One to One:

if $T: V \to W$ is a linear transformation from vector space V to a vector space W Then T is said to be one-to-one if T maps distinct vectors in V into distinct vectors in W

Onto: if $T: V \to W$ is Linear transformation from a vector space V to a vector space W, then T is said to be onto if every vector in W is the image of at least one vector in V

Theorem: if $T: V \to W$ is Linear transformation then following satements are equivalent.

a) T is one-to-one

b) $Ker(t) = \{0\}$

Q Check whether transformation is one-to-one or not

1a) $T: R^2 \to R^3$ where $T(x,y) = (x \cdot y, x+y)$

Sol

$T(x,y) = (x \cdot y, x+y)$

put $T(x,y) = 0$

$T(0,0) = (0\cdot 0, 0+0) = (0,0)$

by Theorem

it $\text{Kernel}(T) = \{0\}$

So $T_1(x,y) = (x\cdot y, x+y)$ is one-to-one.

Q2: $T: R^2 \to R^2$ where

$T(x,y) = (0, 2x+3y)$

for kernel we check $(x,y) = (0,0)$

$T(0,0) = (0, 2(0)+3(0)) = (0,0)$

But if we see here if we put

$(x,y) = \left(-\frac{1}{2}, \frac{1}{3}\right)$

$T\left(-\frac{1}{2}, \frac{1}{3}\right) = \left(0, 2\left(-\frac{1}{2}\right) + 3\left(\frac{1}{3}\right)\right) = (0, -1+1)$

$= (0,0)$

Also $(x,y) = \left(\frac{3}{2}, -1\right)$

$T\left(\frac{3}{2}, -1\right) = \left(0, 2\left(\frac{3}{2}\right) + 3(-1)\right) = (0, 3-3)$

$= (0,0)$ $\quad \text{Ker}(T) = \left\{0, (3/2, -1)\left(-\frac{1}{2}, \frac{1}{3}\right)\right\}$.

So $\text{Ker}(T) \neq \{0\}$ So $T(x,y) = (0, 2x+3y)$ is not one-to-one

3

$$A = \begin{bmatrix} 1 & -2 \\ 2 & -4 \\ -3 & 6 \end{bmatrix}$$

Note:
$T_A(v) = Av$
will be one to one if
$\text{Ker}\,T = \{0\}$ i.e
Nullity = 0

(3)

Sol

solve by Echelon form

$$R_2 - 2R_1 = \begin{bmatrix} 1 & -2 \\ 0 & 0 \\ -3 & 6 \end{bmatrix} \longrightarrow R_3 - 3R_1 = \begin{bmatrix} 1 & -2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Rank(A) = ~~2~~ 1 (No. of Non-zero rows in reduced Echelon form)

Nuallity (A) = No. of columns ~~rows~~ − Rank (A)

Nuallity (A) = 2 − 1 = 1 So not one-to-one.

Q4

$$A = \begin{bmatrix} 1 & 3 & 1 & 7 \\ 2 & 7 & 2 & 4 \\ -1 & -3 & 0 & 0 \end{bmatrix}$$

$T_A(v) = Av$
will be one to one
if $\text{Ker}\,T = \{0\}$
$\Rightarrow$ nullity = 0

$$R_2 - 2R_1 = \begin{bmatrix} 1 & 3 & 1 & 7 \\ 0 & 1 & 0 & -10 \\ -1 & -3 & 0 & 0 \end{bmatrix}$$

$$R_3 + R_1 = \begin{bmatrix} 1 & 3 & 1 & 7 \\ 0 & 1 & 0 & -10 \\ 0 & 0 & 1 & 7 \end{bmatrix}$$

$$R_1 - R_3 = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 1 & 0 & -10 \\ 0 & 0 & 1 & 7 \end{bmatrix}$$

$$R_1 - 3R_2 = \begin{bmatrix} 1 & 0 & 0 & 30 \\ 0 & 1 & 0 & -10 \\ 0 & 0 & 1 & 7 \end{bmatrix}$$

Rank(A) = No. of non-zero rows in echelon form

Rank(A) = 3

Nuallity = No. of ~~rows~~ columns of matrix(A) − Rank (A)

4 − 3 = 1

Nuallity (A) = ~~4 − 3 = 1~~

So its not one-to-one.

<u>Note:</u> If Nullity = 0 then one to one.

Composition

if $T_1 : U \to V$ and $T_2 : V \to W$ are linear transformation then the composition of T_2 with T_1 denoted by $T_2 \circ T_1$ (which is read T_2 circle T_1) is the function defined by formula

$$(T_2 \circ T_1)u = T_2(T_1(u))$$

where u is a vector in U.

