Normal Distribution:

The most important continuous Probability distribution used in the entire field of Statistics is the normal distribution. It's graph, called the normal curve, is a bell-shaped curve that extends (as shown in fig.) indefinitely in both directions. It is also known as Gaussian distribution

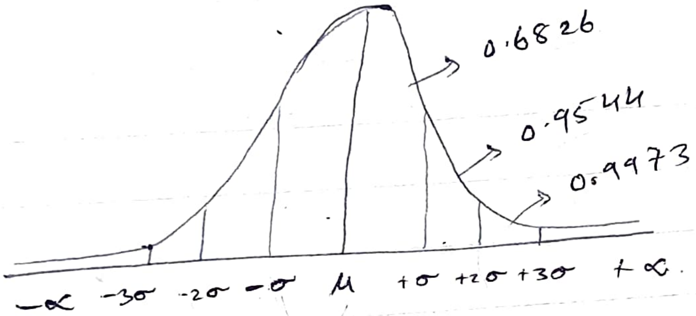
Defⁿ: A continuous random variable X is said to be normally distributed if it has the probability density function represented by

$$f(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \; ; \quad -\infty < x < \infty$$

$$-\infty < \mu < \infty$$

$$\sigma > 0$$

It's



Properties:

i) The curve is bell-shaped and symmetrical about the line $x = \mu$

ii) Mean, Median and mode of the dist coincide

iii) The max. prob occuring at point $x = \mu$ and $= \frac{1}{\sigma\sqrt{2\pi}}$

iv) $\beta_1 = 0, \quad \beta_2 = 3$

v) $\mu_{2n+1} = 0 \qquad (n = 0, 1, 2, \cdots)$

vi) $\mu_{2n} = 1 \cdot 3 \cdot 5 \cdots (2n-1)\,\sigma^{2n} \qquad (n = 1, 2, \cdots)$

vii) Point of inflextion of the curve are given by

$x = \mu \pm \sigma$, $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \bar{e}^{x^2/2}$

N(0,1).

Show that the normal distribution

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} ; \quad -\infty < x < \infty . \quad = f(x;\mu,\sigma$$

is a continuous probability distribution.

Proof :1;

By definition if $f(x)$ is a p.d.f then

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

Substitute $Z = (x-\mu)/\sigma \Rightarrow x = \mu + \sigma z$

$dx = \sigma dz.$

when $x = -\infty \Rightarrow z = -\infty$

$x = +\infty \Rightarrow z = +\infty$

$$I = \frac{1}{\sqrt{2\pi}\,\cancel{\sigma}} \int_{-\infty}^{\infty} e^{-\frac{z^2}{2}} . \cancel{\sigma} dz$$

$$I = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{z^2}{2}} dz$$

$$= 2 . \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-\frac{z^2}{2}} dz.$$

Let $u = \frac{1}{2} z^2$

$du = 2 . \frac{1}{2} z\,dz$

$du = z\,dz.$

$dz = \frac{du}{z}$

$u = \frac{1}{2} z^2$

$z = \sqrt{2u}.$

$$= \frac{\sqrt{2}}{\sqrt{\pi}} \int_{0}^{\infty} e^{-u} . \frac{du}{\sqrt{2u}}$$

$$= \frac{1}{\sqrt{\pi}} \int_{0}^{\infty} e^{-u} . u^{-1/2} du.$$

$$= \frac{1}{\sqrt{\pi}} \int_{0}^{\infty} e^{-u} u^{1/2 - 1} du.$$

$$= \frac{1}{\sqrt{\pi}} . (\Gamma_{1/2})$$

$$= \frac{1}{\sqrt{\pi}} . \sqrt{\pi}$$

$$= 1$$

Proved

Area Under the Normal Curve.

$$P(a \le X \le b) = \int_a^b f(x)\,dx.$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma}\int_a^b e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$ ile

