STATISTICS AND DATA ANALYSIS:-

DATA TYPES:-

Quantitative
Ratio ⟷ Interval

Quantitative		Qualitative	
Discrete	Continuous	Nominal	ORdinal
↓	↓	↓	↓
counting	any value	without order	with order
whole numbers	hight, weight	Hair color	Grade (A,B,C)
etc	etc	favourite animal	Time of day
		etc	Financial status

FREQUENCY DISTRIBUTION:-

Frequency Distribution in statistics is a representation of observations with in a given interval.

Types of frequency Distribution:-

1. Grouped Frequency Distribution.
2. Ungrouped Frequency Distribution.

For Example of both Type Sample data:
Marks obtained by 20 Students: 5, 10, 20, 15, 5, 20, 20, 15, 15, 15, 10, 10, 10, 20, 15, 5, 18, 18, 18, 18

Distribution

Grouped Frequency Table:

To arrange a large no of Observations or data, we use grouped frequency distribution table. In this we form class Intervals to tally the frequency for the data the belongs to that particular class Interval.

★ Number of classes

$N = 20 \dashrightarrow$ No of students

$\because m = 1 + 3.3\,\log_{10} N$

$m = 1 + 3.3\,\log_{10} 20$

$\boxed{m \cong 5}$

★ Range

$\because R = max - min$

$R = 20 - 5$

$\boxed{R = 15}$

★ Class Interval

$CI = R/m = 15/5 =$

$\boxed{CI = 3}$

Obtained Marks (Class Intervals	No of Students frequency
5 - 7	3
8 - 10	4
11 - 13	0
14 - 16	5
17 - 19	4
20 - 22	4
Total	20

Ungrouped Frequency Distibution Table.

In Ungroup Frequency table we dont make classes Intervals. we write the accurate frequency of Individual data. or you can say we write frequency of a category.

Marks Obtained	No Students
5	3
10	4
15	5
18	4
20	4
Total	20

LAB I

a) Collect the data set in a tabular form. Done

b) Construct a frequency distribution of height / weight data

c) Using frequency distribution as construct draw a histogram and Superimpose it on a frequency polygon and frequency curve

d) write comment on the data distribution

Solving

b)

For Height:

★ Number of Classes

$\because N = 67$

$\because m = 1 + 3.3 \log_{10} N$

$\boxed{m \cong 7}$

★ Range

$R = max - min = 191 - 145 = 46$

$R = 46$

* Class Interval

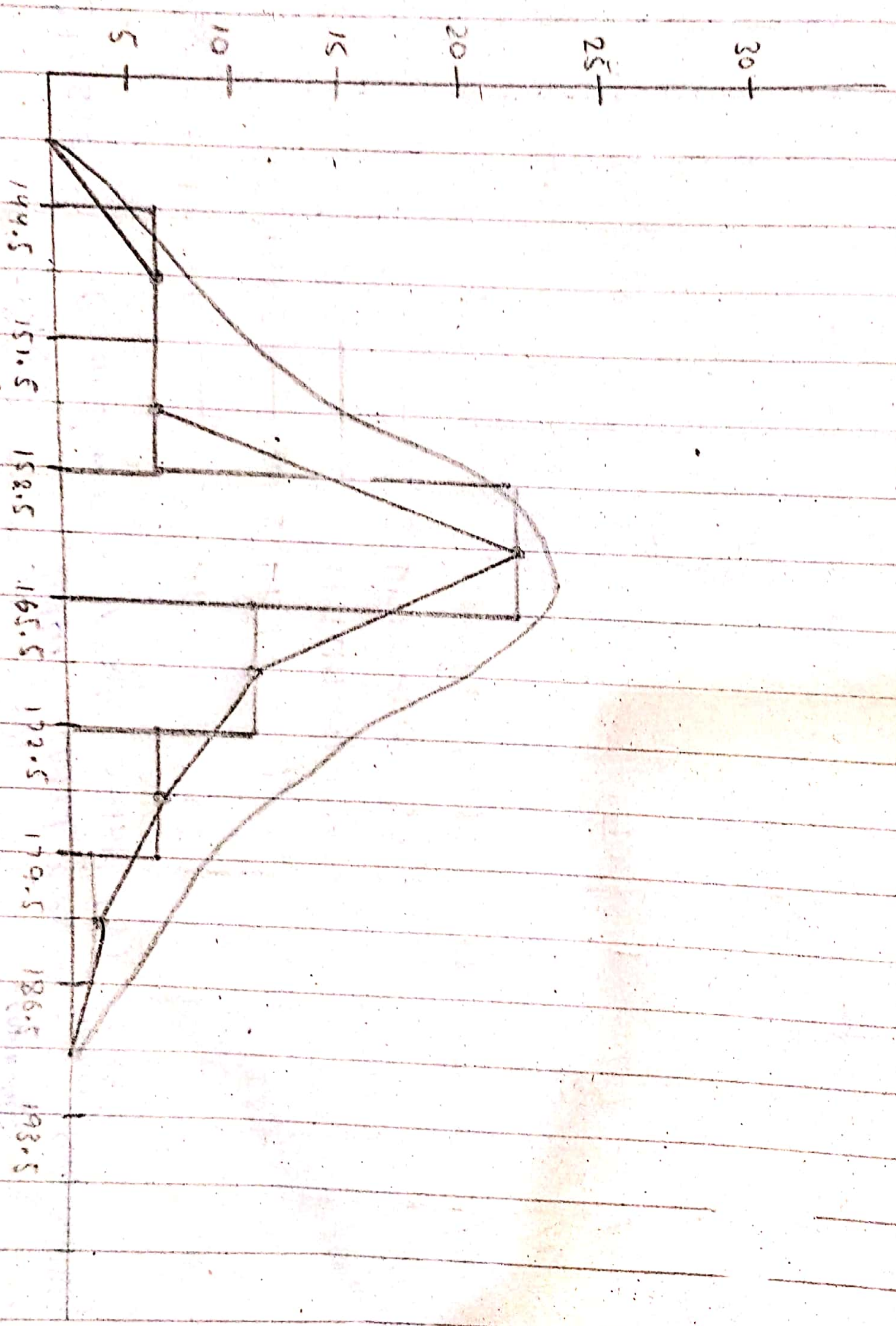
$CI = \frac{\text{Range}}{\text{class size}} = R/m = 46/7 = 7$

$\boxed{CI = 7}$

d)

Classes	Class Boundaries	Tally Mark	Mid point	frequency (f)	commulative frequency (cf)	Probability
145 — 151	144.5 —— 151.5	卌 |	148	6	6	6/67
152 — 158	151.5 —— 158.5	卌 |	155	6	12	6/67
159 — 165	158.5 —— 165.5	卌 卌 卌	162	15	27	15/67
166 — 172	165.5 —— 172.5	卌 卌 卌 卌	169	22	49	22/67
173 — 279	172.5 —— 179.5	卌 卌 |	176	11	60	11/67
180 — 186	179.5 —— 186.5	卌 |	183	6	66	6/67
187 — 193	186.5 —— 193.5	|	190	1	67	1/67

Q c)



LAB II: DATA SUMMARIZATION:-

Probem: Consider the Student height/weight data from table 1, and Summarize the Information.

Mean Median Mode.

DATA Type/ Statistics	MEAN	MEDIAN	MODE
Ungrouped	$\bar{x} = \frac{\Sigma x}{n}$	when n is odd $\tilde{x} = \left(\frac{n+1}{2}\right)^{th}$ when n is even $\tilde{x} = \left\{\left(\frac{n}{2}\right) + \left(\frac{n+2}{2}\right)\right\}/2$	Mode is most frequent value
Grouped	$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$	$\tilde{x} = x_L + \frac{\left(\frac{n}{2} - c\right)}{f} * h$	Mode = $x_L + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right) * h$

Variables:-

Σx = Sum of all Students height

n = Total no of Students = Σf

In Group data

MEDIAN Variables:-

X_L = MEDIAN Class lower boundary

f = MEDIAN Class frequency

h = Class Interval

C = MEDIN Class se piche wali class ki Commulative frequency

MODE Variables:-

f_m = MEDIN Class frequency

f_1 = MEDIAN Class before frequency

f_2 = MEDIN Class after frequency

LAB III

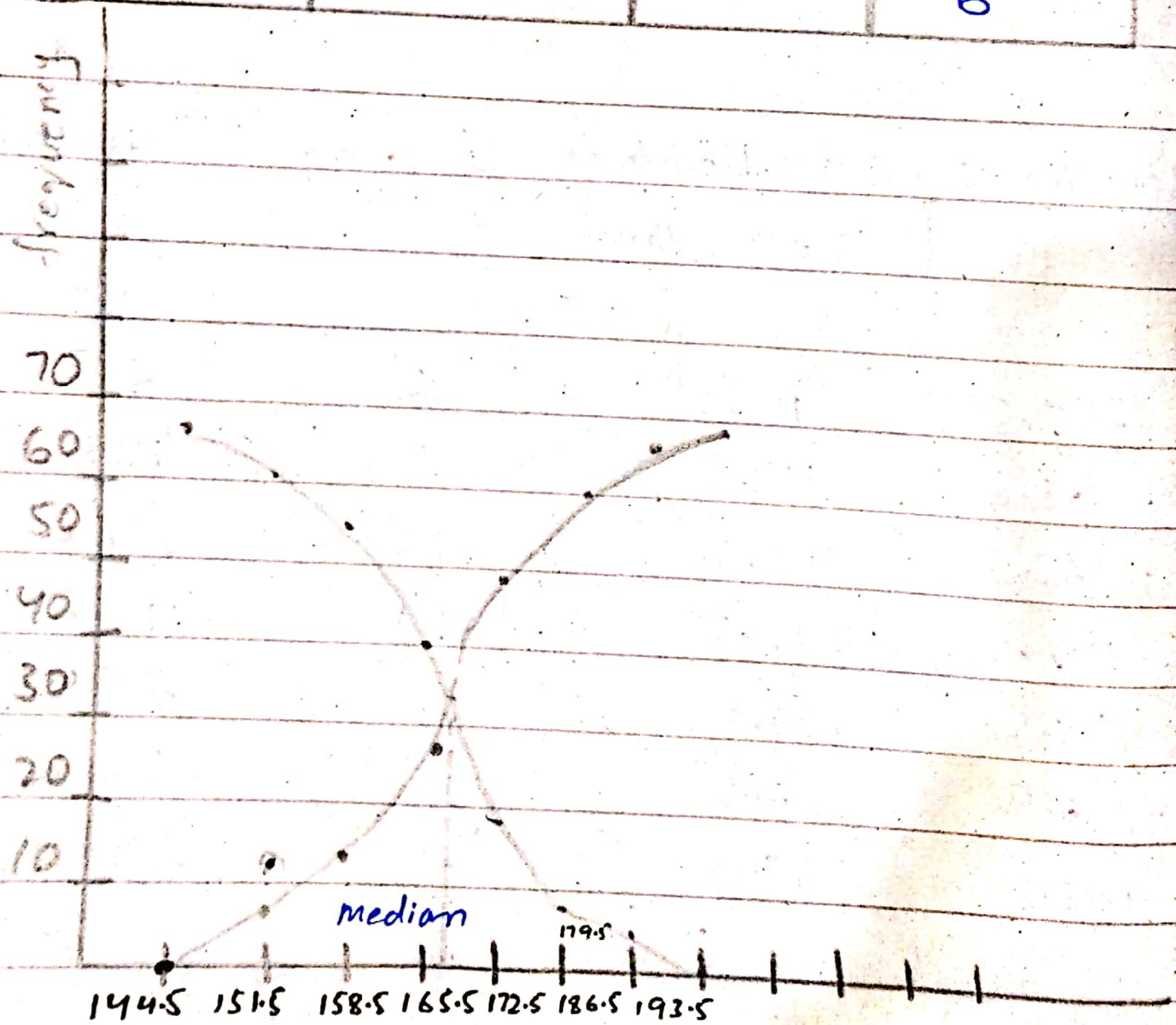
Problem: Consider the frequency distribution of Height/weight Data in Lab 1

a) Obtain the Median graphically
b) Obtain Mode graphically
c) compare values numerically with values completed in lab 2.

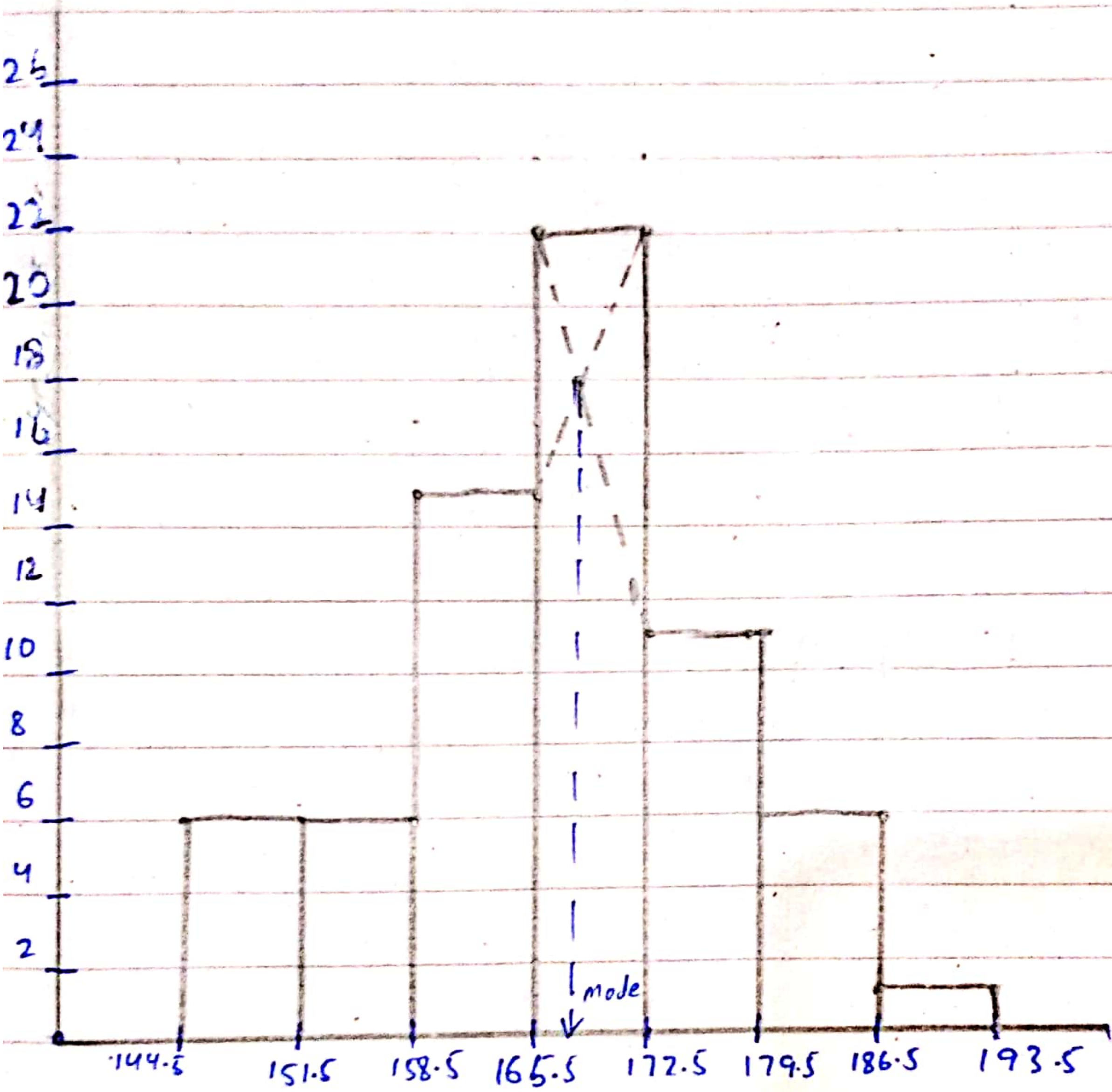
a)

Class Boundaries	Class Boundaries Less than	frequency	frequency less than Type
144.5 — 151.5	151.5	6	6
151.5 — 158.5	158.5	6	12
158.5 — 165.5	165.5	15	27
165.5 — 172.5	172.5	22	49
172.5 — 179.5	179.5	11	60
179.5 — 186.5	186.5	6	66
186.5 — 193.5	193.5	1	67

C.B	C.B More than	f	f more than
144.5 — 151.5	144.5	6	67
151.5 — 158.5	151.5	6	61
158.5 — 165.5	158.5	15	55
165.5 — 172.5	165.5	22	40
172.5 — 179.5	172.5	11	18
179.5 — 186.5	179.5	6	7
186.5 — 193.5	186.5	1	6



Median = 167.5



Mode = 167.8

	MEDIA	MODE
From LAB II	167.014	168.2
From LAB III	167	167.8

LAB IV Shape of Distribution

Problem:

Consider the height/weight data collected in practical # 1

a) calculate all measure of dispersion from grouped and ungrouped data set and write comments

b) Consider the grouped data of height and compute - Skewness and kurtosis during first four moments.

c) consider the ungrouped data of height calculate first four moments and calculate Skewness and kurtosis.

a)

For Group Data:

Range Deviation:-

$$R.D = \frac{Max - Min}{2}$$

$$R.D = \frac{191-145}{2}$$

$$\boxed{R.D = 23}$$

coefficien of Range

$$CRD = \frac{Max - Min}{Max + Min}$$

$$CRD = \frac{191-145}{191+145}$$

$$\boxed{CRD = 0.136}$$

Quartile Deviation:-

$$\because Q.D = \frac{Q_3 - Q_1}{2}$$

Its for ungroup data

$$Q_i = L + \left(\frac{\frac{in}{4} - c}{f}\right)h$$

$$\Rightarrow Q_1 = L + \left(\frac{\frac{n}{4} - C}{f}\right) \times h$$

for Q_1 $n/4 = 67/4$

$$\frac{67}{4} \rightarrow 16.75$$

For group data Max mid point and Min Mid point

Class Boundaries	f	C.f	
144.5 — 151.5	6	6	
151.5 — 158.5	6	12	
158.5 — 165.5	15	27	← Q_1
165.5 — 172.5	22	49	
172.5 — 179.5	11	60	← Q_3
179.5 — 186.5	6	66	
186.5 — 193.5	1	67	

$$Q_1 = 158.5 + \left(\frac{16.75 - 12}{15}\right) * 7$$

$$\boxed{Q_1 = 160.716}$$

Similarly for Q_3

$$Q_3 = L + \left(\frac{3n/4 - C}{f}\right) h$$

$$\frac{3n}{4} \rightarrow \frac{3(67)}{4} \rightarrow 50.25$$

$$Q_3 = 172.5 + \left(\frac{50.25 - 49}{11}\right) * 7$$

$$\boxed{Q_3 = 173.2954}$$

$$Q.D = \frac{Q_3 - Q_1}{2} = \frac{173.2954 - 160.716}{2}$$

$\boxed{Q.D = 6.2897}$ Cofficient of QD

MEAN DEVIATION:-

$$CQD = \frac{Q_3 - Q_1}{Q_3 + Q_1} = 3.7 * 10$$

Deviation = Midpoint − C

Mean

Median

Mode

$$\therefore M.D = \frac{\Sigma f |x_i - \bar{x}|}{\Sigma f}$$

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Meand deviation Table.

Classes	f	Mid point x_i	fx	$\lvert x_i - \bar{x} \rvert$	$f\lvert(x_i - \bar{x})\rvert$
145 — 151	6	148	888	19.014	114.084
152 — 158	6	155	930	12.014	72.084
159 — 165	15	162	2430	5.014	75.21
166 — 172	22	169	3718	1.986	43.692
173 — 179	11	176	1936	8.986	98.846
180 — 186	6	183	1098	15.986	95.916
189 — 193	1	190	190	22.986	22.986
	67		11190	85.986	522.818

$$\text{Mean} = \bar{x} = \frac{\Sigma fu}{n} = \frac{11190}{67}$$

$$\boxed{\bar{x} = 167.014}$$

$$M.D = \frac{\Sigma f\lvert x - x_i \rvert}{\Sigma f} = \frac{522.818}{67}$$

$$\boxed{M.D = 7.803}$$

Standard Deviation:-

$$S.D = \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n}} = \sqrt{\frac{\Sigma D^2}{n}}$$

$D = (x_i - \bar{x})$ → Mid point − Mean

$$S.D = \sqrt{\frac{\Sigma fD^2}{\Sigma f} - \left(\frac{\Sigma fD}{\Sigma f}\right)^2}$$

Variance:-

$$\sigma^2 = (S.D)^2$$

$$\sigma^2 = \frac{\Sigma fD^2}{\Sigma f} - \left(\frac{\Sigma fD}{\Sigma f}\right)^2$$

Cofficient of S.D

$$CSD = \frac{S.D}{\bar{X}}$$

$\bar{X}$ -> mean

Cofficient of variation:-

$$CV = \frac{S.D}{\bar{X}} * 100$$

Ronge Deviation:-

$$R.D = \frac{\text{Max Mid point} - \text{Min Midpoint}}{2}$$

Grouped data.

Ungroup Data:-

Quartile Deviation:-

$$Q.D = \frac{Q_3 - Q_1}{2}$$

$$Q_i = \left(\frac{in}{4}\right)^{th}$$ Orderd data.

MEAN DEVIATION:-

$$MD = \frac{\Sigma|x_i - c|}{n}$$

c → Median, Mean, Mode

Standard Deviation:-

$$S.D = \sqrt{\frac{\Sigma D^2}{n} - \left(\frac{\Sigma D}{n}\right)^2}$$

$D = (x_i - \bar{x})$ = individual data - Mean.

