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ASSIGNMENT (BLOCK II)

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Section : B
Course : Mathematics II (Differential Equations) – BSCS 304

Exercise 4.4

Q) Solve the given differential equation by undetermined coefficients

1) $y'' + 3y' + 2y = 6$

$m^2 + 3m + 2 = 0$

$\boxed{m = -1}$ $\boxed{m = -2}$

$y_c = C_1 e^{m_1 x} + C_2 e^{m_2 x}$

$y_c = C_1 e^{-x} + C_2 e^{-2x}$

$y_p = A$

$y_p' = 0$, $y_p'' = 0$

Put in Original equation we get,

$0 + 0 + 2A = 6$

$\boxed{A = 3}$ $\therefore$ $y_p = 3$

$y = y_c + y_p$

$y = C_1 e^{-x} + C_2 e^{-2x} + 3$

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2) $4y'' + 9y = 15$

$4m^2 + 9 = 0$

$\therefore m = 0 \pm \frac{3}{2} i$

$\therefore \alpha = 0$ and $\beta = \frac{3}{2}$

$y_c = e^{\alpha x}\{C_1 \cos(\beta x) + C_2 \sin(\beta x)\}$

$y_c = e^0\{C_1 \cos(\frac{3}{2}x) + C_2 \sin(\frac{3}{2}x)\}$

$y_c = C_1 \cos(\frac{3}{2}x) + C_2 \sin(\frac{3}{2}x)$

$y_p = A \quad , y_p' = 0 \quad , y_p'' = 0$

$4(0) + 9(A) = 15$

$A = 15/9$

$\therefore y_p = 15/9 = 5/3$

$y = y_c + y_p$

$y = C_1 \cos \frac{3}{2}x + C_2 \sin \frac{3}{2}x + 5/3$

3) $y'' - 10y + 25y = 30x + 3$

$m^2 - 10m + 25 = 0$

$\boxed{m = 5} \qquad \boxed{m = 5}$

$\therefore y_c = C_1 e^{mx} + x C_2 e^{mx}$

$y_c = C_1 e^{5x} + C_2 x e^{5x}$

$y_p = Ax + B \quad , y_p' = A \quad , y_p'' = 0$

So, $0 - 10A + 25(Ax + B) = 30x + 3$

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$\boxed{A = 6/5}$, $\boxed{B = 3/5}$

$\therefore\ y_p = \frac{6}{5}x + \frac{3}{5}$

$y = C_1e^{5x} + xC_2e^{5x} + \frac{6}{5}x + \frac{3}{5}$

4) $y'' + y' - 6y = 2x$

$m^2 + m - 6 = 0$

$\boxed{m = 2}$ $\boxed{m = -3}$

$y_c = C_1e^{2x} + C_2e^{-3x}$

$y_p = Ax + B$, $y_p' = A$, $y_p'' = 0$

$A - 6(Ax + B) = 2x$

$-6A = 2$ $A - 6B = 0$

$\boxed{A = -1/3}$ $\boxed{B = -1/18}$

$y_p = -\frac{1}{3}x - \frac{1}{18}$

$y = C_1e^{2x} + C_2e^{-3x} \mp \frac{1}{3}x - \frac{1}{18}$

5) $\frac{1}{4}y'' + y' + y = x^2 - 2x$

$\frac{1}{4}m^2 + m + 1 = 0$

$\boxed{m = -2}$, $\boxed{m = -2}$

$y_c = C_1e^{-2x} + xC_2e^{-2x}$

$y_p = Ax^2 + Bx + C$

$y_p' = 2Ax + B$

$y_p'' = 2A$

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$\frac{1}{2}A + 2Ax + B + Ax^2 + Bx + C = x^2 - 2x$

$\boxed{A = 1}$ $\qquad 2A + B = -2$

$\boxed{B = -4}$

$\therefore$ ~~$y_p = \frac{1}{2} A +$~~

Equating Coefficients

$\frac{1}{2}A + B + C = 0$

$\boxed{C = 7/2}$

$y_p = x^2 - 4x + 7/2$

$y = C_1e^{-2x} + C_2xe^{-2x} + x^2 - 4x + 7/2$

6) $y'' - 8y' + 20y = 100x^2 - 26xe^x$

$m^2 - 8m + 20 = 0$

$\boxed{m = 4 \pm 2i}$ $\quad \therefore \alpha = 4 \quad , \beta = 2$

$y_c = e^{\alpha x}\{C_1 \cos\beta x + C_2 \sin\beta x\}$

$y_c = e^{4x}\{C_1 \cos 2x + C_2 \sin 2x\}$

$y_p = (Ax^2 + Bx + C)e^x$

$y_p' = e^x(2Ax + B) + e^x(Ax^2 + Bx + C)$

$y_p'' = (2Ax + B)e^x + e^x(2A) + e^x(Ax^2 + Bx + C) + e^x(2Ax + B)$

$y_{p_1} = Ax^2 + Bx + C \quad , y_{p_1}' = 2Ax + B \quad , y_p'' = 2A$

$2A - 8(2Ax + B) + 20(Ax^2 + Bx + C) = 100x^2$

$\boxed{A = 5}$ $\quad \boxed{B = 4}$ $\quad \boxed{C = 11/10}$

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$y_{p_1} = 5x^2 + 4x + 11/10$