Let $z = \frac{X-\mu}{\sigma} \Rightarrow dz = \frac{dx}{\sigma}$

$$= \frac{1}{\sqrt{2\pi}\,\cancel{\sigma}}\int_{\frac{a-\mu}{\sigma}}^{\frac{b-\mu}{\sigma}} e^{-\frac{1}{2}z^2}\cdot\cancel{\sigma}\,dz.$$

$$= \frac{1}{\sqrt{2\pi}}\int_{\frac{a-\mu}{\sigma}}^{\frac{b-\mu}{\sigma}} e^{-\frac{1}{2}z^2}\,dz.$$

By using Probability integral also called Laplace function

$$\phi(z) = \int_0^z f(t)\,dt = \frac{1}{\sqrt{2\pi}}\int_0^z e^{-t^2/2}\,dt.$$

$$\boxed{P(a \le x \le b) = \phi\left(\frac{b-\mu}{\sigma}\right) - \phi\left(\frac{a-\mu}{\sigma}\right)}$$

Features of Laplace's function

i) $\phi(-z) = -\phi(z)$

ii) $\phi(\infty) = \frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-t^2/2}\,dt = \frac{1}{2}$

iii) $F(X) = \int_{-\infty}^{x} f(x)\,dx = \frac{1}{2} + \phi\left(\frac{x-\mu}{\sigma}\right)$

cdf or distribution Function;

$$F(x) = \int_{-\infty}^{\infty} f(x)\,dx.$$

$$= \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx \;; \qquad \sigma > 0.$$

$$-\infty < \mu < \infty.$$

Consider the transformation.

$$Z = \frac{(X-\mu)}{\sigma} \Rightarrow dz = \frac{dx}{\sigma} \quad \text{or} \quad X = \mu + \sigma Z.$$

when $X \to -\infty \Rightarrow Z \to -\infty$

$X \to \infty \Rightarrow Z \to \infty.$

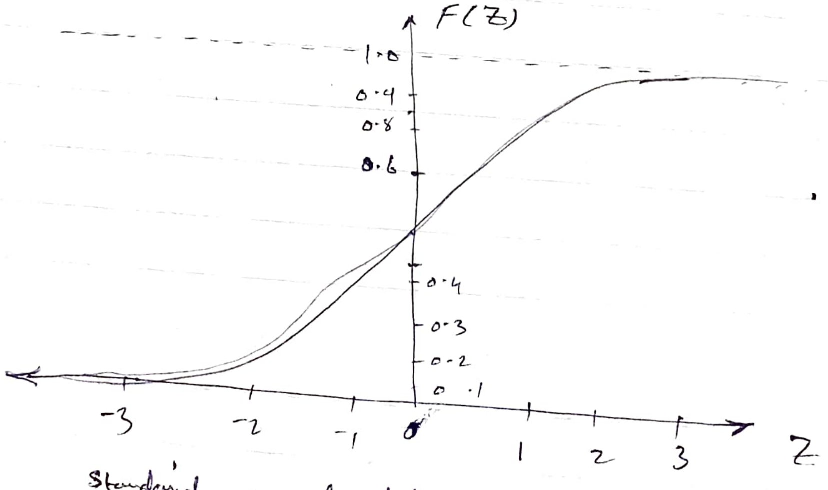
$$\Rightarrow F(Z) = \int_{-\infty}^{Z} \frac{1}{\sqrt{2\pi}\,\cancel{\sigma}} \cdot e^{-\frac{1}{2}z^2} \cdot \cancel{\sigma}\, dz$$

$$\boxed{F(Z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}z^2}\, dz.}$$

The transform function is of the form.

$$f(z) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}z^2} \;; \quad -\infty < Z < \infty.$$

where Z is distributed normally with mean zero and variance unity. i.e $Z = N(0,1)$.



Standard normal dist function.