Variance:-

$$\sigma^2 = (S.D)^2$$

$$\sigma^2 = \frac{\Sigma D^2}{n} - \left(\frac{\Sigma D}{n}\right)^2$$

ABSOLUTE MEASURE OF Dispersion.

Range Deviation:-

~~Ears~~

Problem: Only two observations.

Quartile Deviation:-

Problem:

* It doesnot depends on all values
* It affected by only two observation

Defination:-

It is also called semi-inter quartile range. It is the half of the product of the upper and lower quartile or $\left\{\frac{P_{75} - P_{25}}{2}\right\}$

MEAN DEVIATION:-

It is defined as the average of the absolute deviation taken from some central value such as (Mean, Median and Mode)

Points.

- based on all values.
- Simple to calculate.
- not effected by entreme values

- Not accurate Measure of dispersion
- It is not widely used.

Standard Deviation:-

It is defined as the positive square root of the average of the squared deviations, taken from A.M of the distribution Mathematically.

S.D Formulas.

Ungroup Data

$$S.D = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$$

$$S.D = \sqrt{\left[\frac{\sum D_i^2}{n} - \left(\frac{\sum D}{n}\right)^2\right]}$$

Grouped Data

$$S.D = \sqrt{\frac{\sum f_i D^2}{\sum f_i}}$$

$$S.D = \sqrt{\frac{\sum f_i D^2}{\sum f_i} - \left(\frac{\sum f_i D}{\sum f_i}\right)^2}$$

Standard deviation is independent of change of origin and scale.

ROOT MEAN SQUARED DEVIATION:-

It is defined as positive square root of average of deviation taken from some arbitary constant (A)

$$S = \sqrt{\frac{\sum f_i (x_i - A)^2}{n}}$$

Relation b/w R.M.S.D and S.D

$$S^2 = \sigma^2 + (\bar{x} - A)^2$$

if $\bar{x} = A$ then

$$\underset{\text{R.M.S.D}}{S^2} = \underset{\text{S.D}}{\sigma^2}$$

Points.

- It maybe manipulated Mathematically.
- It is best measure of dispersion
- It is based on all the observations
- It is of greate importance of sampling Theory
- It is least effected by Sampling fluctuations

- It is not good measure when the series of data is short.

Variance:

Variance is not an independent measure of dispersion it is defined as the squared of Standard deviation.

Moments:

moments of the distribution is designated by the power of the deviation, varies to at end, when deviation is taken about mean then moments are called as mean moments where as the deviation about any constant A (when A = 0) then moments are called raw moments or moments about origin. and order of moments designated by the power of deviation.

Raw Moments:-

when A = 0

Group

$\because \mu_r' = \frac{1}{\sum f_i} \cdot \sum f_i x_i^r$

Ungroup

$\because \mu_r' = \frac{1}{n} \sum x_i^r$

x_i = Mid point. ~~x_i = Deviation~~ x_i = Individual osevation

MEAN MOMENTS:-

Group

$\mu_r = \frac{1}{\sum f_i} \cdot \sum (x_i - \bar{x})^r f_i$,

Ungroup

$\mu_r = \frac{1}{n} \sum (x_i - \bar{x})^r$

Points:

Moments are Independent of change of Origin but dependent of change of scale.

Relation b/w Raw Moments and Mean Moments:-

Raw Moments about Origin.

$$\mu_1' = \frac{1}{n} . \Sigma x_i = \text{Mean} = \bar{x}$$

$$\mu_2' = \frac{1}{n} . \Sigma x_i^2$$

$$\mu_3' = \frac{1}{n} . \Sigma x_i^3$$

$$\mu_4 = \frac{1}{n} \Sigma x_i^4$$

Moments about Mean

For μ_1

$$\mu_1 = \frac{1}{n} . \Sigma(x_1 - \bar{x})$$

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$$\mu_1 = \overset{\text{Mean}}{\boxed{\frac{1}{n}.\Sigma(x_i)}} - \overset{\text{also mean}}{\boxed{\frac{1}{n}\Sigma(\bar{x})}}$$

$$\boxed{\mu_1 = 0}$$ Proof

$\bar{x}$ pe summision lag ke n se devide ho to ans mean he hoga.

For μ_2

$$\mu_2 = \frac{1}{n}.\Sigma(x_i - \bar{x})^2$$

$$\mu_2 = \frac{1}{n}\Sigma(x_i^2 - 2x_i\bar{x} + \bar{x}^2)$$

$$\cancel{\mu_2} = \frac{1}{\cancel{n}}.\cancel{(}$$

$$\mu_2 = \frac{1}{n}.\Sigma x_i^2 - 2\bar{x}.\frac{\Sigma x_i}{n} + \bar{x}^2$$

$$\mu_2 = \mu_2' - 2\mu_1'.\mu_1' + \mu_1^2$$

$$\mu_2 = \mu_2' - 2\mu_1^2 + \mu_1^2$$

$$\boxed{\mu_2 = \mu_2' - \mu_1'^2}$$

For μ_3

$$\mu_3 = \frac{1}{n}.\Sigma(x_i - \bar{x})^3$$

$$\mu_3 = \frac{1}{n}\Sigma(x_i^3 - 3x_i^2\bar{x} + 3x_i\bar{x}^2 - \bar{x}^3)$$

$$\mu_3 = \frac{1}{n}.\Sigma x_i^3 - 3\bar{x}\frac{\Sigma x_i^2}{n} + 3\bar{x}\frac{\Sigma x_i}{n} - \bar{x}^3$$

$$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 3\mu_1'\cdot\mu_1' - \mu_1'^3$$

$$\boxed{\mu_3 = \mu_3' - 3\mu_1\mu_2 + 2\mu_1'^2}$$

For μ_4

$$\mu_4 = \frac{1}{n}\cdot\Sigma(x_i - \bar{x})^4$$

$$\mu_4 = \frac{1}{n}\Sigma(x_i^4 - 4x_i^3\bar{x} + 6x_i^2\bar{x}^2 - 4x_i\bar{x}^3 + \bar{x}^4)$$

$$\mu_4 = \frac{1}{n}\cdot\Sigma x_i^4 - 4\bar{x}\frac{\Sigma x_i^3}{n} + 6\bar{x}^2\frac{\Sigma x_i^2}{n} - 4\bar{x}^3\frac{\Sigma x_i}{n} + \bar{x}^4$$

$$\mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6\mu_1'^2\mu_2' - 4\mu_1'^3\cdot\mu_1' + \mu_1'^4$$

$$\mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6\mu_1'^2\mu_2' - 4\mu_1'^4 + \mu_1'^4$$

$$\boxed{\mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6\mu_1'^2\mu_2' - 3\mu_1'^4}$$

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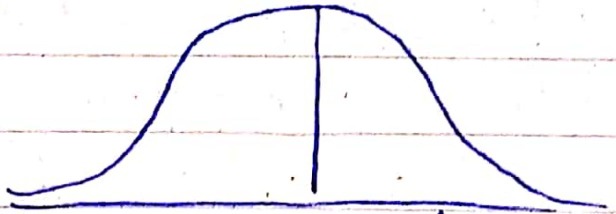
Skewness:-

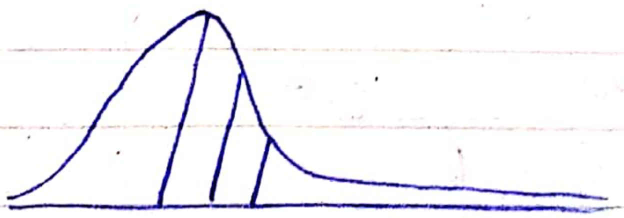
The degree of departure of frequency curve of a distribution from Symmetric nature is called Skewness of the distribution.

1 Symmetric ⇒ Mean = Median = Mode



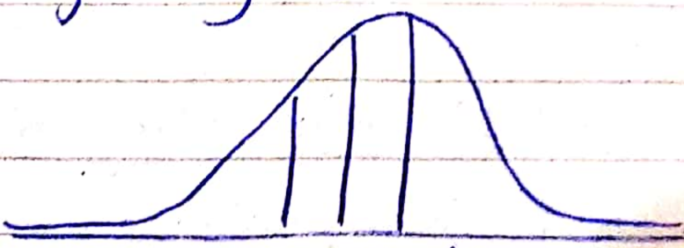
Mean = Median = Mode

2 - Positively skewed ⇒ Mean > Median > Mode



Mean > Median > Mode

3 - Negatively skewed



Mean < Median < Mode.

MEASURE OF SKEWNESS:-

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3},$$

β_1 is absolute measure of skewness

If

$\beta_1 = 0$ distribution is symmetric

$\beta_1 > 0$ distribution is skewed either positively or negatively.

Relative Measure of Skewness.

A/c to karl pearson.

$$Sk = \frac{\text{Mean} - \text{Mode}}{S.D}$$

Using relaation b/w mean, Median and Mode. (Empirical relation)

Hence

$$S.k = \frac{3(\text{Mean} - \text{Median})}{S.D}$$

also

$$S.k = \frac{3(\text{Mean} - Q_2)}{S.D}$$

According to Bowley.

$$S.k = \frac{Q_1 + Q_3 - 2Q_2}{Q_3 - Q_1}$$

$$0 \leq \text{Skewness} \leq +1$$

3.

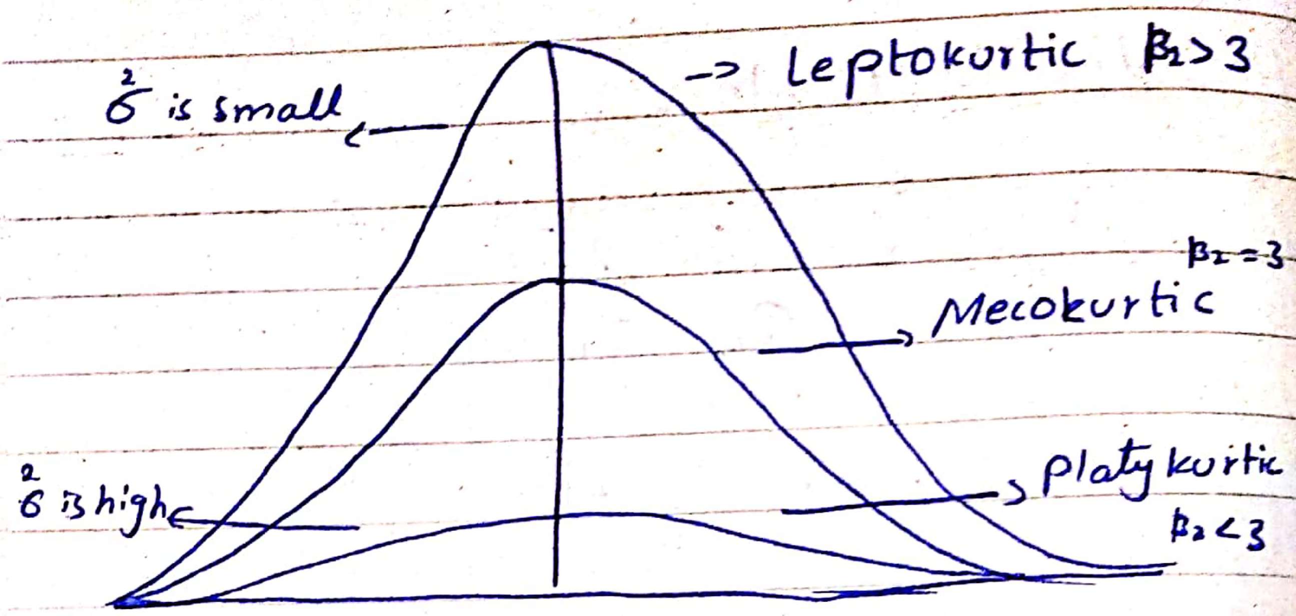
$$\text{Skewness} = \frac{\sqrt{\beta_1} \quad (\beta_2 + 3)}{2[5\beta_2 - 6\beta_1 - 9]}$$

MEASURE OF Kurtosis:-

β_2 is measure of kurtosis

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

★ It is a chracterstic of frequency distribution It measures flatness and peakness of curve of the given distribution.



☆ Show that $\beta_2 > 1$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$\frac{\mu_4}{\mu_2^2} \geq 1$$

$$\mu_4 \geq \mu_2^2$$

$$\because \mu_4 = \frac{\Sigma f(x_i - \bar{x})^4}{\Sigma f}$$

$$\because \mu_2 = \frac{\Sigma f(x_i - \bar{x})^2}{\Sigma f}$$

$$\frac{\Sigma f(x_i-\bar{x})^4}{\Sigma f} \geq \left[\frac{\Sigma f(x_i-\bar{x})^2}{\Sigma f}\right]^2$$

~~Lt~~

Let $d = (x_i - \bar{x})^2$

$$\frac{\Sigma f d^2}{\Sigma f} \geq \left(\frac{\Sigma f d}{\Sigma f}\right)^2$$

$$\frac{\Sigma f d^2}{\Sigma f} - \left(\frac{\Sigma f d}{\Sigma f}\right)^2 \geq 0$$

$$\boxed{\sigma^2 \geq 0}$$

It is always true by using the property of variance

$\cancel{\beta_2}$ $\cancel{4}$ $\boxed{\beta \geq 1}$

Theorem:- Show that cofficient of kurtosis is greater than equals to cofficient of skewness.

$\star \; \beta_2 \geq \beta_1$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}, \qquad \beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$\beta_2 \geq \beta_1$$

$$\rightarrow \frac{\mu_4}{\mu_2^2} \geq \frac{\mu_3^2}{\mu_2^3}$$

$$\rightarrow \boxed{\mu_2 \mu_4 \geq \mu_3^2} \text{ — } ①$$

$$\because \mu_2 = \frac{\sum f d^2}{n}$$

$$\because \mu_3 = \frac{\sum f d^3}{n} \quad \longrightarrow d = (x_i - \bar{x})$$

$$\because \mu_4 = \frac{\sum f d^4}{n}$$

① ⇒

$$\left[\frac{\sum f d^2}{n}\right]\left[\frac{\sum f d^4}{n}\right] \geq \left[\frac{\sum f d^3}{n}\right]^2$$

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$$\Sigma f d^2 \cdot \Sigma f d^4 \geq \left(\Sigma f d^3\right)^2$$

LAB V

Box Plot / Stem & Leaf Plot:-

Stem & Leaf plot is a method for presenting quantitative data in a graphical format, similar to a histogram, to assist in visualising the shape of distribution.

Example data:-

1.17	1.61	1.16	1.38	3.53
1.23	3.76	1.94	0.96	4.75
0.15	2.41	0.71	0.02	1.59
0.19	0.82	0.47	2.16	2.01
0.92	0.75	2.59	3.07	1.40

STEMS	LEAFS
0	15 19 82 71 47 96 02 92 75
1	17 23 61 16 94 38 59 40
2	41 59 16 01
3	76 07 53
4	75

STEMS	LEAFS	Frequency	Probability
0	02 15 19 47 71 75 82 92 96	9	0.36
1	16 17 23 38 40 59 61 94	8	0.32
2	01 16 41 59	4	0.16
3	07 53 76	3	0.12
4	75	1	0.04
		n=25	1

$$Q_i = \left(\frac{in}{4}\right)^{th}$$

$$Q_1 = \frac{25}{4} = (7)^{th} = \cancel{0.82}\ 0.82$$

$$Q_2 = \frac{2(25)}{4} = (13)^{th} = 1.38$$

$$Q_3 = \frac{3(25)}{4} = (19)^{th} = 2.16$$

$$Q_4 = \frac{4(25)}{4} = (25)^{th} = 4.75$$

$$IQR = Q_2 - Q_1 = 2.16 - 0.82$$

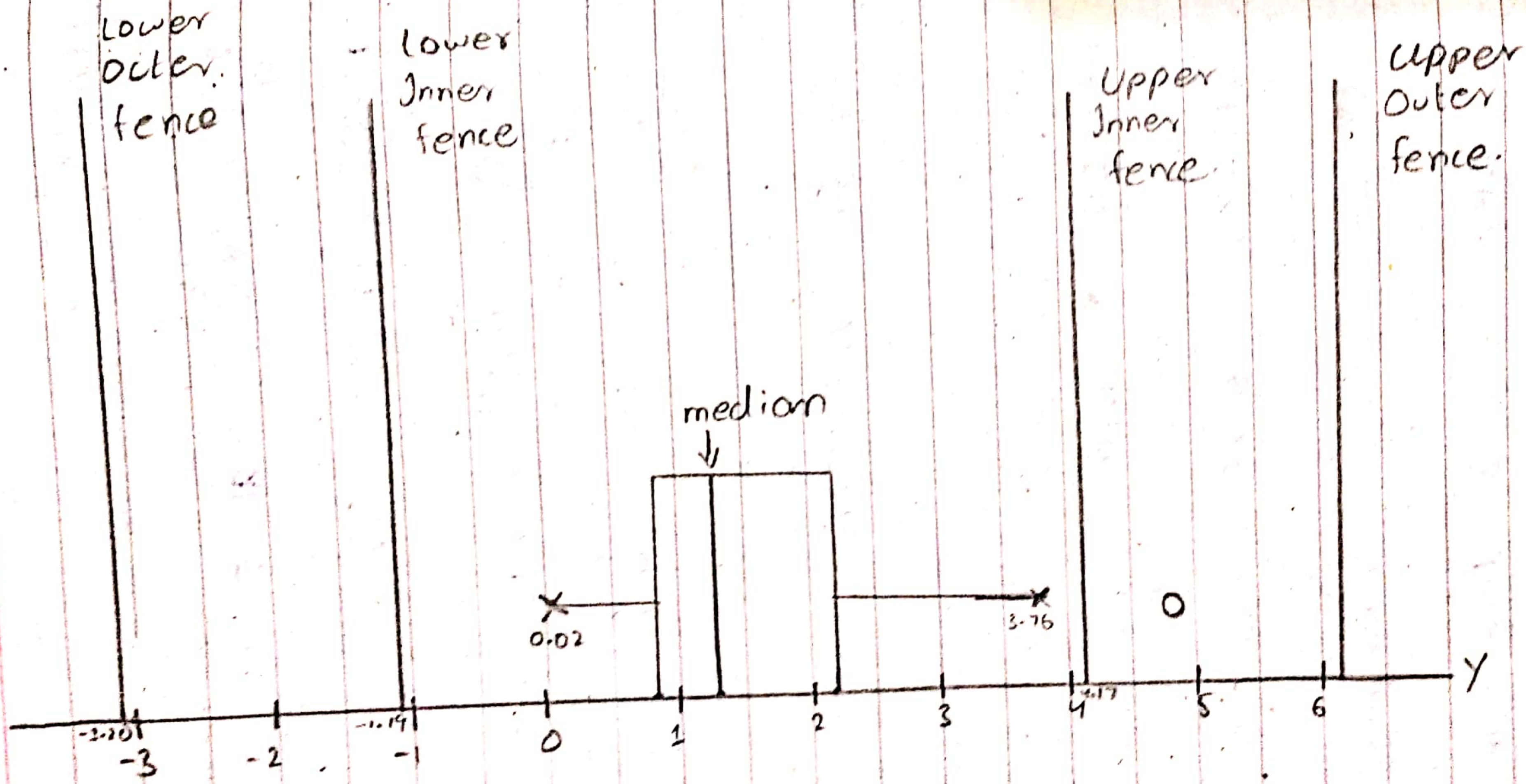
$$\boxed{IQR = 1.34}$$

$Q_2 - 1.5(IQR) = -1.19 \rightarrow$ Lower Inner fence

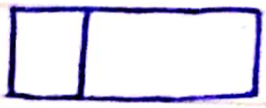
$Q_1 - 3(IQR) = -3.20 \rightarrow$ lower Outer fence

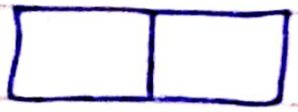
$Q_3 + 1.5(IQR) = 4.17 \rightarrow$ Upper inner fence

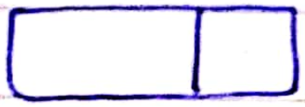
$Q_3 + 3(IQR) = 6.18 \rightarrow$ Upper outer fence.


Lower
Outer
fence
lower
Inner
fence
Upper
Inner
fence
upper
Outer
fence.
median
0.02
3.76
-3.20
-1.19
4.17
-3
-2
-1
0
1
2
3
4
5
6
Y

Hence middle line of box is near to Quartile 1 so we can say the distribution is Positively skewed.

 -> Positively skewed.

 -> Symmetric

 -> Negatively skewed.

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Probability:-

A phenomenon is called random if the enact outcome is uncertain, the mathematical ~~outcome~~ study of such randomness is called the theory of probability.

Event:-

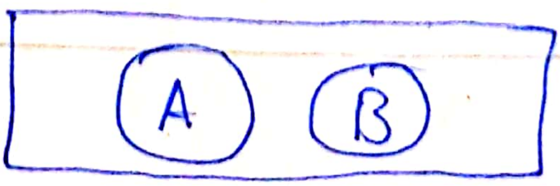
A probability event can be defined as a set of outcomes of an enperiment, in other words, an event in probability is the subset of the respective sample space.

///Sample Space:-

All possible outcomes collection.

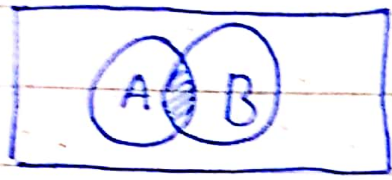
Mutually Enclusive Events:-

Two events are mutually enclusive when no elements are common between them.



Non-Mutually Exclusive Events:-

Two Events are Non-Mutually Exclusive If they Share common Elements.



VEN Diagram:-

A Ven diagram is a diagram which related in set theory in mathematics by which happening of events can be visualize.

