

# Policy Gradient Methods for Reinforcement Learning with Function Approximation

## Reminders

- REINFORCE algorithm performs Monte Carlo sampling of the stochastic policy, and updates the policy using policy gradient calculated from the partial returns ( $G_t$ ) of episodes
  - Unbiased estimate of the gradient, but slow due to slow
  - Improves greatly aided with value function approximation

## Overview

- Explicitly represent the policy with function approximator (parametrised)
- Update the parameters according to the gradient of expected reward
- Previous work:
  - Approximation of value-function + greedy *deterministic* policy
  - Optimal policy often stochastic and policy highly noise (from v-f) dependent
- Function approximators: e.g. NN with state as input and action probabilities as output
  - Update parameters using the policy gradient:  $\Delta\theta = \alpha \frac{\partial J}{\partial \theta}$
- Here, small changes in  $\theta$  cause only small changes in (stochastic) policy and thus state-visitation distribution, while small changes in v-f with deterministic policy have a larger influence
- Then, two different objectives used:
  - Global average reward:

$$J(\pi) = \lim_{n \rightarrow \infty} E_{\pi} \left[ \sum_{i=1}^N r_i \right] = \sum_s d^{\pi}(s) \sum_a \pi(s, a | \theta) E[r | a, s]$$

- Long-term reward from a start state (**prevalent**):

$$J(\pi) = E_{\pi} \left[ \sum_{t=1}^{\infty} \gamma^{t-1} r_t \mid s_o \right]$$

## Key ingredients

- Convergence of the policy iteration with arbitrary differentiable function approximation (local optimum)
- Policy gradient theorem:

$$\frac{\partial J}{\partial \theta} = \sum_s d^{\pi}(s) \sum_a \frac{\partial \pi(s, a | \theta)}{\partial \theta} Q^{\pi}(s, a)$$

- The effect of policy changes on the state distribution  $\frac{\partial d^{\pi}(s)}{\partial \theta}$  does not appear
  - Convenient for sampling: simply draw samples following  $\pi$  to obtain unbiased estimate  $\frac{\partial \log \pi(s, a | \theta)}{\partial \theta} Q^{\pi}(s, a)$  of the gradient

- Good, but still need to approximate Q!
  - Using actual returns leads to REINFORCE
  - Function approximation for  $Q^\pi$  speeds up learning and gives better performance
  - “Convenient” function approximation  $Q^\omega = \frac{\partial \log \pi(s, a | \theta)^T}{\partial \theta} \omega$  (linear in  $\omega$ ) which we estimate from an unbiased  $Q^\pi$  estimate, e.g. the returns

## Comments

- Well written and easy to follow
- Gives a lot of insights and useful comments on the matter
- Especially found useful comparisons with REINFORCE algorithm and insights about sampling
- Can see strong collaboration influences
- Sutton’s book acts as a great supplement for understanding the paper