

Mathematical Modeling

Home Work # 2

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$$\ddot{u} + \omega^2 u - \mu \dot{u} + \alpha \dot{u}^3 = 0$$

$$x_1 = u$$

$$x_2 = \dot{u}$$

$$\boxed{\dot{x}_1 = x_2} \quad (1)$$

$$\dot{x}_2 + \omega^2 x_1 - \mu x_2 + \alpha x_2^3 = 0$$
$$\boxed{\dot{x}_2 = \mu x_2 - \alpha x_2^3 - \omega^2 x_1} \quad (2)$$

Graphs

Let $\omega = 0.4$,

$\alpha = 0.2$

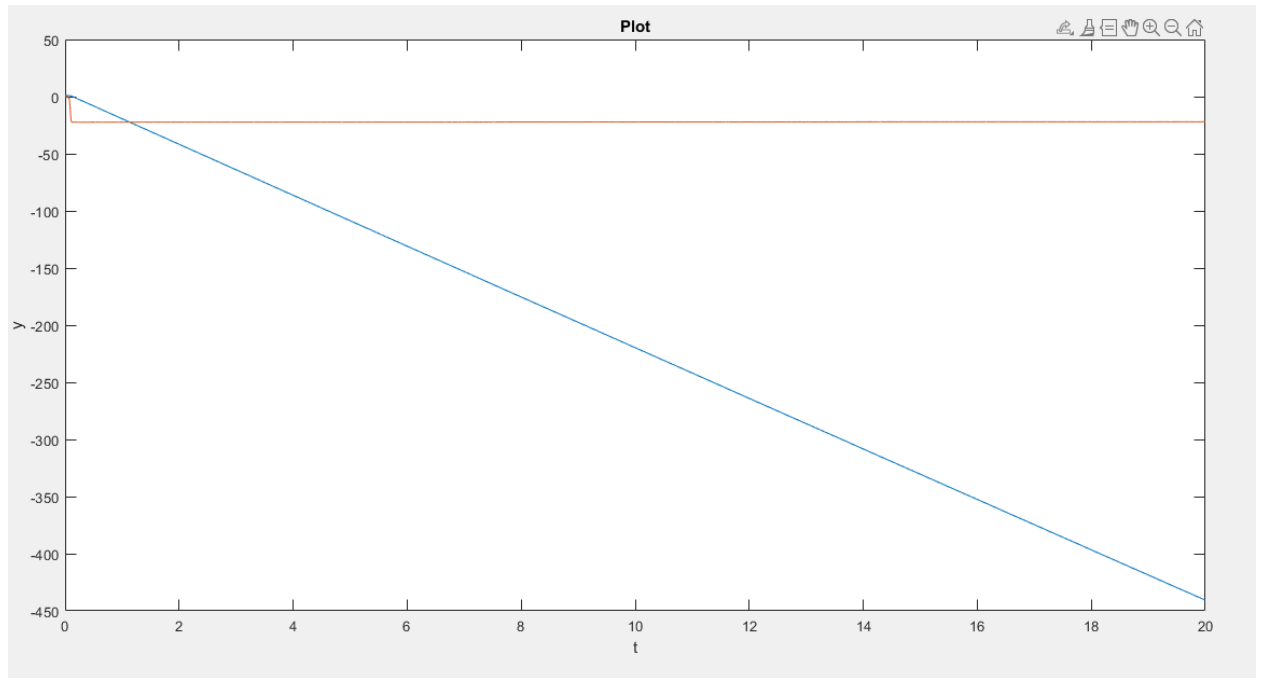
$\mu = 100$

Time Series Plot

Code:

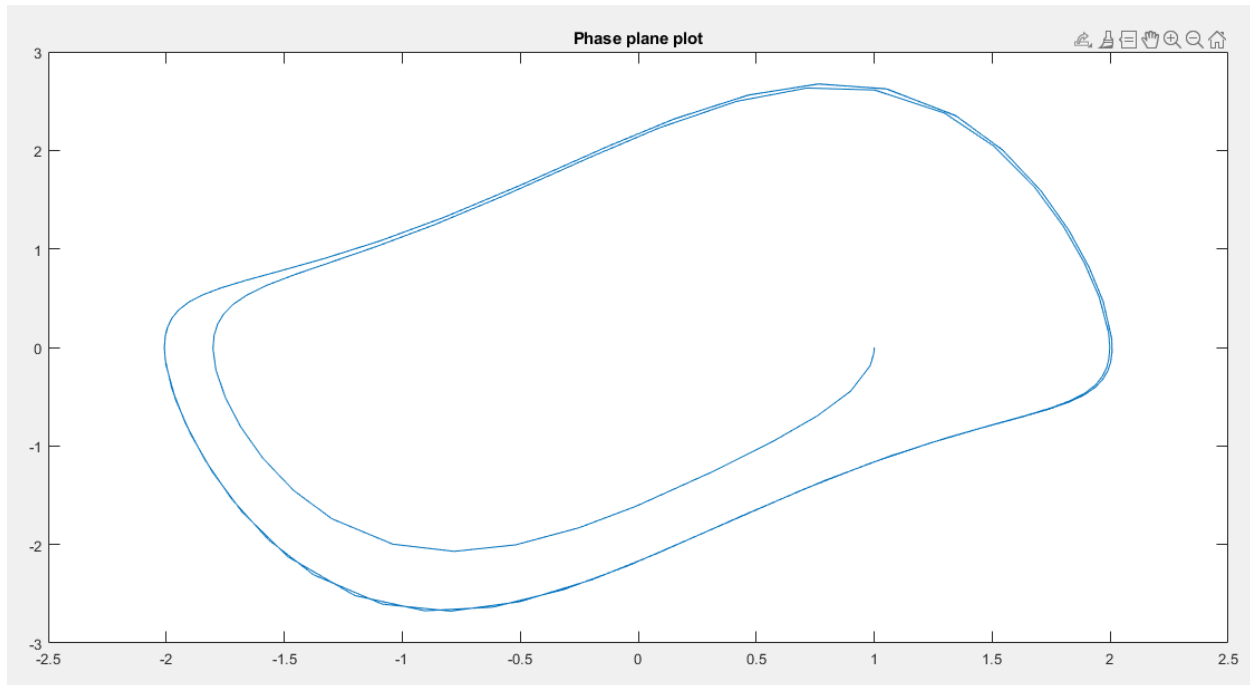
```
[t,y] = ode45(@na,[0 20],[1; 0]);  
plot(t,y(:,1),t,y(:,2))  
title('Plot');  
xlabel('t');  
ylabel('y');
```

Initial conditions 1 and 0



Phase Portrait Plots

Initial conditions 1 and 0



Explanation:

The model is a non-conservative oscillator with non-linear damping. It evolves in time according to the second-order differential equation:

where x is the position coordinate—which is a function of the time t , and μ is a scalar parameter indicating the nonlinearity and the strength of the damping.

Real Life Examples:

- This equation is governed in vocal fold oscillators
- In Seismology it can be used to model two plates in geological fault