$y_{p_2} = (Ax + B)e^x$

$y_{p_2}' = e^x A + e^x(Ax + B)$

$y_{p_2}'' = Ae^x + e^x A + e^x(Ax + B)$

$= 2Ae^x + e^x(Ax + B)$

$2Ae^x + e^x(Ax + B) - 8\{e^x A + e^x(Ax + B)\} + 20\{(Ax + B)e^x\}$

$= -26xe^x$

$\boxed{A = -2}$ $\boxed{B = -12/13}$

$\therefore\ y_{p_2} = (-2x - 12/13)e^x$

$y = C_1 e^{4x}\cos 2x + C_2 e^{4x}\sin 2x + 5x^2 + 4x + 11/10 - 2xe^x - \frac{12}{13}e^x$

7) $y'' + 3y = -48x^2 e^{3x}$

$m^2 + 3 = 0$

$m = 0 \pm \sqrt{3}i$

$\cancel{y_c = C_1 e^{\sqrt{3}x} + C_2 e^{-\sqrt{3}x}}$

$y_c = C_1 \sin(\sqrt{3}x) + C_2 \cos(\sqrt{3}x)$

$y_p = (Ax^2 + Bx + C)e^{3x}$

$y_p' = e^{3x}(2Ax + B) + 3e^{3x}(Ax^2 + Bx + C)$

$y_p'' = e^{3x}(2A) + 3e^{3x}(2Ax + B) + 9e^{3x}(Ax^2 + Bx + C)$

$+ 3e^{3x}(2Ax + B)$

$\therefore$ Original equation becomes

$2Ae^{3x} + 3e^{3x}(2Ax + B) + 9e^{3x}(Ax^2 + Bx + C) + 3e^{3x}(2Ax + B)$

$+ 3\{e^{3x}(Ax^2 + Bx + C)\} = -48x^2 e^{3x}$

Equation coefficients of $x^2 e^{3x}$

$12A = -48 \Rightarrow \boxed{A = -4}$

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Equating xe^{3x}

$12A + 12B = 0$

$\boxed{B = 4}$

Equating Coefficients of e^{3x}

$2A + 6B + 12C = 0$

$\boxed{C = -4/3}$

$\therefore y_p = (-4x^2 + 4x - 4/3)e^{3x}$

$y = C_1 \sin(\sqrt{3}x) + C_2 \cos(\sqrt{3}x) - 4x^2e^{3x} + 4xe^{3x} - 4/3\, e^{3x}$

8) $4y'' - 4y' - 3y = \cos 2x$

$4m^2 - 4m - 3 = 0$

$\boxed{m = 3/2}$ $\boxed{m = -1/2}$

$y_c = C_1 e^{3/2 x} + C_2 e^{-x/2}$.

$y_p = A\cos 2x + B\sin 2x$

$y_p' = -2A\sin 2x + 2B\cos 2x$

$y_p'' = -4A\cos 2x - 4B\sin 2x$

Put in Original Equation we get,

$4(-4A\cos 2x - 4B\sin 2x) - 4(-2A\sin 2x + 2B\cos 2x)$
$- 3(A\cos 2x + B\sin 2x) = \cos 2x$

Equating Coefficients of $\cos 2x$

$-16A - 8B - 3A = 1$

$B = \dfrac{-1 - 19A}{8}$ —— (i)

Equating Coefficients of $\sin 2x$

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$-16B + 8A - 3B = 0$ Put value of B

$\boxed{A = -19/425}$

Put A in eq (i) we get

$\boxed{B = -8/425}$

$\therefore y_p = \frac{-19}{425}\cos 2x - \frac{8}{425}\sin 2x$

$\therefore y = y_c + y_p$

$y = \frac{-19}{425}\cos 2x - \frac{8}{425}\sin 2x + C_1 e^{3/2 x} + C_2 e^{-x/2}$

9) $y'' - y' = -3$

$m^2 - m = 0$

$\boxed{m = 0}$ $\boxed{m = 1}$

$y_c = C_1 e^x + C_2 e^0$

$= C_1 e^x + C_2$

$y_p = Ax$, $y_p' = A$, $y_p'' = 0$

Original Equation becomes

$0 - A = -3 \Rightarrow \boxed{A = 3}$

$\therefore y_p = 3x$

$y = C_1 e^x + 3x + C_1$

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10) $y'' + 2y' = 2x + 5 - e^{-2x}$