Indepent Events:-

If the occurance of events cannot effect occurence of another event

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$$
$$P(E_1 \cap E_2 \cap E_3) = P(E_1) \cdot P(E_2) \cdot P(E_3)$$

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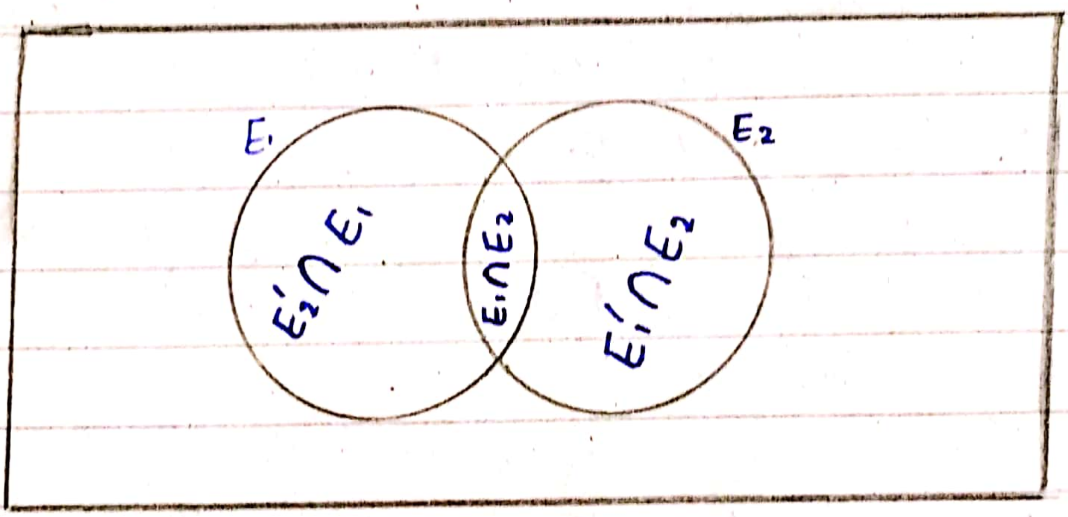
Theorem:-

If E_1 & E_2 be two events define on the sample spaces, then the probabilit that E_1 occurs or E_2 occurs or both of them occurs is given by.

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

Proof:



$$E = (E_1 \cup E_2) = E_1 \cup (E_1' \cap E_2)$$

$$P(E) = P(E_1 \cup E_2) = P[E_1 \cup (E_1' \cap E_2)]$$

$$= P(E_1 \cup E_2) = P(E_1) + P(E_1' \cap E_2)$$

→ Adding and Subtracting $P(E_1 \cap E_2)$

$$= P(E_1 \cup E_2) = P(E_1) + \underline{P(E_1' \cap E_2) + P(E_1 \cap E_2)} - P(E_1 \cap E_2)$$

$$P(E) = P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

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Theorem #2:-

If E_1, E_2 & E_3 be the events define on the sample spaces, then the probability that E_1 occur or E_2 or E_3 or at least one of them occur is given by.

$$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2) - P(E_1 \cap E_3) - P(E_2 \cap E_3) + P(E_1 \cap E_2 \cap E_3)$$

Proof:

Here E and E_3 be two Events define on S, then.

$$P(E \cup E_3) = P(E) + P(E_3) - P(E \cap E_3)$$

In above theorem

$\because P(E) = P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$P(E_1 \cup E_2 \cup E_3) = P(E_1 \cup E_2) + P(E_3) - P((E_1 \cup E_2) \cap E_3)$

$\because [(E_1 \cup E_2) \cap E_3] = (E_1 \cap E_3) \cup (E_2 \cap E_3)$ distributive law

$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) - P[(E_1 \cap E_3) \cup (E_2 \cap E_3)]$

$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) - [P(E_1 \cap E_3) + P(E_2 \cap E_3) - P(E_1 \cap E_2 \cap E_3)]$

Theorem

$$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_3) - P(E_2 \cap E_3) + P(E_1 \cap E_2 \cap E_3)$$

$$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) - \underline{P(E_1 \cap E_2)} - P(E_1 \cap E_3) - P(E_2 \cap E_3) + P(E_1 \cap E_2 \cap E_3)$$

missed it in above steps.

For General Formula:-

$$S_1 = P(E_1) + P(E_2) + \cdots + P(E_n)$$

$$S_2 = P(E_i \cap E_j) \;;\; \text{for } i < j$$

$$S_3 = P(E_i \cap E_j \cap E_k) \;;\; \text{for } i < j < k$$

$\cdot$

$\cdot$

$\cdot$

$$S_n = P(E_i \cap E_j \cap \cdots \cap E_n)$$

$$\boxed{P(E_1 \cup E_2 \cup \cdots \cup E_n) = S_1 - S_2 + S_3 + \cdots + (-1)^{n+1} S_n}$$

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LAB VI : Probability:-

Problem:-

Two Dices are rolled

a) Construct the sample space

b) Let n be the random variable that denotes the sum of the scores on the Two dice

c) Construct the probability distribution for the random variable x

d) repeat it for four sided dice.

D2 \ D1	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$P(n) = \frac{n(E)}{n(S)}$$

Probability Distribution.

n	2	3	4	5	6	7	8	9	10	11	12
P(n)	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

LAB <u>VI</u> : Probability:-

Problem:-

Two Dices are rolled

a) Construct the sample space

b) let x be the random variable that denotes the sum of the scores on the Two dice

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5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$P(x) = \frac{n(E)}{n(S)}$$

Probability Distribution.

x	2	3	4	5	6	7	8	9	10	11	12
P(x)	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

Random Variables:-

It is a real valued function which assign a real number to each sample point in the sample space.

Example:-

Three Coins

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

for Heads

$X(S_1) = 3$
$X(S_2) = X(S_3) = X(S_4) = 2$
$X(S_5) = X(S_6) = X(S_7) = 1$
$X(S_8) = 0$

Random variable $x \in \mathbb{N}$ & $x \geq 0$

Discrete Probability Distribution Function.

X (no of heads)	0	1	2	3	T
P(x)	1/8	3/8	3/8	1/8	1

Probability Mass function:- P.M.F

following Conditions:-

(i) $P(x)_i \geq 0$
(ii) $\sum P(x)_i = 1$

Distribution Function:-

It is define as commulative distribution function.

$$f(x) = \sum P_i$$

$$f(x) = \begin{cases} 1/8 & x \leq 0 \\ 4/8 & x \leq 1 \\ 7/8 & x \leq 2 \\ 1 & x \leq 3 \end{cases}$$

Axioms in Probability:-

i) $P(\emptyset) = 0$; If an event is certain not to occur its probability is zero

ii) $P(S) = 1$; If an event is certain to occur its probability is unity.

iii) $0 \leq P(E) \leq 1$; from (i) and (ii)

iv) $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

Non Mutually Exclusive Events.

PROBLEM:-

The probability that a person visiting his dentist will have his teeth cleaned, or a cavity filled, a tooth extracted, his teeth cleaned and a cavity filled, his teeth cleaned and a tooth extracted, a cavity filled and a tooth extracted, or his teeth cleaned and cavity filled and a tooth extracted are 0.47, 0.29, 0.22, 0.08, 0.06, 0.07 and 0.03 what is the probability that a person will have at least one of them is done?

DATA:-

E_1 = Tooth Cleaned
E_2 = Cavity filled
E_3 = Tooth extracted

$P(E_1) = 0.47$
$P(E_2) = 0.29$
$P(E_3) = 0.22$
$P(E_1 \cap E_2) = 0.08$
$P(E_1 \cap E_3) = 0.06$
$P(E_2 \cap E_3) = 0.07$
$P(E_1 \cap E_2 \cap E_3) = 0.03$

$P(E_1 \cup E_2 \cup E_3) = ?$

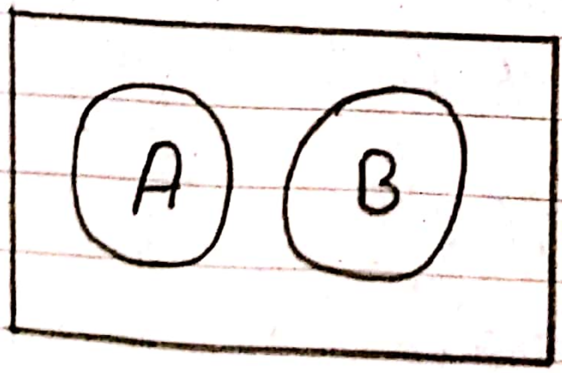
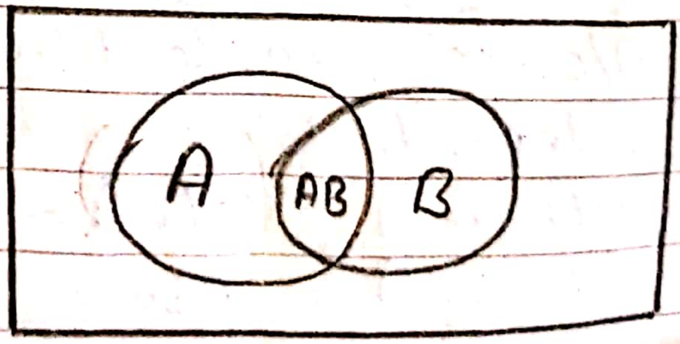
we know that.

$$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2)$$
$$- P(E_1 \cap E_3) - P(E_2 \cap E_3) + P(E_1 \cap E$$

$$P(E_1 \cup E_2 \cup E_3) = 0.47 + 0.29 + 0.22 - 0.08 -$$
$$0.06 - 0.07 + 0.03$$

$$\boxed{P(E_1 \cup E_2 \cup E_3) = 0.8}$$

Difference between Mutually exclusive and Independent events:-

Mutually Exclusive EVENTS:-	INDEPENDENT EVENTS:-
when the occurance is not simultaneous for two events then they are termed as Mutually Exclusive events	when the occurance of one event does not control the happening of the other event. the it is termed as an Independent event
The non-occurance of an event will end up in the occurance of an event.	There is no influence of an occurance with another and they are independent of each other.
Formula: $P(E_1 \cap E_2) = 0$	Formula: $P(E_1 \cap E_2) = P(E_1).P(E_2)$
	

Theorem 1:-

If E_1 & E_2 are Independent then Show that E_1 and E_2' are also Independent:-

Proof:

Given:

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$$

To Prove:

$$P(E_1 \cap E_2') = P(E_1) \cdot P(E_2')$$

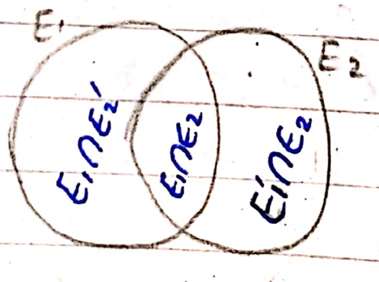


fig: 1

A/c to figure 1

$\rightarrow P(E_1 \cap E_2') = P(E_1) - P(E_1 \cap E_2)$

A/c to given condition that E_1 & E_2 are Independent

$\rightarrow P(E_1 \cap E_2') = P(E_1) - P(E_1) P(E_2)$

$\rightarrow P(E_1 \cap E_2') = P(E_1) [1 - P(E_2)]$

$\because U - A = A'$

$$\boxed{P(E_1 \cap E_2') = P(E_1).P(E_2')}$$ Proved.

Theorem 2:-

If E_1 & E_2 are Indepent than show that E_1' & E_2 are also Indepent:-

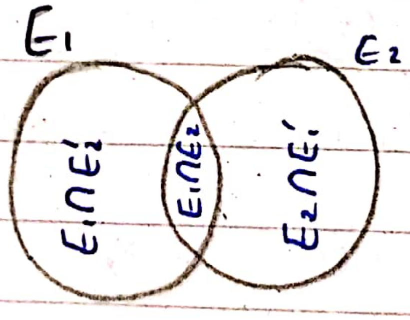
Proof:

Given:

$$P(E_1 \cap E_2) = P(E_1).P(E_2)$$

To Prove:

$$P(E_1' \cap E_2) = P(E_1').P(E_2)$$



A/c to figure

$$P(E_1' \cap E_2) = P(E_2) - P(E_1 \cap E_2)$$

A/c to given Condition E_1 & E_2 are Independent

$$P(E_1' \cap E_2) = P(E_2) - P(E_1)P(E_2)$$

$$P(E_1' \cap E_2) = P(E_2) \cdot [1 - P(E_1)]$$

$\because U - A = A'$

$$\boxed{P(E_1' \cap E_2) = P(E_2) . P(E_1')}$$ Proved.

Theorem 3:-

If E_1 & E_2 are Independent than Show that E_1' & E_2' are also Independent:-

Proof:

Given:

$$P(E_1 \cap E_2) = P(E_1) . P(E_2) \quad — ①$$

To prove:-

$$P(E_1' \cap E_2') = P(E_1') . P(E_2') \quad \rightarrow ②$$

By applyin De Morgan's law on eq ② we have.

$$P(E_1' \cap E_2') = P(E_1 \cup E_2)'$$

$\because U - A = A'$

$\rightarrow P(E_1' \cap E_2') = 1 - P(E_1 \cup E_2)$

we know that

$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\rightarrow P(E_1' \cap E_2') = 1 - [P(E_1) + P(E_2) - P(E_1 \cap E_2)]$

$\rightarrow P(E_1' \cap E_2') = 1 - P(E_1) - P(E_2) + P(E_1 \cap E_2)$

$\rightarrow P(E_1' \cap E_2') = [1 - P(E_1)] - P(E_2) + P(E_1 \cap E_2)$

from eq ①

$\rightarrow P(E_1' \cap E_2') = [1 - P(E_1)] - P(E_2) + P(E_1) . P(E_2)$

$\rightarrow P(E_1' \cap E_2') = [1 - P(E_1)] - P(E_2)[1 - P(E_1)]$

$\rightarrow P(E_1' \cap E_2') = [1 - P(E_1)] [1 - P(E_2)]$

$\because U - A = A'$

$\boxed{P(E_1' \cap E_2') = P(E_1') . P(E_2')}$ Proved.

PROBLEM:-

A firm has two operating systems working independently the probability that a specific system is available when needed is 0.96

a) what is the probability that neither

is available when needed.

b) what is the probability that an OS is available when needed.

Solution:-

CONDITION 1:-

$P(O_1) = P(O_2) = 0.96$

$P(O_1') = P(O_2') = 1 - 0.96$

$P(O_1') = P(O_2') = 0.04$

The probability of neither is availab

$P(O_1' \cap O_2') = P(O_1') \cdot P(O_2')$

$P(O_1' \cap O_2') = 0.04 * 0.04$

$P(O_1' \cap O_2') = (0.04)^2$

Condition 2-

$P(O_1) = P(O_2) = 0.96$

The probability of one of them is available.

$P(O_1 \cup O_2) = P(O_1) + P(O_2) - P(O_1 \cap O_2)$

$P(O_1 \cup O_2) = P(O_1) + P(O_2) - P(O_1) P(O_2)$

A/c to Independent condition

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$P(O_1 \cup O_2) = 0.96 + 0.96 - (0.96)^2$

$\boxed{P(O_1 \cup O_2) = 0.9984}$

x ——— x ——— x

PROBABILITY:-

ADDITION RULE

1. Mutually Exclusive

$P(A \cup B) = P(A) + P(B)$

2. Non Mutually Exclusive

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

MULTIPLICATION RULE

1. Independent

$P(A \cap B) = P(A) \cdot P(B)$

2. Dependent

$P(A \cap B) = P(A) \cdot P(B|A)$

x ——— x ——— x ———

Conditional Probability:-

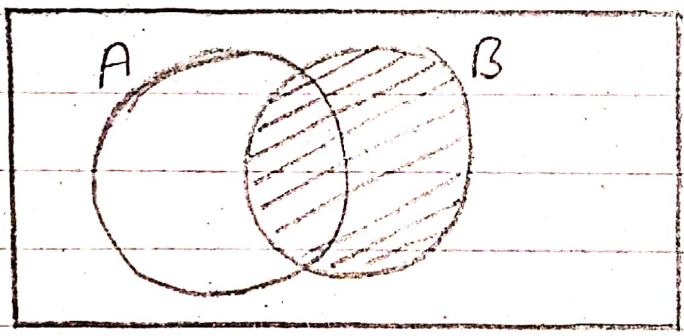
Conditional probability is defined as the likelihood of an event or outcome occuring based on the occurance of a previous event or outcome.

$$\because P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\because P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Let a Condition:

Find Probability of A given that B is occured



Hence it is a conditional probability and Event B is Occurred so the sample space becomes

$$S = \{B\}$$

Now According to this sample space we have to find probability of Event A gives as required answer.

BAYES THEOREM:-

Bayes Theorem is nothing but the application of Conditional Probability.

According to Conditional Probability.

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

$$\boxed{P(A\cap B) = P(A|B) \cdot P(B)} \text{ --- } ①$$

$$P(B|A) = \frac{P(B\cap A)}{P(A)}$$

$$\boxed{P(B\cap A) = P(B|A) \cdot P(A)} \text{ --- } ②$$

comparing eq ① & ②

$$P(A|B) \cdot P(B) = P(B|A) \cdot P(A)$$

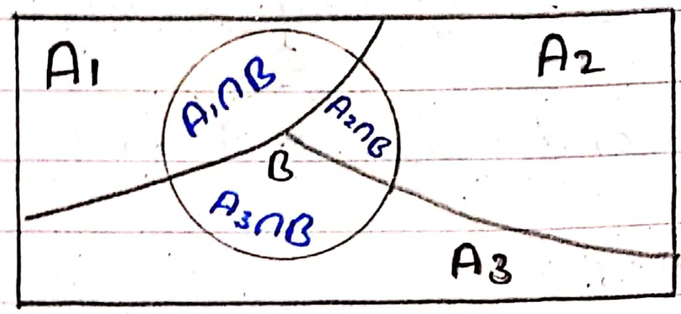
$$\boxed{P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}} \text{ --- } ③$$

↓

BAYES Theorem:-

Now we find a generalized case of Bayes Theorem.

Generalized Case of Bayes Theorem:-



$A_1 + A_2 + A_3 = 100\%$

$P(A_1) + P(A_2) + P(A_3) = 1$

$P(B) = P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B)$ — (4)

we know that A/c to Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$\therefore P(A \cap B) = P(A|B) \cdot P(B)$

(4) =>

$$P(B) = P(B|A_1) \cdot P(A_1) + P(B|A_2) \cdot P(A_2) + P(B|A_3) \cdot P(A_3)$$

$$\boxed{P(B) = \sum_{i=1}^{n} P(B|A_i) \cdot P(A_i)} \quad \text{— (5)}$$

from Eq (3)

$$\rightarrow P(A_k|B) = \frac{P(B|A_k) \cdot P(A_k)}{P(B)}$$

A/c to eq ⑤

$$\boxed{P(A_k|B) = \frac{P(B|A_k) \cdot P(A_k)}{\sum_{i=1}^{n} P(B|A_i) \cdot P(A_i)}}$$

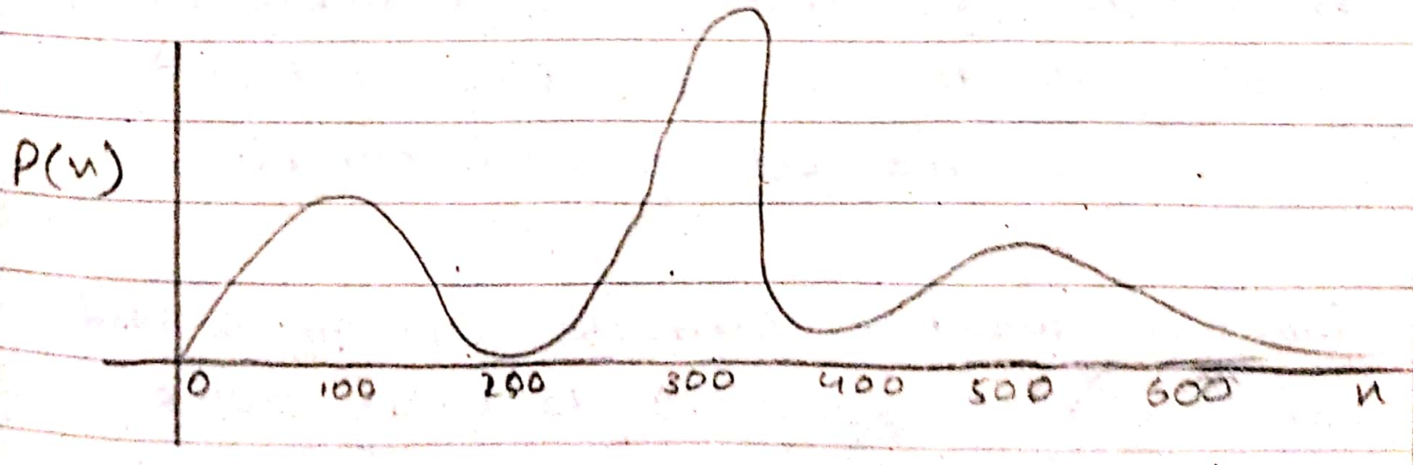
Generallized case of Bayes theorem.

X ———— X ———— X ———— X

PROBABILITY DENSITY FUNCTION:-

Everything that can be known about random variable is Contained in its Probability Density Function.

It allow us to determine the probability that a particular measurement of a random variable might be written some set of values.



Overview: Use Of Probability:

In any statistical analysis we use probability in two ways

1. Probability models describe non-deterministic nature of measurements

2. Probability is used to quantify the uncertamity in the results (conclusions) of our statistical analysis.

Random Variable:-

A Random Variable is a function of sample space, It is basically a device for transferring probability from complicated sample spaces to simple sample space.

