Date:

EXERCISE 4.4

1- $y'' + 3y' + 2y = 6$

SOLUTION:-

$m^2 + 3m + 2 = 0$

$m(m+2) + (m+2) = 0$

$(m+2)(m+1)$

$m = -2,\ m = -1$

$y_c = C_1 e^{-2x} + C_2 e^{-x}$

$y_p = C_3$

$y'_p = 0$

$y''_p = 0$

Now put in question

$(0) + 3(0) + 2(C_3) = 6$

$\cancel{2}C_3 = \cancel{6}$

$\boxed{C_3 = 3}$

$\boxed{y = C_1 e^{-2x} + C_2 e^{-x} + 3}$

2 $4y'' + 9y = 15$

SOLUTION:-

$4m^2 + 9 = 0$

$m^2 = -9/4$

$m = \sqrt{-9/4}$

$m = \pm 3/2\, i$

$y_c = C_1 \sin 3/2\, x + C_2 \cos 3/2\, x$

$y_p = C_3$

$y'_p = 0$

$y''_p = 0$

$4(0) + 9(C_3) = 15$

$C_3 = 5/3$

so,

$\boxed{y = C_1 \sin \frac{3}{2}x + C_2 \cos \frac{3}{2}x + \frac{5}{3}}$

Date:

3- $y'' - 10y' + 25y = 30x + 3$

SOLUTION:

$m^2 - 10m + 25y = 0$

$(m-5)(m-5) = 0$

$m = 5, 5$

so,

$y_c = C_1 e^{5x} + x C_2 e^{5x}$

$y_p = C_3 x + C_4$

$y'_p = C_3$

$y''_p = 0$

Now put in question.

$(0) - 10(C_3) + 25(C_3 x + C_4) = 30x + 3$

$-10C_3 + 25xC_3 + 25C_4 = 30x + 3$

$25C_3 x = 30x$

$\boxed{C_3 = 6/5}$

$-10\left(\frac{6}{5}\right) + 25C_4 = 3$

$-12 + 25C_4 = 3$

$25C_4 = 15$

$C_4 = 15/25$

$\boxed{C_4 = 3/5}$

$$\boxed{y = C_1 e^{5x} + xC_2 e^{5x} + \frac{6}{5}x + \frac{3}{5}}$$

4 $y'' + y' - 6y = 2x$

SOLUTION:-

$m^2 + m - 6 = 0$

$m = 2, m = -3$

$y = C_1 e^{2x} + C_2 e^{-3x}$

$y_p = C_3 x + C_4$

$y'_p = C_3$

$y''_p = 0$

$(0) + (C_3) - 6(C_3 x + C_4) = 2x$

$C_3 - 6C_3 x + 6C_4 = 2x$

$-6C_3 x = 2x$

$\boxed{C_3 = -1/3}$

$C_3 - 6C_4 = 0$

$(-1/3) - 6C_4 = 0$

$-6C_4 = 1/3$

$\boxed{C_4 = -1/18}$

$$\boxed{y = C_1 e^{2x} + C_2 e^{-3x} - \frac{1}{3}x - \frac{1}{18}}$$

Date:

5- $\frac{1}{4}y'' + y' + y = x^2 - 2x$

SOLUTION:-

$\frac{1}{4}m^2 + m + 1 = 0$

$m = -2, \quad m = 1/4$

$y_c = C_1 e^{-2x} + C_2 e^{x/4}$

$y_p = C_3 x^2 + C_4 x + C_5$

$y'_p = C_3 2x + C_4$

$y''_p = 2C_3$

Now,

$\frac{1}{4}(2C_3) + (2C_3 x + C_4) + (C_3 x^2 + C_4 x + C_5) = x^2 - 2x$

$\Rightarrow \frac{1}{2}C_3 + 2C_3 x + C_4 + C_3 x^2 + C_4 x + C_5 = x^2 - 2x$

$C_3 x^2 = x^2$

$\boxed{C_3 = 1}$

$2C_3 x + C_4 x = -2x$

$(2(1) + C_4)\cancel{x} = -2\cancel{x}$

$2 + C_4 = -2$

$C_4 = -2 - 2$

$\boxed{C_4 = -4}$

$\frac{1}{2}C_3 + C_4 + C_5 = 0$

$\frac{1}{2}(1) + (-4) + C_5 = 0$

$-7/2 + C_5 = 0$

$\boxed{C_5 = 7/2}$

$$\boxed{y = C_1 e^{-2x} + C_2 e^{x/4} + x^2 - 4x + \frac{7}{2}}$$

Date:

$y'' - 8y' + 20y = 100x^2 - 26xe^x$

SOLUTION:-

$m^2 - 8m + 20 = 0$

$m = 4 \pm 2i$

$y_c = e^{4x}[C_1 \cos 2x + C_2 \sin 2x]$

$y_{p_1} = 100x^2$

$y_p = Ax^2 + Bx + C$

$y'_p = 2Ax + B$

$y''_p = 2A$

$(2A) - 8(2Ax+B) + 20(Ax^2+Bx+C) = 100x^2$

$2A - 16Ax - 8B + 20Ax^2 + 20Bx + 20C = 100x^2$

$20Ax^2 = 100x^2$ | $2A - 8B + 20C = 0$
$\boxed{A = 5}$ | $2(5) - 8(4) + 20C = 0$
$-16Ax + 20Bx = 0$ | $20C = 22$
$-16(5) + 20B = 0$ | $\boxed{C = 11/10}$
$\boxed{B = 4}$

