Math W4 A1

Date: 16 Sep 24'

Section A:-

Q1. $g(f(4)) = 2\left(\frac{1}{2}x - 3\right) + 5$
$= x - 6 + 5$
$= x - 1 = 4 - 1 = 3 \Rightarrow$ C

Q2. D
Q3. B
Q4. A
Q5. D
Q6. C
Q7. $f(g(\frac{1}{2})) = 4(\frac{1}{x}) - (\frac{1}{x})^2$
$= 4(2) - (2)^2$
$= 8 - 4 = 4 \Rightarrow$ D

Q8. $f(x) = |x-3| + 2 \Rightarrow$ range...

$|0-3| + 2$
$\cancel{\Rightarrow 3 + 2 = 5}$
$x - 3 = 0$
$x = 3$

2 , $\Rightarrow$ B

$9 - x^2 \geq 0$
$9 \geq x^2$
$3 \geq \cancel{x}$
$-3 \leq x \leq +3$

Q9. B
Q10. B
Q11. $\sqrt{9-x^2} \Rightarrow$ domain $\Rightarrow$ $\cancel{(-\infty, 3] \cup [3, \infty)}$ $[-3, 3]$

range $\Rightarrow$ $\cancel{[0, \infty)}$ $[0, 3]$

$\sqrt{9-x^2}$ max
when $x^2 = 0 \Rightarrow x = 0 = 3$
& min when $x^2 = max = 3^2 = 9 \Rightarrow 0$

A

$\cancel{(-\infty, -3] \cup [-3, 3] \cup [3, \infty)}$

Date:

Q12. D

Q13. C

Q14. $y = \frac{3}{\sqrt{x-4}}$ domain $\Rightarrow \{x \mid x > 4\} \Rightarrow (4, \infty)$

range $\Rightarrow (0, \infty)$

A

$\frac{3}{\sqrt{x-4}}$ is max when $\sqrt{x-4}$ is min,

when $x \to 4^+$ but NOT $= 4$

↳ is min when $\sqrt{x-4}$ is max,

when $x \to \infty$

$x \to 4^+ \Rightarrow \infty$ $\Rightarrow (0, \infty)$

$x \to \infty \Rightarrow 0$

Q15. $f(g(x)) = 2(x+5)^2 - 3(x+5) + 1$

$= 2(x^2 + 10x + 25) - 3x - 15 + 1$

$= 2x^2 + 20x + 50 - 3x - 15 + 1$

$= 2x^2 + 17x + 36 \Rightarrow$ A

Q16. $f(x) = -\frac{3}{\sqrt{2-x}} \Rightarrow \sqrt{2-x} > 0$.

$2 - x > 0$ B

$2 > x$

$x < 2$

Q17. $f(a+1) = 4x^2 - x + 1$

$= 4(a+1)^2 - (a+1) + 1$

$= 4(a^2 + 2a + 1) - a - 1 + 1$

$= 4a^2 + 8a + 4 - a$

$= 4a^2 + 7a + 4 \Rightarrow$ D

Q18. Same output on diff inputs:-

C $(1,3) \dots (3,3)$

Q19. $f(g(-2)) = 2(3x-2)^2+1$

$= 2(3(-2)-2)^2+1$

$= 2(-6-2)^2+1$

$= 2(-8)^2+1$

$= 2(64)+1$

$= 128+1$

$= 129 \Rightarrow$ D

Q20. To be a function: Diff outputs on same inputs.

1-1: Only if unique inputs to unique outputs.

↓

$f(x) = x^2$ graph...

m is not 1-1

But is a function

when not 1-1, inverse is not a function ... ∴ m is func, while m^{-1} is not,

$\Rightarrow$ B.