A Random Variable X is a function whose domain is the sample space and whose range is the set of real numbers.

Thus a Random Variable assigns a real value (i.e a number) to every outcome in the sample space the particular value are called realisations and are denoted as x.

If the realisations are coutable, $x_1, x_2, x_3,$ the random variable is said to be discre

If there are many infinite uncountable reliasation, the random variable

Random Variable can be classified according to the types of values they can take on The range of the random variable.

1. Binary
2. Categorical (Ordered or Unordered)
3. Qualitative
4. Quantitative.

Probability Mass Function:-

If X is a discrete random variable the function given by $f(x) = P(X = x)$ for every x with in range of X, is called probability mass function or probability function or probability distribution of x.

A function can serve the probability of a discrete random variable x iff its value $f(x)$ satisfy the condition

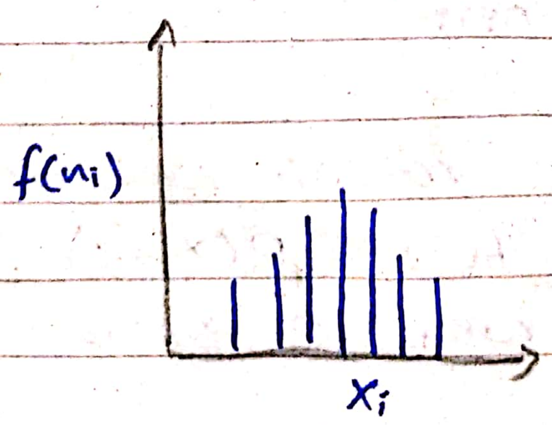
1. $f(x) \geq 0$
2. $\sum f(x) = 1$

Probability Distribution:- (Discrete R.V)

Table, formula or graphical representation of probability of random variable x

X_i	x_1	x_2	......	x_k	Total
$P(x_i)$	$P(x_1)$	$P(x_2)$	....	$P(x_k)$	1

Probability Histogram:-



Probability Distribution:- (Continious R.V)

The probability distribution of a random variable describes how the probabilities are distributed. For a Continious Random Variable outcomes and related probabilities are not defined at specific values, but rather over an Interval of values.

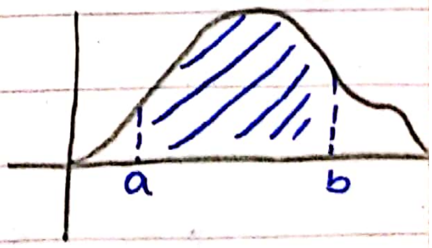
Example:

A random Variable X, the weight of an adult blue crab caught from Ireland, may range of 0.5 to 0.3 kg the probability that an adult blue crab weighs between these values is the area under the curve of the probability density function.

Formulas:

$$P\{X \in (-\infty, \infty)\} = \int_{-\infty}^{+\infty} f(x)\,dx = 1$$

$$P\{x \in (a, b)\} = \int_{a}^{b} f(x)\,dx =$$



$$P\{x < a\} = \int_{-\infty}^{a} f(x)\,dx$$

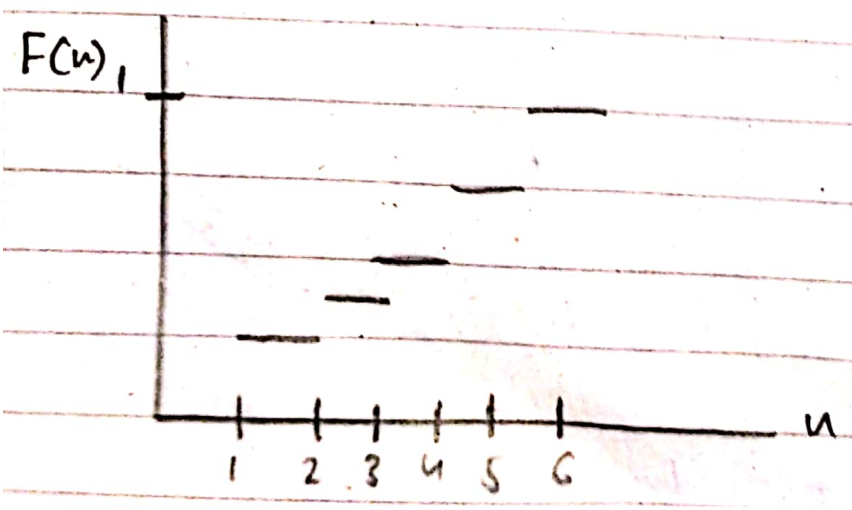
Commulative Distribution Function: (CDF)

The Commulative Distribution Function (cdf) is $F(u)$ of a discrete random variable X with p.m.f, $p(u)$ or $f(u)$ is defined for every number u by

$$F(u) = P(X \leq u) = \sum_{u=0}^{n} P(u)$$

For any number u, $F(u)$ is the probability that the observed value of X will be atmost u,

For X a discrete Random Variable the graph will have jump at every possible value of X and flat between any two possible values of X Such a graph is called Step Function:



For Continious Random Variable.

$$F(a) = P\{x \leq a\} = \int_{-\infty}^{a} f(x)\, dx$$

Example for C.d.f :-

Suppose a P.m.f of y is

Y	1	2	3	4
P(Y)	.4	.3	.2	.1

$$F(Y) = \begin{cases} 0 & \text{if } y < 1 \\ 0.4 & \text{if } y < 2 \\ 0.7 & y < 3 \\ 0.9 & y < 4 \\ 1 & y \leq 4 \end{cases}$$

X ———— X ———— X ———— X

Distribution of Boy's and Girl's having three kids in a family.

$n(\text{sample space}) = 2^3$

$n(S) = 8$

case	1st	2nd	3rd	No of B
1	B	B	B	3
2	B	B	G	2
3	B	G	B	2
4	B	G	G	1
5	G	B	B	2
6	G	B	G	1
7	G	G	B	2
8	G	G	G	0

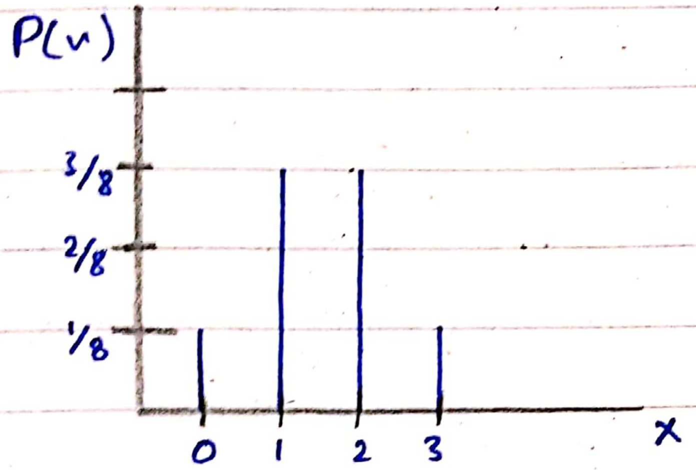
Tabular Form →

X	F	P(x)
0	1	1/8
1	3	3/8
2	3	3/8
3	1	1/8

Functional Form:-

$$P(x) = \frac{\binom{3}{x}}{2^3} \quad ; \quad x = 0, 1, 2, 3$$

Probability Histogram:-



Commulative Density function:-

$P(X \le x)$

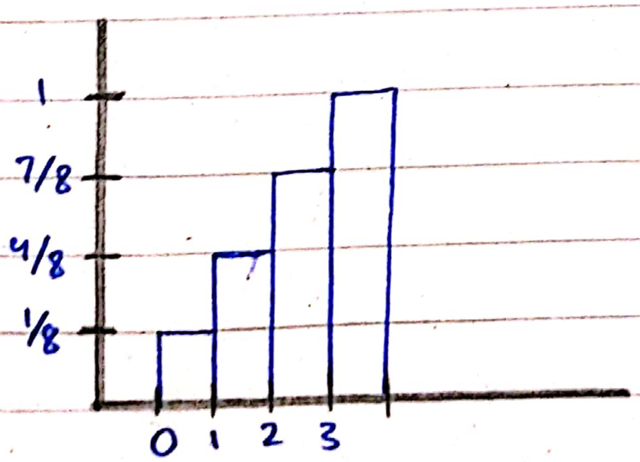
$P(X \le 0) = 1/8$

$P(X \le 1) = 4/8 = 1/2$

$P(X \le 2) = 7/8$

$P(x \le 3) = 1$

STEP FUNCTION:-



Example For Continious Random Variable:-

A Gas Station operates two pumps, each of which can pump up-to 10,000 gallons of gas in a month, The total amount of gas pumped at the station in a month is a random variable y (measured in 10,000 gallons) with a p.d.f given by.

$$f(y) = \begin{cases} y & , \ 0 < y < 1 \\ 2-y & , \ 1 \le y < 2 \\ 0 & , \ \text{else where} \end{cases}$$

a) Graph $f(y)$

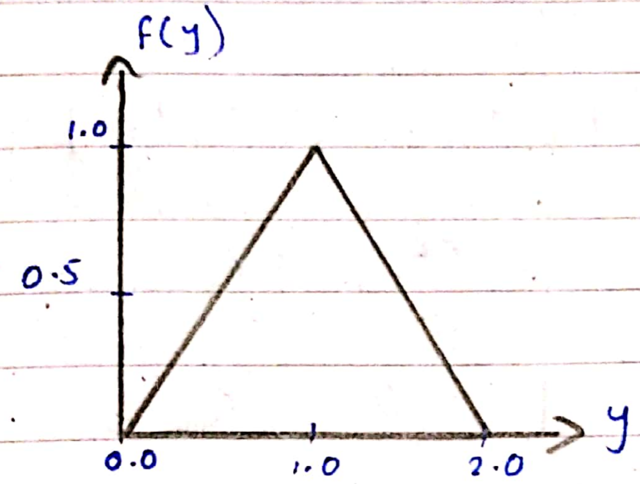
b) Find $F(y)$ and graph it

c) Find the probability that the station will pump b/w 8000 and 12,000 gallons in a particular month.

d) Given that the station pumped more than 10,000 gallons in a month, Find the probability the Station pumped more than 15,000 gallons during the month.

Solution:-

a)



b)

$$F(y) = P(Y \le y) = \int_{-\infty}^{y} f(y)\,dy$$

$$\rightarrow \quad F(y) = \begin{cases} \int_0^y y\,dy & ; \ 0 < y < 1 \\ \int_0^1 y\,dy + \int_1^y (2-y)\,dy & ; \ 1 \le y < 2 \\ 1 & ; \ y \ge 2 \end{cases}$$

$$\rightarrow \quad F(y) = \begin{cases} \frac{y^2}{2} & ; \ 0 < y < 1 \\ \frac{y^2}{2}\Big|_0^1 + \int_1^y 2\,dy - \int_1^y y\,dy & ; \ 1 \le y < 2 \\ 1 & ; \ y \ge 2 \end{cases}$$

$$\rightarrow \quad F(y) = \begin{cases} \frac{y^2}{2} & ; \ 0 < y < 1 \\ \frac{1}{2} + 2y\Big|_1^y - \frac{y^2}{2}\Big|_1^y & ; \ 1 \le y < 2 \\ 1 & ; \ y \ge 2 \end{cases}$$

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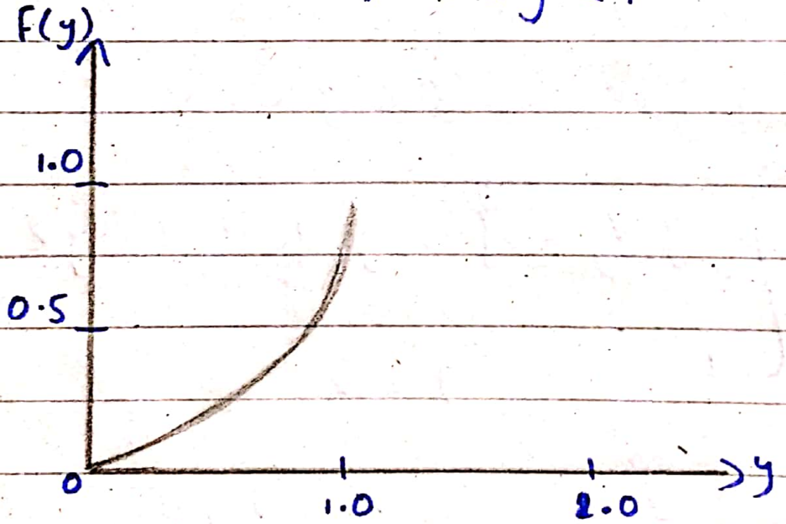
$$F(y) = \begin{cases} \dfrac{y^2}{2} & ; \ 0 < y < 1 \\ \dfrac{1}{2} + 2(y-1) - \dfrac{1}{2}(y^2-1) & ; \ 1 \leq y < 2 \\ 1 & ; \ y \geq 2 \end{cases}$$

$$F(y) = \begin{cases} \dfrac{y^2}{2} & ; \ 0 < y < 1 \\ 2y - \dfrac{y^2}{2} - 1 & ; \ 1 \leq y < 2 \\ 1 & ; \ 0 < y < 1 \end{cases}$$



c) $P(8000 \leq y \leq 12{,}000) = P(0.8 \leq y \leq 1.2)$

$$= \int_{0.8}^{1} y \, dy + \int_{1}^{1.2} (2-y) \, dy$$

$$= \frac{1}{2} \cdot y^2 \Big|_{0.8}^{1} + 2 \cdot y \Big|_{1}^{1.2} - \frac{1}{2} \cdot y^2 \Big|_{1}^{1.2}$$

$$= \frac{1}{2}\cdot\left(1-(.8)^2\right) + 2(1.2-1) - \frac{1}{2}\left((1.2)^2 - 1\right)$$

$$= 0.36$$

d) Using Conditional Probability.

$$\frac{P(Y > 15000 \cap X > 10000)}{P(X > 10{,}000)} = \frac{P(Y > 15000)}{P(X > 10{,}000)}$$

$$\frac{P(Y > 1.5)}{P(X > 1.0)} = \frac{1 - P(Y \le 1.5)}{1 - P(X \le 1.0)} \quad \text{— } 1$$

For $P(Y \le 1.5)$

$$P(Y \le 1.5) = \int_0^1 y\,dy + \int_1^{1.5} (2-y)\,dy$$

$$P(y \le 1.5) = \frac{1}{2}\cdot y\Big|_0^1 + 2y\Big|_1^{1.5} - \frac{1}{2}\left\{y^2\right\}\Big|_1^{1.5}$$

$$P(y \le 1.5) = \frac{1}{2}\cdot(1-0) + 2(1.5-1) - \frac{1}{2}\left((1.5)^2 - 1\right)$$

$$\boxed{P(y \le 1.5) = 0.875}$$

For $P(X \le 1.0)$

$$P(x \le 1.0) = \int_0^1 y\,dy = \frac{1}{2}\cdot y^2\Big|_0^1$$

$$\boxed{P(x \le 1.0) = 0.5}$$

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$$\frac{P(y > 15000)}{P(x > 10000)} = \frac{P(y > 1.5)}{P(x > 1.0)} = \frac{1 - P(y \leq 1.5)}{1 - P(x \leq 1.0)}$$

$$\frac{P(y > 1.5)}{P(x > 1.0)} = \frac{1 - 0.875}{1 - 0.5} = 0.25 \text{ Ans}$$

MATHMATICAL EXPECTATION:-

Let 'X' be the random variable associated with probabilities $f(x)$ or $P(X=x)$.
The p.d.f is

X	x_1	x_2	.....	x_k	Total
$P(X)$	$P(x_1)$	$P(x_2)$	.....	$P(x_k)$	1

The mean or expected value of x is
when x is discrete
$\mu = E(x) = \sum x P(x) =$

when x is continious
$\mu = E(x) = \int_{-\infty}^{\infty} x P(x)\, dx$

PROPERTIES:-

1. $E(x) = \mu = \bar{x}$

Proof

As $\bar{x} = \mu = \dfrac{\sum f_i x_i}{\sum f_i} = \sum x_i \cdot \dfrac{f_i}{N}$

$= \sum x_i P(x_i)$

$\boxed{\bar{x} = E(x)}$

2. $E(\text{constant}) = \text{constant}$

$E(c) = \sum c P(x_i)$

$= c \sum P(x_i)$

$\boxed{E(c) = c}$

3. The variance of a random variable x is $Var(x) = \sigma^2 = E(x^2) - \{E(x)\}^2$

$var(x) = E(x^2) - \mu^2$

$\sigma^2 = E\{(x-\mu)^2\} = E\{x^2 - 2\mu x + \mu^2\}$

$= E(x^2) - 2\mu E(x) + \mu^2$

$= E(x^2) - 2\mu^2 + \mu^2$

$\boxed{\sigma^2 = E(x^2) - \mu^2}$

4. The expected value of the sum or difference of two or more function of random variable.

X and Y is the sum or the difference of the expected values of the functions
i.e

$$E(X \pm Y) = E(X) \pm E(Y)$$

Proof

$$E[X \pm Y] = \sum_{\text{all } x} \sum_{\text{all } y} (x \pm y) \cdot P(x, y)$$

$$= \sum_{x} \sum_{y} (x) \cdot P(x, y) \pm \sum_{x} \sum_{y} y P(x, y)$$

$$= \sum_{x} x P(x) \pm \sum_{y} y P(y)$$

$$E[X \pm Y] = E(X) \pm E(y)$$

5- If X and Y are discerete random variabl and a & b are constant then

$$E[aX \pm bY] = a E(x) \pm b E(Y)$$

Proof

Let

$$U = ax \quad , \quad V = by$$

$$E[U \pm V] = ?$$

A/c to 4th property

$$E[U \pm V] = E(U) \pm E(V)$$

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$\rightarrow E[ax \pm by] = E(ax) \pm E(by)$

$\rightarrow E[ax \pm by] = aE(x) \pm bE(y)$

6. Let X & Y be the two Independent random Variable's then
$E(XY) = E(X) \cdot E(Y)$.
Proof:-

$$E[XY] = \sum_x \sum_y xy \cdot P(x,y)$$

$\because$ X and Y are Independent
$P(x,y) = P(x) \cdot P(y)$

$$E[X \cdot Y] = \sum_x \sum_y xy \cdot P(x) \cdot P(y)$$

$$= \sum x P(x) \cdot \sum_y y P(y)$$

$$E[X \cdot Y] = E(X) \cdot E(Y)$$

7. $E(g(x)) = \sum_{\text{all } x} g(x) \cdot P(x)$

8. $E(x^n) = \sum x^n P(x)$

Raw Moments :- about origin

-- When X is discrete R.V

$$\boxed{E(X^r) = \sum x_i^r P(x_i) = \mu_r'}$$

-- When X is Continious R.V

$$\boxed{E(X^r) = \int_{-\infty}^{\infty} x^r P(x)\, dx = \mu_r'}$$

PROBLEM:-

A Race Car driver wishes to insure his car for the racing Season's for $50,000. The insurance company estimates a total loss may occur with probability 0.002, a 50% loss with probability 0.01, and a 25% loss with probability 0.1, Ignoring all other partial losses, what premium Should the insurance company charge each season to realize an average profit of 500$?

Sol

Let 'X' be the profit to the company

and 'A' be the amount of Premium paid for every season.

Given $E(x) = \$500$

X	P(x)
A - 50,000 \$	0.002
A - 25,000 \$	0.01
A - 12,500 \$	0.1
A	0.888
Total	1

$\because E(x) = \sum x P(x)$

$$E(x) = (A-50000)(0.002) + (A-25000)(0.01) + (A-12500)(0.1) + A(0.888)$$

$$E(x) = 0.002A - 100 + 0.01A - 250 + 0.1A - 1250 + 0.888A$$

$$E(x) = A - 1600 \quad \text{--- (1)}$$

A/c to given data

$\because E(x) = 500$

(1) =>

$$500 = A - 1600$$

$$\boxed{A = 2100\$}$$

Decision:-

The company charge \$ 21,00 every Season in order to get an average profit of 500\$

Problem:-

In a gambling game a man is paid 5$ if he gets all heads or all tails when three coin's are tossed and he pays out $3 if either one or two heads show. what is his expected gain?

Sol:

Let 'x' be the amount the gambler can win

when $x = 5\$$

$E_1 = \{HHH, TTT\}$

when $x = -3\$$

$E_2 = \{HHT, HTH, THH, HTT, TTH, THT\}$

$P(E_1) = \frac{2}{8} = \frac{1}{4}$

$P(E_2) = \frac{6}{8} = \frac{3}{4}$

Thus

x	5	-3	Total
P(x)	1/4	3/4	1

$Mean = \mu = E(x) = \sum x P(x)$

$\sum x P(x) = 5(1/4) + (-3)(3/4)$

$Mean = \frac{5}{4} - \frac{9}{4}$

$\boxed{Mean = -1}$

Decision:-

In this game gambler will on the average loss of 1\$ per toss of the three coins

Problem:

Suppose that the number of cars, x, that pass through a car wash b/w 4:00 pm and 5:00 pm on avg. Sunny Friday has the following Probability distribution.

X	4	5	6	7	8	9
P(x)	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{6}$

Let $g(x) = 2x - 1$ represent the amount of money in dollars, paid to the attendent by the manager. Find the attendent's expected earning's

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For this particular time period.

