Exercise 6.2

Q1-2 : Find the cosine of the angle between the vectors w.r.t Euclidean inner product

(c) $u = (1,0,1,0)$ & $v = (-3,-3,-3,-3)$

$\langle u,v \rangle = (1)(-3) + 0(-3) + 1(-3) + 0(-3)$
$= -3 + 0 - 3 + 0$
$= -6$

$\|u\| = \sqrt{\langle u,u \rangle} = \sqrt{(1)(1)+(0)(0)+(1)(1)+(0)(0)} = \sqrt{2}$

$\|v\| = \sqrt{\langle v,v \rangle} = \sqrt{(-3)(-3)+(-3)(-3)+(-3)(-3)+(-3)(-3)} = \sqrt{9+9+9+9}$
$= 6$

$$\cos\theta = \frac{\langle u,v \rangle}{\|u\| \, \|v\|} = \frac{-6}{\sqrt{2}\,(6)} = \frac{-1}{\sqrt{2}}$$

Q3-4 Find the cosine of the angle between the vectors using standered inner product on P_2

$p = -1 + 5x + 2x^2 \qquad q = 2 + 4x - 9x^2$

$\langle p,q \rangle = (-1)(2) + 5(4) + 2(-9)$
$= -2 + 20 - 18$
$= 0$

$\|p\| = \sqrt{\langle p,p \rangle} = \sqrt{(-1)(-1)+(5)(5)+(2)(2)} = \sqrt{1+25+4} = \sqrt{30}$

$\|q\| = \sqrt{\langle q,q \rangle} = \sqrt{(2)(2)+(4)(4)+(-9)(-9)} = \sqrt{4+16+81} = \sqrt{101}$

$$\cos\theta = \frac{\langle p,q \rangle}{\|p\| \, \|q\|} = \frac{0}{(\sqrt{30})(\sqrt{101})} = 0$$

Q_{5-6}: Find the cosine of the angle between A & B w.r.t standard inner product on M_{22}.

$$A = \begin{bmatrix} 2 & 6 \\ 1 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$$

$$\langle A, B \rangle = (2)(3) + (6)(2) + (1)(1) + (-3)(0)$$
$$= 6 + 12 + 1 - 0$$
$$= 19$$

$$\|A\| = \sqrt{\langle A, A \rangle} = \sqrt{(2)(2) + (6)(6) + (1)(1) + (-3)(-3)}$$
$$= \sqrt{4 + 36 + 1 + 9} = \sqrt{50}$$

$$\|B\| = \sqrt{\langle B, B \rangle} = \sqrt{(3)(3) + (2)(2) + (1)(1) + (0)(0)}$$
$$= \sqrt{9 + 4 + 1 + 0} = \sqrt{14}$$

$$\cos\theta = \frac{\langle A, B \rangle}{\|A\| \|B\|} = \frac{19}{\sqrt{50} \times \sqrt{14}} = \frac{19}{\sqrt{2 \times 5 \times 5}\ \sqrt{2 \times 7}} = \frac{19}{10\sqrt{7}}$$

Q_{7-8} Determine the whether the vectors are orthogonal w.r.t Euclidean inner product

(a) $u = (-1, 3, 2)$ & $v = (4, 2, -1)$

$$\langle u, v \rangle = (-1)(4) + (3)(2) + (2)(-1)$$
$$= -4 + 6 - 2$$
$$= -6 + 6$$
$$= 0$$

Yes $u \perp v$

Q 9-10

Show that the vectors are orthogonal on P_2 w.r.t standard inner product.

$p = -1 - x + 2x^2$ $\quad q = 2x + x^2$

$\langle p, q \rangle = (-1)(0) + (-1)(2) + 2(1)$

$= 0 - 2 + 2$

$= 0$

Yes orthogonal.

Q 11-12 Show that the matrices are orthogonal w.r.t SIP on M_{22}

Q 11 $\quad U = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$ & $V = \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix}$

$\langle U, V \rangle = (2)(-3) + (1)(0) + (-1)(0) + (3)(2)$

$= -6 + 0 + 0 + 6$

$= 0$

$\Rightarrow \quad U \perp V$.

Q 13-14 show that given vectors are not orthogonal by Euclidean inner product.

$u = (1,3)$, $v = (2,-1)$

$\langle u, v \rangle = (1)(2) + (3)(-1)$

$= 2 - 3 = -1 \neq 0$

Yes $u \not\perp v$

Now, Find value of k for which the vectors are orthogonal w.r.t weighted inner product $\langle u, v \rangle = 2u_1v_1 + k u_2 v_2$

Consider

$\langle u, v \rangle = 2(1)(2) + k(3)(-1) = 0$

$4 - 3k = 0$

$-3k = -4$

$k = 4/3$