Q21. D $[2,7]$

Q22. $y = \frac{x}{x^2-9}$ undefined when $x^2-9=0$

$x^2=9$

$x=\pm 3 \Rightarrow$ B

Q23. $f(n)=n^2-1 \Rightarrow$ A

Q24. $f(2)=13\dots \Rightarrow$ D

Q25. $-1 \Rightarrow$ A

Section B:-

Q1.a) i) $\frac{2}{x} < \frac{3}{x-2} \Rightarrow x \neq 0, x \neq 2$

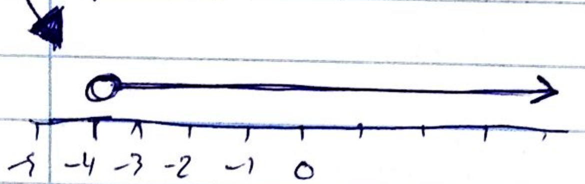
$\Rightarrow 2(x-2) < 3x$

$\Rightarrow 2x-4 < 3x$

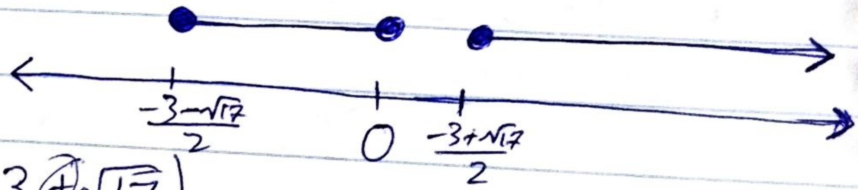
$-4 < x$

$x > -4$

Sketch:-



ii) $x^3+3x^2-2x \geq 0$

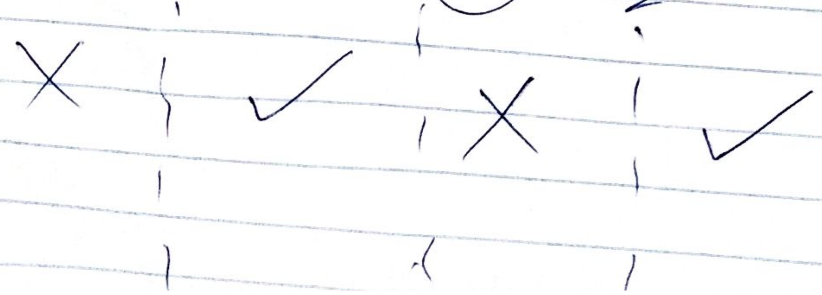


$x(x^2+3x-2) \geq 0$

$x\left(x-\frac{3-\sqrt{17}}{2}\right)\left(x-\frac{3+\sqrt{17}}{2}\right) \geq 0$

$0, \frac{-3\pm\sqrt{17}}{2}$ $\approx 0, 0.562, -3.562.$

-5 $\quad \frac{-3-\sqrt{17}}{2}$ $\quad -1$ $\quad 0$ $\quad 0.2$ $\quad \frac{-3+\sqrt{17}}{2}$ $\quad 3.$



Q1.b) i) $|2x-3| = 2|3x-5|$

$= |2||3x-5|$

$|2x-3| = |6x-10|$

$2x-3 = \pm(6x-10)$

$2x-3 = 6x-10$

$10-3 = 6x-2x$

$7 = 4x$

$x = 7/4$ ✓

$2x-3 = \cancel{6x}\, 10-6x$

$2x+6x = 10+3$

$8x = 13$

$x = 13/8$ ✓

ii) $\dfrac{1}{|2x-3|} \le 3$

$\cancel{\text{Since}}$ $|2x-3| > 0$ ...