Sol

By Property.

$$E[g(x)] = \sum g(x) f(x)$$

$$E[g(x)] = \sum (2x-1) f(x)$$

$$E[g(x)] = (2(4)-1)(1/12) + (2(5)-1)(1/12) + (2(6)-1)(1/4) + (2(7)-1)(1/4) + (2(8)-1)(1/6) + (2(9)-1)(1/6)$$

$$E[g(x)] = 12.67\ \$$$

BERNOULLI DISTRIBUTION:-

Consider an experiment with two possible outcomes call them Success (s) and failure (f) [Alive or dead, Sweet or not sweet, Defective or Non-defective, Solved or Unsolved] with probabilities

$$P(S) = p \quad \& \quad P(f) = 1-p = q$$

Let X be a discrete r.v take value 0 if failure occurs on 1 if success occur with probabilities.

$$P(x=0) = 1-p = q$$
$$P(x=1) = p$$

The probability distribution of the values which the random variable X takes is given by.

$$P(X=x) = p^x(1-p)^{1-x} = p^x \cdot q^{1-x} \ ; x = 0, 1$$

is called a Bernoullie distribution or Point Binomial distribution and the random var is called a Bernoullie variable.

In Bernoulli Distribution's

no of Trial's one $\begin{cases} \text{Sucess } (P) \\ \text{Failure } (1-P) = q \end{cases}$

RAW MOMENTS:-

$\boxed{\because E(x^n) = P}$

Proof 1:

$$E(x^n) = \sum x^n P(x=n)$$

By the defination of Bernoullie distribution

$$E(x^n) = \sum x^n p^x (1-p)^{1-x}$$
$$E(x^n) = 1^n . p^1 (1-p)^{1-1} + 0^n . p^0 (1-p)^{1-0}$$
$$\boxed{E(x^n) = P}$$

Proof 2:

By Moment Generating Function of Bernoulli Distribution

$\because M_x(t) = q + pe^t$

$E(x^r) = M_x^{(r)}(t)$

$M_x^{(r)}(t) = pe^t$

Setting $t=0$

$$\boxed{E(x^r) = pe^0 = p}$$

Moment Generating function:-

$$P(x=x) = \begin{cases} q & : x=0 \\ p & : x=1 \\ 0 & : x \notin \{0,1\} \end{cases}$$

$\because M_x(t) = E(e^{tx}) = \sum e^{tx} P(x=x)$

$M_x(t) = e^{t(0)} P(x=0) + e^{t(1)} \cdot P(x=1)$

$$\boxed{M_x(t) = q + pe^t}$$

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Binomial Distribution:-

Consider an experiment with two possible outcome's call them Sucess (S) and failure (f) with $P(S) = p$ & $P(f) = q$ such that $p + q = 1$ Let 'x' be the random variable denote's the number of Success in n independent repeate trial's e.g

Consider

$$\overbrace{\underbrace{S, S, \ldots, S}_{\text{Success} = x} \qquad \underbrace{f, f, \ldots, f}_{\text{failure} = n - x}}^{n \text{ Trials}}$$

The probability distribution of the particular Sequence (by multiplicative Law of Independent event's) is

$$p^{x} \cdot q^{(n-x)} \quad \text{or} \quad p^{x} \cdot (1-p)^{(n-x)}$$

The number of sequence in which 'x' Success and $n-x$ failures are observed in some order is $\binom{n}{x}$ ways. which is binomial cofficient

Thus the probability distribution that exactly x success and n-x failures occur in n independent trials is

$$b(x; n, p) = \binom{n}{x} p^x \cdot q^{n-x} \; ; \; x = 0, 1, 2, 3, \ldots, n$$

which is known as binomial distribution with index 'n' and parameter p

Q.
Prove That

$\rightarrow \sum_{x=0}^{n} b(x; n, p) = 1$

$\rightarrow \sum_{x=0}^{n} \binom{n}{x} \cdot p^x (1-p)^{n-x}$

$\rightarrow \binom{n}{0} \cdot p^0 (1-p)^{n-0} + \binom{n}{1} \cdot p^1 (1-p)^{n-1} + \binom{n}{2} \cdot p^2 (1-p)^{n-2} + \cdots + \binom{n}{n} \cdot p^n (1-p)^{n-n}$

$\rightarrow (1-p)^n + \binom{n}{1} p (1-p)^{n-1} + \binom{n}{2} \cdot p^2 (1-p)^{n-2} + \cdots + p^n$

$\rightarrow q^n + \binom{n}{1} p \cdot q^{n-1} + \binom{n}{2} p^2 q^{n-2} + \cdots + p^n$

$\rightarrow [q + p]^n$

$\because q + p = 1$

$\rightarrow 1^n = 1$

proved.

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RAW

Moment's In Binomial Distribution:-

Let 'x' be a random variable with the binomial distribution $b(n; n, p)$.

The moments about origin are given by the relation

$$\mu_r' = E(x^r)$$

at $r = 1$

$$\mu_1' = E(x) = \sum_{x=0}^{n} x\, b(n; n, p)$$

$$= \sum_{x=0}^{n} x \cdot \binom{n}{x} \cdot p^x q^{n-x}$$

$$= \cancel{(0) \cdot \binom{n}{0} \cdot p^0 \cdot q^{n-0}} + \cancel{\binom{n}{1} \cdot q^{n-1} \cdot p^1} + \cancel{\binom{n}{2} \cdot p^2 q^{n-2}} + \ldots + \cancel{p} \binom{n}{n} \cdot p^n \cdot q^{n-n}$$

$$= \sum_{x=0}^{n} \cancel{x} \cdot \frac{n(n-1)!}{(n-x)!\, \cancel{x}(x-1)!} \cdot p^x \cdot q^{n-x}$$

$$= np \sum_{x=0}^{n} \frac{(n-1)!}{((n-1)-(x-1))! \cdot (x-1)!} \cdot p^{x-1} \cdot q^{n-x}$$

$$= np \sum_{x=0}^{n} \binom{n-1}{x-1} \cdot p^{x-1} \cdot q^{n-x}$$

$$= np\, [q + p]^{n-1}$$

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$$\boxed{\mu_1' = np}$$

at $r=2$

$$\mu_2' = E[x^2] = \sum_{x=0}^{n} x^2 \cdot \binom{n}{x} \cdot p^x \cdot q^{n-x}$$

$$\mu_2' = E[x^2] = E[x(x-1)+x]$$

$$\mu_2' = E[x(x-1)+x]$$

$$\mu_2' = E[x(x-1)] + E[x] \quad \text{— ①}$$

$$E[x(x-1)] = \sum_{x=0}^{n} x(x-1) \cdot \binom{n}{x} p^x \cdot q^{n-x}$$

$$= \sum_{x=0}^{n} \cancel{x(x-1)} \cdot \frac{n(n-1)(n-2)!}{(n-x)!\,\cancel{x(x-1)}(x-2)!} \cdot p^x \cdot q^{n-x}$$

$$= n(n-1) \cdot p^2 \sum_{x=0}^{n} \frac{(n-2)!}{(n-x)!(x-2)!} p^{x-2} \cdot q^{n-x}$$

$$= n(n-1)p^2 \sum \binom{n-2}{x-2} \cdot p^{x-2} \cdot q^{n-x}$$

$$= n(n-1)p^2 [q+p]^{n-2}$$

$$\boxed{E[x(x-1)] = n(n-1)p^2}$$

① =>

$$\mu_2' = n(n-1)p^2 + np$$

$$\mu_2' = n^2p^2 - np^2 + np$$

$\mu_2' = np[np - p + 1]$

$\mu_2' = np[np + 1 - p]$

$\because q = 1 - p$

$\mu_2' = np[np + q]$

$$\boxed{\mu_2' = npq + (np)^2}$$

at $r = 3$

$\mu_3' = E(x^3) = E[x(x-1)(x-2) + 3x(x-1) + x]$

$\mu_3' = E[x(x-1)(x-2)] + E[3x(x-1)] + E[x]$

$\mu_3' = E[x(x-1)(x-2)] + 3E[x(x-1)] + E[x]$

A/c to above solution's:-

$$\boxed{\mu_3' = E[x(x-1)(x-2)] + 3n(n-1)p^2 + np} \quad (1)$$

For

$E[x(x-1)(x-2)] = \sum_{x=0}^{n} x(x-1)(x-2) \cdot \frac{n!}{(n-x)!\,x!} \cdot p^x \cdot q^{n-x}$

$\Rightarrow \sum_{x=0}^{n} \cancel{x}\cancel{(x-1)}\cancel{(x-2)} \cdot \frac{n(n-1)(n-2)(n-3)!}{(n-x)!\,\cancel{x}\cancel{(x-1)}\cancel{(x-2)}(x-3)!} \cdot p^3 \cdot p^{x-3} \cdot q^{n-x}$

$\Rightarrow n(n-1)(n-2)p^3 \cdot \sum \frac{(n-3)!}{(n-x)!\,(x-3)!} \cdot p^{x-3} \cdot q^{n-x}$

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$$E[x(x-1)(x-2)] = n(n-1)(n-2)p^3 \sum \binom{n-3}{x-3} p^{x-3} q^{n-x}$$

$$\boxed{E[x(x-1)(x-2)] = n(n-1)(n-2)p^3}$$

① ⇒

$$\boxed{\mu_3' = n(n-1)(n-2)p^3 + 3n(n-1)p^2 + np}$$

at $r = 4$

$$\mu_4' = E[x^4] = E[x(x-1)(x-2)(x-3) + 6x(x-1)(x-2) + 7x(x-1) + x]$$

$$\mu_4' = E[x(x-1)(x-2)(x-3)] + 6E[x(x-1)(x-2)] + 7E[x(x-1)] + E[x]$$

A/c to above solutions:-

$$\boxed{\mu_4' = E[x(x-1)(x-2)(x-3)] + 6n(n-1)(n-2)p^3 + 7n(n-1)p^2 + np} \quad \text{— ①}$$

For A/c to above Sequence.

$$\boxed{E[x(x-1)(x-2)(x-3)] = n(n-1)(n-2)(n-3)p^4}$$

① ⇒

$$\boxed{\mu_4' = n(n-1)(n-2)(n-3)p^4 + 6n(n-1)(n-2)p^3 + 7n(n-1)p^2 + np}$$

MOMENTS ABOUT MEAN IN BINOMIAL DISTRIBUTION:-

$$\because \mu_r = \frac{1}{n} \cdot \sum (x_i - \bar{x})^r$$

For $r = 1$

$$\mu_1 = 0$$

For $r = 2$

$$\because \mu_2 = \mu_2' - (\mu_1')^2$$

$$\mu_2 = npq + n^2p^2 - n^2p^2$$

$$\boxed{\mu_2 = npq}$$

For $r = 3$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$\mu_3 = n(n-1)(n-2)p^3 + 3(n(n-1)p^2) + nP - 3\{(npq + (np)^2) \cdot (nP)\} + 2(nP)^3$$

$$\mu_3 = n(n-1)(n-2)p^3 + 3n(n-1)p^2 + np - 3\{(npq + n^2p^2) \cdot (n$$
$$+ 2n^3p^3$$

$$\mu_3 = (n^2 - n)(n-2)p^3 + 3n^2p^2 - 3np^2 + np - 3\{n^2p^2q + n^3p^3\}$$
$$+ 2n^3p^3$$

$$\mu_3 = n^3p^3 - 2n^2p^3 - n^2p^3 + 2np^3 + 3n^2p^2 - 3np^2 + np -$$

$$3n^2p^2q - 3n^3p^3 + 2n^3p^3$$

$$\mu_3 = -3n^2p^3 + 2np^3 + 3n^2p^2 - 3np^2 + np - 3n^2p^2(1-p)$$

$$\mu_3 = np - \cancel{3n^2p^3} + 2np^3 + \cancel{3n^2p^2} - 3np^2 - \cancel{3n^2p^2} + \cancel{3n^2p^3}$$

$$\mu_3 = np + 2np^3 - 3np^2$$

$$\boxed{\mu_3 = np[1 - 3p + 2p^2]}$$

$$\mu_3 = np[1 - 3p + 2p^2]$$

$$\mu_3 = np[2p^2 - p - 2p + 1]$$

$$\mu_3 = np[2p^2 - 2p + 1 - p]$$

$$\mu_3 = np[2p(p-1) - (p-1)]$$

$$\mu_3 = np[(p-1)(2p-1)]$$

$$\mu_3 = -np(1-p)(2p-1)$$

$$\mu_3 = -npq(2p-1)$$

$$\mu_3 = npq(1-2p) = npq(1-p-p)$$

$$\boxed{\mu_3 = npq(q-p)}$$

$3n^2p^2q - 3n^3p^3 + 2n^3p^3$

$\mu_3 = -3n^2p^3 + 2np^3 + 3n^2p^2 - 3np^2 + np - 3n^2p^2(1-p)$

$\mu_3 = np - \cancel{3n^2p^3} + 2np^3 + \cancel{3n^2p^2} - 3np^2 - \cancel{3n^2p^2} + \cancel{3n^2p^3}$

$\mu_3 = np + 2np^3 - 3np^2$

$\boxed{\mu_3 = np[1 - 3p + 2p^2]}$

$\mu_3 = np[1 - 3p + 2p^2]$

$\mu_3 = np[2p^2 - p - 2p + 1]$

$\mu_3 = np[2p^2 - 2p + 1 - p]$

$\mu_3 = np[2p(p-1) - (p-1)]$

$\mu_3 = np[(p-1)(2p-1)]$

$\mu_3 = -np(1-p)(2p-1)$

$\mu_3 = -npq(2p-1)$

$\mu_3 = npq(1-2p) = npq(1-p-p)$

$\boxed{\mu_3 = npq(q-p)}$

for $r = 4$

$\mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6\mu_1'^2\mu_2 - 3\mu_1'^4$

$\mu_4 = (n(n-1)(n-2)(n-3)p^4 + 6n(n-1)(n-2)p^3 + 7n(n-1)p^2 + np) - 4((np)(n(n-1)(n-2)p^3 + 3n(n-1)p^2 + np) + 6(np)^2(npq + (np)^2) - 3n^4p^4$

$\mu_4 = [(n^2-n)(n-2)(n-3)p^4 + 6(n^2-n)(n-2)p^3 + 7(n^2-n)p^2 + np] - 4((np)((n^2-n)(n-2)p^3 + 3(n^2-n)p^2 + np) + 6n^3p^3q + 6n^4p^4 - 3n^4p^4$

$$\mu_4 = [(n^3-2n^2-n^2+2n)(n-3)p^4 + 6(n^3-2n^2-n^2+2n)p^3 + 7(n^2-n)p^2 + np] - [4np(n^3-2n^2-n^2+2n)p^3 + 3(n^2-n)p^2 + (np)] + 6n^3p^3q + 3n^4p^4$$

$$\mu_4 = [(n^3-3n^2+2n)(n-3)p^4 + 6(n^3-3n^2+2n)p^3 + 7(n^2-n)p^2 + np] - 4np(n^3-3n^2+2n)p^3 - 3(n^2-n)p^2 + np + 6n^3p^3q + 3n^4p^4$$

$$\mu_4 = [(n^4-3n^3-3n^3+9n^2+2n^2-6n)p^4 + 6(n^3-3n^2+2n)p^3 + 4(n^2-n)p^2 + 2np] - 4(n^4p - 3n^3p + 2n^2p)p^3 + 6n^3p^3q + 3n^4p^4$$

$$\mu_4 = (n^4-6n^3+11n^2-6n)p^4 + 6n(n^2-3n+2 + 4(n^2-n)p^2 + 2np - 4n(n^3-3n^2+2n)p + 6n^3p^3q + 3n^4p^4$$

$$\cancel{\mu_4 = (n^3-6n^2+11n-6)np^4 + 6n((n-1)(n-2))p^3}$$

$$\mu_4 = n^4p^4 - 6n^3p^4 + 11n^2p^4 - 6np^4 + 6(np)^3 - 18n + 12np^3 + 4n^2p^2 - 4np^2 + 2np - 4(np)^4 + 12n - 8n^2p^4 + 6n^3p^3q + 3n^4p^4$$

$$\mu_4 = n^4p^4 + 3n^4p^4 - 4n^4p^4 - 6n^3p^4 + 12n^3p^4 + 11n^2p^4 - 8n^2p^4 - 6np^4 + 6n^3p^3 + 6n^3p^3q - 18n^2p^3 + 12np^3 + 4n^2p^2 - 4np^2 + 2np$$

$$\mu_4 = -6n^3p^4 + 3n^2p^4 - 6np^4 + 6n^3p^3(1-q) - 18n^2p^3 + 12np^3 + 4n^2p^2 - 4np^2 + 2np$$

$$\mu_4 = -6n^3p^4 + 3n^2p^4 - 6np^4 + 6n^3p^4 - 18n^2p^3 + 12np^3 + 4n^2p^2 - 4np^2 + 2np$$

$$\mu_4 = 3n^2p^4 - 6np^4 - 18n^2p^3 + 12np^3 + 4n^2p^2 - 4np^2 + 2np$$

$$\boxed{\mu_4 = np(3np^3 - 6p^3 - 18np^2 + 12p^2 + 4np - 4p + 2)}$$

$$\boxed{\mu_4 = 3n^2p^2q^2 + npq(1-6pq)}$$ perfect form.

MGF:-

the moment generating function of the binomial distribution $b(x;n;p)$ is derived as.

$$\because M_x(t) = E[e^{tx}]$$

A/c to defination of Binomial Distribution

$$M_x(t) = \sum_{x=0}^{n} e^{tx} \binom{n}{x} p^x q^{(n-x)}$$

$$= \sum_{x=0}^{n} \binom{n}{x} [pe^t]^x q^{(n-x)}$$

$$= \binom{n}{0}(pe^t)^0 q^{n-0} + \binom{n}{1}(pe^t)^1 q^{(n-1)} + \binom{n}{2}(pe^t)^2 q^{(n-2)} + \cdots\cdots + \binom{n}{n}(pe^t)^n q^{(n-n)}$$

$$= q^n + \binom{n}{1}(pe^t) q^{n-1} + \binom{n}{2}(pe^t)^2 q^{n-2} + \cdots \cdots + \binom{n}{n}(pe^t)^n$$

$$M_x(t) = [q + pe^t]^n$$

which is the simplified form of M.g.f of Binomial Distribution.

Moments of Binomial Distri... by (M.G.F)

$$\because \mu_r' = E(x^r) = \frac{d^r}{dt^r}[q + pe^t]^n \Big|_{t=0}$$

Formula To find r^{th} Moment of Binomial Distribution using Moment Generating Function.

at $r=1$

$$\mu_1' = E[x] = \frac{d^{(1)}}{dt^{(1)}}[q+pe^t]^n \Big|_{t=0}$$

$$\mu_1' = n(q+pe^t)^{n-1}\cdot(pe^t)\Big|_{t=0}$$

$$= n(q+pe^0)^{n-1}(pe^0)$$

$$= n(q+p)^{n-1}p$$

$$= np(q+p)^{n-1} \qquad \because p+q = 1$$

$$\boxed{\mu_1' = np}$$

at $r=2$

$$\mu_2' = E[x^2] = \frac{d^{(2)}}{dt^{(2)}}[q+pe^t]^n\Big|_{t=0}$$

$$\mu_2' = \frac{d}{dt}\left[\frac{d}{dt}[q+pe^t]^n\right]\Big|_{t=0}$$

$$\mu_2' = \frac{d}{dt}\left[n(q+pe^t)^{n-1}\cdot pe^t\right]\Big|_{t=0}$$

$$\mu_2' = n(q+pe^t)^{n-1}\cdot pe^t + pe^t n(n-1)(q+pe^t)^{n-2}\cdot pe^t$$

$$\mu_2' = n(q+pe^0)^{n-1}\cdot pe^0 + pe^0\cdot pe^0 n(n-1)(q+pe^0)^{n-2}$$

$$\mu_2' = n(q+p)^{n-1}\cdot p + p^2 n(n-1)(q+p)^{n-1}$$

$$\mu_2' = np + p^2 n(n-1)$$

$$\mu_2' = np + n^2p^2 - np^2 \Rightarrow n^2p^2 + np(1-p)$$

$$\boxed{\mu_2' = n^2p^2 + npq}$$

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at $r=3$

$$\mu_3' = E[x^3] = \frac{d^{(3)}}{dt^{(3)}}[q+pe^t]^n \Big|_{t=0}$$

$$\mu_3' = \frac{d}{dt}\left[\frac{d}{dt}\left\{n(q+pe^t)^{n-1} \cdot pe^t\right\}\right]\Big|_{t=0}$$

$$\mu_3' = \frac{d}{dt}\left[\frac{d}{dt}\left\{n(q+pe^t)^{n-1} \cdot pe^t\right\}\right]\Big|_{t=0}$$

$$\mu_3' = \frac{d}{dt}\left[n(q+pe^t)^{n-1} \cdot pe^t + pe^t \cdot n(n-1)(q+pe^t)^{(n-2)} \cdot p\right.$$

$$\mu_3' = \frac{d}{dt}\left[pe^t n(q+pe^t)^{n-1} + (pe^t)^2 n(n-1)(q+pe^t)^{n-2}\right]$$

$$\mu_3' = \frac{d}{dt}\left[pe^t n(q+pe^t)^{n-1}\right]_{t=0} + \frac{d}{dt}\left[(pe^t)^2 n(n-1)(q+pe\right.$$

$$\mu_3' = \left\{n(q+pe^t)^{n-1} \cdot pe^t + pe^t n(n-1)(q+pe^t)^{n-2} \cdot pe^t\right\}$$

$$+ \left\{n(n-1)(q+pe^t)^{n-2} \cdot 2(pe^t)^2 + (pe^t)^2 \cdot n(n-1)(n-\right.$$

$$\left.(q+pe^t)^{n-3} \cdot pe^t\right\}$$

$$\mu_3' = pe^t n(q+pe^t)^{n-1} + (pe^t)^2 n(n-1)(q+pe^t)^{n-2} + 2(pe^t)^2 n(n-1)(q+pe^t)^{n-2} + (pe^t)^3 n(n-1)(n-2)(q+pe^t)^{n-3} \quad \text{— (1)}$$

by putting $t=0$

$$\mu_3' = pn(q+p)^{n-1} + p^2 n(n-1)(q+p)^{n-2} + 2(p)^2 n(n-1)(q+p)^{n-2} + p^3 n(n-1)(n-2)(q+p)^{n-3}$$

$$\mu_3' = np + n(n-1)p^2 + 2np^2(n-1) + n(n-1)(n-2)p^3$$

$$\mu_3' = np + n^2p^2 - np^2 + 2n^2p^2 - 2np^2 + (n^2-n)(n-2)p^3$$

$$\mu_3' = np + 3n^2p^2 - 3np^2 + (n^3p^3 - 2n^2p^3 - n^2p^3 + 2np^3$$

$$\mu_3' = np + 3n^2p^2 - 3np^2 + n^3p^3 - 3n^2p^3 + 2np^3$$

at $r=4$

$$\mu_4' = E[x^4] = \frac{d^{(4)}}{dt^{(4)}}[q+pe^t]^n \Big|_{t=0}$$

Skewness & Kurtosis In Binomial Distribution:-

$$\because \beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$$\beta_1 = \frac{[npq(q-p)]^2}{(npq)^3} = \frac{(q-p)^2}{npq}$$

$$\boxed{\beta_1 = \frac{(1-2p)^2}{npq}}$$

$$\because \beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{3n^2p^2q^2 + npq(1-6pq)}{(npq)^2}$$

$$\beta_2 = 3 + \frac{npq(1-6pq)}{(npq)^2}$$

$$\boxed{\beta_2 = 3 + \frac{(1-6pq)}{npq}}$$

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Problem:

The probability that a patient recovers from a rare disease is 0.4. If 15 people are known to contracted this rare blood disease what is the probability that.

a) at least 10 Survive?
b) from 3 to 8 survive?
c) Exactly 5 survive?
d) Fewer than 5 survive?