Moments of the Normal Distribution :

Mean

$$\mu_1' = E(x) = \int_{-\infty}^{\infty} x \cdot \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Let $z = \frac{x-\mu}{\sigma} \Rightarrow dx = \sigma dz$ or $x = \mu + \sigma z$

$$\therefore \mu_1' = \int_{-\infty}^{\infty} (\mu + \sigma z) \cdot \frac{1}{\sqrt{2\pi}\,\cancel{\sigma}} e^{-\frac{z^2}{2}} \cdot \cancel{\sigma} dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty} \mu \cdot e^{-\frac{z^2}{2}} dz + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sigma z\, e^{-\frac{1}{2}z^2} dz$$

$$= \mu \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz + \frac{\sigma}{\sqrt{2\pi}} \underbrace{\int_{-\infty}^{\infty} z\, e^{-\frac{1}{2}z^2} dz}_{\text{integral is odd.}}$$

$$= \mu \cdot 1 + 0.$$

$$\boxed{\mu_1' = \mu. = \text{Mean}}$$

$$\mu_2' = E(x^2) = \int_{-\infty}^{\infty} x^2 \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx.$$

$$= \int_{-\infty}^{\infty} (\mu + \sigma z)^2 \cdot \frac{1}{\sqrt{2\pi}\,\cancel{\sigma}} e^{-\frac{1}{2}z^2} \cdot \cancel{\sigma} dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (\mu^2 + 2\mu\sigma z + \sigma^2 z^2)\, e^{-\frac{1}{2}z^2} dz.$$

$$= \mu^2 \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz \qquad \sim \mu^2 \cdot 1$$

$$+ \frac{2\mu\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z\, e^{-\frac{1}{2}z^2} dz \qquad -(i) \qquad \sim 0$$

$$+ \frac{\sigma^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^2 e^{-\frac{1}{2}z^2} dz \qquad -(ii)$$

$\int_{-\infty}^{+\infty} z^2 e^{-1/2 z^2} dz$ let $y = \frac{z^2}{2}$

$dy = \frac{1}{2} \cdot 2z dz$

$= z dz.$

$= \int_{-\infty}^{\infty} (2y) e^{-y} \frac{dy}{\sqrt{2y}}$

$= 2\sqrt{2}\int_{0}^{\infty} \sqrt{y}\, e^{-y} dy.$

$= 2\sqrt{2} \int_{0}^{\infty} y^{\frac{3}{2}-1} e^{-y} dz.$ $\because \int_{0}^{\infty} x^{\alpha-1} e^{-x} dx = \Gamma\alpha.$

$= 2\sqrt{2}\, \Gamma\frac{3}{2}$ $\Gamma\frac{3}{2} = \frac{1}{2}\Gamma\frac{1}{2} = \frac{1}{2} \cdot \sqrt{\pi}$

$= \frac{\sqrt{\pi}}{2}.$

$= \cancel{2}\sqrt{2} \cdot \frac{\sqrt{\pi}}{\cancel{2}}$

$= \sqrt{2\pi}$

$\Rightarrow \mu_2' = \mu^2 + 0 + \frac{\sigma^2}{\sqrt{2\pi}} \cdot \sqrt{2\pi}$

$\boxed{\mu_2' = \mu^2 + \sigma^2}$

$Var(x) = \sigma^2 = \mu_2' - \mu_1'^2$

$= \mu^2 + \sigma^2 - \mu^2$

$\boxed{Var(x) = \sigma^2}$

n Moments :

odd order moments about mean :

$$\mu_{2n+1} = \int_{-\infty}^{\infty} (x-\mu)^{2n+1} f(x)\,dx$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{\infty} (x-\mu)^{2n+1} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

let $z = \frac{x-\mu}{\sigma}$

$x = \mu + \sigma z$

$dx = \sigma dz.$

$$= \frac{1}{\sqrt{2\pi}\,\cancel{\sigma}} \int_{\infty}^{\infty} (\sigma z)^{2n+1} e^{-\frac{z^2}{2}} \cdot \cancel{\sigma} dz.$$

$$= \frac{\sigma^{2n+1}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^{2n+1} e^{-\frac{z^2}{2}} dz.$$