$m^2 + 2m = 0$

$\boxed{m = 0}$ $\boxed{m = -2}$

$y_c = C_1 e^{-2x} + C_2$

$y_{p_1} = A x e^{-2x}$

$y_{p_1}' = A e^{-2x} - 2x A e^{-2x}$ $y_{p_1}'' = -4A e^{-2x} + 4A x e^{-2x}$

Put in Original equation

$-4Ae^{-2x} + 4Axe^{-2x} + 2(Ae^{-2x} - 2xAe^{-2x}) = -e^{-2x}$

Equating Coefficients of e^{-2x} we get,

$-4A + 2A = -1$

$\boxed{A = 1/2}$

$\therefore y_{p_1} = \frac{1}{2} x e^{-2x}$

~~Equating Constants~~

$y_{p_2} = x(Ax + B) = Ax^2 + Bx$

$y_{p_2}' = 2Ax + B$, $y_{p_2}'' = 2A$

Put in Original equation

$2A + 2(2Ax + B) = 2x + 5$

$\boxed{A = 1/2}$ $\boxed{B = 2}$

$\therefore y_{p_2} = \frac{x^2}{2} + 2x$

$y = C_1 e^{-2x} + \frac{1}{2} x e^{-2x} + \frac{x^2}{2} + 2x + C_2$

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11) $y'' - y' + \frac{1}{4}y = 3 + e^{x/2}$

$m^2 - m - \frac{1}{4} = 0$

$\boxed{m = \frac{1}{2}}$

$y_c = C_1 e^{x/2} + C_2 x e^{x/2}$

$y_{p_1} = A \quad , \quad y_{p_1}' = 0 \quad , \quad y_{p_1}'' = 0$

$\therefore$ Original equation becomes

$\frac{1}{4} A = 3 \quad \Rightarrow \quad \boxed{A = 12}$

$y_{p_2} = A x^2 e^{x/2} \quad , \quad y_{p_2}' = \frac{1}{2} A x^2 e^{x/2} + 2 A x e^{x/2}$

$y_{p_2}'' = \frac{1}{2} A x e^{x/2} + \frac{1}{4} A x^2 e^{x/2} + 2 A e^{x/2} + A x e^{x/2}$

Put in Original equation

$$\frac{3}{2} A x e^{x/2} + \frac{1}{4} A x^2 e^{x/2} + A e^{x/2} - \frac{1}{2} A x^2 e^{x/2} - A x e^{x/2} + \frac{1}{4} A x^2 e^{x/2} = e^{x/2}$$

Equating Coefficients of $e^{x/2}$

$2A = 1$

$\boxed{A = \frac{1}{2}}$

$\therefore \quad y_{p_2} = \frac{x^2 e^{x/2}}{2}$

$y = C_1 e^{x/2} + C_2 x e^{x/2} + \frac{x^2 e^{x/2}}{2} + 12$

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12) $y'' - 16y = 2e^{4x}$

$m^2 - 16 = 0$

$\boxed{m = \pm 4}$

$y_c = C_1 e^{4x} + C_2 e^{-4x}$

$y_p = Axe^{4x}$, $y_p' = Ae^{4x} + 4Axe^{4x}$

$y_p'' = 4Ae^{4x} + 16Axe^{4x} + 4Ae^{4x}$

$y_p'' = 8Ae^{4x} + 16Axe^{4x}$

Put in original equation

$8Ae^{4x} + \cancel{16Axe^{4x}} - \cancel{16Axe^{4x}} = 2e^{4x}$

$8A = 2$

$\boxed{A = 1/4}$

$\therefore$ $y_p = 1/4\, xe^{4x}$

$y = C_1 e^{4x} + C_2 e^{-4x} + 1/4\, xe^{4x}$

13) $y'' + 4y = 3\sin 2x$

$m^2 + 4 = 0$

$m = 0 \pm 2i$

$y_c = C_1 \cos 2x + C_2 \sin 2x$

$y_p = A\cos 2x + B\sin 2x$

$y_p' = -2A \sin 2x + 2B\cos 2x$

$y_p'' = -4A\cos 2x - 4B\sin 2x$

$-4A\cos 2x - 4B\sin 2x + 4(A\cos 2x + B\sin 2x) = 3\sin 2x$

$\boxed{A = -3/4}$ $\boxed{B = 0}$

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$\therefore \quad y_p = -\frac{3}{4} x \cos 2x$

$y = C_1 \cos 2x + C_2 \sin 2x - \frac{3}{4} x \cos 2x$

14) $y'' - 4y = (x^2-3)\sin 2x$

$m^2 - 4 = 0$

$\boxed{m = \pm 2}$

$y_c = C_1 e^{2x} + C_2 x e^{2x}$

$y_p = (Ax^2 + Bx + C)\cos 2x + \sin 2x(Dx^2 + Ex + F)$ — (1)