$y_{p_1} = 5x^2 + 4x + 11/10$

$y_{p_2} = -26xe^x$

$y_{p_2} = (Ax+B)e^x$

$y'_p = Ae^x + (Ax+B)e^x$

$y'_p = e^x(A + Ax + B)$

$y''_p = Ae^x + (A + Ax + B)e^x$

$= e^x(2A + Ax + B)$

$= [e^x(2A+Ax+B)] + 20[e^x(Ax+B)] - 8[e^x(A+B+Ax)] = -26xe^x$

$\Rightarrow e^x[Ax + 2A + B + 20Ax + 20B - 8A - 8B - 8Ax] = -26xe^x$

$\Rightarrow e^x[13Ax - 6A + 13B] = -26xe^x$

$13A = -26$ | $-6A + 13B = 0$
$\boxed{A = -2}$ | $-6(-2) + 13B = 0$
| $\boxed{B = -12/13}$

$y_{p_2} = \left(-2x - \frac{12}{13}\right)e^x$

So,

$$\boxed{y = e^{4x}[C_1 \cos 2x + C_2 \sin 2x] + 5x^2 + 4x + 11/10 + \left[-2x - \frac{12}{13}\right]e^x}$$

Date:

1- $y'' + 3y = -48x^2e^{3x}$ — ①

SOLUTION:-

$m^2 + 3 = 0$

$m = \pm\sqrt{3}i$

$y_c = C_1\cos\sqrt{3}x + C_2\cos\sqrt{3}x$

$y_p = (Ax^2 + Bx + C)e^{3x}$

$y'_p = A[2x(e^{3x}) + x^2(e^{3x})(3)] + B[e^{3x} + xe^{3x}(3)] + C[e^{3x}(3)]$

$= 2Axe^{3x} + 3Ax^2e^{3x} + Be^{3x} + 3Bxe^{3x} + 3Ce^{3x}$

$y''_p = 2A[e^{3x} + xe^{3x}(3)] + 3A[2x(e^{3x}) + x^2(e^{3x})(3)] + 3Be^{3x} + 3B[e^{3x} + xe^{3x}(3)] + 9Ce^{3x}$

$= 2Ae^{3x} + 6Axe^{3x} + 6Axe^{3x} + 9Ax^2e^{3x} + 3Be^{3x} + 3Be^{3x} + 9Bxe^{3x} + 9Ce^{3x}$

$= 2Ae^{3x} + 12Axe^{3x} + 9Ax^2e^{3x} + 6Be^{3x} + 9Bxe^{3x} + 9Ce^{3x}$

Now put in eq ①:-

$\Rightarrow (2Ae^{3x} + 12Axe^{3x} + 9Ax^2e^{3x} + 6Be^{3x} + 9Bxe^{3x} + 9Ce^{3x})$

$+ 3(Ax^2e^{3x} + 3Bxe^{3x} + 3Ce^{3x}) = -48x^2e^{3x}$

$\Rightarrow 12Ax^2e^{3x} + 12Bxe^{3x} + 12Axe^{3x} + 2Ae^{3x} + 6Be^{3x} + 12Ce^{3x} = -48x^2e^{3x}$

$12A\cancel{x^2e^{3x}} = -48\cancel{x^2e^{3x}}$

$A = -48/12$

$\boxed{A = -4}$

$x(12Be^{3x} + 12Ae^{3x}) = 0$

$12Be^{3x} + 12(-4)e^{3x} = 0$

$12B\cancel{e^{3x}} = +48\cancel{e^{3x}}$

$B = 48/12$

$\boxed{B = 4}$

$6Be^{3x} + 2Ae^{3x} + 12Ce^{3x} = 0$

$6(4)e^{3x} + 2(-4)e^{3x} + 12Ce^{3x} = 0$

$16e^{3x} + 12Ce^{3x} = 0$

$12C\cancel{e^{3x}} = -16\cancel{e^{3x}}$

$C = -16/12$

$\boxed{C = -4/3}$

So,

$\boxed{y = C_1\cos\sqrt{3}x + C_2\sin\sqrt{3}x + \left(-4x^2 + 4x - 4/3\right)e^{3x}}$ Ans

Date:

6 $4y'' - 4y' - 3y = \cos 2x$

SOLUTION:

$4m^2 - 4m - 3 = 0$

$m = 3/2 \,,\, -1/2$

$y = C_1 e^{3/2 x} + C_2 e^{-1/2 x}$

$y_p = A\cos 2x + B\sin 2x$

$y'_p = -2A\sin 2x + 2B\cos 2x$

$y''_p = -4A\cos 2x - 4B\sin 2x$

$\Rightarrow 4(-4A\cos 2x - 4B\sin 2x) - 4(-2A\sin 2x + 2B\cos 2x)$
$\quad -3(A\cos 2x + B\sin 2x) = \cos 2x$

$\Rightarrow -16A\cos 2x - 16B\sin 2x + 8A\sin 2x - 8B\cos 2x - 3A\cos 2x$
$\quad -3B\sin 2x = \cos 2x$

$\quad -19A\cos 2x - 19B\sin 2x + 8A\sin 2x - 8B\cos 2x = \cos 2x$

$(-19A - 8B)\cancel{\cos 2x} = \cancel{\cos 2x}$

$\Rightarrow -19A = 1 + 8B \Rightarrow A = \dfrac{1+8B}{-19}$ —— ①

$\Rightarrow (-19B + 8A)\sin 2x = 0$

$A = \dfrac{19B}{8}$ —— ②

compare ① & ②

$\dfrac{19B}{8} = \dfrac{1+8B}{-19}$

$-361B = 8 + 64B$

$425B = -8$

$\boxed{B = -8/425}$

Now

$A = \dfrac{1+8B}{-19}$

$A = \dfrac{1+8(-8/425)}{-19}$

$\boxed{A = -19/425}$

So,

$$\boxed{y = C_1 e^{3/2 x} + C_2 e^{-1/2 x} - \frac{19}{425}\cos 2x - \frac{8}{425}\sin 2x}$$ Ans

Date:

9- $y'' - y' = -3$

SOLUTION:-

$m^2 - m = 0$

$m = 1, \ m = 0$

$y = C_1 e^{x} + C_2$

$y_p = Ax$

$y'_p = A$

$y''_p = 0$

$0 - (A) = -3$

$\boxed{A = 3}$

$\boxed{y = C_1 e^{x} + C_2 + 3x}$

Ans

10 $y'' + 2y' = 2x + 5 - e^{-2x}$

SOLUTION:-

$m^2 + 2m = 0$

$m = 0, \ m = -2$

$y = C_1 + C_2 e^{-2x}$

$y_p = Ax^2 + Bx + xCe^{-2x}$

$y'_p = 2Ax + B + C[(1)(e^{-2x}) + (e^{-2x})(-2)(x)]$

$y'_p = 2Ax + B + Ce^{-2x} - 2Cxe^{-2x}$

$y''_p = 2A - 2Ce^{-2x} - 2C[e^{-2x} + e^{-2x}(-2)(x)]$

$y''_p = 2A - 2Ce^{-2x} - 2Ce^{-2x} + 4Ce^{-2x}x$

$\Rightarrow 2A \cancel{-2Ce^{-2x}} \cancel{+4Ce^{-2x}x} \cancel{-2Ce^{-2x}} + 4Ax + 2B + 2Ce^{-2x} - 4Cxe^{-2x}$

$\Rightarrow 4Ax + 2A + 2B - 2Ce^{-2x} = 2x + 5 - e^{-2x}$

$4A = 2$

$\boxed{A = 1/2}$

$2A + 2B = 5$

$2(1/2) + 2B = 5$

$\boxed{B = 2}$

$-2C = -1$

$\boxed{C = 1/2}$

$\boxed{y = C_1 + C_2 e^{-2x} + \frac{1}{2}x^2 + 2x + \frac{1}{2}xe^{-2x}}$

Ans

Date:

11 $y'' - y' + \frac{1}{4}y = 3 + e^{x/2}$ —①

SOLUTION:-

$m^2 - m + 1/4 = 0$

$m = \frac{1}{2}, m = 1/2$

$y = C_1 e^{1/2 x} + x C_2 e^{1/2 x}$

$y_p = A + Be^{x/2} x^2$

$y'_p = B[\frac{1}{2} e^{x/2}(x^2) + e^{x/2}(2x)]$

$y'_p = \frac{1}{2} B e^{x/2} x^2 + 2Be^{x/2} x$

$y''_p = \frac{1}{2} B[\frac{1}{2} e^{x/2}(x^2) + e^{x/2}(2x)] + 2B[\frac{1}{2} e^{x/2}(x) + e^{x/2}(1)]$

$= \frac{1}{4} Be^{x/2} x^2 + \frac{1}{2} e^{x/2} B 2x + \frac{1}{2} Be^{x/2} 2x + 2Be^{x/2}$

$\Rightarrow \frac{1}{4} Be^{x/2} x^2 + 2Be^{x/2} x + 2Be^{x/2}$

Now put in eq ①.

$\Rightarrow [\cancel{\frac{1}{4} Be^{x/2} x^2} + \cancel{Be^{x/2}(2x)} + 2Be^{x/2} - \cancel{\frac{1}{2} Be^{x/2} x^2} - \cancel{2Be^{x/2} x}$

$+ 1/4 A + \cancel{\frac{1}{4} Be^{x/2} x^2} = 3 + e^{x/2}$

$\frac{1}{4} A + 2Be^{x/2} = 3 + e^{x/2}$

$2B = 1$

$\boxed{B = 1/2}$

$\frac{1}{4} A = 3$

$\boxed{A = 12}$

$y = C_1 e^{x/2} + x C e^{x/2} + 12 + \frac{1}{2} e^{x/2} x^2$

Date:

12 $y'' - 16y = 2e^{4x}$ — (1)

SOLUTION:-

$m^2 - 16 = 0$

$m^2 = 16$

$m = \pm 4$

$y_c = C_1e^{4x} + C_2e^{-4x}$

$y_p = Ae^{4x}x$

$y'p = A[4xe^{4x} + e^{4x}]$

$y'p = 4Axe^{4x} + Ae^{4x}$

$y''p = 16Axe^{4x} + 4Ae^{4x} + 4Ae^{4x}$

Now putting in eq (1)

$[16Axe^{4x} + 4Ae^{4x} + 4Ae^{4x}] - 16Ae^{4x}x = 2e^{4x}$

$16A\cancel{xe^{4x}} - 16\cancel{A}\cancel{e^{4x}}x + 8Ae^{4x} = 2e^{4x}$

$8A = 2$

$\boxed{A = 1/4}$

$$\boxed{y = C_1e^{4x} + C_2e^{-4x} + \frac{1}{4}e^{4x}x}$$

Ans

13 $y'' + 4y = 3\sin 2x$ — (1)

SOLUTION:-

$m^2 + 4 = 0$

$m^2 = -4$

$m = 2i, -2i$

$y = C_1\sin 2x + C_2\cos 2x$

$y_p = Ax\sin 2x + Bx\cos 2x$

$y'p = A[\sin 2x + 2x\cos 2x] + B[\cos 2x - 2x\sin 2x]$

$y'p = A\sin 2x + 2Ax\cos 2x + B\cos 2x - 2Bx\sin 2x$

$y'' = 2A\cos 2x + 2A\cos 2x - 4Ax\sin 2x$
$- 2B\sin 2x - 2B\sin 2x - 4Bx\cos 2x$

$y'' = 4A\cos 2x - 4B\sin 2x - 4Ax\sin 2x$
$- 4Bx\cos 2x$

Now eq (1) =>

$\Rightarrow (4A\cos 2x - 4B\sin 2x - \cancel{4Ax\sin 2x} - \cancel{4Bx\cos 2x}$
$+ \cancel{4Ax\sin 2x} + \cancel{4Bx\cos 2x} = 3\sin 2x$

$\Rightarrow 4A\cos 2x - 4B\sin 2x = 3\sin 2x$

$-4B = 3$

$\boxed{B = -3/4}$

$4A = 0$

$\boxed{A = 0}$

So,

$$\boxed{y = C_1\sin 2x + C_2\cos 2x - \frac{3}{4}x\cos 2x}$$

Ans

14 $y'' - 4y = (x^2-3)\sin 2x \longrightarrow (1)$

SOLUTION:-

$m^2 - 4 = 0$

$m = 2, -2$

$y = C_1 e^{2x} + C_2 e^{-2x}$

$y_p = (Ax^2+Bx+C)\sin 2x + (Dx^2+Ex+F)\cos 2x$.