$1 \le |3||2x-3|$

$(1)^2 \le (|6x-9|)^2$

$(1)^2 - (6x-9)^2 \le 0$

$(1-6x+9)(1+6x-9) \le 0$

$(-1)\ (10-6x)(6x-8) \le 0\ \ (-1)$

$(6x-10)(6x-8) \ge 0$

$6x-10 = 0$; $6x-8 = 0$

$x = 10/6 = 5/3$; $x = 8/6 = 4/3$

$x \le 4/3\ ,\ x \ge 5/3$

Q2 a) $f(x)=\frac{x}{1+x^2}$, $g(x)=\frac{1}{x}$

$f \circ g(x) = \frac{1}{x(1+(\frac{1}{x})^2)} = \frac{1}{x+\frac{1}{x}}$

$(-\infty,0) \cup (0,\infty)$ ← domain ⇒ $x+\frac{1}{x} \neq 0$ & $x \neq 0$ ⇒

↳ ~~$x+\frac{1}{x}=0$ at $x=-1$~~

~~$x=-\frac{1}{x}$~~ ~~$-1+\frac{1}{-1}=-1-1$~~

~~$x^2=-1$~~

~~$x=$~~

~~Range ⇒ $(-\infty,\infty)$~~

$g \circ f(x) = \frac{1+x^2}{x} = \frac{1}{x}+x$ ⇒ Domain ⇒ $x \neq 0$
$(-\infty,0) \cup (0,\infty)$

~~⇒ Range ⇒ $(-\infty,\infty)$~~

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b) $y=\frac{7}{2}-x$ ⇒ $x=\frac{7}{2}-y$ ↔ $y^{-1}=\frac{7}{2}-x$

~~$y^{-1}=7$~~

$y=\frac{3}{x}$ $y^{-1}=\frac{3}{x}$

$x=\frac{3}{y}$ ⇒ $y^{-1}=\frac{3}{x}$

$$f(x)=\begin{cases} \frac{7}{2}-x & x<2 \\ \frac{3}{x} & x \geq 2 \end{cases}$$

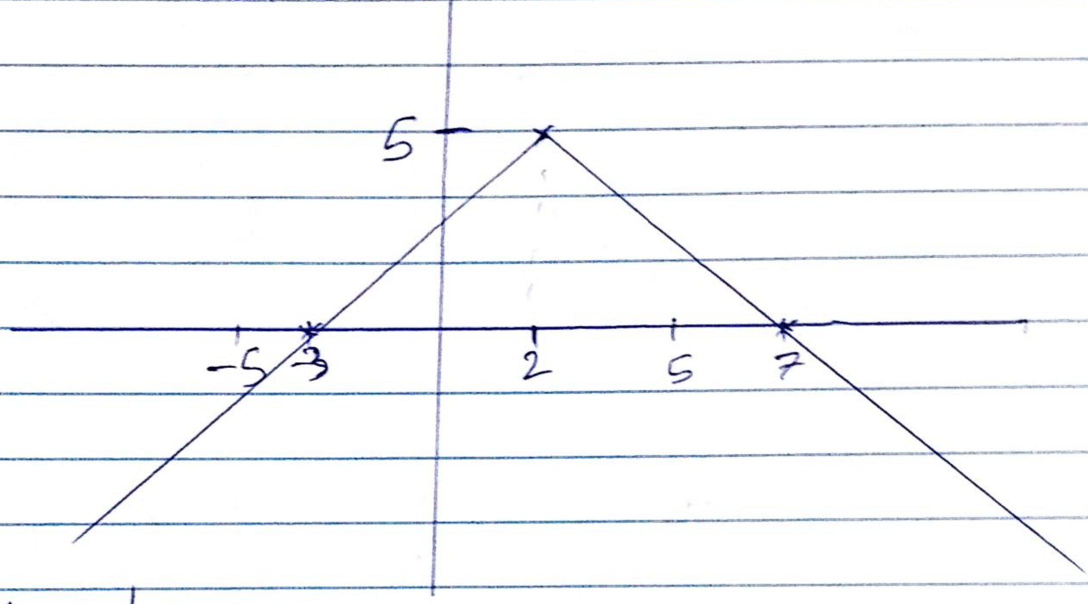
$$f^{-1}(x)=\begin{cases} \frac{7}{2}-x & x<2 \\ \frac{3}{x} & x \geq 2 \end{cases}$$

Date:

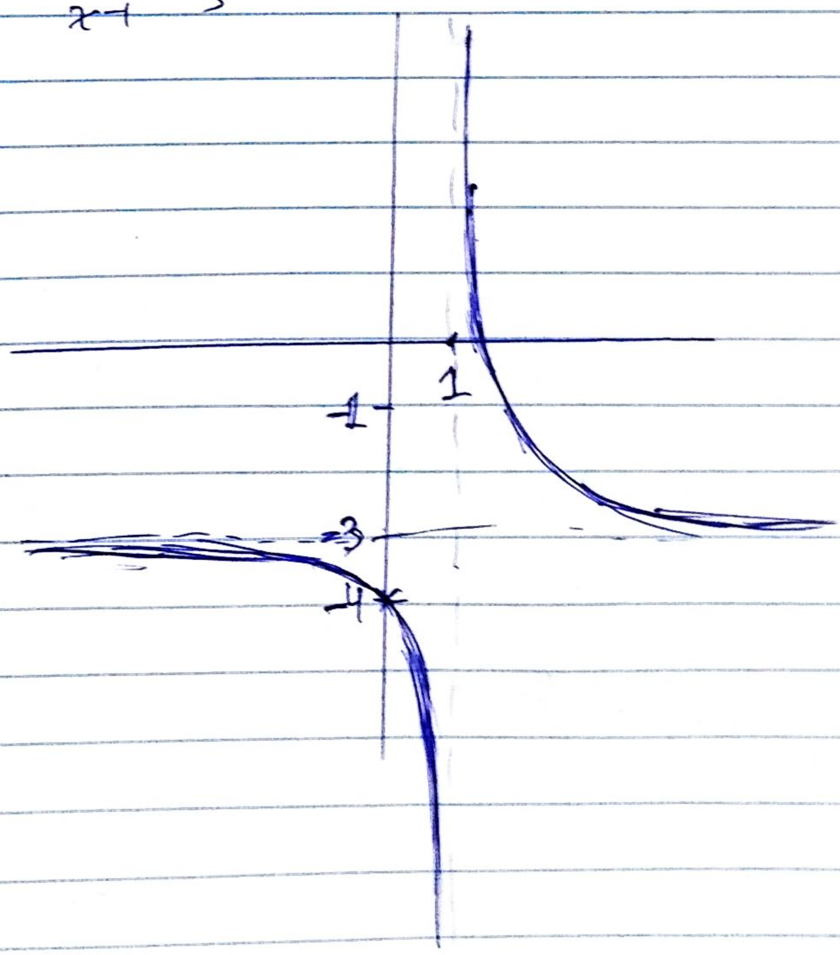
Q3. i) $f(x) = -|x-2|+5$

$x-2=0$

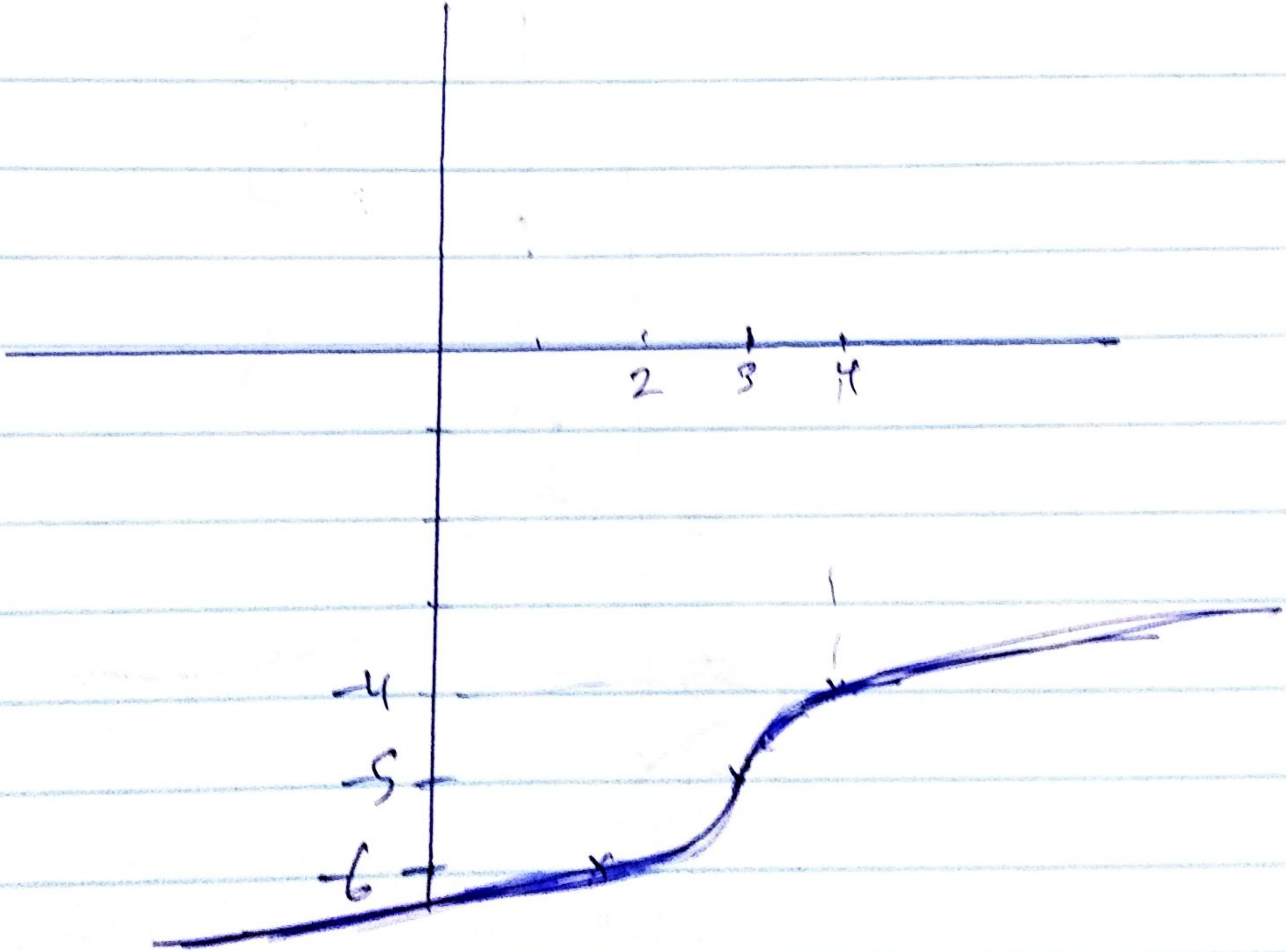
when $x=2$



ii) $f(x) = \frac{1}{x-1} - 3$



iii) $f(x) = \sqrt[3]{x-3} - 5$



Q4. $\lim_{x\to -1} f(x) \Rightarrow \lim_{x\to -1^+} f(x) = 1$

$\lim_{x\to -1^-} f(x) = 0$

$\therefore \lim_{x\to -1} f(x) = \boxed{DNE}$

$f(-1) = 1$

$\lim_{x\to 1} f(x) \Rightarrow \lim_{x\to 1^+} f(x) = 2$

$\lim_{x\to 1^-} f(x) = 1$

$\therefore \lim_{x\to 1} f(x) = \boxed{DNE}$

$f(1) = -2$

$\lim_{x\to 3} f(x) \Rightarrow \lim_{x\to 3^+} f(x) = 1$

$\lim_{x\to 3^-} f(x) = 1$

$\lim_{x\to 3} f(x) = 1$.