Sol

Let 'x' be the random variable denote the number of people's survive.

a) $P(x \geq 10) = 1 - P(x < 10)$

$= 1 - \sum_{x=0}^{9} b(x; n; p)$

from commulative binomial table

$= 1 - 0.9662$

$= 0.338$

b) $P(3 \leq x \leq 8) = P(x \leq 8) - P(x \leq 3)$

$= \sum_{x=0}^{8} P(x; n; P) - \sum_{x=0}^{3} P(x; n; P)$

from commulative binomial table

$= 0.905 - 0.027 = 0.8779$

c) $P(X=5) = P(X<6) - P(X<5)$

$$= \sum_{x=0}^{5} b(x;n;P) - \sum_{x=0}^{4} b(x;n;P)$$

from commulative binomial table

$$= .403 - .217 = .1859$$

d) $P(X<5) = \sum_{x=0}^{4} b(x;n;P) = .217$

POISSON DISTRIBUTION:-

Suppose that an experiment with two possible outcomes S & f and $P(S) = P$ and $P(f) = 1-P$ is repeated independently and indefinitely let P be small $(P \to 0)$. Such that $n \to \infty$ as $np \to \lambda$. Then the Probability distribution of the number of Success is

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \; ; \; x = 0, 1, 2, \dots, \infty$$

λ = mean = np -> Expectation / avg / weighted a

First moment about origin

n = No of Trials

x = No Succes Trials, P = Success Probabili

Derivation:

According to binomial distribution

$$P(X=x) = {}^nC_x \cdot p^x q^{n-x}$$

$$P(X=x) = \frac{n!}{x!(n-x)!} \cdot p^x q^{n-x}$$

$$P(X=x) = \frac{n(n-1)\cdots(n-x+1)(n-x)!}{x!\,(n-x)!} \cdot p^x q^{n-x}$$

$$P(X=x) = \frac{n(n-1)\cdots(n-x+1)}{x!}\, p^x \cdot q^{n-x}$$

Let $np = \lambda \Rightarrow p = \frac{\lambda}{n}$ and $q = \left(1-\frac{\lambda}{n}\right)$

$$P(X=x) = \frac{n(n-1)\cdots(n-x+1)}{x!}\left(\frac{\lambda}{n}\right)^x \cdot \left(1-\frac{\lambda}{n}\right)^{n-x}$$

$$P(X=x) = \frac{\lambda^x}{x!} \cdot \frac{n(n-1)\cdots(n-x+1)}{n^x} \cdot \left(1-\frac{\lambda}{n}\right)^n \cdot \left(1-\frac{\lambda}{n}\right)^{-x}$$

$$P(X=x) = \frac{\lambda^x}{x!}\left(1-1/n\right)\left(1-2/n\right)\cdots\left(1-(x+1)/n\right)\left(1-\frac{\lambda}{n}\right)^n\left(1-\frac{\lambda}{n}\right)^{-x}$$

Since we know that

$n \to \infty$

$$P(X=x) = \frac{\lambda^x}{x!}\left(1\cdot 1\cdot 1\cdots\cdots 1\right) \cdot \left(1-\frac{\lambda}{n}\right)^n \left(1-0\right)^{-x}$$

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$$P(X=n) = \frac{\lambda^x}{x!} \cdot \left(1-\frac{\lambda}{n}\right)^n \quad \text{— ①}$$

the term : $\left(1-\frac{\lambda}{n}\right)^n = \left[\left(1-\frac{\lambda}{n}\right)^n\right]^{\lambda/\lambda}$

$$\Rightarrow \left[\left(1-\frac{\lambda}{n}\right)^{n/\lambda}\right]^{\lambda}$$

$$\Rightarrow \left[\left(1-\frac{1}{(n/\lambda)}\right)^{n/\lambda}\right]^{\lambda}$$

Let $k = \frac{n}{\lambda}$

If n increase so for each k

$n \to \infty$

$k \to \infty$

$$\Rightarrow \left[\left(1-\frac{1}{k}\right)^{k}\right]^{\lambda} \to e^{-\lambda}$$

① $\Rightarrow$

$$P(X=x) = \frac{\lambda^x}{x!} \cdot \left(1-\frac{\lambda}{n}\right)^n$$

$$\lim_{n\to\infty} P(X=n) = \frac{\lambda^x}{x!} \lim_{n\to\infty} \left[\left(1-\frac{1}{k}\right)^{k}\right]^{\lambda}$$

$$\boxed{P(X=n) = \frac{\lambda^x}{x!} \cdot e^{-\lambda}}$$

Moments:

Let X be a random variable with the poisson distribution $P(x;\lambda)$ then

$$\mu_r' = E(x^r) = \sum_{x=0}^{\infty} x^r P(x;\lambda)$$

$r = 1$

$$\mu_1' = \sum_{x=0}^{\infty} x^1 \cdot P(x;\lambda)$$

$$\mu_1' = \sum_{x=0}^{\infty} x \cdot \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{x\lambda^x}{x!}$$

$$= e^{-\lambda} \cdot \left[0 + \frac{\lambda}{1!} + \frac{2\lambda^2}{2!} + \frac{3\lambda^3}{3!} + \cdots\cdots \right]$$

$$= e^{-\lambda} \cdot \left[\lambda + \lambda^2 + \frac{\lambda^3}{2!} + \frac{\lambda^4}{3!} + \frac{\lambda^5}{4!} + \cdots \right]$$

$$\because e^{\theta} = 1 + \theta + \frac{\theta^2}{2!} + \frac{\theta^3}{3!} + \cdots\cdots$$

$$= e^{-\lambda} \cdot \lambda \left[1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots\cdots \right]$$

$$= e^{-\lambda} \cdot \lambda \cdot e^{\lambda}$$

$$\boxed{\mu_1' = \lambda} = \text{Mean} = E[x]$$

$$\mu_2' = E[x^2] = E[x(x-1)+x] = E[x(x-1)] + E$$

$$\mu_2' = E[x(x-1)] + \lambda$$

$$E[x(x-1)] = \sum_{x=0}^{\infty} x(x-1) \frac{e^{-\lambda} \cdot \lambda^x}{x!}$$

$$E[x(x-1)] = e^{-\lambda} \sum_{x=0}^{\infty} x(x-1) \frac{\cancel{e}\, \lambda^x}{x!}$$

$$E[x(x-1)] = e^{-\lambda} \left[0 + 0 + \frac{2 \cdot 1 \cdot \cancel{e}\, \lambda^2}{2!} + \frac{3 \cdot 2 \cancel{e}\, \lambda^3}{3!} + \right.$$

$$E[x(x-1)] \quad \left. \frac{4 \cdot 3 \cdot \lambda^4}{4!} + \frac{5 \cdot 4 \cdot \lambda^5}{5!} + \cdots \right]$$

$$E[x(x-1)] = e^{-\lambda} \cdot \lambda^2 \left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots \right]$$

$$\because e^{\theta} = 1 + \frac{\theta}{1!} + \frac{\theta^2}{2!} + \frac{\theta^3}{3!} + \cdots$$

$$E[x(x-1)] = e^{-\lambda} \lambda^2 \cdot e^{\lambda}$$

$$\boxed{E[x(x-1)] = \lambda^2}$$

$$\boxed{\mu_2' = E[x^2] = \lambda^2 + \lambda}$$

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$$\mu_3' = E[x^3] = E[x(x-1)(x-2) + 3x(x-1) + x]$$

$$\mu_3' = E[x(x-1)(x-2)] + 3E[x(x-1)] + E[x]$$

$$\mu_3' = E[x(x-1)(x-2)] + 3(\lambda^2) + \lambda \quad - \text{①}$$

$$E[x(x-1)(x-2)] = \sum_{x=0}^{\infty} x(x-1)(x-2)\frac{e^{-\lambda}\lambda^x}{x!}$$

$$\Rightarrow e^{-\lambda}\left[0 + 0 + 0 + 3.2.1.\frac{\lambda^3}{3!} + 4.3.2.\frac{\lambda^4}{4!} + 5.4.3.\frac{\lambda^5}{5!} + \cdots\right]$$

$$\Rightarrow e^{-\lambda}\left[\lambda^3 + \frac{\lambda^4}{1!} + \frac{\lambda^5}{2!} + \frac{\lambda^6}{3!} + \cdots\right]$$

$$\Rightarrow e^{-\lambda}.\lambda^3.\left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots\right]$$

$$\Rightarrow E[x(x-1)(x-2)] = e^{-\lambda}.\lambda^3.e^{\lambda}$$

$$\Rightarrow E[x(x-1)(x-2)] = \lambda^3$$

① $\Rightarrow$

$$\mu_3' = \lambda^3 + 3(\lambda^2) + \lambda$$

$$\boxed{\mu_3' = \lambda^3 + 3\lambda^2 + \lambda}$$

$\mu_4' = E[x^4] = E[x(x-1)(x-2)(x-3) + 6x(x-1)(x-2) +$
$7x(x-1) + x]$

$\mu_4' = E[x(x-1)(x-2)(x-3)] + 6E[x(x-1)(x-2)] + 7E[x(x-1)]$
$+ E[x]$

$\mu_4' = E[x(x-1)(x-2)(x-3)] + 6\lambda^3 + 7\lambda^2 + \lambda$ —(

$$E[x(x-1)(x-2)(x-3)] = \sum_{x=0}^{\infty} x(x-1)(x-2)(x-3) \cdot \frac{e^{-\lambda}\lambda^x}{x!}$$

$$\Rightarrow e^{-\lambda} \cdot \sum_{x=0}^{\infty} x(x-1)(x-2)(x-3) \cdot \frac{\lambda^x}{x!}$$

$$\Rightarrow e^{-\lambda} \cdot \left[0 + 0 + 0 + 4\cdot3\cdot2\cdot1\frac{\lambda^4}{4!} + 5\cdot4\cdot3\cdot2\cdot\frac{\lambda^5}{5!} + 6\cdot5\cdot4\cdot3\cdot\frac{\lambda^6}{6!} + 7\cdot6\cdot5\cdot4\cdot\frac{\lambda^7}{7!} + \cdots\right]$$

$$\Rightarrow e^{-\lambda}\left[\lambda^4 + \frac{\lambda^5}{1!} + \frac{\lambda^6}{2!} + \frac{\lambda^7}{3!} + \cdots\right]$$

$$\Rightarrow e^{-\lambda} \cdot \lambda^4 \cdot e^{\lambda}$$

$$E[x(x-1)(x-2)(x-3)] = \lambda^4$$

$$\boxed{\mu_4' = \lambda^4 + 6\lambda^3 + 7\lambda^2 + \lambda}$$

Moments About Mean:-

$\because \mu_1 = \mu_1' - \mu_1' = 0$

$\because \mu_2 = \mu_2' - (\mu_1')^2$

$\mu_2 = \lambda^2 + \lambda - \lambda^2$

$\boxed{\mu_2 = \lambda} = Var(x) \Rightarrow S.D = \sqrt{\lambda}$

$\because \mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$\mu_3 = \lambda^3 + 3\lambda^2 + \lambda - 3(\lambda^2+\lambda)\lambda + 2\lambda^3$

$\mu_3 = 3\lambda^3 + 3\lambda^2 + \lambda - 3\lambda^3 - 3\lambda^2$

$\boxed{\mu_3 = \lambda}$

$\because \mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$\mu_4 = (\lambda^4 + 6\lambda^3 + 7\lambda^2 + \lambda) - 4(\lambda^3 + 3\lambda^2 + \lambda)\lambda +$
$6(\lambda^2 + \lambda)\lambda^2 - 3\lambda^4$

$\mu_4 = -2\lambda^4 + 6\lambda^3 + 7\lambda^2 + \lambda - 4\lambda^4 - 12\lambda^3 - 4\lambda^2$
$6\lambda^4 + 6\lambda^3$

$\mu_4 = 4\lambda^4 - 4\lambda^4 + 3\lambda^2 + \lambda$

$\boxed{\mu_4 = 3\lambda^2 + \lambda}$

Measure of Skewness:-

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{\lambda^2}{\lambda^3} = \frac{1}{\lambda}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{3\lambda^2 + \lambda}{\lambda^2} = 3 + \frac{1}{\lambda}$$

Measure of kurtosis:-

GEOMETRIC DISTRIBUTION:-

In a series of trials, if you assume that the probability of either sucess or failure of a random variable in each trial is the same, Geomotric distribution gires the probability of acheiving success after N number of failures. The distribution is essentially a set of probabilities that presents the chance of success after zero failures, one failure and so on.

Proof:

consider an experiment with two possible outcomes, Success (s) and failure (f).
with,

$P(S) = P$

$P(f) = q = 1-P$

The experiment is repeated until first success appears.
Let X be the number of independent trials required to obtain one Success, It means last trials must end in Success

$$\underbrace{f \cdot f \cdot f \cdot f \cdot f \cdot f}_{X-1} \cdot \underset{1}{S}$$

The probability distribution of $(n-1)$ failures and a success in last trial appear as

$$P(X=n) = (1-P)^{X-1} \cdot P = q^{X-1} \cdot P \; ; \; n = 1, 2, \ldots$$

$$g(n, P) = q^{n-1} \cdot P \; ; \; n = 1, 2, 3, \ldots$$

This is called geomatric distribution with parameter P.

Moments: About Origin.

$$\mu^r = E[x^r] = \sum_{x=1}^{\infty} x^r g(x;P)$$

$$\mu^r = \sum_{x=1}^{\infty} x^r \cdot q^{x-1} \cdot P$$

at $r = 1$

$$\mu_1' = E[x] = \sum_{x=1}^{\infty} x \cdot q^{x-1} \cdot P$$

$$\mu_1' = P + 2Pq + 3Pq^2 + 4Pq^3 + \cdots$$

$$= P(1 + 2q + 3q + 4q^3)$$

$$= P[1-q]^{-2} = P \cdot P^{-2} = 1/P$$

$$\boxed{\mu_1' = 1/P}$$

at $r = 2$

$$\mu_2' = E[x^2] = \sum_{x=1}^{\infty} x^2 \cdot q^{x-1} \cdot P$$

$$\mu_2' = P + 4Pq^1 + 9Pq^2 + 16Pq^3 + \cdots$$

$$\mu_2' = P(1 + 4q + 9q^2 + 16q^3 + \cdots)$$

$$\mu_2' = P(1 + 4q + 9q^2 + 16q^3 + \cdots]$$

$$\mu_2' = P[(1 + 3q + 6q^2 + 10q^3) + (q + 3q^2 + 6q^3)$$

$$\mu_2' = P[(1-q)^{-3} + q(1-q)^{-3}]$$

$$\mu_2' = \frac{P}{P^3} + \frac{Pq}{P^3}$$

$$\boxed{\mu_2' = \frac{1+q}{P^2}}$$

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Moments About Mean:

$$\boxed{\mu_1 = \mu_1' - \mu_1' = 0}$$

$$\mu_2 = var(x) = \mu_2' - \mu_1'^2$$

$$var(n) = \frac{1+q}{p^2} - \frac{1}{p^2}$$

$$var(n) = \frac{1+q-1}{p^2}$$

$$\boxed{var(n) = \mu_2 = \frac{1-p}{p^2}}$$

Moment Generating Function:- M.g.f

The m.g.f of the geometric distribution is derived as.

$$\because M_x(t) = E[e^{tx}] = \sum^{\infty} e^{tx} \cdot q^{x-1} \cdot p$$

$$M_x(t) = \sum_{x=1}^{\infty} e^{t(x-1+1)} \cdot q^{x-1} \cdot p$$

$$M_x(t) = \sum_{x=1}^{\infty} e^{t(x-1)+t} \cdot q^{x-1} \cdot p$$

$$M_x(t) = pe^t \sum_{x=1}^{\infty} e^{t(x-1)} \cdot q^{(x-1)} \cdot \cancel{p}$$

$$M_x(t) = pe^t \sum_{x=1}^{\infty} (e^t q)^{x-1}$$

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$$M_x(e^t) = pe^t[1 + qe^t + (qe^t)^2 + (qe^t)^3 + \dots$$

$$M_x(e^t) = pe^t[1 - qe^t]^{-1}$$

$$\boxed{M_x(e^t) = \frac{pe^t}{1 - qe^t}}$$

For Moments:

$$\mu_r' = M_x(t)^{(r)}\Big|_{t=0} = \frac{d^r}{dt^r}\left(\frac{pe^t}{1-qe^t}\right)\Bigg|_{t=0}$$

$$M_x(t) = \frac{pe^t}{1-qe^t} = \left(\frac{p}{e^{-t}-q}\right) = p(e^{-t}-q)^{-1}$$

$$\boxed{M_x(t) = p(e^{-t}-q)^{-1}}$$

at $r=1$

$$\mu_1' = \frac{d}{dt}\left\{p(e^{-t}-q)^{-1}\right\}$$

$$\mu_1' = p\left\{(-1)(e^{-t}-q)^{-2}.(-e^{-t})\right\}$$

$$\mu_1' = pe^{-t}(e^{-t}-q)^{-2}\Big|_{t=0} \quad ①$$

$$\mu_1' = p(1-q)^{-2}$$

$$\boxed{\mu_1' = \frac{1}{p}}$$

at $r=2$

$$\mu_2' = \frac{d^2}{dt^2} = P(e^{-t}-q)^{-1} = \frac{d}{dt}\left[Pe^{-t}(e^{-t}-q)^{-2}\right]\Big|_{t=0}$$

$$\mu_2' = P\frac{d}{dt}\, e^{-t}(e^{-t}-q)^{-2}$$

$$\mu_2' = P\left\{e^{-t}\left((-2)(e^{-t}-q)^{-3}(-e^{-t})\right) + (e^{-t}-q)^{-2}(-e^{-t})\right\}$$

$$\mu_2' = P\left\{2e^{-2t}(e^{-t}-q)^{-3} - e^{-t}(e^{-t}-q)^{-2}\right\}\Big|_{t=0}$$

$$\mu_2' = P\left\{2(1-q)^{-3} - (1-q)^{-2}\right\}$$

$$\mu_2' = \frac{2P}{p^3} - \frac{P}{p^2}$$

$$\boxed{\mu_2' = \frac{2}{p^2} - \frac{1}{p}}$$

Similarly:

$$\boxed{\mu_3' = \frac{6q^2 + 6pq + p^2}{p^3}}$$

$$\boxed{\mu_4' = \frac{24q^3}{p^4} + \frac{36q^2}{p^3} + \frac{14q}{p^2} + \frac{1}{p}}$$

Recurrence Relation for geometric distribution:

$$\mu_{r+1} = q\left[\frac{r}{p^2}\mu_{r-1} - \frac{d\mu_r}{dp}\right]$$

at $r=1$

$$\mu_2 = q\left[\frac{\mu_0}{p^2} - \frac{d(0)}{dp}\right]$$

$$\boxed{\mu_2 = q/p^2}$$

at $r=2$

$$\mu_3 = q\left[\frac{2\mu_1}{p^2} - \frac{d\mu_2}{dp}\right]$$

$$\mu_3 = q\left[2(0) - \frac{d}{dp}\frac{q}{p^2}\right] = q\frac{d}{dt}\frac{1-p}{p^2}$$

$$\mu_3 = -q\left\{\frac{d}{dp}\left(\frac{1}{p^2} - \frac{1}{p}\right)\right\}$$

$$\mu_3 = -q\left(-2p^{-3} + p^{-2}\right)$$

$$\mu_3 = \frac{2q}{p^3} - \frac{q}{p^2} = \frac{2q - pq}{p^3}$$

$$\mu_3 = \frac{q(2-p)}{p^3}$$

$$\boxed{\mu_3 = \frac{q(2-p)}{p^3}}$$

dt

at $r = 3$

$$\mu_4 = q\left[\frac{3\mu_2}{p^2} - \frac{d\mu_3}{dp}\right]$$

$$\mu_4 = q\left[\frac{3\,q/p^2}{p^2} - \frac{d}{dp}\left(\frac{q(2-p)}{p^3}\right)\right]$$

$$\mu_4 = q\left[\frac{3q}{p^4} - \frac{d}{dp}\left(\frac{2q - pq}{p^3}\right)\right]$$

$$\mu_4 = q\left[\frac{3q}{p^4} - \frac{d}{dp}\left(\frac{2(1-p) - (1-p)p}{p^3}\right)\right]$$

$$\mu_4 = \frac{3q^2}{p^4} - q\frac{d}{dp}\left[\left(\frac{2 - 2p - p + p^2}{p^3}\right)\right]$$

$$\mu_4 = \frac{3q^2}{p^4} - q\left\{\frac{d}{dp}\left(\frac{2 - 3p + p^2}{p^3}\right)\right\}$$

$$\mu_4 = \frac{3q^2}{p^4} - q\left\{\frac{d}{dp}\left(\frac{2}{p^3} - \frac{3}{p^2} + \frac{1}{p}\right)\right\}$$

$$\mu_4 = \frac{3q^2}{p^4} - q\left\{\frac{-6}{p^4} - \frac{6}{p^3} - \frac{1}{p^2}\right\}$$

$$\mu_4 = \frac{3q^2}{p^4} + \frac{6q}{p^4} + \frac{6q}{p^3} + \frac{q}{p^2}$$

$$\mu_4 = \frac{3q^2}{p^4} + \frac{6q}{p^4} + \frac{6pq}{p^4} + \frac{p^2q}{p^4}$$

$$\boxed{\mu_4 = \frac{3q^2}{p^4} + \frac{q}{p^4}\left\{6 + 6p + p^2\right\}}$$

Skewness:

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{\left(q(2-p)/p^{3}\right)^2}{(q/p^2)^3} = \frac{q^2(2-p)^2 p^6}{q^3 \cdot p^6}$$

$$\beta_1 = \frac{p^6(2-p)^2}{q \cdot p^6}$$

$$\beta_1 = \frac{(2-p)^2}{q} = \frac{(1+q)^2}{q}$$

Kurtosis:

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \left\{\frac{3q^2}{p^4} + \frac{q}{p^4}\left(6+6p+p^2\right)\right\} \Big/ \left(\frac{q}{p^2}\right)^2$$

$$\beta_2 = \frac{3q^2 + q(6+6p+p^2)}{p^4} \times \frac{p^4}{q^2}$$

$$\boxed{\beta_2 = \frac{3q+6+6p+p^2}{q}}$$

Problem:-

If the probability is 0.75 that an applicant pass the road test on any try, what is the probability that an application will finally pass the test on the fourth try.