$y_p' = [(2Ax+B)\sin 2x + (Ax^2+Bx+C)2\cos 2x] + (2Dx+E)\cos 2x$
$\quad + [Dx^2+Ex+F](-2\sin 2x)$

$\Rightarrow (2Ax+B-2Dx^2-2Ex-2F)\sin 2x + (2Ax^2+2Bx+2C+2Dx+E)\cos 2x$

$y_p'' = (2A-4Dx-2E)\sin 2x + (2Ax+B-2Dx^2-2Ex-2F)2\cos 2x +$
$\quad (4Ax+2B+2D)\cos 2x - 2(2Ax^2+2Bx+2C+2Dx+E)\sin 2x$

$\Rightarrow (2A-4Dx-2E-4Ax^2-4Bx-4C-4Dx-2E)\sin 2x +$
$\quad (4Ax+2B-4Dx^2-4Ex-4F+4Ax+2B+2D)\cos 2x$

putting in eq (1)

$\Rightarrow (-4Ax^2-8Dx-4Bx+2A-4C-4E)\sin 2x +$
$\quad (-4Dx^2+8Ax-4Ex+4B-4F+2D)\cos 2x$
$\quad -4[(Ax^2+Bx+C)\sin 2x - (Dx^2+Ex+F)\cos 2x] = (x^2-3)\sin 2x$

comparing sin2x terms:-

$\sin 2x[-4Ax^2 - 4(2D+B)x + 2A - 4C - 4E - 4Ax^2 - 4Bx - 4C] = (x^2-3)\sin 2x$

$\Rightarrow -8Ax^2 - 4(2D+2B)x + 2A - 4E - 8C = x^2 - 3 \longrightarrow (2)$

comparing cos2x terms

$(-4Dx^2 + 8Ax - 4Ex + 4B - 4F + 2D - 4Dx^2 - 4Ex - 4F)\cos 2x = 0$

$-8Dx^2 - 4(2E-2A)x - 8F + 4B + 2D = 0$

Now comparing x^2 values with 1 of sin2x of eq (2):

$-8Ax^2 = x^2$ | Now x term with 0.

$-8A = 1$ | $-4(2D+2B)x = 0$

$\boxed{A = -1/8}$ | $2D + 2B = 0$

$B = -D \longrightarrow (3)$

Now comparing constants with -3.

$2A - 4E - 8C = -3$

$2(-1/8) - 4E - 8C = -3$

$-\frac{1}{4} - 4E - 8C = -3 \longrightarrow (4)$

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Now comparing cos2x terms with zero:-

$-8Dx^2=0$

$-8D=0$

$\boxed{D=0}$

Now ③ =>

$B=-D$

$\boxed{B=0}$

$-4(2E-2A)\cancel{x}=0\cancel{x}$

$2E-2A=0$

$2E-2(-1/8)=0$

$2E+1/4=0$

$\boxed{E=-1/8}$

Now eq ④ =>

$-\frac{1}{4}-4E-8C=-3$

$-1/4-4(-1/8)-8C=-3$

$\frac{1}{4}-8C=-3$

$-8C=-3-1/4$

$\cancel{-}8C=\cancel{-}13/4$

$\boxed{C=13/32}$

For F:-

Comparing constant of cos2x:

$-8F+4B+2D=0$

$-8F+4(0)+2(0)=0$

$-8F=0$

$\boxed{F=0}$

So,

$y_c=C_1e^{2x}+C_2e^{-2x}$

$y_p=\left(-\frac{1}{8}x^2+13/32\right)\sin 2x+\left(-\frac{1}{8}x\right)\cos 2x$

$y_p=-\frac{1}{8}x^2\sin 2x+\frac{13}{32}\sin 2x-\frac{1}{8}x\cos 2x$

$\boxed{y=C_1e^{2x}+C_1e^{-2x}-\frac{1}{8}x^2\sin 2x+\frac{13}{32}\sin 2x-\frac{1}{8}\cos 2x}$ Ans

Date:

15- $y'' + y = 2x\sin x$ — (1)

SOLUTION:

$m^2 + 1 = 0$

$m = i, -i$

$y_c = C_1\cos x + C_2\sin x$

$y_p = (Ax^2 + Bx)\cos x + (Cx^2 + Dx)\sin x$

$y_p' = (2Ax + B)\cos x - \sin x(Ax^2 + Bx) + (2Cx + D)\sin x + \cos x(Cx^2 + Dx)$

$y_p'' = \left[(2A)\cos x - \sin x(2Ax + B)\right] + \left[-(2Ax + B)\sin x - (Ax^2 + Bx)\cos x\right]$
$+ \left[(2C)\sin x + (2Cx + D)\cos x\right] + \left[(2Cx + D)\cos x - (Cx^2 + Dx)\sin x\right]$

$= 2A\cos x - 2(2Ax + B)\sin x - (Ax^2 + Bx)\cos x + 2C\sin x$
$+ 2(2Cx + D)\cos x - (Cx^2 + Dx)\sin x$

Now put y_p' & y_p'' in eq (1): -

(1) $\Rightarrow 2A\cos x - 2(2Ax + B)\sin x - \cancel{(Ax^2 + Bx)\cos x} + (2C\sin x) +$
$+ 2(2Cx + D)\cos x - (Cx^2 + Dx)\sin x + \cancel{(Ax^2 + Bx)\cos x} + (Cx^2 + Dx)\sin x$
$= 2x\sin x$.