Q5. a) at $x=1$: $f(x) = a(x-2)^2$

is continuous so $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(a)$

$\Rightarrow a(x-2)^2 = b-x$

$a(1-2)^2 = b-1$

$a(-1)^2 = b-1$

$\boxed{a = b-1}$

b) if $a=-1$, $a=b-1$

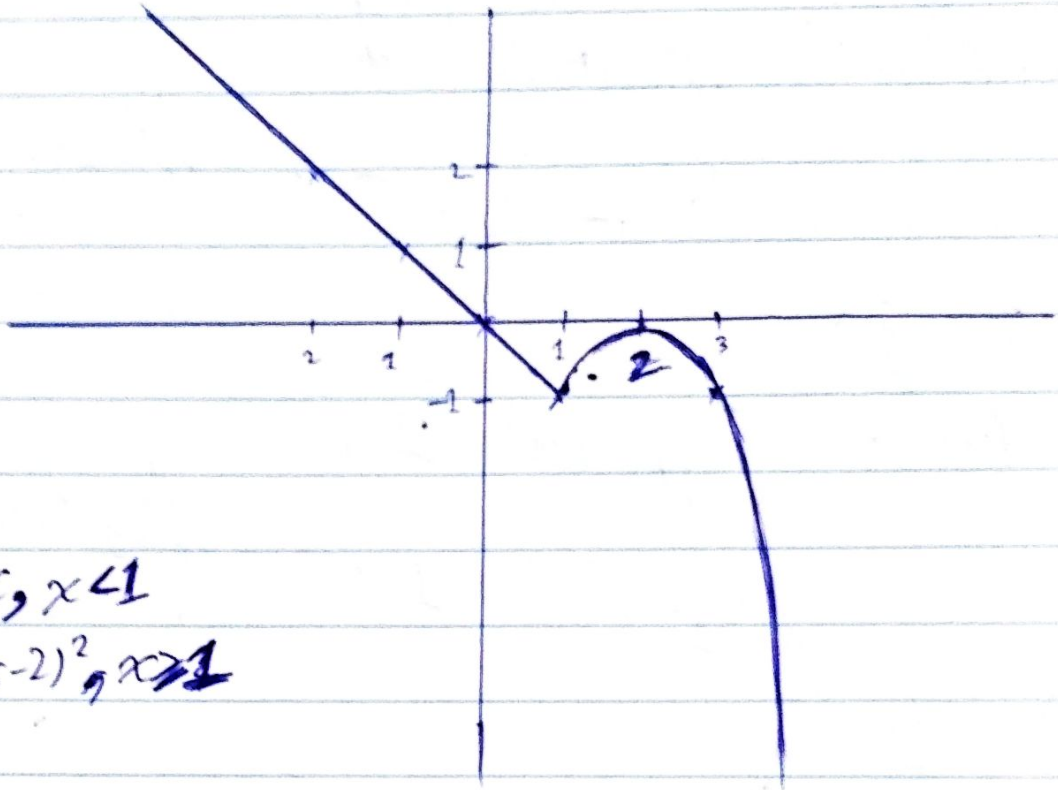
$\Rightarrow -1 = b-1$

$b=0$

$f(x) = \begin{cases} -x, & x<1 \\ -(x-2)^2, & x\geq 1 \end{cases}$

Q5.b) $a = -1$

$b = 0$

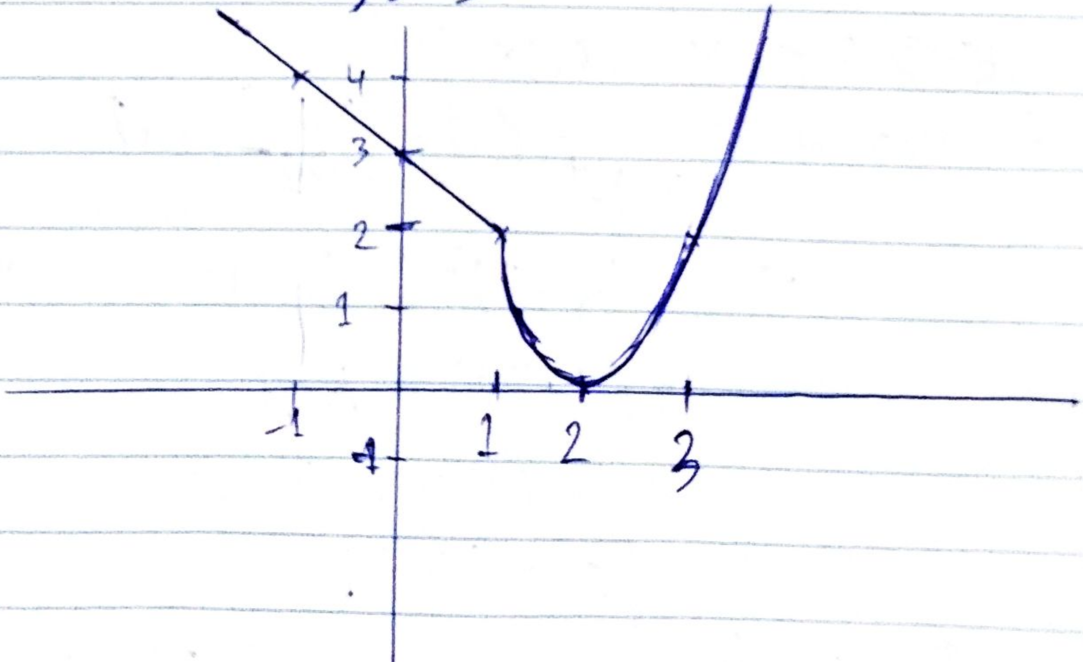


$$f(x) = \begin{cases} -x, & x < 1 \\ -(x-2)^2, & x \geq 1 \end{cases}$$

c) $a = b - 1$

~~$4 = 5 - 1 \Rightarrow a = 4, b = 5$~~

$\Rightarrow 2 = 3 - 1 \Rightarrow a = 2, b = 3$



Q6. i) at $t=10$, continuous when... $\lim_{t \to 10^+} T(t) = \lim_{t \to 10^-} T(t)$

$$C^2 - 15C - 3t = 2t$$
$$C^2 - 15C - 30 = 20$$
$$C^2 - 15C - 50 = 0$$
$$\cancel{C^2 - 10C - 5C - 50 = 0}$$
$$\cancel{C(C-10)}$$

$$C = \frac{15 \pm 5\sqrt{17}}{2}$$
$$C \approx -2.807764064\ldots,\ 17.80776406\ldots$$