$y_p' = 2Ax\cos 2x - 2Ax^2 \sin 2x + B\cos 2x - 2Bx\sin 2x - 2C\sin 2x$
$+ 2Dx\sin 2x + 2Dx^2\cos 2x + E\sin 2x + 2Ex\cos 2x + 2F\cos x$

$y_p'' = (-4Dx^2 + x(-4E-8A) - 4F + 2D - 4B)\sin 2x) + \cos 2x(-4Ax^2$
$+ (8D-4B)x + 4E - 4C + 2A)$

Substitute in original equation

$-8Dx^2\sin(2x) + (-8E-8A)x\sin 2x + (-8F + 2D - 4B)\sin 2x$
$-8Ax^2\cos 2x + (8D - 8B)x\cos 2x + (4E - 8C + 2A)\cos 2x$
$= (x^2-3)\sin 2x$

Equating coefficients we get

$-8D = 1 \Rightarrow D = -1/8$

$-8A = 0 \Rightarrow A = 0$

$-8D - 8B = 0 \Rightarrow B = -1/8$

$-8F + 2D - 4B = -3 \Rightarrow F = 13/32$

$-8E - 8A = 0 \Rightarrow E = 0$

$4E - 8C + 2A = 0 \Rightarrow C = 0$

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Substitute in (i) we get,

$$y_p = \left(\frac{13}{32} - \frac{x^2}{8}\right)\sin 2x - \frac{x\cos(2x)}{8}$$

$$\therefore\ y = \frac{-x^2\sin 2x}{8} + \frac{13\sin 2x}{32} - \frac{x\cos 2x}{8} + Ce^{2x} + C_1e^{-2x}$$

15) $y'' + y = 2x\sin x$

$m^2 + 1 = \cancel{2x}\ 0$

$m = 0 \pm i$

$$y_c = C_1\sin(x) + C_2(\cos x)$$

$y_p = ((Ax+B)\cos x + (Cx+D)\sin x)x$

$y_p = Ax^2\cos x + Bx\cos x + Cx^2\sin x + Dx\sin x$

$y_p' = 2Ax\cos x - Ax^2\sin x - Bx\sin x + B\cos x + 2Cx\sin x + Cx^2\cos x + D\sin x + Dx\cos x$

$y_p'' = 2A\cos x - 4Ax\sin x - Ax^2\cos x - 2B\sin x - Bx\cos x + 2C\sin x + 4Cx\cos x - Cx^2\sin x + 2D\cos x - Dx\sin x$

Substitute in Original Equation

$2A\cos x - 4Ax\sin x - \cancel{Ax^2\cos x} - 2B\sin x - \cancel{Bx\cos x} + 2C\sin x + 4Cx\cos x - \cancel{Cx^2\sin x} + 2D\cos x - \cancel{Dx\sin x} + \cancel{Ax^2\cos x} + \cancel{Bx\cos x} + \cancel{Cx^2\sin x} + \cancel{Dx\sin x} = 2x\sin x$

Equating Coefficients of $x\sin x$

$-4A = 2 \Rightarrow \boxed{A = -\tfrac{1}{2}}$

Equating $\cos x$

$2A + 2D = 0 \Rightarrow \boxed{D = \tfrac{1}{2}}$

Equating $x\cos x$

$\cancel{B} + 4C = 0$

$\boxed{C = 0}$

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$2C - 2B = 0 \Rightarrow \boxed{B = 0}$

$\therefore\ y_p = x\left(\frac{\sin x}{2} - \frac{x\cos x}{2}\right)$

$y = C_1 \sin x + C_2 \cos x + \frac{x \sin x}{2} - \frac{x^2 \cos x}{2}$

16) $y'' - 5y' = 2x^3 - 4x^2 - x + 6$

let $m^2 - 5m = 0$

$\boxed{m = 0}$ $\boxed{m = 5}$

$y_c = C_1 e^{5x} + C_2$

$y_p = x(Ax^3 + Bx^2 + Cx + D)$ — (i)

$y_p = Ax^4 + Bx^3 + Cx^2 + Dx$

$y_p' = 4Ax^3 + 3Bx^2 + 2Cx + D$

$y_p'' = 12Ax^2 + 6Bx + 2C$

Substitute in ~~eq (i)~~ we get

$-20Ax^3 + (12A - 15B)x^2 + (6B - 10C)x - 5D + 2C = 2x^3 - 4x^2 - x + 6$

Equating x^3

$-20A = 2 \Rightarrow \boxed{A = -1/10}$

Equating x^2

$12A - 15B = -4 \Rightarrow \boxed{B = -14/75}$

x

$6B - 10C = -1 \Rightarrow \boxed{C = 53/250}$

Constants

$-5D + 2C = 6 \Rightarrow \boxed{D = \frac{-697}{625}}$

Substitute (i)

