Ex 3.2 Total Q:[1-23] 10/4/2020 ①

Q8:-

let $v = (1,1,2,-3,1)$. Find all scalars 'k' such that $\|kv\| = 4$. Since

$$\|kv\| = 4$$

$$\Rightarrow \|k(1,1,2,-3,1)\| = 4$$

$$\|(k,k,2k,-3k,k)\| = 4$$

$$\sqrt{(k)^2+(k)^2+(2k)^2+(-3k)^2+(k)^2} = 4$$

$$k^2+k^2+4k^2+9k^2+k^2 = 16$$

$$16k^2 = 16$$

$$k^2 = \frac{16}{16} = 1 \Rightarrow k^2 = 1 \Rightarrow k = \pm 1$$

Q9 (a) $\vec{u} = (3,1,4)$ $\vec{v} = (2,2,-4)$

① $\vec{u}\cdot\vec{v} = (3)(2)+(1)(2)+(4)(-4)$

② $\vec{u}\cdot\vec{u} = (3,1,4)\cdot(3,1,4)$

$= (3)(3)+(1)(1)+(4)(4)$

$= 9+1+16$

$= 26$

③ $\vec{v}\cdot\vec{v} = (2,2,-4)\cdot(2,2,-4)$

$= (2)(2)+(2)(2)+(-4)(-4)$

$= 4+4+16 = 24$

④ $d(\vec{u},\vec{v}) = \|\vec{u}-\vec{v}\| = \sqrt{(3-2)^2+(1-2)^2+(4+4)^2}$

$= \sqrt{(1)^2+(-1)^2+(8)^2}$

$= \sqrt{1+1+64} = \sqrt{66}$

Formulas

If $U = (u_1,u_2,u_3)$

$V = (v_1,v_2,v_3)$

$\underline{u}\cdot\underline{v} = u_1v_1+u_2v_2+u_3v_3$

Dot product,

Distance

$d(u,v) = \|\underline{u}-\underline{v}\|$

$= \sqrt{(u_1-v_1)^2+(u_2-v_2)^2+(u_3-v_3)^2}$

Do: Q10 ⓑ and Q11 ⓒ

Q12 ⓒ

Q_{13}:- $\underline{u} = (3,3,3)$, $V = (1,0,4)$

<u>Find Cosine of the angle between $\vec{u}$ & $\vec{v}$</u>

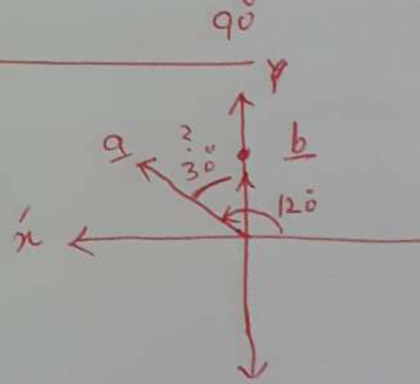
Let 'θ' be the angle between $\vec{u}$ and $\vec{v}$, then Cosine of θ is

$$\cos\theta = \frac{\underline{u}\cdot\underline{v}}{\|\underline{u}\|\|\underline{v}\|} = \frac{(3,3,3)\cdot(1,0,4)}{\sqrt{(3)^2+(3)^2+(3)^2}\sqrt{(1)^2+(0)^2+(4)^2}}$$

$$\cos\theta = \frac{(3)(1)+(3)(0)+(3)(4)}{\sqrt{27}\ \sqrt{17}} = \frac{15}{\sqrt{27}\sqrt{17}} \Rightarrow \cos\theta = \frac{15}{\sqrt{27}\ \sqrt{17}}$$ (Cosine of angle)

$\theta = \cos^{-1}\left(\frac{15}{\sqrt{27}\sqrt{17}}\right) < 90 \Rightarrow \theta$ is acute.

<u>Do Q_{14}:-</u>

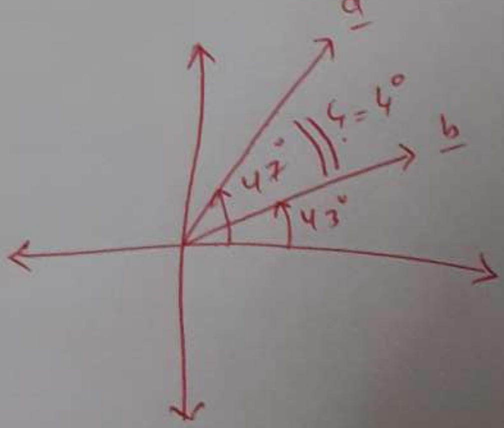


Q_{15}:-

$$\cos\theta = \frac{\underline{a}\cdot\underline{b}}{\|\underline{a}\|\|\underline{b}\|}$$

$\underline{a}\cdot\underline{b} = \|\underline{a}\|\|\underline{b}\|\cos\theta = (9)(5)\cos 30 = 45\frac{\sqrt{3}}{2}$

Q_{16}



$\underline{a}\cdot\underline{b} = \|\underline{a}\|\|\underline{b}\|\cos\theta$
$= ab\cos 4°$

Q_{20}:- (d) Find a unit vector that is oppositely directed to the given vector

$$\underline{v} = (-3, 1, \sqrt{6}, 3)$$

Let $\underline{u}$ be the required unit vector.

