrightarrow{\text{put in } ②}$ ③

$\left(\frac{4}{3}y\right)^2 + y^2 = 1$

$\frac{16}{9}y^2 + y^2 = 1$

$\frac{25}{9}y^2 = 1 \Rightarrow y^2 = \frac{9}{25} \Rightarrow \boxed{y = \pm \frac{3}{5}}$ put in ③

If $y = \frac{3}{5}$ then ③ $\Rightarrow x = \frac{4}{3}\left(\frac{3}{5}\right) = \frac{4}{5} \Rightarrow u = \left(\frac{4}{5}, \frac{3}{5}\right)$

If $y = -\frac{3}{5}$ then ③ $\Rightarrow x = \frac{4}{3}\left(-\frac{3}{5}\right) = -\frac{4}{5} \Rightarrow u = \left(-\frac{4}{5}, -\frac{3}{5}\right)$

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⑦ $A(1,1,1)$
$B(-2,0,3)$
$C(-3,-1,1)$

Form right angle triangle.

$\vec{AB} = (-2-1, 0-1, 3-1) = (-3,-1,2) \Rightarrow |\vec{AB}| = \sqrt{9+1+4} = \sqrt{14}$

$\vec{BC} = (-3+2, -1-0, 1-3) = (-1,-1,-2) \Rightarrow |\vec{BC}| = \sqrt{1+1+4} = \sqrt{6}$

$\vec{CA} = (+3+1, +1+1, 1-1) = (+4, +2, 0) \Rightarrow |\vec{CA}| = \sqrt{16+4+0} = \sqrt{20}$

① $\vec{AB} \cdot \vec{BC} = (-3,-1,2) \cdot (-1,-1,-2) = 3+1-4 = 4-4 = 0$

$\vec{AB} \perp \vec{BC}$

② $|\vec{CA}|^2 = |\vec{AB}|^2 + |BC|^2$

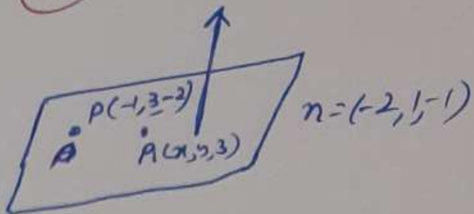
$(\sqrt{20})^2 = (\sqrt{14})^2 + (\sqrt{6})^2$

$\Rightarrow 20 = 14 + 6$

$20 = 20$ holds.

Yes | perform right Angle triangle.

Q9:



let $A(x,y,z)$ be some point on the plane, then

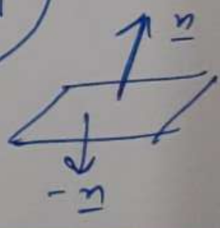
$\vec{PA} = (x+1,\ y-3,\ z+2)$. Moreover $\vec{PA} \perp n$

therefore $\vec{PA} \cdot \bar{n} = 0$

$$-2(x+1) + 1(y-3) + (-1)(z+2) = 0$$
$$-2x - 2 + y - 3 - z - 2 = 0$$
$$-2x + y - z - 7 = 0 \qquad \boxed{-2x+y-z=7}$$
$$\text{or}$$
$$-2x + y - z = 7 \Rightarrow \boxed{2x - y + z = -7}$$

Coeffici

<u>Note</u> $\underline{n} = (-2,1,-1)$
$n = (2,-1,1)$



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(15) Given planes are parallel?

$$2y = 8x - 4z + 5 \qquad\qquad x = \tfrac{1}{2}z + \tfrac{1}{4}y$$
$$\Rightarrow 8x - 2y - 4z + 5 = 0 \qquad\qquad 4x = 2z + y$$
$$8x - 2y - 4z = -5 \qquad\qquad 4x - y - 2z = 0$$
$$\vec{n}_1 = (8,-2,-4) \qquad\qquad \vec{n}_2 = (4,-1,-2)$$

<u>Consider</u>

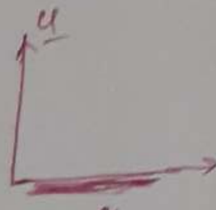
* $\vec{n}_1 \cdot \vec{n}_2 = (8,-2,-4) \cdot (4,-1,-2)$
 $= 32 + 2 + 8 = 42 \neq 0 \quad n_1 \not\perp n_2$ (not perpendicular)

* $n_1 = (8,-2,-4) = 2(4,-1,-2) = 2n_2$
 $\Rightarrow n_1 = 2n_2 \Rightarrow n_1\ \&\ n_2$ are parallel.

Projection of $\underline{u}$ along $\underline{a}$ $= \| Proj_{\vec{a}} \vec{u} \|$

$$= \frac{\vec{u} \cdot \vec{a}}{\|\vec{a}\|} = \frac{(1,-2)\cdot(-4,-3)}{\sqrt{16+9}}$$

$$= \frac{-4+6}{\sqrt{25}} = \frac{2}{5}$$



Extra

Projection of $\underline{a}$ along $\underline{u}$ $= \dfrac{\underline{a}\cdot\underline{u}}{\|\underline{u}\|} = \| Proj_{\vec{u}} \vec{a} \|$

$$= \frac{(-4,-3)\cdot(1,-2)}{\sqrt{(1)^2+(-2)^2}} = \frac{-4+6}{\sqrt{5}}$$

$$= \frac{2}{\sqrt{5}}$$

Note: Both were scalar projections

Vector projection of $\underline{u}$ along $\underline{a}$ $= Proj_{\vec{a}} \vec{u}$

$$= \left(\frac{\vec{u}\cdot\vec{a}}{\|a\|}\right) \underline{a}$$

$$= \left(\frac{(1,-2)\cdot(-4,-3)}{\sqrt{16+9}}\right)(-4,-3)$$

$$= \frac{2}{5}(-4,-3) = \left(\frac{-8}{5}, \frac{-6}{5}\right)$$

Vector projection of $\underline{a}$ along $\underline{u}$ $= proj_{\vec{u}} \vec{a}$

$$= \left(\frac{\vec{a}\cdot\vec{u}}{\|u\|}\right)\vec{u}$$

$$= \frac{2}{\sqrt{5}}(1,-2) = \left(\frac{2}{\sqrt{5}}, \frac{-4}{\sqrt{5}}\right)$$