$\Rightarrow$ Comparing sinx terms:

$\Rightarrow [-2(2Ax + B) + 2C - \cancel{(Cx^2 + Dx)} + \cancel{(Cx^2 + Dx)}]\cancel{\sin x} = 2x\cancel{\sin x}$

$\Rightarrow -4Ax - 2B + 2C = 2x$

$\Rightarrow -4A = 2$

$\boxed{A = -1/2}$ comparing cosx terms: -

$2A\cos x + 2(2Cx + D)\cos x = 0$

$2A + 4Cx + 2D = 0$

$4Cx = 0$

$\boxed{C = 0}$

$2A + 2D = 0$

$2(-1/2) + 2D = 0$

$\boxed{D = 1/2}$

$-2B + 2C = 0$

$-2B + 2(0) = 0$

$\boxed{B = 0}$

Date:

$y_c = C_1 \cos x + C_2 \sin x.$

$y_p = \left(-\frac{1}{2}x^2\right)\cos x + \left(\frac{1}{2}x\right)\sin x$

So,

$$\boxed{y = C_1 \cos x + C_2 \sin x - \frac{1}{2}x^2 \cos x + \frac{1}{2} x \sin x}$$

16- $y'' - 5y' = 2x^3 - 4x^2 - x + 6$ — (i)

SOLUTION:-

$m^2 - 5m = 0$

$m = 5, \; m = 0$

$y_c = C_1 e^{5x} + C_2$

$y_p = (Ax^4 + Bx^3 + Cx^2 + Dx)$

$y'_p = 4Ax^3 + 3Bx^2 + 2Cx + D$

$y''_p = 12Ax^2 + 6Bx + 2C$

Now putting y_p, y'_p & y''_p in eq (i)

$\Rightarrow (12Ax^2 + 6Bx + 2C) - 5(4Ax^3 + 3Bx^2 + 2Cx + D) = 2x^3 - 4x^2 - x + 6$

$\Rightarrow -20Ax^3 + 12Ax^2 - 15Bx^2 + 6Bx - 10Cx + 2C - 5D = 2x^3 - 4x^2 - x + 6$

$-20A = 2$	$(12A - 15B) = -4$	$-10C + 6B = -1$
$\boxed{A = -1/10}$	$12(-1/10) - 15B = -4$	$-10C + 6(14/75) = -1$
	$\boxed{B = 14/75}$	$\boxed{C = 53/250}$

$2C - 5D = 6$

$2\left(\frac{53}{250}\right) - 5D = 6$

$\boxed{D = -697/625}$

$y_c = C_1 e^{5x} + C_2$, $y_p = -\frac{1}{10}x^4 + \frac{14}{75}x^3 + \frac{53}{250}x^2 - \frac{697}{625}x.$

So

$$\boxed{y = C_1 e^{5x} + C_2 - \frac{1}{10}x^4 + \frac{14}{75}x^3 + \frac{53}{250}x^2 - \frac{697}{625}x}$$

Date:

17. $y'' - 2y' + 5y = e^x \cos 2x$

SOLUTION:-

$m^2 - 2m + 5 = 0$

$m = 1+2i,\ 1-2i$

$y_c = e^x[C_1 \cos 2x + C_2 \sin 2x]$

$y_p = e^x(Ax\cos 2x + Bx\sin 2x)$

$y_p = Axe^x\cos 2x + Bxe^x \sin 2x$

Now

$y'_p = Ae^x[\cos 2x - 2x\sin 2x] + A[x\cos 2x]e^x + Be^x(\sin 2x + 2x\cos 2x)$
$\quad + B[x\sin 2x]e^x$

$y'_p = Ae^x\cos 2x - 2Ae^x x\sin 2x + Ae^x x\cos 2x + Be^x\sin 2x$
$\quad + 2Be^x x\cos 2x + Be^x x\sin 2x.$

$y''_p = [Ae^x\cos 2x - 2Ae^x\sin 2x] - 2[Ae^x(\sin 2x + 2x\cos 2x)$
$\quad + A(x\sin 2x)e^x] + [Ae^x(\cos 2x - 2x\sin 2x) + Ae^x(x\cos 2x)]$
$\quad + [Be^x\sin 2x + 2Be^x\cos 2x] + [2Be^x(\cos 2x - 2x\sin 2x)$
$\quad + (2B(x\cos 2x)e^x] + [Be^x(\sin 2x + 2x\cos 2x) + Be^x(x\sin 2x)]$

$y''_p = Ae^x\cos 2x - 2Ae^2\sin 2x - 2Ae^x\sin 2x - 4Ae^x x\cos 2x$
$- 2Ae^x x\sin 2x + Ae^x\cos 2x - 2Ae^x x\sin 2x + Ae^x x\cos 2x$
$+ Be^x\sin 2x + 2Be^x\cos 2x + 2Be^x\cos 2x - 4Be^x x\sin 2x$
$+ 2Be^x x\cos 2x + Be^x\sin 2x + 2Be^x x\cos 2x + Be^x x\sin 2x$

$\Rightarrow 2Ae^x\cos 2x - 4Ae^x\sin 2x - 3Ae^x x\cos 2x - 4Ae^x x\sin 2x$
$+ 2Be^x\sin 2x + 4Be^x\cos 2x - 3Be^x x\cdot\sin 2x + 4Be^x x\cos 2x.$

Put in question:-

$\Rightarrow (\cancel{2Ae^x\cos 2x} - 4Ae^x\sin 2x - \cancel{3Ae^x x\cos 2x} - \cancel{4Ae^x x\sin 2x}$
$+ \cancel{2Be^x\sin 2x} + 4Be^x\cos 2x - \cancel{3Be^x x\sin 2x} + \cancel{4Be^x x\cos 2x}$
$- \cancel{2Ae^x\cos 2x} + \cancel{4Ae^x x\sin x} - \cancel{2Ae^x x\cos 2x} - \cancel{2Be^x\sin 2x}$
$- \cancel{4Be^x x\cos 2x} - \cancel{2Be^x x\sin 2x} + \cancel{5Axe^x\cos 2x} + \cancel{5Be^x x\sin 2x}$
$= e^x\cos 2x$

