Discrete Uniform Distribution:

A random variable 'X' has the discrete uniform distribution if it has a finite number of possible values $x_1, x_2, \ldots, x_n$ and

$$f(x_i) = f(X = x_i) = \frac{1}{n} \; ; \; i = 1, 2, \ldots, n.$$

If 'X' has discrete Uniform distribution on the consecutive integers $a, a+1, a+2, \ldots, b$, then

$$\mu = E(X) = \frac{b+a}{2}$$

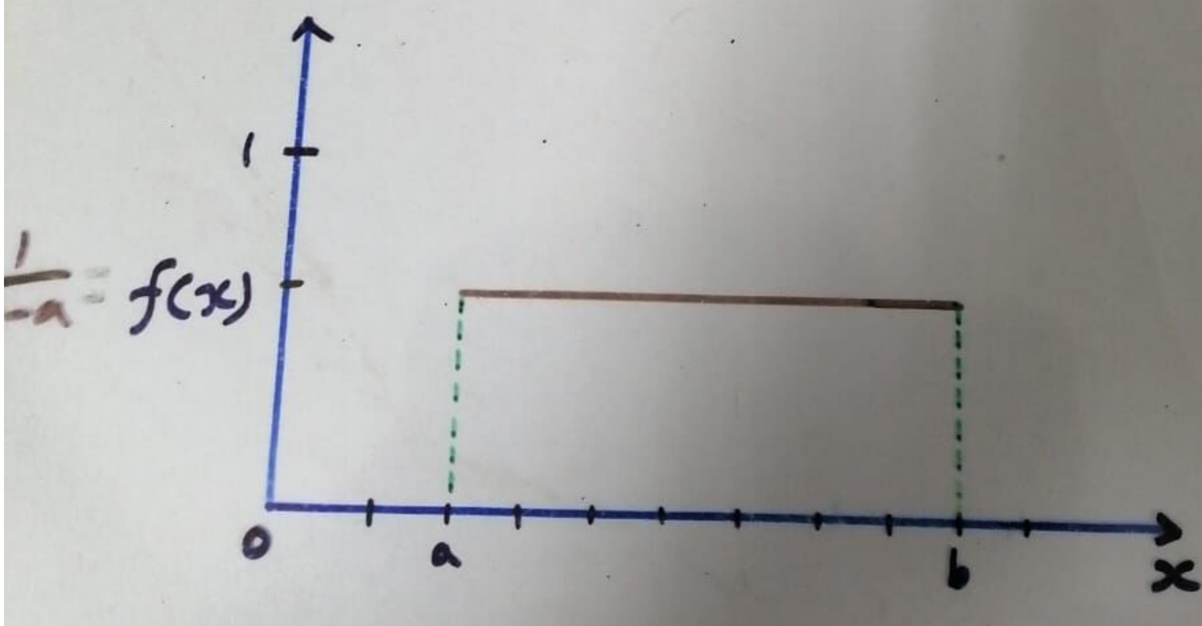
and

$$\sigma^2 = V(X) = \frac{(b-a+1)^2 - 1}{12}.$$

Continious Uniform Distribution:

A random variable 'X' has a Uniform distribution if its p.d.f is

$$f(x; a, b) = f(x) = \frac{1}{b-a} \; ; \quad a \leq x \leq b, \; a < b \in R$$



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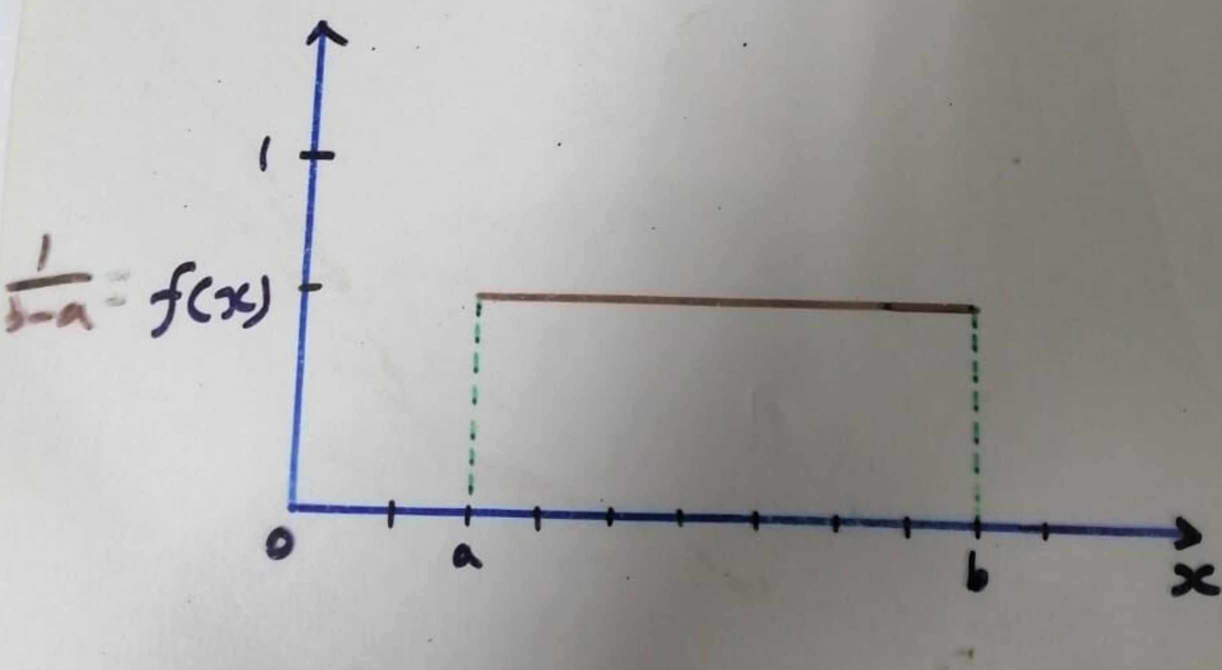


Different forms of Uniform Distribution:

1. $f(x) = \frac{1}{b-a}$; $a < x < b$ ②

2. $f(x) = \begin{cases} \frac{1}{\theta} & ; \quad 0 < x < \theta \\ 0 & ; \quad e.w \end{cases}$ Put $b=\theta,\ a=0$

3. $f(x) = \begin{cases} \frac{1}{2\theta} & ; \quad -\theta < x < \theta \\ 0 & ; \quad e.w \end{cases}$ Put $b=\theta;\ a=-\theta$

4. $f(x) = \begin{cases} 1 & ; \quad 0 < x < 1 \\ 0 & ; \quad e.w \end{cases}$ Put $b=1,\ a=0$

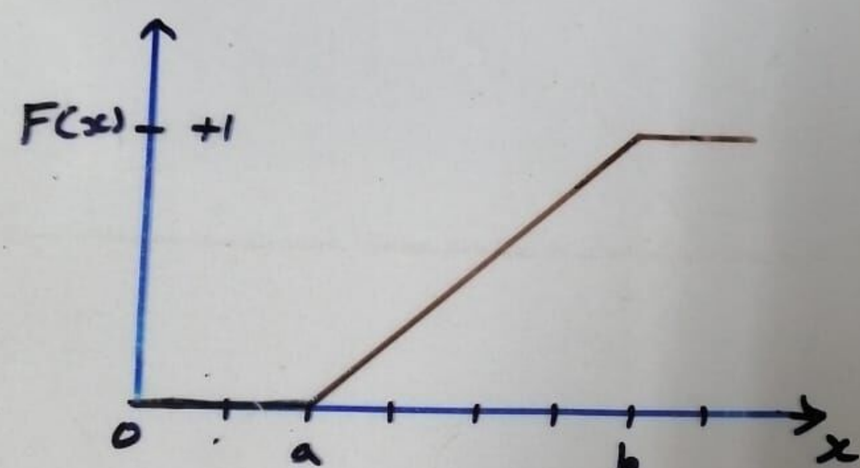
Cumulative Distribution Function: (cdf)

$$F(x) = \int_a^x f(x)\,dx = \frac{1}{b-a}\int_a^x dx = \frac{1}{b-a}[x]_a^x \quad (3)$$

$$= \frac{x-a}{b-a};$$

Thus

$$F(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \le x \le b \\ 1, & x \geq b \end{cases}$$



Moments: (about origin):

r^{th} moment about origin is

$$\mu_r' = E[X^r] = \frac{1}{b-a}\int_a^b x^r\,dx = \frac{1}{b-a}\left[\frac{x^{r+1}}{r+1}\right]_a^b$$

$$= \frac{[b^{r+1} - a^{r+1}]}{(b-a)(r+1)}$$

$r=1$

$$\mu_1' = \text{mean} = E[x] = \frac{b^2-a^2}{2(b-a)} = \frac{\cancel{(b-a)}(b+a)}{2\cancel{(b-a)}} = \frac{a+b}{2}$$