$y_p = x\left(-\frac{x^3}{10} + \frac{14x^2}{75} + \frac{53x}{250} - \frac{697}{625}\right)$

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$$y = C_1 e^{5x} + x\left(\frac{-x^3}{10} + \frac{14x^2}{75} + \frac{53x}{250} - \frac{697}{625}\right) + C_2$$

17) $y'' - 2y' + 5y = e^x \cos 2x$

Homogenous Solution

$m^2 - 2m + 5 = 0$

$\boxed{m = 1 \pm 2i}$

$y_c = e^x \{ C_1 \cos 2x + C_2 \sin 2x \}$

$y_p = x e^x \{ B \sin 2x + A \cos 2x \}$

$y_p = B x e^x \sin 2x + A x e^x \cos 2x$

$y_p' = B(e^x \sin 2x + x e^x \sin 2x + 2x e^x \cos 2x) + A(\cancel{e} x e^x \cos(2x)$
$- 2x e^x \sin 2x + e^x \cos(2x))$

$\cancel{e^{-x}(2x \sin(2x) + (-x - 1)\cos(2x))}$

$y_p'' = \{(-3B - 4A)x + 2B - 4A) e^x \sin(2x) + (4B - 3A)x$
$+ 4B + 2A) e^x \cos 2x \}$

Put in Original Equation we get

$4Be^x \cos(2x) - 4Ae^x \sin(2x) = e^x \cos 2x$

$4B = 1 \Rightarrow \boxed{B = 1/4}$

$-4A = 0 \Rightarrow \boxed{A = 0}$

$\therefore y_p = \frac{x e^x \sin(2x)}{4}$

$$y = \frac{x e^x \sin(2x)}{4} + C_1 e^x \sin(2x) + C e^x \cos(2x)$$

18) $y'' - 2y' + 2y = e^{2x}(\cos x - 3\sin x)$

$m^2 - 2m + 2 = 0$

$m = 1 \pm i$

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$$y_c = e^{x}\{C_1 \sin x + C_2 \cos x\}$$
$$y_c = C_1 e^{x} \sin x + C_2 e^{x} \cos x$$

$$y_p = e^{2x}\{B \sin x + A \cos x\}$$
$$y_p = Be^{2x} \sin x + Ae^{2x} \cos x$$
$$\cancel{y_p' = 2Be^{2x} \cos x +}$$
$$y_p' = 2Be^{2x} \sin x + Be^{2x} \cos x + 2Ae^{2x} \cos x - Ae^{2x} \sin x$$
$$y_p'' = 4Be^{2x} \sin x + 2Be^{2x} \cos x + 2Be^{2x} \cos x - Be^{2x} \sin x$$
$$- 2Ae^{2x} \sin x + 4Ae^{2x} \cos x - Ae^{2x} \cos x - 2Ae^{2x} \sin x$$

Put in Original Equation we get

$$\cancel{4Be^{2x}}\ 3Be^{2x} \sin x + 4Be^{2x} \cos x + 3Ae^{2x} \cos x - 4Ae^{2x} \sin x -$$
$$2\{2Be^{2x} \sin x + Be^{2x} \cos x + 2Ae^{2x} \cos x - Ae^{2x} \sin x\}$$
$$+ 2\{Be^{2x} \sin x + Ae^{2x} \cos x\} = e^{2x}(\cos x - 3 \sin x)$$

$$3Be^{2x} \sin x + 4Be^{2x} \cos x + 3Ae^{2x} \cos x - 4Ae^{2x} \sin x - 4Be^{2x} \sin x$$
$$- 2Be^{2x} \cos x - 4Ae^{2x} \cos x + 2Ae^{2x} \sin x + 2Be^{2x} \sin x +$$
$$2Ae^{2x} \cos x = e^{2x} \cos x - 3e^{2x} \sin x$$

$$Be^{2x} \sin x + 2Be^{2x} \cos x + Ae^{2x} \cos x - 2Ae^{2x} \sin x$$
$$= e^{2x} \cos x - 3e^{2x} \sin x$$

$$(B - 2A)e^{2x} \sin(x) + (2B + A)e^{2x} \cos x = e^{2x}(\cos x - 3 \sin x)$$

Equating $e^{2x} \sin x$

$$B - 2A = -3$$
$$B = -3 + 2A$$

Equation $e^{2x} \cos x$

$$2B + A = 1$$
$$2(-3 + 2A) + A = 1 \Rightarrow -6 + 5A = 1$$
$$\boxed{A = \cancel{5/} \ 7/5}$$

$$\therefore \boxed{B = -1/5}$$

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$$\therefore y_p = \frac{-e^{2x}\sin x}{5} + \frac{7\cos x}{5}$$

$$y = C_1 e^{x}\sin(x) + e^{2x}\left(\frac{7\cos x}{5} - \frac{\sin x}{5}\right) + C_2 e^{x}\cos x$$

