Modelling a Parameterized ODE for predicting USD to PKR Exchange Rate

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***Abstract*—The paper focuses on developing a parameterized ordinary differential equation that can effectively predict the exchange rate of USD to PKR. Now-a-days neural networks are a common approach for predicting and forecasting exchange rates. But we would stick to the general development technique of parameterized Ordinary Differential Equation that would help in forecasting our rates.**

***Keywords*—component, formatting, style, styling, insert (*key words*)**

I. Introduction

Foreign exchange rate is one of the topics that interests most of the investors and board managers of banks. It is vital in developing an effective plan for the years to come. The more accurate the prediction is the more profit investors and banks can make of it. So, the development of an effective model that can accurately predict the prices in the near future are vital in most cases.

Many different techniques have been developed that can help in predicting the future foreign exchange prices. For example, with more and more data modelling being shift to Artificial intelligence, many neural networks model are deployed that can effectively predict the exchange rates in the upcoming days [1]. Similarly, there are many other ways through which foreign exchange rates are being predicted such as ARIMA Model [2].

However, the approached developed in this paper mainly depends on the logistic growth model. However, the technique used is specifically a dynamic logistic model.

II. Literature Review

Time series forecasting model are based on analysis of historical data. These methods support the assumption that past patterns in data can be used to forecast future data points. There are many researches about forecasting foreign exchange rate was carried out in the world. Until now, there are a lot of forecasting model, each model possesses private strong-point as well as private weak-point, models can be listed as using neutral networks, Arima model, Least Squared model (Hongxing et al., 2007) or Purchasing Power Parity model and Balassa-Samuelson channel (David et al., 2010). (Meyler, 1998) used Arima model for forecasting inflation in Irish, (Mondal, 2014) used Arima model for forecasting stock price. Arima is also used for predicting stock price in the research of (Jarrett, 2011), (Adebiyi, 2014), (Isenah, 2014). It is also used for forecasting the price of gold (Guha & Bandyopadhyay, 2016). With object is foreign exchange rate, Arima is also a good solution for prediction, some authors used it for forecasting such as (Appiah & Adetunde, 2011), (Nwankwo, 2014), (Tlegenova, 2014). After examining the results of these studies above, we decide to choose Arima model as the main methodology for forecasting foreign exchange rate between Vietnam Dong and US Dollar. There are two issues that we used in our research: Arima model: Arima model is one method for forecasting time series, it is assumed that past value of the series plus previous error terms contain information for the purpose of forecasting. The main advantage of Arima forecasting is that it requires data on time series in question only. However, Arima model are essentially backward looking, they are generally poor at predicting turning points, unless the turning point represents a return to a long-run equilibrium (Meyler et al., 1998). Exchange rate forecasting: Exchange rate forecasting means estimating the rate which will be any of future time.

III. Methodology

Before starting to model our Differential equation, we would first take a look at the exchange rates of USD to PKR over the past one year. The data has been gathered and a graph with a simple trendline is shown.

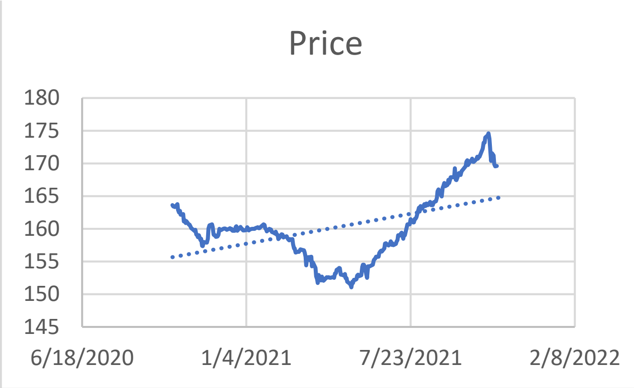
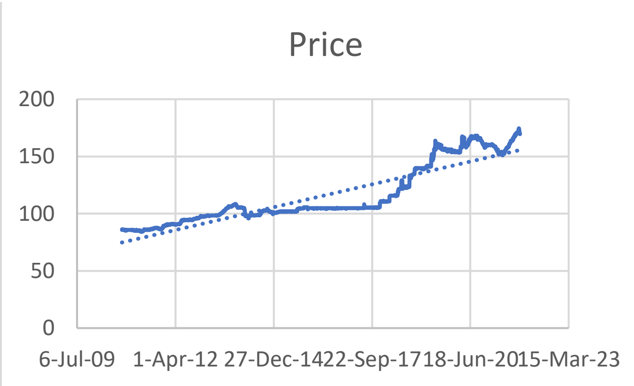


Figure 1: USD to PKR from (10/2020 – 11/2021)

We can see the trend line that the price gradually increases so we can predict that the price would rise in the coming future as well however, the model is not of any use. We would consider another trend in which we extend the data to over 5 years (Figure 2).

Now we can see the trend line is more steep towards the rise and there are many surges in the way but we would discuss about that in a later time.