$\because$ integral is an odd function

$$= 0.$$

Even order moments about mean :

$$\mu_{2n} = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{\infty} (x-\mu)^{2n} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$z = \frac{x-\mu}{\sigma}$

$dz = \frac{dx}{\sigma}$

$$= \int_{-\infty}^{\infty} (\sigma z)^{2n} \frac{e^{-\frac{z^2}{2}}}{\sqrt{2\pi}\,\cancel{\sigma}} \cancel{\sigma} dz.$$

$$= \frac{\sigma^{2n}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^{2n} e^{-\frac{1}{2}z^2} dz$$

Let $y = \frac{z^2}{2}$

$dy = z\,dz$

$z = \sqrt{2y}.$

$\boxed{z^2 = 2y}$

$$= \frac{\sigma^{2n}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (2y)^n e^{-y} \cdot \frac{dy}{\sqrt{2y}}$$

$$= \frac{\sigma^{2n} \cdot 2^n}{\sqrt{\pi}\, 2} \int_{-\infty}^{\infty} \cdot y^{(n+\frac{1}{2})-1} \cdot e^{-y} dy$$

$$= \frac{\sigma^{2n}\, 2^n \cdot \cancel{2}}{\sqrt{\pi}\, \cancel{2}} \int_{0}^{\infty} y^{(n+\frac{1}{2})-1} e^{-y} dy \quad \text{—} \ ①$$

$$\boxed{\mu_{2n} = \frac{\sigma^{2n}\, 2^n}{\sqrt{\pi}} \cdot \Gamma\left(n+\tfrac{1}{2}\right)}$$

$$\mu_{2n} = \frac{\sigma^{2n}\, 2^{n}}{\sqrt{\pi}} \Gamma\left(n+\tfrac{1}{2}\right)$$

hanging n to $(n-1)$, we get

$$\mu_{2n-2} = \frac{2^{n-1}\sigma^{2n-2}}{\sqrt{\pi}} \Gamma\left(n-\tfrac{1}{2}\right) \quad — ②$$

Then

$$\frac{\mu_{2n}}{\mu_{2n-2}} = 2\sigma^{2}\cdot\frac{\Gamma\left(n+\frac{1}{2}\right)}{\Gamma\left(n-\frac{1}{2}\right)} = 2\sigma^{2}\cdot\left(n-\tfrac{1}{2}\right)$$

$$\mu_{2n} = 2\sigma^{2}\left(n-\tfrac{1}{2}\right)\cdot\mu_{2n-2}$$

$$\boxed{\mu_{2n} = \sigma^{2}(n-1)\cdot\mu_{2n-2}}$$

Which is the recurrence relation for mean moments of normal dist.

M.g.f :

$$M_X(t) = E[e^{tx}] = \int_{-\infty}^{\infty} e^{tx} f(x)\, dx.$$

$$M_X(t) = \int_{-\infty}^{\infty} e^{tx}\cdot\frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^{2}} dx$$

let $z = \frac{x-\mu}{\sigma}$

$x = \mu + \sigma z$

$dx = \sigma dz$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\cancel{\sigma}}\cdot e^{(\mu+\sigma z)t}\, e^{-\frac{1}{2}z^{2}}\cdot\cancel{\sigma}\, dz$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{\mu t}\cdot e^{-\frac{z^{2}}{2}+\sigma z t}\, dz$$

$$= \frac{e^{\mu t}}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{1}{2}(z^{2}-2\sigma z t)}\, dz$$

$$M_x(t) = \frac{e^{\mu t}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left[(z-\sigma t)^2-\sigma^2t^2\right]} dz$$

$$= \frac{e^{\mu t+\frac{\sigma^2t^2}{2}}}{\sqrt{2\pi}} \cdot \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z-\sigma t)^2} dz$$

$$= \frac{e^{\mu t+\frac{1}{2}\sigma^2t^2}}{\sqrt{2\pi}} \cdot \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z-\sigma t)^2} dz.$$