19) $y'' + 2y' + y = \sin x + 3\cos 2x$

$m^2 + 2m + 1 = 0$

$\boxed{m = -1}$

$$y_c = C_1 e^{-x} + C_2 x e^{-x}$$

$y_{p_1} = B\sin 2x + A\cos 2x$

$y_{p_1}' = 2B\cos 2x \mp 2A\sin 2x$

$y_{p_1}'' = -4B\sin 2x - 4A\,\cancel{\sin}\cos 2x$

Put in Original eqiation

$-4B\sin 2x - 4A\cos 2x + 2\{2B\cos 2x - 2A\sin 2x\} + B\sin 2x$
$+ A\cos 2x = 3\cos 2x$

$-3B\sin 2x - 3A\cos 2x + 4B\cos 2x - 4A\sin 2x = 3\cos 2x$

Equating Coefficients of $\cos 2x$

$-3A + 4B = 3$

$4B = 3 + 3A$

$B = \frac{3+3A}{4}$

Equating Coefficients of $\sin 2x$

$-3B - 4A = 0$

$-\frac{9-9A}{4} - 4A = 0 \Rightarrow -4A = \frac{9+9A}{4}$

$A = \frac{-9+9A}{16}$

$16A = -9 \mp 9A \Rightarrow \cancel{8}25A = -9 \Rightarrow \boxed{A = \frac{-9}{25}}$

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$\therefore \boxed{B = \frac{12}{25}}$ $\therefore y_{p_1} = \frac{12\sin 2x}{25} - \frac{9\cos 2x}{25}$

$y_{p_2} = B\sin x + A\cos x$

$y_{p_2}' = B\cos x - A\sin x$, $y_{p_2}'' = -B\sin x - A\cos x$

Substitute in original equation, we get

$-\cancel{B\sin x} - \cancel{A\cos x} + 2\{B\cos x - A\sin x\} + \cancel{B\sin x} + \cancel{A\cos x}$
$= \sin x$

$2B\cos x - 2A\sin x = \sin x$

Equating coefficients of $\sin x$

$-2A = 1$

$\boxed{A = -1/2}$

Equating coefficients of $\cos x$

$2B = 0 \Rightarrow \boxed{B = 0}$

$\therefore y_{p_2} = -\frac{\cos x}{2}$

$y = \frac{12\sin(2x)}{25} - \frac{9\cos(2x)}{25} - \frac{\cos x}{2} + \frac{C_2 x + C_1}{e^x}$

20) $y'' + 2y' - 24y = 16 - (x+2)e^{4x}$

$m^2 + 2m - 24 = 0$

$\boxed{m = 4}$ $\boxed{m = -6}$

$y_c = C_1 e^{4x} + C_2 e^{-6x}$

$y_{p_1} = A$, $y_{p_1}' = 0$, $y_{p_1}'' = 0$

Substitute in original equation we get

$-24A = 16$

$\boxed{A = -2/3}$ $\therefore y_{p_1} = -2/3$

Date:

$y_{p2} = x(Ax+B)e^{4x} \Rightarrow y_{p2} = (Ax^2 + Bx)e^{4x}$

$y_{p2}' = 2Axe^{4x} + 4Ax^2e^{4x} + Be^{4x} + 4Bxe^{4x}$

$y_{p2}'' = 16Ax^2 + (16B + 16A)x + 8B + 2A)e^{4x}$

Substitute in Original equation

$20Axe^{4x} + (10B + 2A)e^{4x} = (-x-2)e^{4x}$

Equating Coefficients of e^{4x}

$10B + 2A = \cancel{-1} -2$

$A = \frac{-2-10B}{2} \Rightarrow A = \frac{-1-5B}{1}$

Equating Coefficients of xe^{4x}

$20A = -1 \Rightarrow A = -1/20$

$\therefore B = \frac{-19}{100}$

$\therefore y_{p2} = \left(\frac{-x}{20} - \frac{19}{100}\right)xe^{4x}$

$y = \frac{-x^2e^{4x}}{20} - \frac{19xe^{4x}}{100} + C_1e^{4x} + C_2e^{-6x} - \frac{2}{3}$

Q21) $y''' - 6y'' = 3 - \cos x$

$m^3 - 6m^2 = 0$

$\boxed{m=0}$ $\cancel{\boxed{m = 1/6}}$ $\boxed{m=6}$

$\boxed{m=0}$

$y_c = C_1e^{6x} + C_2x + C_3$

$y_{p1} = Ax^2$, $y_{p1}' = \cancel{0}\ 2Ax$, $y_{p1}'' = \cancel{0}\ 2A$, $y_{p1}''' = 0$