ii) A function at a point is continuous when the limit as x approaches that point exists, and is equal to the output of the function at that point.

$$\lim_{t \to 10^+} T(t) = \lim_{t \to 10^-} T(t) = \lim_{t \to 10} T(t) = f(10) = 2(10) = 20$$

$$\left(\frac{15 \pm 5\sqrt{17}}{2}\right)^2 - 15\left(\frac{15 \pm 5\sqrt{17}}{2}\right) - 3(10) = 20 = \lim_{t \to 10^-} T(t)$$

$$2(10) = 20$$

$$\therefore \lim_{t \to 10} T(t) = 20$$

$=$ equal; therefore, continuous at $t=10$.

Q7. a) $f(x) = x^3 + 3^x \Rightarrow \boxed{\text{No discontinuity}}$

b) $f(x) = 5/(x^2 - 81)$ at $\boxed{x = 9, -9}$

c) $f(x) = \dfrac{x^2 + 2x - 24}{x^2 - 36} = \dfrac{x^2 + 6x - 4x - 24}{x^2 - 6^2} = \dfrac{x(x+6) - 4(x+6)}{(x+6)(x-6)}$

$$= \frac{(x-4)(x+6)}{(x+6)(x-6)}$$

$x = 6$, $x = -6$

c) $\frac{(x-4)\cancel{(x+6)}}{(x-6)\cancel{(x+6)}} = \frac{x-4}{x-6}$

So discont. at $x = \pm 6$
BUT Removable so now
only $x = +6$

d) $f(x) = \frac{2x+1}{x^2+6x+9} = \frac{2x+1}{x^2+3x+3x+9} = \frac{2x+1}{x(x+3)+3(x+3)}$

$= \frac{2x+1}{(x+3)^2}$

discont at $\boxed{x = -3}$