Date:

$-4Ae^{x}\sin 2x + 4Be^{x}\cos 2x = e^{x}\cos 2x$

$-4A = 0$ $\qquad$ $4B = 1$

$\boxed{A = 0}$ $\qquad$ $\boxed{B = 1/4}$

$y_p = \frac{1}{4}e^{x}x\sin 2x$

$$\boxed{y = e^{x}\left[C_1\cos 2x + C_2\sin 2x\right] + \frac{1}{4}e^{x}x\sin 2x}$$

18 $y'' - 2y' + 2y = e^{2x}(\cos x - 3\sin x)$

SOLUTION:-

$m^2 - 2m + 2 = 0$

$m = 1+i, 1-i.$

$y = e^{x}[C_1\cos x + C_2\sin x]$

$y_p = e^{2x}(A\cos x + B\sin x)$

$y_p' = e^{2x}((-A\sin x) + B(\cos x)) + 2(A\cos x + B\sin x)e^{2x}$

$= 2e^{2x}A\cos x - e^{2x}A\sin x + Be^{2x}\cos x + 2Be^{2x}\sin x.$

$y_p'' = 2A[e^{2x}(-\sin x) + 2(\cos x)e^{2x}] - A[e^{2x}(\cos x) + 2(\sin x)e^{2x}]$
$+ B[e^{2x}(-\sin x) + 2(\cos x)e^{2x}] + 2B[e^{2x}(\cos x) +$
$e^{2x}(\sin x)2].$

$\Rightarrow = -2Ae^{2x}\sin x + 4Ae^{2x}\cos x - Ae^{2x}\cos x - 2Ae^{2x}\sin x$
$-Be^{2x}\sin x + 2Be^{2x}\cos x + 2Be^{2x}\cos x + 4Be^{2x}\sin x$

$\Rightarrow 3Ae^{2x}\cos x - 4Ae^{2x}\sin x + 3Be^{2x}\sin x + 4Be^{2x}\cos x.$

Now put in question:-

$\Rightarrow (3Ae^{2x}\cos x - 4Ae^{2x}\sin x + 3Be^{2x}\sin x + 4Be^{2x}\cos x]$
$-4e^{2x}A\cos x + 2e^{2x}A\sin x - 2Be^{2x}\cos x - 4Be^{2x}\sin x$
$+ 2Ae^{2x}\cos x + 2Be^{2x}\sin x = e^{2x}(\cos - 3\sin x)$

Date:

$\Rightarrow 3Ae^{2x}\cos x - 4Ae^{2x}\cos x + 2Ae^{2x}\cos x - 4Ae^{2x}\sin x$
$- 2Ae^{2x}\sin x + 4Be^{2x}\cos x - 2Be^{2x}\cos x + 3Be^{2x}\sin x$
$- 4Be^{2x}\sin x + 2Be^{2x}\sin x = e^{2x}(\cos x - 3\sin x)$

$\Rightarrow Ae^{2x}\cos x - 2Ae^{2x}\sin x + 2Be^{2x}\cos + Be^{2x}\sin x = e^{2x}(\cos x - 3\sin x)$

Now comparing:-

$(A\cos x + 2B\cos x)\cancel{e^{2x}} = \cancel{e^{2x}}\cos x$

$A + 2B = 1$

$A = 1 - 2B \longrightarrow (2)$

$(-2A + B)e^{2x}\sin x = -3e^{2x}\sin x$

$B = -3 + 2A \longrightarrow (3)$

put (3) in (2)

$A = 1 - 2(-3 + 2A)$

$A = 1 + 6 - 4A$

$5A = 7$

$\boxed{A = 7/5}$

$B = -3 + 2(7/5)$

$\boxed{B = -1/5}$

so,

$\boxed{y = e^{x}[C_1\cos x + C_2\sin x] + e^{2x}\left[\frac{7}{5}\cos x - \frac{1}{5}\sin x\right]}$ Ans

"Date:"

4- $y'' + 2y' + y = \sin x + 3\cos 2x$ —(1)

SOLUTION:-

$m^2 + 2m + 1 = 0$

$m = -1, -1$

$y = C_1 e^{-x} + C_2 x e^{-x}$

$y_p = A\cos x + B\sin x + C\cos 2x + D\sin 2x.$

$y'_p = -A\sin x + B\cos x - 2C\sin 2x + 2D\cos 2x$

$y''_p = -A\cos x - B\sin x - 4C\cos 2x - 4D\sin 2x.$

(1) $\Rightarrow (-A\cancel{\cos x} - B\cancel{\sin x} - 4C\cos 2x - 4D\sin 2x) +$
$(-2A\sin x + 2B\cos x - 4C\sin 2x + 4D\cos x) +$
$(A\cancel{\cos x} + B\cancel{\sin x} + C\cos 2x + D\sin 2x) =$
$\sin x + 3\cos 3x$

$\Rightarrow -4C\cos 2x - 4D\sin 2x - 2A\sin x + 2B\cos x - 4C\sin 2x$
$+ 4D\cos 2x + C\cos 2x + D\sin 2x = \sin x + 3\cos x$

Comparing:-

$-2A = 1$ | $2B = 0$ | $-3D - 4C = 0$

$\boxed{A = -1/2}$ | $\boxed{B = 0}$ | $-3D = 4C$

$D = 4C/-3$ —(2)

$4D - 3C = 3$

$4\left(\frac{4C}{-3}\right) - 3C = 3$

$\frac{-16C - 9C}{+3} = 3$

$-25C = 9$

$\boxed{C = -9/25}$

Now putting value of C

$D = 4(-9/25)/3$

$\boxed{D = 12/25}$

$$\boxed{y = C_1 e^{-x} + C_2 x e^{-x} - \frac{1}{2}\cos x - \frac{9}{25}\cos 2x + \frac{12}{25}\sin 2x}$$