Substitute in eq above

$0 - 6(2A) = 3$

$-12A = 3$

$\boxed{A = -1/4}$ $\therefore y_{p1} = \frac{-x^2}{4}$

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$y_{p2} = -\cos x$

$y_{p2} = B\sin x + A\cos x$

$y_{p2}' = B\cos x - A\sin x$

$y_{p2}'' = -B\sin x - A\cos x$

$y_{p2}''' = -B\cos x + A\sin x$

Substitute in Original equation

$-B\cos x + A\sin x - 6(-B\sin x - A\cos x) = -\cos x$

$-B\cos x + A\sin x + 6B\sin x + 6A\cos x = -\cos x$

Equating Coefficients we get

$\boxed{A = \frac{-6}{37}}$ $\boxed{B = \frac{1}{37}}$

$$y_{p2} = \frac{\sin x}{37} - \frac{6\cos x}{37}$$

$$y = \frac{\sin x}{37} - \frac{6\cos x}{37} + C_1 e^{6x} - \frac{x^2}{4} + C_2 x + C_3$$

(Q22) $y''' - 2y'' - 4y' + 8y = 6xe^{2x}$

$m^3 - 2m^2 - 4m + 8 = 0$

$\boxed{m = -2}$ $\boxed{m = 2}$

$$y_c = (C_1 x + C_3)e^{2x} + C_2 e^{-2x}$$

$y_p = x^2 e^{2x}(Ax + B)$

$y_p = Ax^3 e^{2x} + Bx^2 e^{2x}$

$y_p' = 3Ax^2 e^{2x} + 2Ax^3 e^{2x} + 2Bxe^{2x} + 2Bx^2 e^{x}$

~~$y_p'' = 6Axe^{x} + Ax^3 e^{x} + 2Be^{x}$~~

$y_p'' = \cancel{6Axe^{2x} + 6Ax^2 e^{2x}} + 3Ax^2 e^{2x} + Ax^3 e^{2x} + \cancel{2Be^{2x}} + 2Bxe^{2x}$

$\cancel{+ 2Bxe^{2x} + Bx^2 e^{2x}}$

Date:

Substitute in Original equation we get

$y_p'' = (4Ax^3 + (4B+12A)x^2 + (8B+6A)x + 2B)e^{2x}$

$y_p''' = 8Ax^3 + (8B+36A)x^2 + (24B+36A)x + 12B + 6A)e^{2x}$

Substitute in Original eq

$24Axe^{2x} + (8B+6A)e^{2x} = 6xe^{2x}$

Equating Coefficients of xe^{2x}

$24A = 6 \Rightarrow \boxed{A = 1/4}$

Equating Coefficients of e^{2x}

$8B + 6A = 0 \Rightarrow \boxed{B = -3/16}$

$\therefore y_p = \left(\frac{x}{4} - \frac{3}{16}\right)x^2e^{2x}$

$y = \left(\frac{x}{4} - \frac{3}{16}\right)x^2e^{2x} + (C_1x + C_3)e^{2x} + C_2e^{2x}$

23) $y''' - 3y'' + 3y' - y = x - 4e^x$

$m^3 - 3m^2 + 3m - 1 = 0$

$\boxed{m = 1}$

All 3 roots are $m = 1$, so

$y_c = (C_2x^2 + C_1x + C)e^x$

$y_{p_1} = Ax + B$, $y_{p_1}' = A$, $y_{p_1}'' = 0$, $y_{p_1}''' = 0$

$-3A - Ax - B = x$

Comparing Coefficients, we get

$-A = 1 \Rightarrow \boxed{A = -1}$

$-3A - B = 0 \Rightarrow \boxed{B = -3}$

$\therefore y_{p_1} = -x - 3$

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$y_{P_2} = x^3(Ae^x)$, $y_{P_2}' = 3Ax^2e^x + Ax^3e^x$

$y_{P_2}'' = 6Axe^x + 3Ax^2e^x + 3Ax^2e^x + Ax^3e^x = 6Axe^x + 6Ax^2e^x + Ax^3e^x$

$y_{P_2}''' = 6Ae^x + 6Axe^x + 12Axe^x + 6Ax^2e^x + 3Ax^2e^x + Ax^3e^x$

$y_{P_2}''' = Ax^3e^x + 9Ax^2e^x + 18Axe^x + 6Ae^x$

Substitute in Original Equation, we get

$Ax^3e^x + 9Ax^2e^x + 18Axe^x + 6Ae^x - 3\{6Axe^x + 6Ax^2e^x + Ax^3e^x\} + 3\{3Ax^2e^x + Ax^3e^x\} - Ax^3e^x = -4e^x$

