EXPONENTIAL

Problem: Customers arrive in a certain shop according to an approximate poisson process at a mean rate of 20 per hour. What is probability that shopkeeper will have to wait more than 5 minutes for arrival of customers?

Sol: let x denote the waiting time in minutes until the first customer arrives and note that $\lambda = \frac{20}{60} \Rightarrow \frac{1}{3}$ is expected number of arrival per minutes.

$$\alpha = \frac{1}{3}$$

$$f(x) = \frac{1}{3} e^{-\frac{x}{3}} \qquad \because x \geq 0$$

$$P(x>5) = \int_5^\infty f(x)\,dx = \frac{1}{3}\int_5^\infty e^{-\frac{1}{3}x}\,dx$$

$$= \frac{1}{3}\left\{\frac{e^{-\frac{1}{3}x}}{-\frac{1}{3}}\right]_5^\infty$$

$$= \left(e^{-\frac{5}{3}} - 0\right)$$

$$= e^{-5/3}$$

$$\boxed{P(x>5) = 0.189}$$

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UNIFORM !!

Problem: In a certain experiments, the error made in determining the density of a substance is in random variable having a uniform distribution with $a = -0.015$ and $b = 0.015$

Find the probablities that such an error will:

a) be between -0.002 and 0.003

b) Exceed 0.005 in absolute value.

Sol:

$$X \sim U\left(\frac{a+b}{2}, \frac{(b-a)^2}{12}\right)$$

$$p.d.f \quad f(x) = \frac{1}{b-a} = \frac{1}{0.015+0.015} = \frac{1}{0.03}$$

a) $$P(-0.002 \leq x \leq 0.003) = \frac{1}{0.03}\int_{-0.002}^{0.003} dx$$

$$= \frac{1}{0.03}\left[x\right]_{-0.002}^{0.003}$$

$$= \frac{0.005}{0.03} \Rightarrow \frac{5}{30} \Rightarrow 0.16$$

b) $$P(|x| \geq 0.005) = P(-0.005 \leq x \leq 0.005)$$

$$= \frac{1}{0.03}\int_{-0.005}^{0.005} dx \Rightarrow \frac{1}{0.03}\left[x\right]_{-0.005}^{0.005}$$

$$= \frac{0.010}{0.03} \Rightarrow \frac{1}{3}$$

$$= 0.333\ldots$$

Hypergeometric:

Problem: Among 120 applicants for the post of Computer Programmer only 80 are actually qualified. If 5 of these applicants are randomly selected for an "indepth interview", find the probablity that only 2 of 5 will be qualified for job.

Solution:

Let $x=2$; $n=5$; $N=120$; $k=80$

$$h(2,5,120,80) = \frac{\binom{80}{2}\binom{40}{3}}{\binom{120}{5}} \Rightarrow 0.164$$

Using Binomial Distribution:

$$b\left(2,5,\frac{2}{3}\right) = \binom{5}{2}\left(\frac{2}{3}\right)^2\left(1-\frac{2}{3}\right)^3$$

$$\Rightarrow 0.165$$

Geometric

Problem: Three people toss a coin and the odd man pays for coffee. If coins all turn up same, they are tossed again. Find the probablity that fewer than 4 tosses are needed.

Solution:

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

$$p = \frac{6}{8} \Rightarrow \frac{3}{4}$$

$$\boxed{1-p = q}$$

$$q = \frac{2}{8} \Rightarrow \frac{1}{4}$$

$$P(x<4) = \sum_{x=1}^{3} g(x;p) \Rightarrow \sum_{x=1}^{3} p \cdot q^{x-1} \Rightarrow \sum_{x=1}^{3} \left(\frac{3}{4}\right) \cdot \left(\frac{1}{4}\right)^{x-1}$$

$$= \frac{3}{4}\left[1 + \frac{1}{4} + \frac{1}{16}\right]$$

$$= \frac{21 \times 3}{64}$$

$$\boxed{P(x<4) = \frac{63}{64}}$$

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Poisson Distribution

Q. A secretary makes 2 typing errors per page on average. What is probablity that on next page she makes:

a) 4 or more errors c) no errors.

b) Atleast 2 errors.

Sol:

a) $P(x \geq 4)$

Mean: $\lambda = 2$

$$= \sum_{x=4}^{\infty} P(x;2)$$

$$= 1 - P(x<4)$$

$$= 1 - \sum_{x=0}^{3} P(x;2)$$

$$= 1 - 0.8571$$

$$= 0.1429$$

b) $P(x=0)$

$$P(0;2) = \frac{e^{-\lambda}\lambda^{2}}{\lambda!} \Rightarrow \frac{e^{-2}\,2^{0}}{0!}$$

$$= e^{-2}$$

$$= 0.1353$$

c) $P(x \geq 2)$

$$= 1 - P(x<2)$$

$$= \sum_{x=2}^{\infty} P(X;\lambda)$$

$$= 1 - \sum_{x=0}^{1} P(x;2)$$

$$= 1 - \frac{e^{-2}\,2^{0}}{0!} - \frac{e^{-2}\,2^{1}}{1!}$$

$$= 1 - 0.1353 - 0.2707$$

$$= 0.5940$$

BINOMIAL

Q. The probablity that a computer recovers from a virus attack is 0.4. If 15 computers are known to contracted this disease, what is the probablity:

a) at least 10 survive?
b) From 3 to 8 survive?
c) Exactly 5 survive.
d) Fewer than 5 survive?