Q) Compute $(T_2 \circ T_1)(x,y)$

if $T_1(x,y) = (2x, 3y)$, $T_2(x,y) = (x-y, x+y)$

Sol

$$(T_2 \circ T_1)(x,y) = ?$$

by defi $(T_2 \circ T_1)(x,y) = T_2(T_1(x,y))$

$$= T_2(2x, 3y)$$

$$= (2x-3y, 2x+3y)$$

Q) Compute $(T_2 \circ T_1)(x,y)$

if $T_1(x,y) = (2x, -3y, x+y)$, $T_2(x,y,z) = (x-y, y+z)$

Sol

$$(T_2 \circ T_1)(x,y) = T_2(T_1(x,y)) = T_2(2x, -3y, x+y)$$

$$= ((2x-(-3y)), (-3y+(x+y)))$$

$$= (2x+3y, x-2y)$$

$(T_1 \circ T_1)_{xy} = T_1(T_1(x,y)$ ⑥

$= T_1(T_1(x,y)) = T_1(2x, -3y, x+y)$

$= (2(2x), -3(-3y), 2x-3y)$

$= (4x, 9y, 2x-3y)$

$(T_1 \circ T_2)(x,y) = T_1(T_2(x,y)$ $(2x, -3y, x+y$

$= T_1(x-y, y+z)$

$= (2(x-y), -3(y+z), (x-y)+(y+z))$

$= (2x-2y, -3y-3z, x+z)$

$(T_2 \circ T_2)(x,y) = T_2(T_2(x,y))$

$T_2(x,y) = (x-y, y+z)$

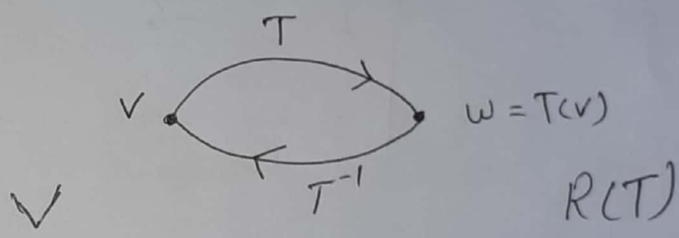
$= T_2(x-y, y+z)$

$= ((x-y)-(y+z), y+z+z)$

$= (x-y-y+z, y+2z) = (x-2y+z, y+2z)$

Inverse Linear Transformation:

if $T: V \to W$ is one-to-one Linear transformation with range $R(t)$ and if w is any vector in $R(T)$. Then The Fact T is one-to-one means that There exists exactly one vector v in V for which $T(v) = w$, This Fact allows us to define a new function called the inverse of T (and denoted by T^{-1}) that is defined on the range of T and that maps w back into v.



Q let

$T: R^3 \to R^3$

be the linear operator defined by formula

$$T(x_1, x_2, x_3) = (3x_1 + x_2, -2x_1 - 4x_2 + 3x_3, 5x_1 + 4x_2 - 2x_3)$$

$$[T] = \begin{bmatrix} 3 & 1 & 0 \\ -2 & -4 & 3 \\ 5 & 4 & -2 \end{bmatrix}$$

we find its inrese by Adjoint Method

$$|T| = 3\begin{vmatrix} -4 & 3 \\ 4 & -2 \end{vmatrix} - 1\begin{vmatrix} -2 & 3 \\ 5 & -2 \end{vmatrix} + 0\begin{vmatrix} -2 & -4 \\ 5 & 4 \end{vmatrix}$$

$$= -12 + 11 + 0$$

$$|T| = -1$$

$$Adj(T) = Adj\begin{bmatrix} 3 & 1 & 0 \\ -2 & -4 & 3 \\ 5 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} \begin{vmatrix} -4 & 3 \\ 4 & -2 \end{vmatrix} & \begin{vmatrix} -2 & 3 \\ 5 & -2 \end{vmatrix} & \begin{vmatrix} -2 & -4 \\ 5 & 4 \end{vmatrix} \\ \begin{vmatrix} 1 & 0 \\ 4 & -2 \end{vmatrix} & \begin{vmatrix} 3 & 0 \\ 5 & -2 \end{vmatrix} & \begin{vmatrix} 3 & 1 \\ 5 & 4 \end{vmatrix} \\ \begin{vmatrix} 1 & 0 \\ -4 & 3 \end{vmatrix} & \begin{vmatrix} 3 & 0 \\ -2 & 3 \end{vmatrix} & \begin{vmatrix} 3 & 1 \\ -2 & -4 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} -4 & 11 & 12 \\ 2 & -6 & -7 \\ 3 & -9 & -10 \end{bmatrix}^T$$

$$Adj(T) = \begin{bmatrix} -4 & 2 & 3 \\ 11 & -6 & -9 \\ 12 & -7 & -10 \end{bmatrix}$$

$$[T^{-1}] = \frac{Adj(T)}{|T|} = \frac{1}{-1}\begin{bmatrix} -4 & 2 & 3 \\ 11 & -6 & -9 \\ 12 & -7 & -10 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -2 & -3 \\ -11 & 6 & 9 \\ -12 & 7 & 10 \end{bmatrix}$$

follows that

$$T^{-1}\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = [T^{-1}]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -2 & -3 \\ -11 & 6 & 9 \\ -12 & 7 & 10 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= \begin{bmatrix} 4x_1 - 2x_2 - 3x_3 \\ -11x_1 + 6x_2 + 9x_3 \\ -12x_1 + 7x_2 + 10x_3 \end{bmatrix}$$

Expressing this result in horizontal notation yields.

$$T^{-1}(x_1, x_2, x_3) = (4x_1 - 2x_2 - 3x_3, -11x_1 + 6x_2 + 9x_3, -12x_1 + 7x_2 + 10x_3)$$

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