Assignment 1

Q1. $x = [5, -3, -1, 2]^T$

1. $\|x\|_2^2 = 5^2 + (-3)^2 + (-1)^2 + 2^2$
$= 25 + 9 + 1 + 4$
$= 39$

2. $\|x\|_1 = |5| + |-3| + |-1| + |2|$
$= 5 + 3 + 1 + 2$
$= 11$

3. $a = [4, -2, 6, -1]^T$

$a^T x = [4, -2, 6, -1] \begin{bmatrix} 5 \\ -3 \\ -1 \\ 2 \end{bmatrix}$

$= (5 \times 4) + (-2 \times -3) + (6 \times -1) + (-1 \times 2)$
$= 20 + 6 - 6 - 2$
$= 18$

Q2.

$$A = \begin{bmatrix} 6 & 1 & -2 \\ -5 & 7 & 9 \end{bmatrix} \qquad b = \begin{bmatrix} -4 \\ 5 \\ 2 \end{bmatrix}$$

1.

$$Ab = \begin{bmatrix} 6 & 1 & -2 \\ -5 & 7 & 9 \end{bmatrix} \begin{bmatrix} -4 \\ 5 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} (6\times-4) + (1\times5) + (-2\times2) \\ (-5\times-4) + (7\times5) + (9\times2) \end{bmatrix}$$

$$= \begin{bmatrix} -24+5-4 \\ 20+35+18 \end{bmatrix}$$

$$= \begin{bmatrix} -23 \\ 73 \end{bmatrix}$$

2.

$$AA^T = \begin{bmatrix} 6 & 1 & -2 \\ -5 & 7 & 9 \end{bmatrix} \begin{bmatrix} 6 & -5 \\ 1 & 7 \\ -2 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} (6\times6)+(1\times1)+(-2\times-2) & (6\times-5)+(1\times7)+(-2\times9) \\ (-5\times6)+(7\times1)+(9\times-2) & (-5\times-5)+(7\times7)+(9\times9) \end{bmatrix}$$

$$= \begin{bmatrix} 36+1+4 & -30+7-18 \\ -30+7-18 & 25+49+81 \end{bmatrix}$$

$$= \begin{bmatrix} 41 & -41 \\ -41 & 155 \end{bmatrix}$$

Q3.

$$x = [x_1, x_2, x_3]$$

$$y = \frac{x_1^2}{2} + \log_e x_2 - \frac{x_1}{x_3}$$

$$\frac{dy}{dx} = \left[\frac{dy}{dx_1}, \frac{dy}{dx_2}, \frac{dy}{dx_3} \right]$$

$$\frac{dy}{dx_1} = \frac{d}{dx_1}\left(\frac{x_1^2}{2} + \log_e x_2 - \frac{x_1}{x_3} \right)$$

$$= x_1 + 0 - \frac{1}{x_3}$$

At $x = [9, 1, \frac{1}{2}]$, $\frac{dy}{dx_1} = 9 + 0 - \frac{1}{1/2}$

$$= 9 - 2$$
$$= 7$$

$$\frac{dy}{dx_2} = \frac{d}{dx_2}\left(\frac{x_1^2}{2} + \log_e x_2 - \frac{x_1}{x_3} \right)$$

$$= \frac{1}{x_2}$$

At $x = [9, 1, \frac{1}{2}]$, $\frac{dy}{dx_2} = \frac{1}{1} = 1$

$$\frac{dy}{dx_3} = \frac{d}{dx_3}\left(\frac{x_1^2}{2} + \log_e x_2 + \frac{x_1}{x_3}\right)$$

$$= \frac{x_1}{x_3^2}$$

At $x = [9, 1, \frac{1}{2}]$, $\frac{dy}{dx_3} = \frac{9}{(1/2)^2} = 9 \times 4 = 36$

$\therefore$ At $x = [9, 1, \frac{1}{2}]$, $\frac{dy}{dx} = [7, 1, 36]$

Q4.

$$f(w) = \|Xw - y\|_2^2 + \lambda\|w\|_2^2$$

$$\frac{\partial}{\partial w} f(w) = \frac{\partial}{\partial w}\left(\|Xw - y\|_2^2 + \lambda\|w\|_2^2\right)$$

$$= \frac{\partial}{\partial w}\left(\|Xw - y\|_2^2\right) + \frac{\partial}{\partial w}\left(\lambda\|w\|_2^2\right)$$

$$= 2(Xw - y)\frac{\partial}{\partial w}(Xw - y) + 2\lambda w \frac{\partial}{\partial w}(w)$$

$$= 2X[Xw - y] + 2\lambda w$$

$$= 2[X(Xw - y) + \lambda w]$$