Sol:

a) $P(x \geq 10) = 1 - P(x < 10)$

$$= 1 - \sum_{x=0}^{9} b(x; n=15, P=0)$$

$$= 1 - 0.9662$$

$$= 0.033$$

b) $P(3 \leq x \leq 8) = \sum_{x=3}^{8} b(x; n=15, P=0.4)$

$$= \sum_{x=3}^{8} b(x; n=15; P=0.4) - \sum_{x=0}^{2} b(x; n; p=0.4)$$

$$= 0.9050 - 0.027$$

$$= 0.8779$$

c) $P(x=5) = b(x; n=15, p=0.4)$

$$= \sum_{x=0}^{5} b(x; n=15, p=0.4) - \sum_{x=0}^{4} b(x; n, p=0.4)$$

$$= 0.4032 - 0.2173$$

$$= 0.1859$$

d) $P(x < 5) = \sum_{x=0}^{4} b(x; n=15, p=0.4)$

$$= 0.2173$$

Math-Exp:

<u>Problem</u>: Suppose that no. of cars, X, that pass through a car wash between 4:00 pm and 5 pm on any sunny Friday has the following probablity distribution.

X	4	5	6	7	8	9
$P(x)$	$1/12$	$1/12$	$1/4$	$1/4$	$1/6$	$1/6$

Let $g(x) = 2x-1$ represented the amount of money in dollars, paid to the attendent by manager. Find the attendent is expected earning for particular time period.

<u>Sol</u>: By property:

$$E[g(x)] = \sum_{\text{all } x} g(x)\, f(x)$$

$$= \sum_{x=4}^{9} (2x-1)\, f(x)$$

$$= 7\left(\frac{1}{12}\right) + 9\left(\frac{1}{12}\right) + 11\left(\frac{1}{4}\right) + 13\left(\frac{1}{4}\right) + 15\left(\frac{1}{6}\right) + 17\left(\frac{1}{6}\right)$$

$$E[g(x)] = \$12.67$$

RVS - pdf - cdf

Problem: The sample space of a random experiment is (a, b, c, d, e, f) and each outcome is equally likely. A random variable is defined as follows:

outcome	a	b	c	d	e	f
x	0	0	1.5	1.5	2	3

a) $P(X=1.5)$ b) $P(0.5<X<2.7)$ c) $P(x>3)$ d) $P(0\leq x<2)$

e) $P(X=0 \text{ or } X=2)$

Solution:

Probablity mass function is:

$P(X=0) = P(\{a,b\}) = \frac{1}{6} + \frac{1}{6} \Rightarrow \frac{2}{6}$;

$P(X=1.5) = \frac{2}{6}$;

$P(x=2) = \frac{1}{6}$; $P(X=3) = \frac{1}{6}$;

a) $P(X=1.5) = \frac{2}{6}$

b) $P(0.5<X<2.7) = P(X=1.5) + P(X=2) = \frac{3}{6}$

c) $P(X>3) = 0$

d) $P(0\leq X<2) = P(X=0) + P(X=1.5) = \frac{4}{6}$

e) $P(X=0 \text{ or } X=2) = \frac{3}{6}$

NORMAL - DISTRIBUTION

Problem: Suppose the force acting on a column that helps to support a building is normally distributed with mean 15.0 kips and S.D is 1.25 kips, what is probablity that, the force is:

a) at most 17 kips   b) b/w 12 & 17 kips

Solution:

$\mu = 15$; $\sigma = 1.25$

i) $Pr(x \leq 17) = ?$

$$Pr\left(z \leq \frac{17-15}{1.25}\right) = Pr(z \leq 1.6)$$

$$P(z \leq 1.6) = 0.9452$$

ii) $Pr(12 < x < 17) = ?$

$$Pr\left(\frac{12-15}{1.25} < z < \frac{17-15}{1.25}\right)$$

$$Pr(-2.4 < z < 1.6)$$

$$(0.9452 - 0.0082) = 0.937$$

$$Pr(12 < x < 17) = 0.937$$

PROBABLITY DISTRIBUTION

Problem: Suppose that customers arrive at a check-out according to a Poison process with mean $\lambda = 12$ per hour. What is the probablity that we will have to wait longer than 10 minutes to see first customer?