$\underline{u} = \|\underline{u}\| \hat{u}$ (By definition vector = magnitude × Direction

$\underline{u} = (1)(-\hat{v})$ ($\because \|\underline{u}\| = 1$ as unit, $\hat{u} = -\hat{v}$ (oppositely to $\underline{v}$))

$\underline{u} = -\hat{v}$

$= -\dfrac{\underline{v}}{\|\underline{v}\|}$ (Formula

$$= -\frac{(-3, 1, \sqrt{6}, 3)}{\sqrt{(-3)^2 + (1)^2 + (\sqrt{6})^2 + (3)^2}} = \frac{(-3, 1, \sqrt{6}, 3)}{\sqrt{9+1+6+9}} =$$

$$= -\frac{(-3, 1, \sqrt{6}, 3)}{\sqrt{25}} = \left(+\frac{3}{5}, -\frac{1}{5}, -\frac{\sqrt{6}}{5}, -\frac{3}{5}\right)$$

DO: Q_{19}, Q_{18}

Q_{21} Let $\vec{u}$ be the required vector of magnitude m and in the direction of the vector $\vec{v}$

Then $\vec{u} = \|\vec{u}\| \hat{u}$

$= \|u\| \hat{v}$

$= m \dfrac{\underline{v}}{\|\underline{v}\|}$

$$\vec{u} = \frac{m\underline{v}}{\|\underline{v}\|}$$

(22) Given $\|\underline{v}\| = 2$

$\|\underline{w}\| = 3$

Largest value of $\|\underline{v} - \underline{w}\| = ?$

Consider

$\|\underline{x}\|^2 = \underline{x} \cdot \underline{x}$ Formula

$$\|\underline{v} - \underline{w}\|^2 = (\underline{v} - \underline{w}) \cdot (\underline{v} - \underline{w})$$
$$= \underline{v} \cdot \underline{v} - \underline{v} \cdot \underline{w} - \underline{w} \cdot \underline{v} + \underline{w} \cdot \underline{w}$$
$$= \|\underline{v}\|^2 - 2\underline{v} \cdot \underline{w} + \|\underline{w}\|^2$$
$$= (2)^2 + (3)^2 - 2\underline{v} \cdot \underline{w}$$
$$= 4 + 9 - 2\|\underline{v}\|\|\underline{w}\| \cos\theta$$
$$= 13 - 2(2)(3)\cos\theta$$
$$= 13 - 12\cos\theta$$

$$\Rightarrow \|\underline{v} - \underline{w}\| = \sqrt{13 - 12\cos\theta}$$

$$-1 \leq \cos\theta \leq 1$$

Largest value of $\|\underline{v} - \underline{w}\| = \sqrt{13 - 12(-1)}$
$$= \sqrt{13 + 12}$$
$$= \sqrt{25} = 5$$

Smallest Value $= \sqrt{13 - 12(1)} = \sqrt{13 - 12} = \sqrt{1}$
$$= 1$$

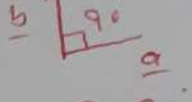
DO: Q23

Exercise 3.3

Total [1–40]

Definition:- Two vectors $\underline{a}$ & $\underline{b}$ are said to be orthogonal if $\underline{a}\cdot\underline{b} = 0$ (i.e $\underline{a} \perp \underline{b}$)

a is perpendicular to $\underline{b}$



$\theta = 90°$

Q1: Is the set of vectors orthogonal?

$S = \{\underline{v}_1 = (2,-2,1),\ \underline{v}_2 = (2,1,-2),\ \underline{v}_3 = (1,2,2)\}$

$\underline{v}_1\cdot\underline{v}_2 = (2,-2,1)\cdot(2,1,-2) = (2)(2)+(-2)(1)+(1)(-2) = 4-4=0$

$\underline{v}_1\cdot\underline{v}_3 = (2,-2,1)\cdot(1,2,2) = (2)(1)+(-2)(2)+(1)(2) = 2-4+2=0$

$\underline{v}_2\cdot\underline{v}_3 = (2,1,-2)\cdot(1,2,2) = (2)(1)+(1)(2)+(-2)(2) = 2+2-4=0$

$\Rightarrow\ v_1 \perp v_2\ ,\ v_1 \perp v_3$ & $v_2 \perp v_3$

$\Rightarrow$ S is set of orthogonal vectors

Do: Q1 & Q3

Q5: Find a unit vector that is orthogonal to both $\underline{u} = (1,0,1)$ & $\underline{v} = (0,1,1)$

Let $\underline{x} = (x_1, x_2, x_3)$ is required vector

$\underline{x}\cdot\underline{u} = 0 \Rightarrow (x_1,x_2,x_3)\cdot(1,0,1) = 0 \Rightarrow x_1 + x_3 = 0$

$\underline{x}\cdot\underline{v} = 0 \Rightarrow (x_1,x_2,x_3)\cdot(0,1,1) = 0 \Rightarrow x_2 + x_3 = 0$

$$\Rightarrow \begin{aligned} 1\cdot x_1 + 0x_2 + 1\cdot x_3 &= 0 \\ 0\cdot x_1 + 1\cdot x_2 + 1\cdot x_3 &= 0 \end{aligned}$$

$$\left|\begin{array}{ccc:c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right|$$

Yes / Yes

Echelon / Reduced Echelon

Leading variables : x_1 & x_2

Free variable : x_3

Put $x_3 = t$

$x_1 + x_3 = 0$

$x_1 + t = 0$

$x_1 = -t$

$x_2 + x_3 = 0$

$x_2 + t = 0$

$x_2 = -t$

$\underline{x} = (x_1, x_2, x_3)$

$= (-t, -t, t) \; : \; t \overset{\neq 0}{\in} R$

<u>Take</u> $t = -1$

$\underline{x} = (1, 1, -1)$

unit vector $= \dfrac{x}{\|\underline{x}\|} = \dfrac{(1,1,-1)}{\sqrt{(1)^2+(1)^2+(-1)^2}} = \dfrac{(1,1,-1)}{\sqrt{3}}$

$= \left(\dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}, \dfrac{-1}{\sqrt{3}}\right)$

<u>Take</u> $t = 1$ Do:YS optional