To begin developing our model we would take use of the logistic growth model and use dynamic integration to build our model.

$$\frac{dN}{dt} = rN(1 - \frac{N}{K})$$

In the above-mentioned logistic growth model [3-5] equation. The variable N represents the Population size. The variable r represents the growth rate and the variable K represents the capacity which is how much population can sustain in a given environment.

We would consider that our foreign exchange rate is just a function of time without any other parameter involved in it. Now, let us begin developing our equation

Now, let in our foreign exchange case we would define E as the exchange rate. $\alpha\ and\ \beta\ are\ paramenters$ that would be tuned to fit the model. Our dynamic equation as a logistic growth model would look as follows:

$$\frac{dE}{dt} = \beta(t)E(t)(1 - \frac{E(t)}{-\beta(t)/\alpha(t)}$$

However, we would describe the relationship between the foreign exchange rate, its derivative and acceleration to define our model. Our model is defined as follows.

Dynamic Transformed Logistic Model

$$\boldsymbol{\frac{d^2E(t)}{dt^2} = \alpha \times \left(\frac{dE(t)}{dt}\right)^2 + \beta \times \left(\frac{dE(t)}{dt}\right), (1)}$$

Where E(t) is the exchange rate at time t.

Assuming that the coefficients $\alpha\ and\ \beta$ are constant during a very short time, we consider the parameterized differential equation with given values of two points at time $to\ and\ t1$:

$$\boldsymbol{\frac{d^2E(t)}{dt^2} = \alpha \times \left(\frac{dE(t)}{dt}\right)^2 + \beta \times \left(\frac{dE(t)}{dt}\right),}$$

$$\boldsymbol{S(to) = So}$$

$$\boldsymbol{S(t1) = S1}$$

$$\boldsymbol{to < t1}$$

And it can be solved as follows. Let,

$$v(t) = \frac{dE(t)}{dt}$$

And

$$h = |v(to) + \frac{\frac{\beta}{\alpha}}{v(to)}|$$

then the solution is,

$$E(t) = E(to) - \frac{1}{\alpha}\ln\left|\frac{e^{(\beta(t-to)} - h}{1-h}\right|,$$
$$if\left(v(t) + \frac{\beta}{\alpha}\right) \times v(t) > 0, v(t) > v(to), E(t) > E(to);$$

$$E(t) = E(to) - \frac{1}{\alpha}\frac{|\beta|}{\beta}\ln\left|\frac{e^{(\beta(t-to)} - h}{1-h}\right|,$$
$$if\left(v(t) + \frac{\beta}{\alpha}\right) \times v(t) > 0, v(t) > v(to), E(t) < E(to);$$

$$E(t) = E(to) - \frac{1}{\alpha}\ln\left|\frac{e^{(\beta(t-to)} + h}{1+h}\right|,$$
$$if\left(v(t) + \frac{\beta}{\alpha}\right) \times v(t) < 0, v(t) > v(to), E(t) > E(to);$$

$$E(t) = E(to) - \frac{1}{\alpha}\frac{|\beta|}{\beta}\ln\left|\frac{e^{(\beta(t-to)} + h}{1+h}\right|,$$
$$if\left(v(t) + \frac{\beta}{\alpha}\right) \times v(t) < 0, v(t) > v(to), E(t) < E(to);$$

$$E(t) = E(to) + \frac{1}{\alpha}\frac{|\beta|}{\beta}\ln\left|\frac{e^{-(\beta(t-to)} - h}{1-h}\right|,$$
$$if\left(v(t) + \frac{\beta}{\alpha}\right) \times v(t) > 0, v(t) < v(to), E(t) > E(to);$$

$$E(t) = E(to) - \frac{1}{\alpha}\ln\left|\frac{e^{-(\beta(t-to)} + h}{1+h}\right|,$$
$$if\left(v(t) + \frac{\beta}{\alpha}\right) \times v(t) > 0, v(t) < v(to), E(t) < E(to);$$

$$E(t) = E(to) + \frac{1}{\alpha}\frac{|\beta|}{\beta}\ln\left|\frac{e^{-(\beta(t-to)} + h}{1+h}\right|,$$
$$if\left(v(t) + \frac{\beta}{\alpha}\right) \times v(t) < 0, v(t) < v(to), E(t) > E(to);$$

$$E(t) = E(to) + |\frac{1}{\alpha}|\ln\left|\frac{e^{-(\beta(t-to)} + h}{1+h}\right|,$$
$$if\left(v(t) + \frac{\beta}{\alpha}\right) \times v(t) < 0, v(t) < v(to), E(t) < E(to)$$

Our model depicts the relationship between velocity and acceleration of the foreign exchange rate.

$$\frac{dE}{dt}\ (velocity)$$

$$\frac{dE^2}{dt^2}\ (acceleration)$$

The relationship is based on the logistic growth model. We can see that when α is negative and β is positive, the velocity moves in the way of as depicted by the logistic growth model. That is the velocity reverses its direction to an implied equilibrium velocity so that the foreign exchange rate goes up or down with an approximately constant speed. While in all of the other cases of combinations of $\alpha\ and\ \beta$, the velocity diverges and hence the foreign exchange rate fluctuates more dynamically.