Solution:

$\lambda = 12$ per hr $\qquad \because X =$ number of customers

$T = 10$ minutes $\qquad \Rightarrow X \sim \text{Poisson}\left(\frac{12}{10}\right)$ or $\left(\frac{6}{5}\right)$

Let X be number of arrivals:

$$P_X(k) = \frac{e^{-x} x^k}{k!}$$

$$= \frac{e^{-6/5}\left(\frac{6}{5}\right)^1}{1!}$$

$$\boxed{P(X = 1) \cong 0.36}$$

Test Of Hypothasis

Problem:

A shirt issued for militory has an average life of 85 washings, when used in a moderate climate but will a tropical climate reduces its useful life? A sample of 60 such shirts worn by soldiers in a tropical climate indicates an average life of 76.4 washings, with a standard deviation of 12.8. At 0.01 level of significance can we conclude that the use of shirts in a tropical climate reduces their average useful life?

Sol:

1. Assumption:

2. Hypothesis => $H_0 : \mu = 85$ washings
$H_A : \mu < 85$ washings

3. Level of Significance => $\alpha = 0.01$

4. Test stastic used:

$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

5. Critical Region:

Reject if $Z_{cal} < Z_{\alpha} = -2.31$

6. Computation:

$$Z = \frac{76.4 - 85}{12.8 / \sqrt{60}} = -5.2$$

7. Conclusion:

Reject H_0 and conclude that those shirts in a tropical climate reduces their average useful life.

Gamma Distribution

Problem:

Suppose that on average 30 customer per hour arrive at a shop in accordance with a poisson process. That is, if minute is our unit, then $\lambda = \frac{1}{2}$. What is probablity that shopkeeper will wait more than 5 minutes before both of the first two customers arrive?

Sol:

Let 'X' denotes the waiting time in minutes until second customer arrives. Then

$$X \sim \text{gamma}\left(\alpha = 2, \beta = \frac{1}{\lambda} = 2\right)$$

$$P(x>5) = \int_5^\infty \frac{x^{\alpha-1} e^{-x/\beta}}{\sqrt{\alpha}\, \beta^\alpha} dx$$

$$= \frac{1}{\sqrt{2} \cdot 2^2} \int_5^\infty x^{2-1} e^{-x/2} dx$$

$$= \frac{1}{4} \int_5^\infty x e^{-x/2} dx$$

$$= \frac{1}{4} \left[\frac{xe^{-x/2}}{-\frac{1}{2}} - \frac{e^{-x/2}}{(-\frac{1}{2})(-\frac{1}{2})} \right]_5^\infty$$

$$= \frac{1}{4} \left[-2x e^{-x/2} - 4e^{-x/2} \right]_5^\infty$$

$$\boxed{P(x>5) = \frac{7}{2} e^{-5/2} = 0.287}$$

DISCRETE PROBABLITY DISTRIBUTION

Problem:

A major oil company has decided to drill independent test wells in a region at Sindh. The probablity of any well producing oil is 0.3. Find probablity that fifth well is first to produce oil.

Sol:

$\because P(x=5) = (1-p)^2 p$

$\because p = 0.3$

$= (1-0.3)^2 \times (0.3)$

$= 0.49 \times 0.3$

$$\boxed{P(x=5) = 0.147}$$

Two-Sample Hypothesis Test

Problem:

In a test of reliability of products produced by two machines, machine A produced 15 defective parts in a run of 280, while machine B produced 10 defective parts ports in a run of 200. Do these results imply a difference in reliability of these two machines? (Use $\alpha = 0.01$)

Step-1 → Check Assumptions:

$n_A p_A = y_A = 15 \geq 10$ and $n_A(1-p_A) = n_A - y_A = 265 \geq 10$

$n_B p_B = y_B = 10 \geq 10$ and $n_B(1-p_B) = n_B - y_B = 190 \geq 10$

Step-2 → Hypothesis:

$H_0: \pi_A - \pi_B = 0$

$H_a = \pi_A - \pi_B \neq 0$

Step-3 ⇒ Significance level:

$\alpha = 0.01$

Step-4 ⇒ Test Statistic:

$$p_c = \frac{y_A + y_B}{n_A + n_B} = \frac{15+10}{280+200} = \frac{25}{480}$$

$$Z = \frac{(p_A - p_B) - \delta_0}{\sqrt{p_c(1-p_c)\left(\frac{1}{n_A} + \frac{1}{n_B}\right)}} = \frac{\left(\frac{15}{280} - \frac{10}{200}\right) - 0}{\sqrt{\left(\frac{25}{480}\right)\left(\frac{455}{480}\right)\left(\frac{1}{280} + \frac{1}{200}\right)}} = 0.1736$$

$p\text{-value} = 2 * P(z \geq 0.1736) = 2 * P(z \geq 0.17) = 0.8650$

Step-5 ⇒ Conclusion:

Since $-2.58 \leq 0.1736 \leq 2.58$, we failed to reject null hypothesis.