

Mathematical Modeling (Home Work # 2)

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$$u'' + w^2 u - \mu u' + \alpha u'^3 = 0$$

Solution

Let

$$x_1 = u$$

$$x_2 = u'$$

$$x_1' = x_2 \dots (1)$$

$$x_2' + w^2 x_1 - \mu x_2 + \alpha x_2^3 = 0$$

$$x_2' + w^2 x_1 - \mu x_2 + \alpha x_2^3 = 0 \dots (2)$$

Graphs

Let $w = 0.1$,

$\alpha = 0.5$

$\mu = 1$

Time Series Plot

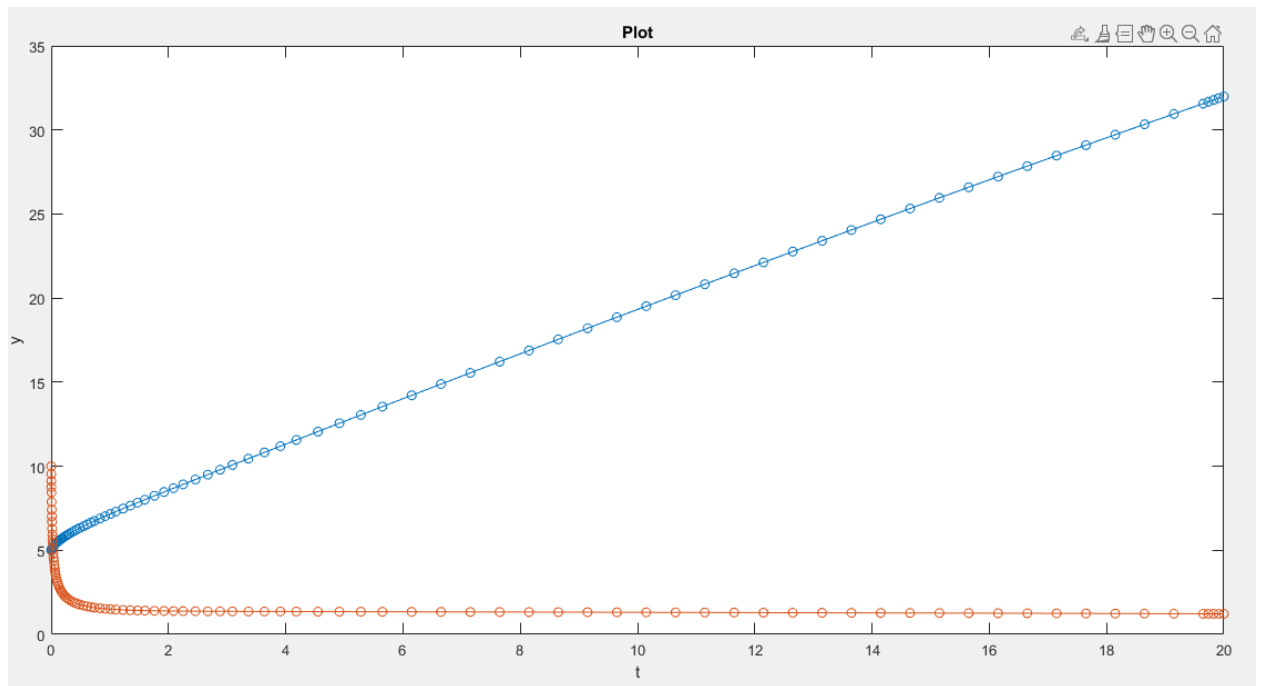
Code:

```
[t,y] = ode45(@eqn,[0 20],[5; 10]);  
plot(t,y(:,1),'-o',t,y(:,2),'-o')  
title('Plot');  
xlabel('t');  
ylabel('y');
```

```
function dydt = eqn(t,y)
```

```
w = 0.1;  
mu = 1;  
alpha = 0.5;  
dydt = [y(2); mu*y(2) - alpha*y(2)^3 - w^2*y(1)];
```

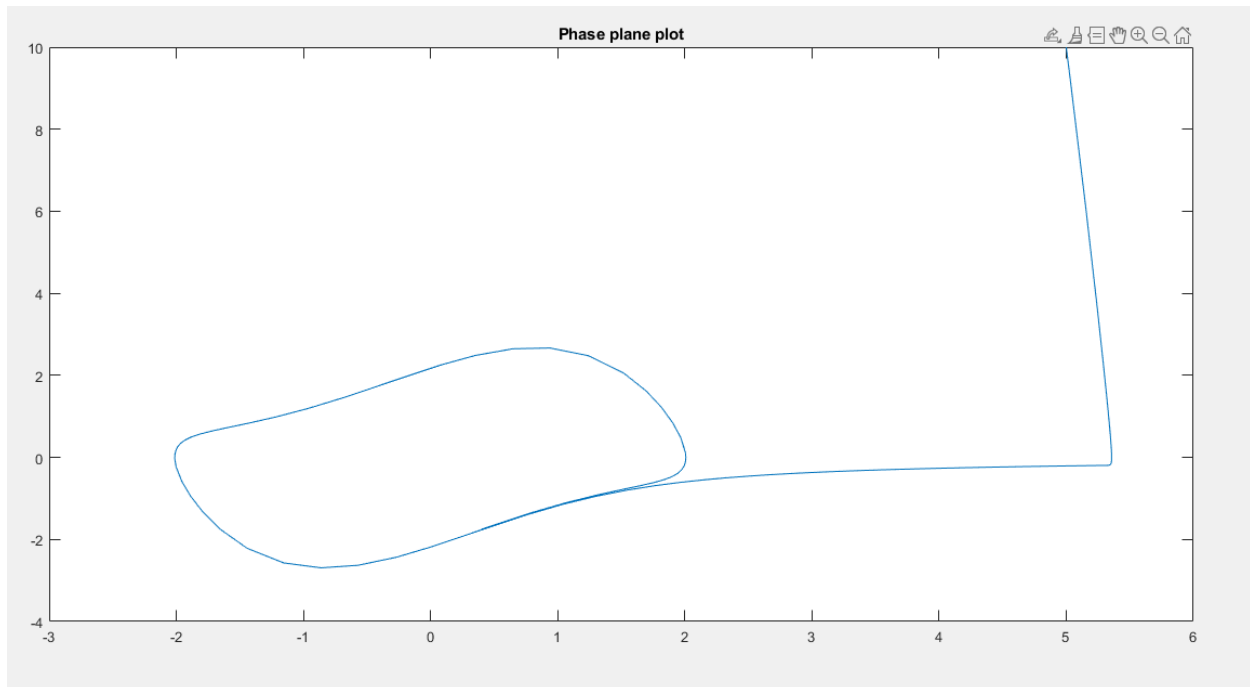
Initial conditions 5 and 10



Phase Portrait Plots

Code:

Initial conditions 5 and 10



DISCUSSION

The equation models a non-conservative system in which energy is added to and subtracted from the system, resulting in a periodic motion called a limit cycle. The parameter μ is a positive scalar indicating the nonlinearity and the strength of the damping.

Real Life Examples:

- Model for action potential of Neurons
- An RLC Circuit involving a triode