Let $z-\sigma t = w$

$dz = dw$

$$= \frac{e^{\mu t+\frac{1}{2}\sigma^2t^2}}{\sqrt{2\pi}} \cdot \int_{-\infty}^{\infty} e^{-\frac{1}{2}w^2} dw$$

$$= \frac{e^{\mu t+\frac{1}{2}\sigma^2t^2}}{\cancel{\sqrt{2\pi}}} \cdot \cancel{\sqrt{2\pi}}$$

$$\boxed{M_x(t) = e^{\mu t+\frac{1}{2}\sigma^2t^2}}$$

$$\mu_r' = \frac{d^r}{dt^r}\left[M_x(t)\right]_{t=0}$$

$$\mu_1' = e^{\mu t+\frac{1}{2}\sigma^2t^2}\left(\mu+\sigma^2t\right)\Big|_{t=0}$$

$$\boxed{\mu_1' = \mu}$$

$$\mu_2' = \frac{d^2M_x(t)}{dt^2} = e^{\mu t+\frac{\sigma^2}{2}t^2} \cdot \sigma^2 + (\mu+\sigma^2t)\,e^{\mu t+\frac{\sigma^2}{2}t^2}\Big|_{t=}$$

$$\boxed{\mu_2' = \sigma^2+\mu^2}$$

$$\mu_2 = \mu_2' - \mu_1'^2 = \sigma^2+\mu^2-\mu^2 = \boxed{\sigma^2 = \mu_2}\ \mathrm{Var}(x)$$

ulant Generating Function of Normal Dist

$K_X(t) = \log_e M_x(t)$

$= \log_e \left[e^{\mu t + \frac{1}{2}\sigma^2 t^2} \right]$

$$\boxed{K_x(t) = \mu t + \frac{1}{2}\sigma^2 t^2}$$

Comparing the coefficients of $\frac{t^r}{r!}$ we get.

$K_1(t) = K_1 = \mu$

$K_2(t) = \boxed{K_2 = \sigma^2}$

$K_3(t) = 0.$

$K_4(t) = 0.$

$\mu_4 =$ ~~$K_4(t)$~~ $K_4 + 3K_2^2$

$$\boxed{\mu_4 = 3\sigma^4}$$

lem : Show that the normal distribution has a relative maximum at $x = \mu$ and inflection Points at $x = \mu - \sigma$ and $x = \mu + \sigma$.

Sol :

$\because$ we know that mode is the solution of

$$f'(x) = 0 \quad \text{and} \quad f''(x) < 0.$$

As P.d.f of $N(\mu, \sigma^2)$ is

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad ; \quad \begin{array}{l} -\infty \leq x \leq \infty \\ -\infty \leq \mu \leq \infty \\ \sigma > 0 \end{array}$$

Taking logarithms.

$$\log f(x) = \log \frac{1}{\sqrt{2\pi}\,\sigma} - \frac{1}{2\sigma^2}(x-\mu)^2$$

$$\log f(x) = c - \frac{1}{2\sigma^2}(x-\mu)^2$$

Differentiating w.r.t $x \Rightarrow$.

$$\frac{1}{f(x)} \cdot f'(x) = -\frac{2}{2\sigma^2}(x-\mu) \times 1 = 0.$$

$$f'(x) = -\frac{1}{\sigma^2}(x-\mu) f(x) = 0. \qquad \text{—— ①}$$

$$= (x-\mu) = 0 \Rightarrow \boxed{x = \mu}$$

Differentiating Again ① $\Rightarrow$.