$r=2$

$$\mu_2' = E[x^2] = \frac{1}{(b-a)}\left[\frac{b^3-a^3}{3}\right] = \frac{(b-a)(b^2+ab+a^2)}{3(b-a)}$$

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$$\mu_2' = \frac{b^2+ab+a^2}{3}$$

$$\mu_2 = \sigma^2 = E[x^2] - \{E(x)\}^2$$

$$= \frac{b^2+ab+a^2}{3} - \frac{a^2+2ab+b^2}{4}$$

Second Mean Moment

$$\boxed{\mu_2 = \frac{(b-a)^2}{12} = Var(X) = \sigma^2}$$

2nd moment about Mean

Similarly

$$\mu_3' = \frac{a^3+a^2b+ab^2+b^3}{4}$$

$$\mu_4' = \frac{a^4+a^3b+a^2b^2+ab^3+b^4}{5}$$

skewness $\beta_1 = 0$

Kurtosis $\beta_2 = 9/5$

m.g.f $M_x(t) = \dfrac{e^{bt} - e^{at}}{(b-a)t} = E[e^{tx}]$

characteristic function $\phi(t) = \dfrac{e^{bit} - e^{ait}}{(b-a)it} = E[e^{itx}]$

In general m.g.f of X about any point a is define as

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$$M_X(t)_{(\text{about } a)} = E\left[e^{t(x-a)}\right]$$

$$= E\left[1 + t(x-a) + \frac{t^2}{2!}(x-a)^2 + \dots + \frac{t^r}{r!}(x-a)^r + \dots\right]$$

$$= 1 + t\mu_1' + \frac{t^2}{2!}\mu_2' + \dots + \frac{t^r}{r!}\mu_r' + \dots$$

where

$\mu_r' = E\left[(x-a)^r\right]$ is the r^{th} moment about the point $x = a$.

e.g.,

Uniform Dist (mgf):

$$M_X(t) = E[e^{tx}] = \frac{1}{b-a}\int_a^b e^{tx}\,dx = \frac{1}{t(b-a)}\left[e^{tx}\right]_a^b$$

$$\boxed{M_X(t) = \frac{e^{bt} - e^{at}}{t(b-a)}}$$

We know that

$$e^{\theta} = 1 + \theta + \frac{\theta^2}{2!} + \frac{\theta^3}{3!} + \dots + \frac{\theta^r}{r!} + \dots$$

$$\Rightarrow M_X(t) = \frac{1}{t(b-a)}\left[\left\{1 + tb + \frac{(tb)^2}{2!} + \frac{(tb)^3}{3!} + \frac{(tb)^4}{4!} + \dots\right\} - \left\{1 + ta + \frac{(ta)^2}{2!} + \frac{(ta)^3}{3!} + \frac{(ta)^4}{4!} + \dots\right\}\right]$$

$$= \frac{1}{t(b-a)}\left[t(b-a) + \frac{t^2(b^2-a^2)}{2!} + \frac{t^3(b^3-a^3)}{3!} + \dots\right.$$

$$= \frac{1}{\cancel{t(b-a)}} \cancel{t(b-a)} \left[1 + \frac{t(b+a)}{2!} + \frac{t^2(b^2+ab+a^2)}{3!} + \frac{t^3(b+a)(b^2+a^2)}{4!} + \cdots \right]$$ 6

$$= 1 + t\frac{(b+a)}{2!} + t^2\frac{(b^2+ab+a^2)}{3!} + \frac{t^3(b+a)(b^2+a^2)}{4!} + \cdots$$

which is the m.g.f of the continuous uniform dist.

Now compare the

coefficient of $\frac{t}{1!}$ $\quad \mu_1' = \frac{b+a}{2}$

coeff of $\frac{t^2}{2!}$ $\quad \mu_2' = \frac{b^2+ab+a^2}{3}$

coeff of $\frac{t^3}{3!}$ $\quad \mu_3' = \frac{(b+a)\cdot(b^2+a^2)}{4}$

coeff of $\frac{t^4}{4!}$ $\quad \mu_4' =$ ______

Median :

$$F(M) = \int_a^M f(x)\,dx = \frac{1}{2}$$

$$= \frac{1}{b-a}\int_a^M dx = \frac{1}{b-a}[x]_a^M = \frac{M-a}{b-a} = \frac{1}{2}$$

$$\boxed{M = \frac{b+a}{2} = \mu} \Rightarrow \text{Mean} = \text{Median}$$

From the p.d.f it is evident that it is a symmetric distribution i.e

Mean = Median = Mode.