Date:

20- $y'' + 2y' - 24y = 16 - (x+2)e^{4x}$ —— (1)

SOLUTION:-

$m^2 + 2m - 24 = 0$

$m = 4, -6$

$y_c = C_1 e^{4x} + C_2 e^{-6x}$

$y_p = A + (Bx^2 + Cx)e^{4x}$

$y_p' = 0 + (2Bx + C)e^{4x} + 4(Bx^2 + Cx)e^{4x}$

$y_p'' = (2B)e^{4x} + 4(2Bx + C)e^{4x} + 4(2Bx + C)e^{4x} + 16(Bx^2 + Cx)e^{4x}$

$= 16(Bx^2 + Cx)e^{4x} + 8(2Bx + C)e^{4x} + (2Be^{4x})$

Now put in 1:

$\Rightarrow \cancel{16(Bx^2 + Cx)e^{4x}} + 8(2Bx + C)e^{4x} + (2Be^{4x}) + 2(2Bx + C)e^{4x}$
$+ \cancel{8(Bx^2 + Cx)e^{4x}} - 24A - \cancel{24(Bx^2 + Cx)e^{4x}}$

$\Rightarrow 10(2Bx + C)e^{4x} + 2Be^{4x} - 24A = 16 - (x+2)e^{4x}$

Now comparing:

$-24A = 16$	$20Bx = -1x$
$\boxed{A = -2/3}$	$\boxed{B = -1/20}$

$2B + 10C = -2$

$2B = -2 - 10C$

$B = -1 - 5C$

$(-1/20) = -1 - 5C$

$\boxed{C = -19/100}$

$$\boxed{y = C_1 e^{4x} + C_2 e^{-6x} + \left(-\frac{2}{3}\right) + \left[-\frac{1}{20} x^2 - \frac{19}{100} x\right] e^{4x}}$$

Ans

$y''' - 6y'' = 3 - \cos x$ Date:

SOLUTION:

$m^3 - 6m^2 = 0$

$m = 6, 0, 0$

$y_c = C_1 + C_2 x + C_3 e^{6x}$

$y_p = Ax^2 + B\cos x + C\sin x$

$y_p' = 2Ax - B\sin x + C\cos x$

$y_p'' = 2A - B\cos x - C\sin x$

$y_p''' = 0 + B\sin x - C\cos x$

$\Rightarrow (0 + B\sin x - C\cos x' - 12A + 6B\cos x + 6C\sin x = 3 - \cos x$

$\Rightarrow 6B\cos x - C\cos x + B\sin x + 6C\sin x - 12A = 3 - \cos x$

$(6B - C)\cancel{\cos x} = -\cancel{\cos x}$

$6B = C - 1$

$B = C - 1/6$

$B = \frac{C-1}{6}$

$B = \frac{(1/37) - 1}{6}$

$B + 6C = 0$

$\left(\frac{C-1}{6}\right) + 6C = 0$

$\frac{-1 + 37C}{6} = 0$

$\boxed{B = \frac{-6}{37}}$

$37C = 1$

$\boxed{C = 1/37}$

$y_p = -\frac{1}{4}x^2 - \frac{6}{37} B\cos x + \frac{1}{37}\sin x$

$$\boxed{y = C_1 + C_2 x + C_3 e^{6x} - \frac{1}{4}x^2 - \frac{6}{37} B\cos x + \frac{1}{37}\sin x}$$

Date:

22 $y''' - 2y'' - 4y' + 8y = 6xe^{2x}$ —①

SOLUTION:

$m^3 - 2m^2 - 4m + 8 = 0$

$m = -2, 2, 2$

$y = C_1 e^{-2x} + xC_1 e^{2x} + C_3 e^{2x}$

$y_p = (Ax^3 + Bx^2)e^{2x}$

$y'_p = 2(Ax^3 + Bx^2)e^{2x} + (3Ax^2 + 2Bx)e^{2x}$

$y'_p = (2Ax^3 + 2Bx^2)e^{2x} + (3Ax^2 + 2Bx)e^{2x}$

$y''_p = 2(2Ax^3 + 2Bx^2)e^{2x} + e^{2x}(6Ax^2 + 4Bx) + 2(3Ax^2 + 2Bx)e^{2x}$
$+ e^{2x}(6Ax + 2B)$.

$y''_p = 4Ax^3e^{2x} + 4Bx^2e^{2x} + 6Axe^{2x} + 12x^2e^{2x} + 8Bxe^{2x} + 12Be^{2x}$

$y'''_p = 4A[3x^2(e^{2x}) + 2e^{2x}(x^3)] + 12A[2xe^{2x} + 2e^{2x}x^2] + 6A[e^{2x} + 2e^{2x}x]$
$+ 4B[2xe^{2x} + 2e^{2x}(x^2)] + 8B[e^{2x} + 2e^{2x}x] + 2B[2e^{2x}]$

$y'''_p = 8Ax^3e^{2x} + 36Ax^2e^{2x} + 8Be^{2x}x^2 + 24e^{2x}x + 36e^{2x}x$
$+ 12Be^{2x} + 6Ae^{2x}$.