$\cancel{Ax^3e^x} + \cancel{9Ax^2e^x} + \cancel{18Axe^x} + 6Ae^x - \cancel{18Axe^x} - \cancel{18Ax^2e^x} - \cancel{3Ax^3e^x} + \cancel{9Ax^2e^x} + \cancel{3Ax^3e^x} - \cancel{Ax^3e^x} = -4e^x$

Comparing Coefficients

$6A\cancel{e^x} = -4\cancel{e^x} \Rightarrow \boxed{A = -2/3}$

$\therefore y_{P_2} = -\dfrac{2x^3e^x}{3}$

$\therefore y = e^x(C_2x^2 + C_1x + C) - \dfrac{2x^3e^x}{3} - x - 3$

24) $y''' - y'' - 4y' + 4y = 5 - e^x + e^{2x}$

Homogenous Solution

$m^3 - m^2 - 4m + 4 = 0$

$\boxed{m = 2}$, $\boxed{m = -2}$, $\boxed{m = 1}$

$y_c = C_1e^{2x} + C_2e^{-2x} + C_3e^x$

$y_{P_1} = A$, $y_{P_1}', y_{P_1}'', y_{P_1}''' = 0$

Substitute in Original Equation we get

$\boxed{y_{P_1} = 5/4}$

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$y_{p_2} = Axe^x \quad , \quad y_{p_2}' = Ae^x + Axe^x$

$y_{p_2}'' = Ae^x + Ae^x + Axe^x = 2Ae^x + Axe^x$

$y_{p_2}''' = 2Ae^x + Ae^x + Axe^x = 3Ae^x + Axe^x$

Substitute in Original equation, we get

$3Ae^x + A\cancel{x}e^x - 2Ae^x - Ax\cancel{e^x} - 4Ae^x - 4A\cancel{x}e^x + 4Axe^x = -e^x$

$-3Ae^x = -e^x$

Comparing coefficients, we get

$-3A = -1 \quad \Rightarrow \quad A = 1/3$

$\therefore \; y_{p_2} = \dfrac{xe^x}{3}$

Now for e^{2x}

$y_{p_3} = (Ae^{2x})x = Axe^{2x}$

$y_{p_3}' = Ae^{2x} + 2Axe^{2x}$

$y_{p_3}'' = 2Ae^{2x} + 4Axe^{2x} + 2Ae^{2x} = 4Ae^{2x} + 4Axe^{2x}$

$y_{p_3}''' = 8Ae^{2x} + 4Ae^{2x} + 8Axe^{2x} = 12Ae^{2x} + 8Axe^{2x}$

Substitute in Original equation, we get

$12Ae^{2x} + 8A\cancel{x}e^{2x} - 4Ae^{2x} - 4A\cancel{x}e^{2x} - 4Ae^{2x} - 8\cancel{A}xe^{2x} +$

$4A\cancel{x}e^{2x} = e^{2x}$

$4Ae^{2x} = e^{2x}$

Comparing Coefficients, we get

$4A = 1 \quad \Rightarrow \quad \boxed{A = 1/4}$

$\therefore \; y_{p_3} = \dfrac{xe^{2x}}{4}$

$\therefore \; y = y_c + y_{p_1} + y_{p_2} + y_{p_3}$

$y = C_1e^{2x} + C_2e^{-2x} + C_3e^x + \dfrac{xe^{2x}}{4} + \dfrac{xe^x}{3} + \dfrac{5}{4}$

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25) $y^{(4)} + 2y'' + y = (x-1)^2$

$m^4 + 2m^2 + 1 = 0$

$m^4 + m^2 + m^2 + 1 = 0$

$m^2(m^2+1) + 1(m^2+1) = 0$

$(m^2+1)(m^2+1) = 0 \Rightarrow m = 0 \pm i$

$\therefore \alpha = 0 \quad , \beta = 1$

$y_c = C_1\cos x + C_2 \sin x + C_3 x\cos x + C_4 x \sin x$

$y_c = (C_1 x + C_3)\cos x + (C_4 x + C_2)\sin x$

$y_p = Ax^2 + Bx + C \qquad \because (x-1)^2 = x^2 - 2x + 1$

$y_p' = 2Ax + B \quad , y_p'' = 2A$

$y_p''' = 0 \quad , y_p^{IV} = 0$

Substitute in Original equation

$Ax^2 + Bx + C + 4A = x^2 - 2x + 1$

Comparing Coefficients, we get

$\boxed{A = 1} \quad \boxed{B = -2} \quad \boxed{C = -3}$

$\therefore y_p = x^2 - 2x - 3$

$y = (C_1 x + C_3)\cos x + (C_4 x + C_2)\sin x + x^2 - 2x - 3$