Vector projection of $\vec{u}$ orthogonal to $\vec{a}$

$$= \vec{u} - \text{proj}_{\vec{a}}\, \vec{u}$$

$$= \vec{u} - \left(\frac{\vec{u}\cdot\vec{a}}{\|\vec{a}\|}\right)\vec{a}$$

$$= (1,-2) - (-8/5, -6/5)$$

$$= (1,-2) + (8/5, 6/5)$$

$$= (1+8/5, -2+6/5)$$

$$= \left(\frac{13}{8}, -4/5\right)$$

Vector projection of $\vec{a}$ orthogonal to $\vec{u}$

$$= \vec{a} - \left(\frac{\vec{a}\cdot\vec{u}}{\|\vec{u}\|}\right)\vec{u}$$

$$= (-4,-3) - \left(\frac{2}{\sqrt{5}}, -\frac{4}{\sqrt{5}}\right) = \left(-4-\frac{2}{\sqrt{5}}, -3+\frac{4}{\sqrt{5}}\right)$$

(1) Distance between two points

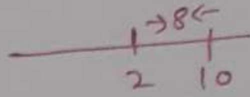
$P(x_1, y_1)$ & $Q(x_2, y_2)$

$$d(P, Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$P(x)$, $Q(y)$

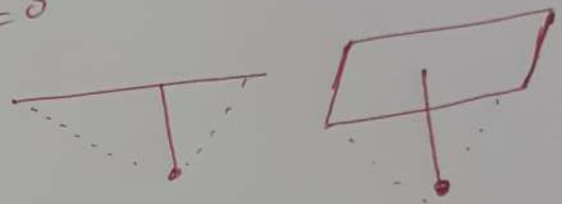
$|PQ| = |y - x|$

$d(P, Q) =$



(2) Distance between a point $P(x_1, y_1)$ and A ~~plan~~ line/plane

$$Ax + By + C = 0$$



$$D = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

$P(x_1, y_1, z_1)$

$Ax + By + Cz + d = 0$

$$D = \frac{|Ax_1 + By_1 + Cz_1 + d|}{\sqrt{A^2 + B^2 + C^2}}$$

(29)

$4x + 3y + 4 = 0$ $\quad$ $P(-3, 1)$

$\underset{A}{4}x + \underset{B}{3}y + \underset{C}{4} = 0$ $\quad$ $P(\underset{x_1}{-3}, \underset{y_1}{1})$

$$D = \frac{|(4)(-3) + (3)(1) + 4|}{\sqrt{(4)^2 + (3)^2}} = \frac{|-12 + 3 + 4|}{\sqrt{16 + 9}} = \frac{|-5|}{\sqrt{25}} = \frac{5}{5} = 1$$

(34)

$2x + 5y - 6z = 4$ & $(-1, -1, 2)$

$\Rightarrow 2x + 5y - 6z - 4 = 0$

$$D = \frac{|(-1)(2) + (-1)(5) + (2)(-6) - 4|}{\sqrt{4 + 25 + 36}} = \frac{|-2 - 5 + 48|}{\sqrt{65}} = \frac{41}{\sqrt{65}}$$

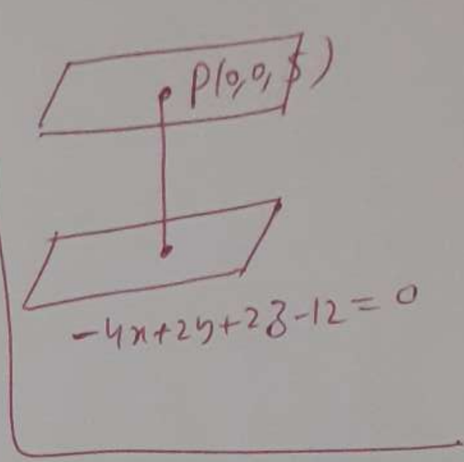
37

8

Distance between two planes

$$2x - y - z = 5 \longrightarrow (1)$$

$$-4x + 2y + 2z = 12 \longrightarrow (2)$$



Point Put $x=0, y=0$ in (1)

$$2(0) - 0 - z = 5 \Rightarrow z = -5$$

$$P(\underset{x_1}{0}, \underset{y_1}{0}, \underset{z_1}{-5})$$

$$(2) \Rightarrow \underset{A}{-4}x + \underset{B}{2}y + \underset{C}{2}z \underset{d}{-12} = 0$$

$$D = \frac{|Ax_1 + By_1 + Cz_1 + d|}{\sqrt{A^2 + B^2 + C^2}}$$

$$= \frac{|(-4)(0) + (2)(0) + (2)(-5) - 12|}{\sqrt{(-4)^2 + (2)^2 + (2)^2}}$$

$$= \frac{|0 + 0 - 10 - 12|}{\sqrt{16 + 4 + 4}} = \frac{|-22|}{\sqrt{24}} = \frac{22}{\sqrt{24}} = \frac{22}{2\sqrt{6}}$$

$$= \frac{11}{\sqrt{6}}$$