Sol $p = 0.75$, $q = 1-0.75$, $q = 0.25$
$n = 4$

$\because g(n,p) = g(4,0.75) = q^{x-1} \cdot p$

$g(4,0.75) = (.25)^{4-1}(.75) = 0.0117$

Problem:-

three people toss a coin and the odd man pays for the coffee, If coins all turn up same, they are tossed again. Find the probability that fewer than 4 tosses are needed.

Sample Space = {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}

$p = 6/8 = 3/4$, $q = 1-P = 1-3/4$

$p = 3/4$, $q = 1/4$

$P(x<4) = \sum_{x=1}^{3} g(n,p) = \sum_{x=1}^{3} p q^{n-1} = \sum_{x=1}^{3} \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^{x-1}$

$P(x<4) = \frac{3}{4}\left\{\left(\frac{1}{4}\right)^0 + \left(\frac{1}{4}\right)^1 + \left(\frac{1}{4}\right)^2\right\}$

$\boxed{P(x<4) = \frac{63}{64}}$

Hyper-Geometric Distribution:-

A random variable X has a hyper-geometric distribution if its probability distribution is given by.

$$h(x; n, N, k) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}} \quad ; \quad \begin{array}{l} x = 0, 1, \ldots, k \\ x \leq k \\ n-x \leq N-K \end{array}$$

Proof:

Suppose a box of N oranges contains k sweet oranges. A sample of n oranges $(n<N)$ is selected. the probability that the sample contains x sweet oranges $(x \leq k)$
let Sample Space $\binom{N}{n}$

Population

$$\underbrace{\underbrace{S \cdot S \cdot S \cdots S}_{k \text{ sweet}} \quad \underbrace{\bar{S} \cdot \bar{S} \cdot \bar{S} \cdots \bar{S}}_{\substack{N-k \\ \text{not sweet}}}}_{N \text{ orranges}}$$

and the sample contains

$$\underbrace{\underbrace{S \cdot S \cdot S \cdot S \cdots S}_{n\text{-sweet}} \quad \underbrace{\bar{S} \cdot \bar{S} \cdot \bar{S} \cdots \bar{S}}_{n-x \text{ not sweet}}}_{n \text{ orranges}}$$

the number of possible ways of selecting n orranges from k is $\binom{k}{n}$ and the number of ways of selecting $(n-n)$ not sweet oranges out of $N-k$ is $\binom{N-k}{n-n}$ ways.

$$h(n; n, N, k) = \frac{\binom{k}{n}\binom{N-k}{n-x}}{\binom{N}{n}}$$

Moments About Origin:

$$\mu_r' = E[x^r] = \sum_{x=0}^{n} n^r h(n; n, N, k)$$

$$\boxed{\mu_r' = \sum_{x=0}^{n} n^r \frac{\binom{k}{n}\binom{N-k}{n-n}}{\binom{N}{n}}}$$

at $r=1$

$$\mu_1' = \sum_{x=1}^{n} n \cdot \frac{\binom{k}{n}\binom{N-k}{n-n}}{\binom{N}{n}}$$

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$$\mu_i' = \frac{1}{\binom{N}{n}} \sum_{x=1}^{n} x \binom{k}{x}\binom{N-k}{n-x}$$

$$\mu_i' = \frac{1}{\binom{N}{n}} \sum_{x=1}^{n} x \cdot \frac{k(k-1)!}{x(x-1)!(k-x)!} \cdot \frac{(N-k)!}{(n-x)!(N-k-(n-x))!}$$

$$\mu_i' = \frac{k}{\binom{N}{n}} \sum_{x=1}^{n} \frac{(k-1)!}{(x-1)!(k-x)!} \cdot \frac{(N-k)!}{[N-k-(n-x)]!\,(n-x)!}$$

Let

$z = x-1 \Rightarrow x = z+1$ at $x=1$, $z=0$

at $x=n$, $z=n-1$

$$\mu_i' = \frac{k}{\frac{N!}{n!(N-n)!}} \sum_{z=0}^{n-1} \frac{(k-1)!}{z!\,(k-z-1)!} \cdot \frac{(N-k)!}{(N-k-n-z-1)!\,(n-z-1)!}$$

$$\mu_i' = \frac{k(n)(n-1)!\,(N-n)!}{N(N-1)!} \sum_{z=0}^{n-1} \frac{(k-1)!}{z!\,(k-1-z)!} \cdot \frac{(N-k)!}{(N-k-\overline{n-z-1})!\,(n-z-1)!}$$

$$\mu_i' = \frac{kn}{N} \cdot \frac{1}{\binom{N-1}{n-1}} \cdot \underbrace{\sum_{z=0}^{n-1} \binom{k-1}{z}\binom{N-k}{n-z-1}}$$

$$\boxed{\mu_i' = \frac{kn}{N}}$$

this part is HG-D Total sum so A/c to P.D.F it is equals to one

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at $r = 2$

$$\mu_2' = E[x^2] = E[x(x-1) + x] = E[x(x-1)] + E[x]$$

$$\mu_2' = E[x(x-1)] + \frac{kn}{N} \quad \text{—} \ (1)$$

(1) $\Rightarrow$

$$E[x(x-1)] = \sum_{x=2}^{n} \frac{x(x-1)}{\binom{N}{n}} \cdot \frac{k!}{x!(k-x)!} \cdot \frac{(N-k)!}{(n-x)!(N-k-\overline{n-x})!}$$

$$E[x(x-1)] = \frac{n(n-1)(n-2)!(N-n)!}{N(N-1)(N-2)!} \sum_{x=1}^{n} \frac{k(k-1)(k-2)!\, x(x-1)}{(k-x)!\, x(x-1)(x-2)!}$$
$$\frac{(N-k)!}{(n-x)!(N-k-\overline{n-x})!}$$

$$E[x(x-1)] = \frac{n(n-1)}{N(N-1)} \cdot k(k-1) \cdot$$
$$\sum_{x=1}^{n} \frac{\dfrac{(k-2)!}{(x-2)!(k-x)!} \cdot \dfrac{(N-k)!}{(n-x)!(N-k-\overline{n-x})!}}{\dfrac{(N-2)!}{(N-n)!(n-2)!}}$$

Let

$z = x-2 \Rightarrow x = z+2$ at $x = 2$, $z = 0$

at $x = n$, $z = n-2$

$$E[X(X-1)] = E[(Z+2)(Z+1)] =$$

$$\frac{n(n-1)}{N(N-1)} \cdot K(K-1) \sum_{z=0}^{n-2} \left(\frac{\frac{(K-2)!}{2!(K-z-2)!} \frac{(N-K)!}{(n-z-2)!(N-K-n)!}}{\frac{(N-2)!}{z!(N-z-2)!}} \right)$$

$$E[X(X-1)] = \frac{n(n-1)}{N(n-1)} \cdot K(K-1) \sum_{z=0}^{n-2} \frac{\binom{K-2}{z}\binom{N-K}{n-z-2}}{\binom{N-2}{n-2}}$$

$$\sum_{z=0}^{n-2} \frac{\binom{K-2}{z}\binom{N-K}{n-z-2}}{\binom{N-2}{n-2}} = 1$$

Since the expression is p.d.f for hyper-geometric for $z = 0, 1, 2, \ldots, n-2$

$$\mu_2' = K(K-1) \cdot \frac{n(n-1)}{N(N-1)} + K \cdot \frac{n}{N}$$

$$\boxed{\mu_2' = \cancel{\text{var}(x)} = \frac{Kn}{N}\left(\frac{(K-1)(n-1)}{N-1} + 1\right)}$$

Moments About Mean

$$\mu_1 = 0$$

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$\mu_2 = \frac{kn}{N}\left(\frac{(k-1)(n-1)}{N-1} + 1\right) + \left(\frac{kn}{N}\right)^2$$

$$\boxed{\mu_2 = \frac{kn}{N}\left(\frac{(k-1)(n-1)}{N-1} + 1 - \frac{kn}{N}\right) = var(x)}$$

Uniform Distribution:-

Discerete:-

A random variable 'x' has the discrete uniform distribution if it has a finite number of possible values $x_1, x_2, \dots, x_n$ and

$$f(x_i) = f(X = x_i) = \frac{1}{n} \; ; \; x = 1, 2, \dots, n$$

If 'x' has discrete UD on the consecutive integers $a, a+1, a+2, \dots, b$ then

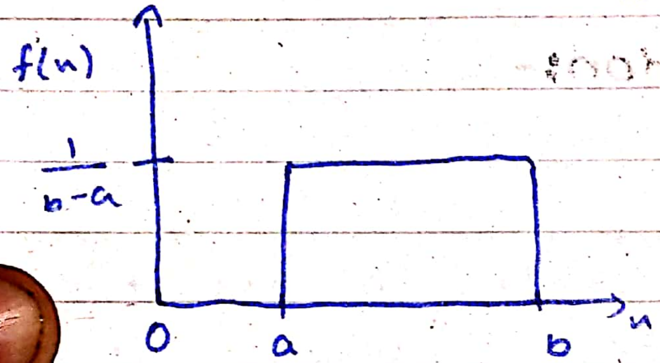
$$\mu = E[x] = \frac{b+a}{2}$$

$$\sigma^2 = var(x) = \frac{(b-a+1)^2 - 1}{12}$$

Continious:

A random variable 'x' has a continious UD if it p.d.f is

$$f(x; a, b) = f(x) = \frac{1}{b-a} ; \quad a \leq x \leq b, a<b$$



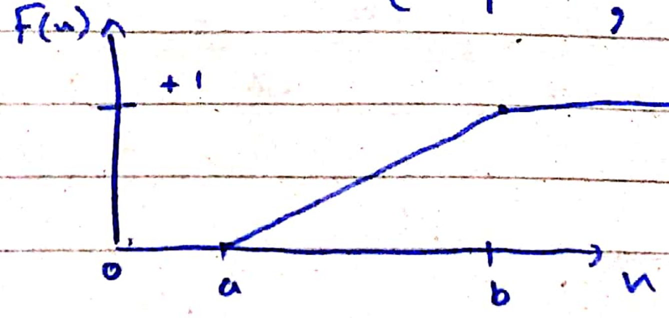
Cumulative Distribution Function: (cdf)

$$F(x) = \int_a^x f(x)\,dx = \int_a^x \left(\frac{1}{b-a}\right) dx = \frac{1}{b-a} \cdot [x]_a^x$$

$$F(x) = \frac{x-a}{b-a} ;$$

Thus

$$F(x) = \begin{cases} 0 & , x < a \\ \frac{-a+x}{b-a} & , a \leq x \leq b \\ 1 & , x \geq b \end{cases}$$



Moments: (about origin)

$$\mu_r' = E[x^r] = \frac{1}{b-a}\int_a^b x^r\,dx = \frac{1}{(b-a)} \cdot \frac{[x^{r+1}]_a^b}{(r+1)}$$

$$\mu_r' = \frac{1}{b-a} \cdot \frac{(b^{r+1} - a^{r+1})}{(r+1)}$$

$$\boxed{\mu_r' = \frac{b^{r+1} - a^{r+1}}{(b-a)(r+1)}}$$

$r=1$

$$\mu_1' = E[x] = \text{mean} = \frac{b^2 - a^2}{2(b-a)} = \frac{(b-a)(b+a)}{2(b-a)}$$

$$\boxed{\mu_1' = \frac{b+a}{2}}$$

$r=2$

$$\mu_2' = E[x^2] = \frac{b^3 - a^3}{(b-a)3} = \frac{(b-a)(b^2+ab+a^2)}{3(b-a)}$$

$$\boxed{\mu_2' = \frac{b^2+ab+a^2}{3}}$$

Similarly

$$\mu_3' = \frac{a^3 + a^2b + ab^2 + b^3}{4}$$

$$\mu_4' = \frac{a^4 + a^3b + a^2b^2 + ab^3 + b^4}{5}$$

$\beta_1 = 0$, $\beta_2 = 9/5$

m.g.f $M_x(t) = \dfrac{e^{bt} - e^{at}}{(b-a)t} = E[e^{tx}]$

$$M_x(t) = E[e^{t(x-a)}]$$

$$= E\left[1 + t(x-a) + \frac{t^2}{2!}(x-a)^2 + \cdots\cdots + \frac{t^r}{r!}(x-a)^r + \cdots\right]$$

where

$$\boxed{\mu'_r = E[(x-a)^r]}$$

$$M_{(x)}(t) = \frac{e^{bt} - e^{at}}{(b-a)t}$$

$$\because e^{\theta} = 1 + \theta + \frac{\theta^2}{2!} + \frac{\theta^3}{3!} + \cdots \frac{\theta^r}{r!} + \cdots$$

$$M_{(x)}(t) = \frac{1}{t(b-a)} \cdot \left[\left\{1 + tb + \frac{(tb)^2}{2!} + \cdots\right\} - \left\{1 + ta + \frac{(ta)^2}{2!} + \cdots\right\}\right]$$

$$M_{(x)}(t) = \frac{1}{t(b-a)} \cdot \left[t(b-a) + \frac{t^2(b^2-a^2)}{2!} + \frac{t^3(b^3-a^3)}{3!} + \cdots\right]$$

$$M_{(x)}(t) = \frac{t(b-a)}{t(b-a)}\left[1 + \frac{t(b+a)}{2!} + \frac{t^2(b^2+ab+a^2)}{2!} + \frac{t^3(b+a)(b^2+a^2)}{4!} + \cdots\right]$$

$$= 1 + \frac{t(b+a)}{2!} + \frac{t^2(b^2+ab+a^2)}{3!} + \frac{t^3(b+a)(b^2+a^2)}{4!} + \cdots$$

this sequence is mgf of the continious uniform dist by comparing the cofficients

$$\frac{t}{2!} \Rightarrow \mu_1' = \frac{b+a}{2}$$

$$\frac{t^2}{3!} \Rightarrow \mu_2' = \frac{b^2+ab+a^2}{6}$$

$$\frac{t^3}{3!} \Rightarrow \mu_3' = \frac{(b+a)(b^2+a^2)}{24}$$

in Uniform Distribution

$$\text{Mean} = \text{Median} = \text{Mode} = \frac{a+b}{2}$$

The Exponential Distribution:-

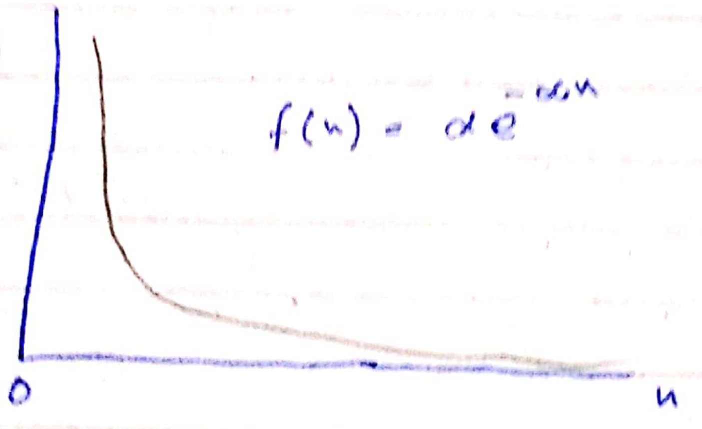
a continious random variable x has an exponential distribution if and only if it p-density is given as.

$$f(x) = \begin{cases} \frac{1}{\theta} \cdot e^{\frac{-x}{\theta}} & , x > 0 \\ 0 & , \text{elsewhere} \end{cases}$$

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & , x > 0 \\ 0 & , \text{elsewhere} \end{cases}$$

$\alpha > 0$

P.d.f



The distribution function is.

$$F(x) = \int_0^x f(x)\,dx = \alpha \int_0^x e^{-\alpha x}\,dx = \alpha \frac{e^{-\alpha x}}{-\alpha}\Big|_0^x$$

$$F(x) = \left[-e^{-\alpha x}\right]_0^x = \left[1 - e^{-\alpha x}\right]$$

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P.d.f

$$\int_0^{\infty} f(x)\, dx = 1$$

$$\alpha \int_0^{\infty} \cancel{e^{-\alpha x}} e^{-\alpha x}\, dx = 1$$

$$\alpha \frac{\left[e^{-\alpha x} \right]_0^{\infty}}{-\alpha} = 1$$

$$\frac{\alpha}{-\alpha} \left[e^{-\alpha(\infty)} - e^{-\alpha(0)} \right] = 1$$

$$-\left[e^{-\infty} - 1 \right] = 1$$

$$-(0-1) = 1$$

$$\boxed{1 = 1}$$

Moments about origin:-

$$E[x^r] = \int_0^{\infty} x^r \alpha e^{-\alpha x}\, dx$$

$$\because \frac{a\Gamma n}{\alpha^n} = \alpha \int_0^{\infty} x^{n-1} \cdot e^{-\alpha x}\, dx$$

$$E[x^r] = \alpha \int_0^{\infty} x^{(r+1)-1} \cdot e^{-\alpha x}\, dx$$

$$E[x^r] = \cancel{\alpha} \cancel{\int} \frac{\alpha \cdot \Gamma r+1}{\alpha^{r+1}}$$

$$E[x^r] = \mu_r' = \frac{r!}{\alpha^r}$$

at $r=1$

$$\mu_1' = \frac{1!}{\alpha^1} = \cancel{\mu_1'}$$

$$\boxed{\mu_1' = \frac{1}{\alpha}}$$

at $r=2$

$$\mu_2' = \frac{2!}{\alpha^2}$$

at $r=3$

$$\mu_3' = \frac{6}{\alpha^3}$$

Moments about Mean:-

$$\mu_1 = 0$$

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$\mu_2 = \frac{2}{\alpha^2} - \frac{1}{\alpha^2}$$

$$\boxed{\mu_2 = Var(x) = \frac{1}{\alpha^2}}$$

Median:-

$$F(M) = \frac{1}{2} \Rightarrow 1 - e^{\alpha M} = \frac{1}{2}$$

$$\alpha \int_0^M e^{-\alpha x}\, dx = \frac{1}{2}$$

$$\alpha \frac{\left[e^{-\alpha x}\right]_0^M}{-\alpha} = \frac{1}{2}$$

$$-\left[e^{-\alpha M} - 1\right] = 1/2$$

$$1 - e^{-\alpha M} = \frac{1}{2}$$

$$e^{-\alpha M} = 1 - \frac{1}{2}$$

$$-\alpha M = \ln(1/2)$$

$$\boxed{M = \frac{1}{\alpha} \ln(2)}$$

Quartile:-

$$F(Q_1) = \frac{1}{4} = 1 - e^{-\alpha Q_1} = \frac{1}{4}$$

$$e^{-\alpha Q_1} = 1 - 1/4$$

$$-\alpha Q_1 = \ln(3/4)$$

$$\boxed{Q_1 = \frac{1}{\alpha} \cdot \ln(4/3)}$$

Similarly $\boxed{Q_3 = \frac{1}{\alpha} \ln(1/4)}$

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Mode:

$$f(x) = \alpha e^{-\alpha x}$$
$$f'(x) = -\alpha^2 e^{-\alpha x} = 0$$

$$e^{-\alpha x} = 0$$
$$x = \infty$$

M.G.F

$$M_x(t) = E[e^{tx}] = \int_0^\infty e^{tx} f(x)\, dx$$
$$M_x(t) = \alpha \int_0^\infty e^{tx} . e^{-\alpha x}\, dx$$

$$M_x(t) = \alpha \int_0^\infty e^{x(t-\alpha)}\, dx$$

$$M_x(t) = \alpha \frac{\left[e^{x(t-\alpha)} \right]_0^\infty}{(t-\alpha)}$$

$$M_x(t) = \frac{\alpha}{t-\alpha} \left[e^{-x(\alpha-t)} \right]_0^\infty$$

$$M_x(t) = \frac{\alpha}{t-\alpha} \left[e^{-\infty(\alpha-t)} - e^0 \right]$$

$$M_x(t) = \frac{\alpha}{t-\alpha} [0-1]$$

$$M_x(t) = \frac{\alpha}{\alpha-t} = \frac{1}{1-t/\alpha}$$

$$M_x(t) = \frac{1}{1-t/\alpha} \quad \alpha > 0$$

Gamma Distribution:-

Gamma distribution is one of the distributions. which is widely used in the field of business, Science and Enigneering. in order to model the continious variable that should have a positive and skewed distribution.

$$f(x;\beta;\alpha) = \frac{1}{\Gamma\alpha\, \beta^{\alpha}} \cdot x^{\alpha-1} \cdot e^{-x/\beta}$$

$0 \leq x < \infty$

$\alpha, \beta > 0$

1. P.D.F

$$\int_0^{\infty} f(x)\, dx = 1$$

$$\Rightarrow \int_0^{\infty} \frac{x^{\alpha-1} e^{-x/\beta} dx}{\Gamma\alpha\, \beta^{\alpha}}$$

$$\Rightarrow \frac{1}{\Gamma\alpha \cdot \beta^{\alpha}} \int_0^{\infty} x^{\alpha-1} \cdot e^{-x/\beta}\, dx$$

Let $y = x/\beta \Rightarrow x = \beta y$

$\beta dy = dx$

when.