Put in eq①:

$\Rightarrow \cancel{8Ax^3e^{2x}} + \cancel{36Ax^2e^{2x}} + \cancel{8Be^{2x}x^2} + \cancel{24B}e^{2x}x + 36Ae^{2x}x +$
$12Be^{2x} + 6Ae^{2x} \cancel{- 8Ax^3e^{2x}} \cancel{- 24Ax^2e^{2x}} - 12Axe^{2x} \cancel{- 8Bx^2e^{2x}} \cancel{- 16Bxe^{2x}}$
$- 4Be^{2x} \cancel{- 8Ax^3e^{2x}} \cancel{- 8Bx^2e^{2x}} \cancel{- 12Ax^2e^{2x}} \cancel{- 8Bxe^{2x}}$
$\cancel{+ 8Ax^3e^{2x}} + \cancel{8Bx^2e^{2x}} = 6xe^{2x}$

$\Rightarrow 24Axe^{2x} + 8Be^{2x} + 6Ae^{2x} = 6xe^{2x}$

$24A = 6 \qquad 8B + 6A = 0$

$\boxed{A = 1/4} \qquad 8B + 6(1/4) = 0$

$\boxed{B = -3/16}$

$$\boxed{y = C_1e^{2x} + C_2xe^{2x} + C_3e^{-2x} + \left(\frac{1}{4}x^3 - \frac{3}{16}x^2\right)e^{2x}}$$

Ans

3 $y''' - 3y'' + 3y' - y = x - 4e^x$ —— (1)

Date:

SOLUTION:

$m^3 - 3m^2 + 3m - 1 = 0$

$m = 1, 1, 1.$

$y_c = C_1e^x + xC_2e^x + x^2C_3e^x.$

$y_p = Ax + B + x^3Ce^x$

$y'_p = A + C[3x^2e^x + Cx^3e^x]$

$y'_p = A + 3Cx^2e^x + Cx^3e^x.$

$y''_p = +3C[2xe^x + e^xx^2] + C[3x^2e^x + x^3e^x]$

$y''_p = Cx^3e^x + 6Cx^2e^x + 6Cxe^x$

$y'''_p = C[3x^2e^x + xe^x] + 6C[2xe^x + x^2e^x] + 6C[e^x + xe^x]$

$y'''_p = Cx^3e^x + 9Cx^2e^x + 18Cxe^x + 6Ce^x$

Now put in eq (1)

$\Rightarrow Cx^3e^x + 9Cx^2e^x + 18xe^x + 6Ce^x - 3Cx^3e^x - 18Cx^2e^x - 18Cxe^x$
$+ 3A + 9Cx^2e^x + 3Cx^3e^x - Ax - B - Ce^xx^3 = x - 4e^x$

$-Ax = x$

$\boxed{+A = -1}$

$3A - B = 0$

$3(-1) - B = 0$

$B = -3$

$6Ce^x = -4e^x$

$C = -4/6$

$\boxed{C = -2/3}$

So,

$$\boxed{y = C_1e^x + xC_2e^x + x^2C_3e^x - x - 3 - \frac{2}{3}x^3e^x}$$

Date:

24- $y''' - y'' - 4y' + 4y = 5 - e^{x} + e^{2x}$ —— (1)

SOLUTION:-

$m^3 - m^2 - 4m + 4 = 0$

$m = -2, 2, 1$

$y_c = C_1 e^{x} + C_2 e^{2x} + C_3 e^{-2x}$

$y_p = A + xBe^{x} + xCe^{2x}$

$y'_p = B[e^{x} + xe^{x}] + C[e^{2x} + 2e^{2x}x]$

$y'_p = Be^{x} + Bxe^{x} + Ce^{2x} + 2Ce^{2x}x.$

$y''_p = Be^{x} + Be^{x} + Bxe^{x} + 2Ce^{2x} + 2Ce^{2x} + 4Ce^{2x}x$

$y''_p = 2Be^{x} + Bxe^{x} + 4Ce^{2x} + 4Ce^{2x}x.$

$y'''_p = 2Be^{x} + Be^{x} + Be^{x}x + 8Ce^{2x} + 4Ce^{2x} + 8Ce^{2x}x.$

$y'''_p = 3Be^{x} + Bxe^{x} + 12Ce^{2x} + 8Ce^{2x}x.$

Now put in eq (1).

$\Rightarrow 3Be^{x} + \cancel{Bxe^{x}} + 12Ce^{2x} + \cancel{8Ce^{2x}x} - 2Be^{x} - \cancel{Bxe^{x}} - 4Ce^{2x}$
$- \cancel{4Ce^{2x}x} - 4Be^{x} - \cancel{4Bxe^{x}} - 4Ce^{2x} - \cancel{8Ce^{2x}x} + 4A$
$+ \cancel{4Bxe^{x}} + \cancel{4Cxe^{2x}}$ ~~$-3Be^{x} + 4Ce^{2x} + 4A$~~ $= 5 - e^{x} + e^{2x}$

$\Rightarrow -3Be^{x} + 4Ce^{2x} + 4A = 5 - e^{x} + e^{2x}$

$4A = 5$	$-3B = -1$	$4C = 1$
$\boxed{A = 5/4}$	$\boxed{B = 1/3}$	$\boxed{C = 1/4}$

So,

$$\boxed{y = C_1e^{x} + C_2e^{2x} + C_3e^{-2x} + \frac{5}{4} + \frac{1}{3}xe^{x} + \frac{1}{4}xe^{2x}}$$

Date:

25- $y'''' + 2y'' + y = (x-1)^2$ —— (1)

SOLUTION:-

$m^4 + 2m^2 + 1 = 0$

$m_1, m_3 = i \quad , m_2, m_4 = -i$

$y_c = C_1 \cos x + C_2 \sin x + x C_3 \cos x + x C_4 \sin x$

$y_p = Ax^2 + Bx + C$

$y'_p = 2Ax + B$

$y''_p = 2A$

$y'''_p = 0$

$y''''_p = 0$

Now put in eq (1).

$\Rightarrow 0 + 2(2A) + Ax^2 + Bx + C = (x-1)^2$

$\Rightarrow \quad 4A + Ax^2 + Bx + C = x^2 - 2x + 1$

$A\cancel{x^2} = \cancel{x^2}$ $\qquad B\cancel{x} = -2\cancel{x}$ $\qquad 4A + C = 1$

$\boxed{A = 1}$ $\qquad \boxed{B = -2}$ $\qquad 4(1) + C = 1$

$\boxed{C = -3}$

So,

$$\boxed{y = C_1 \cos x + C_2 \sin x + x C_3 \cos x + x C_4 \sin x + x^2 - 2x - 3.}$$