IV. Numerical Results and Discussion

The first step is that we gathered data over the past 15 years and then plot the data to show our trend the code and the result are given in the table below

Code
import numpy as np import matplotlib.pyplot as plt from matplotlib.collections import LineCollection import pandas as pd from sklearn.metrics import accuracy_score, confusion_matrix, classification_report n = 1896 x = np.arange(n)+1 df = pd.read_csv('data2.csv') df.head() y = df['Price'] n = y.size x = np.arange(n)+ yn = [] for i in range(y.size-1,-1,-1):   yn.append(y[i]) plt.plot(x,yn)

Table 1: Code for Plotting the graph

The graph is plotted in the figure as shown in figure 3.

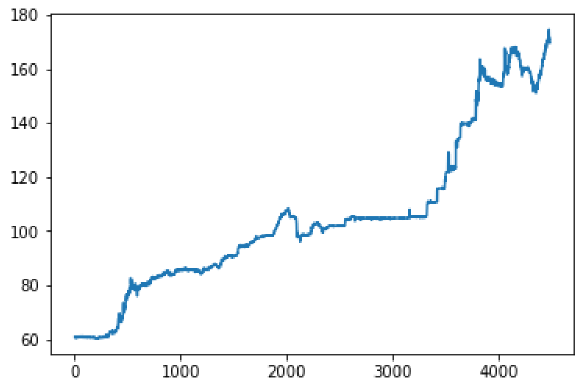


Figure 3: Graph of USD-PKR over the last 15 years from October 2006 – October 2021

The next step was to train our model over this trend which is shown below

Code
ir = IsotonicRegression(out_of_bounds="clip") y_ = ir.fit_transform(x, yn) plt.plot(x,yn,x,y_)

Table 2: Code for plotting with our model

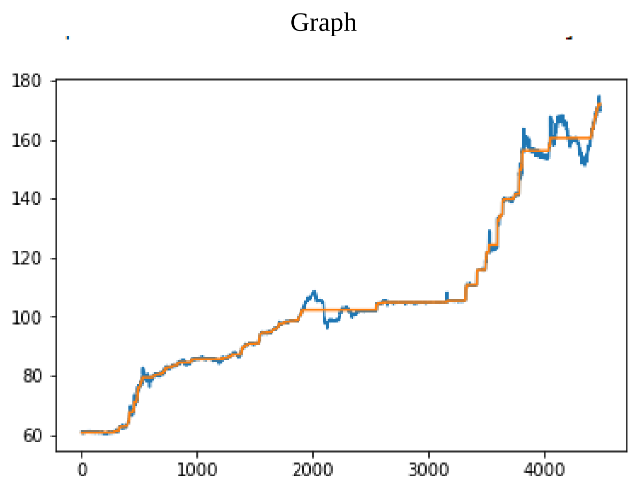


Figure 4: Comparison between Original data and data obtained through Isotonic model

Now, as we can see in the figure that the data predicted by the model is very close to the original data so we could use it to effectively make prediction for the future as well.

Now, we would show some of the metrics obtained through our model.

Root_Mean_Squared_Error	Mean_Absolute_Error
1.7826	0.8384

Table 3: Metrics obtained for the model

As seen by the model we have obtained a good metrics for the model, therefore it would be good to make predictions with it.

V. Recommendations to the Company

First, we would look at the result of the past 20 years of the USD-PKR exchange rate and that we would make a prediction.

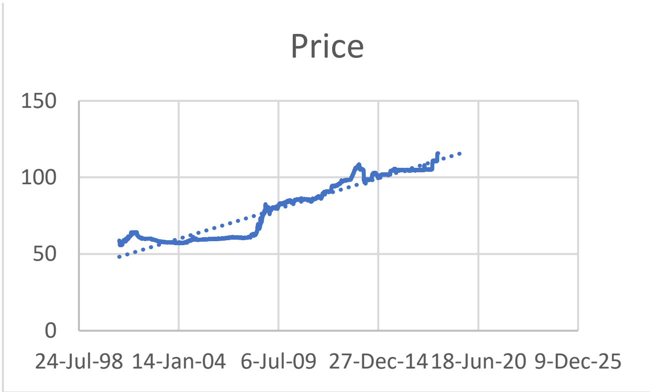


Figure 5: USD-PKR 2000-2021

First, we will make a prediction for October 30, 2022 using our model.

```
: ir.predict([x.size+365])
: array([171.923125])
```

Figure 6: Prediction for October 30, 2022

As seen above our model predicted the rate to be PKR 171.92. Therefore, the company should make decisions in the future considering all its capabilities.

Discussion Regarding Price Surges

If we look at the trend carefully, we can see that there are certain spikes in the chart where the price of dollar has been increasing drastically. These are basically the time during an economical outbreak or political crisis. Therefore, we can see that the price in those certain areas changes drastically.

However, to cope this kind of issues in the model we need to make it artificially intelligent. It should include various different parameters. Such as time of the year, time for next election and various different factors as well.

So, final recommendation to the company would be to do finance while keeping all the factors of the economy in the mind and as our prediction the price would be around **RS. 179.5**. in the end of this fiscal year.

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