Problem:

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In a certain experiments, the error made in determining the density of a substance is a random variable having a uniform distribution with $a = -0.015$ and $b = 0.015$. Find the probabilities that such an error will

a) be between -0.002 and 0.003

b) exceed 0.005 in absolute value.

Sol:

$$X \sim U\left(\frac{a+b}{2}, \frac{(b-a)^2}{12}\right)$$

$\therefore$ p.d.f : $f(x) = \frac{1}{b-a} = \frac{1}{0.015+0.015} = \frac{1}{0.030}$

a) $P(-0.002 \leq X \leq 0.003) = \frac{1}{0.030}\int_{-0.002}^{0.003} dx$

$$= \frac{1}{0.030}\left[x\right]_{-0.002}^{0.003}$$

$$= \frac{1}{0.030}\left[0.003 + 0.002\right]$$

$$= \frac{0.0050}{0.030} = \frac{5}{30} = \frac{1}{6} = 0.16$$

b) $P(|X| \geq 0.005) = P(-0.005 \leq X \leq 0.005)$

$$= \frac{1}{0.030}\int_{-0.005}^{0.005} dx = \frac{1}{0.030}\left[x\right]_{-0.005}^{0.005}$$

$$= \frac{0.010}{0.030} = \frac{1}{3} = 0.333\ldots$$

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Problem:

Suppose the research department of a steel manufacturer believes that one of the company's rolling machines is producing sheets ~~of~~ of varying thickness. The thickness is a uniform random variable with values between 150 and 200 millimeters. Any sheet less than 160 millimeters thick must be scrapped since they are unacceptable to buyers.

a) Calculate the mean and S.D of Y, the thickness of the sheets produced by this machine. Then, graph the probability distribution, and show the mean on the horizontal axis. Als show 1 and 2 SD intervals arround the mean.

b) Calculate the fraction of steel sheets produced by this machine that have to be scrapped.

Sol

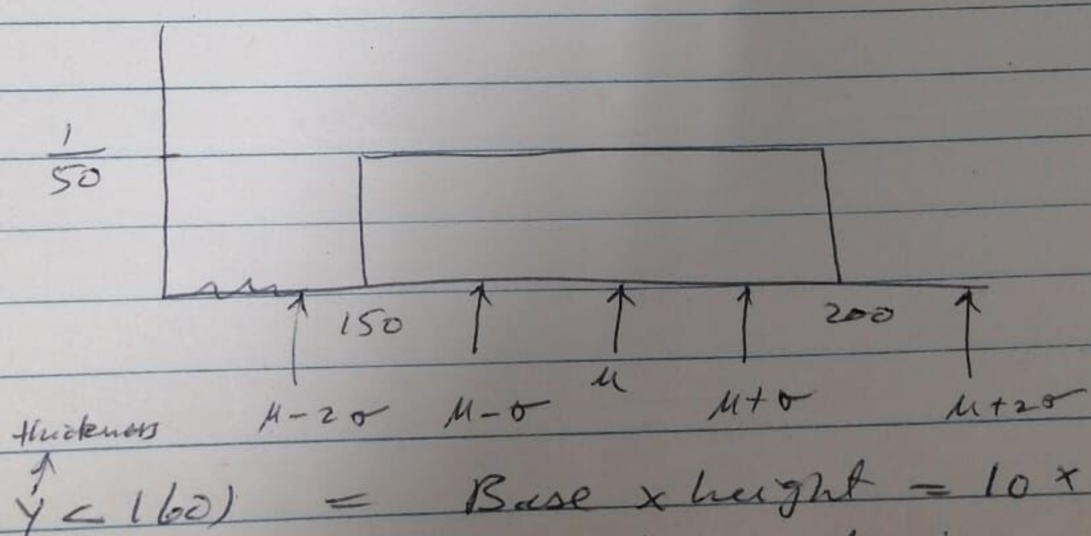
$$\text{Mean} = \frac{a+b}{2} = \frac{150+200}{2} = 175 \text{ millimeters}$$

and

$$\sigma = \frac{b-a}{\sqrt{12}} = \frac{200-150}{\sqrt{12}} = 14.43 \text{ millimeters}$$

P.d.f

$$f(Y) = \frac{1}{b-a} = \frac{1}{200-150} = \frac{1}{50}$$



b) $P(Y < 160) = \text{Base} \times \text{height} = 10 \times \frac{1}{50} = \frac{1}{5}$

20% of all the sheets made by this machine must be scrapped.