$$f''(x) = \frac{d}{dx}[f'(x)] = \frac{d}{dx}\left[-\frac{1}{\sigma^2}(x-\mu) f(x)\right]$$

$$= -\frac{1}{\sigma^2}\left[f(x) + f'(x)(x-\mu)\right]$$

$$= -\frac{1}{\sigma^2}\left[f(x) - \frac{1}{\sigma^2}(x-\mu)^2 f(x)\right]$$

$$f''(x) = -\frac{f(x)}{\sigma^2}\left[1 - \frac{(x-\mu)^2}{\sigma^2}\right] < 0 \qquad \text{—— ②}$$

Point of inflection are at.

$$(x-\mu)^2 = \sigma^2$$

$$\boxed{x = \mu \pm \sigma}$$

stributing the value of X (= μ) from eq ① in eq ②

$$\Rightarrow f''(x) = \frac{-f(x;\mu,\sigma)}{\sigma^2}$$

$$= -\frac{1}{\sqrt{2\pi}\,\sigma^3}\, e^{-0}$$

$$= -\frac{1}{\sqrt{2\pi}\,\sigma^3} < 0$$

Hence $X = \mu$ is the mode of the normal dist.

Median:

$$\int_{-\infty}^{M} f(x)\,dx = \frac{1}{2} \qquad \text{(By definition)}$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma}\int_{-\infty}^{M} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = \frac{1}{2}$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma}\left[\int_{-\infty}^{\mu} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx + \int_{\mu}^{M} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx\right] = \frac{1}{2}$$

But.

$$\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = \frac{1}{\sqrt{2\pi}}\int_{0}^{\infty} e^{-\frac{1}{2}z^2} dz. \Leftarrow$$

$$\therefore \quad = \frac{1}{2} + \frac{1}{\sqrt{2\pi}\,\sigma}\int_{\mu}^{M} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{2} + 0 \qquad \because \frac{1}{\sqrt{2\pi}\,\sigma}\int_{\mu}^{M} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = 0$$

$$= \frac{1}{2} // \qquad \Rightarrow \boxed{\mu = M}$$

Hence $\boxed{\text{Mean} = \text{Median} = \text{Mode}}$

Deviation about Mean:

$$M.D_{\bar{x}} = \int_{-\infty}^{\infty} |x-\mu| \, f(x) \, dx.$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{\infty} |x-\mu| \, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Let $z = \frac{x-\mu}{\sigma}$

$dx = \sigma dz$

$$= \frac{1}{\sqrt{2\pi}\,\cancel{\sigma}} \int_{-\infty}^{\infty} |\sigma z| \, e^{-\frac{1}{2}z^2} \cancel{\sigma} \, dz$$

$$= \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} |z| \, e^{-\frac{1}{2}z^2} dz.$$

(as $|z| e^{-\frac{1}{2}z^2}$ is an even function

$|z| = z$; $0 < z < \infty$.

$$= \frac{\sigma}{\sqrt{2\pi}} \, 2\int_{0}^{\infty} z e^{-\frac{1}{2}z^2} dz.$$

$$= \sqrt{\frac{2}{\pi}} \, \sigma \left(- e^{-y} \Big|_0^{\infty} \right).$$

$$= \sqrt{\frac{2}{\pi}} \, \sigma$$

$$\boxed{M.D_{\bar{x}} = \frac{4}{5} \sigma}$$

lem: An electrical firm manufactures light bulbs hat have a length of life that is approximately normally distributed, with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a random sample of 16 bulbs will have an average life less tha 775 hours.

Sol:

The sampling dist of $\bar{X} \sim N(\mu_{\bar{X}}, \sigma_{\bar{X}}^2)$

$\mu_{\bar{X}} = 800$ hours

$\sigma_X = 40 \qquad \sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{40}{\sqrt{16}} = 10$

Thus $Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{775 - 800}{10} = -2.5$

$P(\bar{X} < 775) = P(Z < -2.5)$

$= 0.0062$