$x = 0 \Rightarrow y = 0$

$x = \infty \Rightarrow y = \infty$

$$\Rightarrow \frac{1}{\Gamma\alpha\, \beta^{\alpha}} \int_0^{\infty} (\beta y)^{\alpha-1} \cdot e^{-y}\, \beta\, dy$$

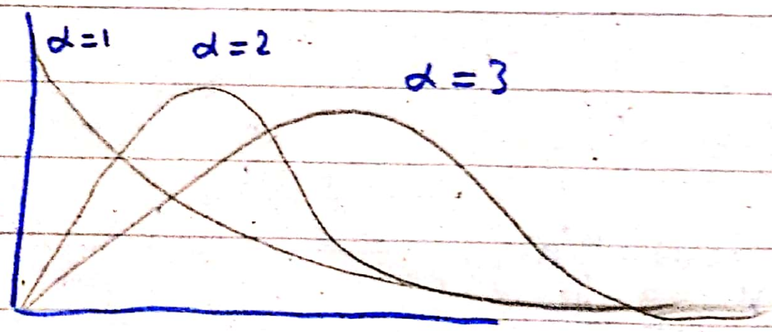
$$\Rightarrow \frac{\beta^{\alpha}}{\Gamma\alpha\, \beta^{\alpha}} \int_0^{\infty} y^{\alpha-1} \cdot e^{-y}\, dy$$

$$\because \int_0^{\infty} x^{\alpha-1}\, e^{-x}\, dx = \Gamma\alpha$$

$$\Rightarrow \frac{1}{\Gamma\alpha} \cdot \Gamma\alpha$$

$$\Rightarrow 1$$

2. Shape:



Incomplete Gamma distribution: Function

we know that

$$F(x) = \frac{1}{\Gamma\alpha} \int_0^{x} x^{\alpha-1} \cdot e^{-x}\, dx$$

$$F(x) = \frac{1}{\Gamma\alpha} \int_0^x x^{\alpha-1} \cdot e^{-x} \, dx$$

where alpha is +ve

$$\because e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots -$$

$$F(x) = \frac{1}{\Gamma\alpha} \int_0^x x^{\alpha-1} \left[1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots - \right]$$

$$F(x) = \frac{1}{\Gamma\alpha} \cdot \left[\frac{x^\alpha}{\alpha} - \frac{x^{\alpha+1}}{\alpha+1} - \frac{1}{2!} \cdot \frac{x^{\alpha+2}}{(\alpha+2)} + \cdots \right]$$

$$\boxed{F(x) = \frac{1}{\Gamma\alpha} \sum_{z=0}^{\infty} (-1)^z \frac{x^{\alpha+z}}{(\alpha+z)} \cdot \frac{1}{z!}}$$

Incomplete Gamma Function.

4- C.d.f

it is called incomplete gamma function

$$F(x) = \begin{cases} \frac{1}{\Gamma\alpha \, \beta^\alpha} \int_0^x x^{\alpha-1} \cdot e^{-x/\beta} \, dx & ; x > 0 \\ 0 & ; \text{e.w} \end{cases}$$

5. Moments About origin.

$$\mu_r' = E[x^r] = \int_{-\infty}^{\infty} x^r f(x)\, dx$$

$$\mu_r' = \int_0^{\infty} x^r \left[\frac{1}{\Gamma\alpha\, \beta^\alpha} \left(x^{\alpha-1} \cdot e^{-\frac{x}{\beta}}\right)\right] dx$$

$$\mu_r' = \frac{1}{\Gamma\alpha\, \beta^\alpha} \int_0^{\infty} x^r \cdot x^{\alpha-1} \cdot e^{-x/\beta}\, dx$$

let $y = \frac{x}{\beta} \Rightarrow x = \beta y \Rightarrow \beta\, dy = dx$

when $x = 0,\ y = 0$

$x = \infty,\ y = \infty$

$$\mu_r' = \frac{1}{\Gamma\alpha\, \beta^\alpha} \int_0^{\infty} (\beta y)^{r+\alpha-1} \cdot e^{-y} \beta\, dy$$

$$\mu_r' = \frac{\beta^{r+\alpha}}{\Gamma\alpha\, \beta^\alpha} \int_0^{\infty} y^{\alpha+r-1} \cdot e^{-y}\, dy$$

$$\mu_r' = \frac{\beta^r}{\Gamma\alpha} \cdot \Gamma\alpha+r \ ; \quad r = 1, 2, 3, \ldots$$

Note when $\beta = 1 \Rightarrow$ Mean = variance

6. M.G.F

$$M_x(t) = E[e^{tx}] = \int_{-\infty}^{\infty} e^{tx} f(x)\, dx$$

$$\Rightarrow \int_0^\infty e^{tx} \left[\left(\frac{1}{\Gamma\alpha\, \beta^\alpha} \right) . x^{\alpha-1} . e^{-x/\beta} \right] dx$$

$$\Rightarrow \int_0^\infty \frac{1}{\Gamma\alpha\, \beta^\alpha} . x^{\alpha-1} . e^{-(1/\beta - t)x} \, dx$$

$$\because \int_0^\infty x^{\alpha-1} . e^{-x/\beta} \, dx = \frac{\Gamma\alpha}{\beta^\alpha}$$

$$\Rightarrow \frac{1}{\Gamma\alpha\, \beta^\alpha} . \frac{\Gamma\alpha}{(1/\beta - t)^\alpha}$$

$$\Rightarrow \frac{1}{\beta^\alpha . \cancel{\Gamma\alpha} (1/\beta - t)^\alpha}$$

$$\Rightarrow \frac{1}{\beta^\alpha . 1/\beta^\alpha (1 - \beta t)^\alpha}$$

$$\Rightarrow \boxed{M_x(t) = (1 - \beta t)^{-\alpha}}$$

Mean $= \alpha\beta$

~~variance = E[X]~~

Variance $= \alpha\beta^2$

Mean = Median = Mode

Normal Distribution:-

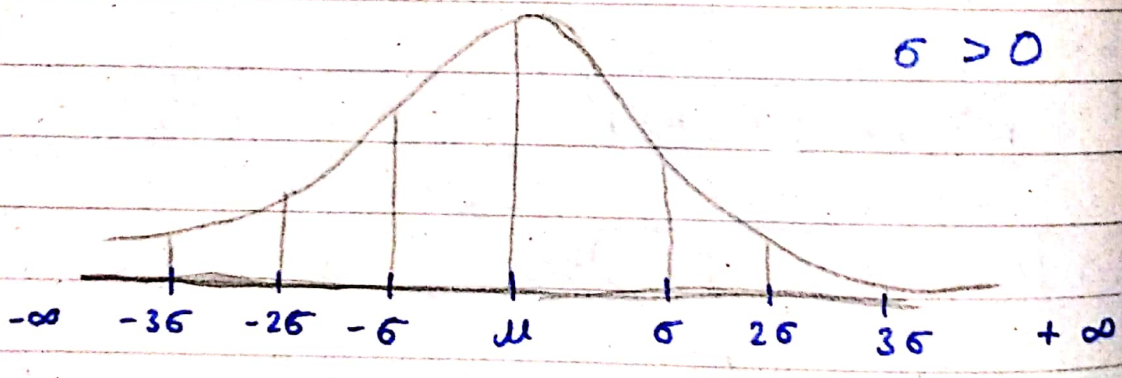
The most important continious proba[illegible] distribution used in the entire field of statistics is the normal distributi[illegible] It's graph called the normal curve It is also known as Gaussion dist[illegible]

$$f(x;\mu;\sigma) = \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$-\infty < x < \infty$

$-\infty < \mu < \infty$

$\sigma > 0$



Properties:-

1-) The curve is bell-shaped and symmetri[illegible] about the line $x : \mu$

II.) Mean, Median and Mode of the dist[illegible] coincide

iii.) The max. prob occuring at point $x = \mu$

iv) $\beta_1 = 0$, $\beta_2 = 3$

v) $\mu_{2n+1} = 0$ $(n: 0, 1, 2, \ldots)$

vi) $\mu_{2n} = (1.3.5 \ldots (2n-1)\sigma^{2n})$ $n: 1, 2, \ldots$

vii) Point of inflection of the curve are given by

$$x = \mu \pm \sigma, \quad f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-x/2}$$

1. P.d.f

$$= \int_{-\infty}^{+\infty} f(x)\, dx = 1$$

$$= \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} = 1$$

$\Rightarrow$ Let $z = (x-\mu)/\sigma$ $\Rightarrow$ $x = \mu + \sigma z$

$dx = \sigma dz$

when

$\left[\begin{array}{l} x = -\infty \\ z = -\infty \end{array}\right.$

$\left[\begin{array}{l} x = \infty \\ z = \infty \end{array}\right.$

$$= \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{+\infty} e^{-\frac{z^2}{2}} \sigma\, dz$$

$$= \boxed{\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-z^2/2}\, dz}$$

$\Rightarrow$ Let $u^2 = \frac{1}{2} z^2$

$\boxed{du = 2dz}$ $\Rightarrow$ $\cancel{z = \sqrt{2u}}$

$\downarrow$

$\cancel{\frac{du}{\sqrt{2u}} = dz}$

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-u} \frac{du}{\sqrt{2u}}$$ somehow in Sir's pdf it became 1

$u = \frac{z}{\sqrt{2}} \Rightarrow \boxed{\sqrt{2}\,du = dz}$

$\boxed{z = \sqrt{2}\,u}$

$$\frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{+\infty} e^{-u^2} \sqrt{2}\,du$$

Gaussian integral

$$\frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-u^2}\,du \quad \because \int_{-\infty}^{+\infty} e^{-y^2}\,dy = \sqrt{\pi}$$

$$\frac{1}{\sqrt{\pi}} \cdot \sqrt{\pi}$$

$1 \Rightarrow$ Hence Proved.

Area Under the Normal Curve:-

$$P(a \le x \le b) = \int_a^b f(x)\,dx$$

$$P(a \le x \le b) = \frac{1}{\sqrt{2\pi}\,\sigma} \int_a^b e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx$$

by using probability integral also called Laplace Function.

$$\phi(z) = \int_0^z f(t)\,dt = \frac{1}{\sqrt{2\pi}} \int_z^z e^{-t^2/2}\,dt$$

$$\boxed{P(a \le x \le b) = \phi\left(\frac{b-\mu}{\sigma}\right) - \phi\left(\frac{a-\mu}{\sigma}\right)}$$

Features of Laplaces Function:-

1) $\phi(-z) = -\phi(z)$

11) $\phi(\infty) = \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-t^2/t}\,dt = \frac{1}{2}$

111) $F(x) = \int_{-\infty}^{x} f(x)\,dx = \frac{1}{2} + \phi\left(\frac{x-\mu}{\sigma}\right)$

<u>CDF</u>

$$F(x) = \int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx$$

Let

$$Z = \frac{x-\mu}{\sigma} \Rightarrow dz = \frac{dx}{\sigma} \text{ or}$$

$$F(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-z^2/2} \cdot \sigma\,dz \qquad x = \mu + \sigma z$$

when

$x \to -\infty \quad z \to -\infty$

$x \to \infty \quad z \to \infty$

$$\boxed{F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-z^2/2} \cdot dz}$$

Moments about origin:-

$$\mu_1' = E[x] = \int_{-\infty}^{\infty} x \cdot \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Let $z = \frac{x-\mu}{\sigma} \Rightarrow dx = \sigma\, dz$ or
$x = \mu + \sigma z$

$$\mu_1' = \int_{-\infty}^{\infty} (\mu + \sigma z) \cdot \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-z^2/2} \sigma\, dz$$

$$\mu_1' = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mu\, e^{-z^2/2} dz + \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z\, e^{-\frac{1}{2} \cdot z^2} dz$$

integral is o

$$\mu_1' = \mu \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \cdot e^{-z^2/2} \cdot dz + 0$$

$$\boxed{\mu_1' = \mu}$$

$$\mu_2' = E[x^2] = \int_{-\infty}^{\infty} x^2 \cdot \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Let $z = \frac{x-\mu}{\sigma} \Rightarrow dx = \sigma\, dz$

$x = \mu + \sigma z$

$$\mu_2' = \int_{-\infty}^{\infty} (\mu + \sigma z)^2 \cdot \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-z^2/2} \cdot \sigma\, dz$$

$$\mu_2' = \int_{-\infty}^{\infty} (\mu + \sigma z)^2 \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-z^2/2} \, dz$$

$$\mu_2' = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (\mu^2 + 2\mu\sigma z + \sigma^2 z^2) \, e^{-z^2/2} \, dz$$

$$\mu_2' = \mu^2 \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \cdot e^{-z^2/2} \, dz \quad \sim \mu^2 \cdot 1$$

$$+ \frac{2\mu\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z \, e^{-z^2/2} \, dz \quad \sim 0$$

$$+ \frac{\sigma^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^2 e^{-z^2/2} \, dz \quad \text{sii}$$

$$\boxed{\mu_2' = \mu^2 + \frac{\sigma^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^2 e^{-z^2/2} \, dz} \quad \text{— i}$$

$$= \int_{-\infty}^{\infty} z^2 e^{-z^2/2} \, dz \qquad \text{Let } y = \frac{z^2}{2}$$

$$\int_{-\infty}^{\infty} (2y) \, e^{-y} \frac{dy}{\sqrt{2y}} \qquad \boxed{dy = z\,dz}$$

$$= 2\sqrt{2} \int_{-\infty}^{\infty} \sqrt{y}\, e^{-y}\, dy$$

$$= 2\sqrt{2} \int_{-\infty}^{\infty} y^{3/2 - 1} \cdot e^{-y}\, dy$$

$$\because \int_{0}^{\infty} x^{\alpha - 1} \cdot e^{-x}\, dx = \Gamma \alpha$$

$$= 2\sqrt{2}\, \Gamma 3/2 \qquad \Gamma 3/2 = \frac{1}{2} \Gamma 1/2 = \frac{\sqrt{\pi}}{2}$$

$$= 2\sqrt{2} \cdot \frac{\sqrt{\pi}}{2}$$

$$= \sqrt{2\pi}$$

① =>

$$\mu_2' = \mu^2 + \frac{\sigma^2}{\sqrt{2\pi}} \cdot \sqrt{2\pi}$$

$$\boxed{\mu_2' = \mu^2 + \sigma^2}$$

$$var(x) = \mu_2' - \mu_1'^2$$

$$var(x) = \mu^2 + \sigma^2 - \mu^2$$

$$\boxed{var(x) = \sigma^2}$$

Mean Moments:-

Odd Order Moments:

$$\mu_{2n+1} = \int_{-\infty}^{\infty} (x-\mu)^{2n+1} f(x)\, dx$$

$$\mu_{2n+1} = \int_{-\infty}^{\infty} (x-\mu)^{2n+1} \cdot \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Let

$$z = \frac{x-\mu}{\sigma}$$

$$x = \mu + \sigma z$$

$$dx = \sigma dz$$

$$\mu_{2x+1} = \frac{1}{\sqrt{2\pi}\,\sigma} \cdot \int_{-\infty}^{\infty} (\sigma z)^{2x+1} \cdot e^{-z^2/2} \sigma dz$$

$$\mu_{2x+1} = \frac{\sigma^{2x+1}}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} z^{(2x+1)} \cdot e^{-z^2/2} dz$$

↓

integral is an odd function.

$$\boxed{\mu_{2x+1} = 0}$$

Even Order Moments:

$$\mu_{2x} = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{-\infty}^{\infty} (x-\mu)^{2x} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Let $z = \frac{x-\mu}{\sigma}$

$$dz = \frac{dx}{\sigma}$$

$$\mu_{2x} = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{\infty} (\sigma z)^{2x} \cdot e^{-z^2/2}$$

Let

$$\Rightarrow \int_{-\infty}^{\infty} \frac{\sigma^{2x}}{\sqrt{2\pi}} \cdot z^{2x} \cdot e^{-z^2/2} \, dz$$

$\frac{z^2}{2} = y \Rightarrow z = \sqrt{}$

$dy = z\,dz$

$\frac{dy}{z} = dz$

$$\Rightarrow \frac{\sigma^{2x}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (2y)^{x} \cdot e^{-y} \frac{dy}{\sqrt{2y}}$$

$$\Rightarrow \frac{\sigma^{2x} \cdot 2^{x}}{2\sqrt{\pi}} \int_{-\infty}^{\infty} y^{x-1/2} \cdot e^{-y} \, dy$$

$$\Rightarrow \frac{\sigma^{2x} \cdot 2^{x-1}}{\sqrt{\pi}} \cdot 2 \int_{0}^{\infty} y^{(x+1/2)-1} \cdot e^{-y} \, dy$$

$$\because \int_{0}^{\infty} x^{n-1} \cdot e^{-x} \, dx = \Gamma n$$

$$\boxed{\Rightarrow \frac{\sigma^{2x} \cdot 2^{x}}{\sqrt{\pi}} \cdot \Gamma\left(x + 1/2\right)} = \mu_{2x}$$

$$\mu_{2x} = \frac{\sigma^{2x} \cdot 2^{x}}{\sqrt{\pi}} \Gamma\left(x + 1/2\right) \quad \text{— ①}$$

For $(x-1)$ we got

$$\mu_{2x-2} = \frac{2^{x-1} \cdot \sigma^{2x-2}}{\sqrt{\pi}} \cdot \Gamma\left(x - 1/2\right) \quad \text{— ②}$$

① / ② =>

$$\frac{\mu_{2x}}{\mu_{2x-2}} = \frac{2^{x} \cdot \sigma^{2x} \cdot \Gamma(x+1/2)}{2^{x-1} \cdot \sigma^{2x-2} \cdot \Gamma(x-1/2)}$$

$$\boxed{\mu_{2x} = 2 \cdot \sigma^{2} \cdot (x-1/2) \cdot \mu_{2x-2}}$$

M.G.F :-

$$M_x(t) = E[e^{xt}] = \int_{-\infty}^{\infty} e^{tx} \cdot f(x)\, dx$$

$$M_x(t) = \frac{1}{\sqrt{2\pi}\,\sigma} \cdot \int_{-\infty}^{\infty} e^{tx} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)} dx$$

Let

$$M_x(t) = \frac{1}{\sqrt{2\pi}\,\sigma} \cdot \int_{-\infty}^{\infty} e^{t(\mu+\sigma z)} \cdot e^{-z^2/2}\, \sigma\, dz \qquad z = \frac{x-\mu}{\sigma}$$

$$x = \mu + \sigma z$$

$$\boxed{dx = \sigma\, dz}$$

$$M_x(t) = \frac{e^{\mu t}}{\sqrt{2\pi}\,\sigma} \cdot \int_{-\infty}^{\infty} e^{\sigma t z + (-z^2/2)}\, dz$$

$$M_x(t) = \frac{e^{\mu t}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-1/2(z^2 - 2\sigma t z)}\, dz$$

$$M_x(t) = \frac{e^{\mu t}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}[z^2 - 2z(\sigma t) + (\sigma t)^2 - (\sigma t)^2]}\, dz$$

$$M_X(t) = \frac{e^{\mu t}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}[(z-\sigma t)^2 - (\sigma t)^2]} dz$$

Let $w = z - \sigma t$

$dw = dz$

$$M_X(t) = \frac{e^{\mu t + \frac{1}{2}\sigma^2 t^2}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(w)^2} dw$$

$$M_X(t) = \frac{e^{\mu t + 1/2\sigma^2 t^2}}{\sqrt{2\pi}} \cdot \sqrt{2\pi}$$

$$\boxed{M_X(t) = e^{\mu t + 1/2\sigma^2 t^2}}$$