# Policy Gradient Methods for Reinforcement Learning with Function Approximation

## Reminders

- REINFORCE algorithm performs Monte Carlo sampling of the stochastic policy, and updates the policy using policy gradient calculated from the partial returns  $(G_t)$  of episodes
  - Unbiased estimate of the gradient, but slow convergence since it uses rollouts instead of function approximation
  - Improves greatly aided with value function approximation

#### Overview

- More focused on discrete actions (examples with softmax)
- Explicitly represent the policy with function approximator (parametrised)
- Update the parameters according to the gradient of expected reward
- Previous work:
  - Approximation of value-function + greedy deterministic policy
  - Optimal policy often stochastic and policy highly noise (from v-f) dependent
- Function approximators: e.g. NN with state as input and action probabilities as output
  - Update parameters using the policy gradient:  $\triangle \theta = \alpha \frac{\partial J}{\partial \theta}$
- Here, small changes in  $\theta$  cause only small changes in (stochastic) policy and thus state-visitation distribution, while small changes in v-f with deterministic policy have a larger influence
- Then, two different objectives used:
  - Global average reward:

$$J(\pi) = \lim_{n \to \infty} \mathbb{E}_{\pi} \left[ \sum_{s=1}^{N} r_{i} \right] = \sum_{s=1}^{N} d^{\pi}(s) \sum_{a=1}^{N} \pi(s, a|\theta) \mathbb{E}[r|a, s]$$

- Long-term reward from a start state (**prevalent**):

$$J(\pi) = \mathbb{E}_{\pi} \left[ \sum_{t=1}^{\infty} \gamma^{t-1} r_t \middle| s_o \right]$$

### **Key ingredients**

- Convergence of the policy iteration with arbitrary differentiable function approximation (local optimum)
- Policy gradient theorem:

$$\frac{\partial J}{\partial \theta} = \sum_{s} d^{\pi}(s) \sum_{a} \frac{\partial \pi(s, a|\theta)}{\partial \theta} Q^{\pi}(s, a)$$

- The effect of policy changes on the state distribution  $\frac{\partial d^{\pi}(s)}{\partial \theta}$  does not appear
  - Convenient for sampling: simply draw samples following  $\pi$  to obtain unbiased estimate  $\frac{\partial \log \pi(s,a|\theta)}{\partial \theta}Q^{\pi}(s,a)$  of the gradient
- Good, but still need to approximate Q!
  - Using actual returns leads to REINFORCE
  - Function approximation for  $Q^{\pi}$  speeds up learning and gives better performance
  - "Convenient" function approximation  $Q^{\omega} = \frac{\partial \log \pi(s, a|\theta)^T}{\partial \theta} \omega$  (linear in  $\omega$ ) which we estimate from an unbiased  $Q^{\pi}$  estimate, e.g. the returns

### Comments

- Well written and easy to follow
- $\bullet\,$  Gives a lot of insights and useful comments on the matter
- Especially found useful comparisons with REINFORCE algorithm and insights about sampling
- Can see strong collaboration influences
- Sutton's book acts as a great supplement for understanding the paper