lecture 14:- Relation

$R \subseteq A \times B$.

$|pow(A \times B)| = 2^{|A \times B|}$

$= 2^{|A| \times |B|}$.

Ternary. Relation.

$A \times B \times C$.

$R \subseteq A \times A$.

$|pow(A \times A)| = 2^{|A \times A|}$

$= 2^{|A|^2}$.

$A \times B \times C$

Ex:- $A = \{1, 2\}$.

$B = \{a, b\}$.

$C = \{\text{II}, \text{III}\}$.

$A \times B \times C = \{ (1, a, \text{II}), (1, a, \text{III}), (1, b, \text{II}), (1, b, \text{III}), (2, a, \text{II}), (2, a, \text{III}), (2, b, \text{II}), (2, b, \text{III}) \}$.

$|pow(A \times B \times C)| = 2^{|A \times B \times C|}$

$= 2^{|A| \times |B| \times |C|}$

.

Ex1 :- 469 R $N \times N \times N$

$R = \{(a, b, c) \mid a < b < c\}$.

$(1, 2, 3) \in R$? ✓

$(2, 4, 3) \in R$. ✗.

Ex2 :- 469 R $Z \times Z \times Z$.

$R = \{(a, b, c) \mid b = a + k \wedge c = a + 2k\}$. $k \in Z$.

$(1, 3, 5) \in R$ ✓

$\downarrow$ a, $\downarrow$ b, $\downarrow$ c.

$3 = 1 + k \Rightarrow k = 2$.

$5 = 1 + 2 \times 2$

$5 = 1 + 4$

$5 = 5$

a b c.

$(2, 5, 9) \notin R$. (H.W.)

$\overset{a}{\uparrow}\ \overset{b}{\uparrow}\ \overset{c}{\uparrow}$

$(2, 5, 9) \notin R$. (HW.)

Rough work.

Ex3 $\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}^+$.

469.

$5 \div 2$

$5 \bmod 2 \equiv 1$.

$$\begin{array}{r} 2 \\ 2\overline{)5} \\ 4 \\ \hline 1 \end{array}$$

$R = \{(a, b, c) \mid a \equiv b \bmod c\}$.

$-5 \div 4$

$$\begin{array}{r} -2 \\ 4\overline{)-5} \\ \mp 8 \\ \hline +3 \end{array}$$

$a \bmod c$

$b \bmod c$

$8 \bmod 4 \equiv 0$

$12 \bmod 4 \equiv 0$.

$15 \bmod 4 \equiv 3$

$(3 \bmod 4 \equiv 3)$.

$8 \equiv 12 \bmod 4$.

$15 \equiv 3 \bmod 4$.

$(8, 2, 3) \in R$. ✓

$8 \equiv 2 \bmod 3$.

$$\begin{array}{r} 2 \\ 3\overline{)8} \\ \pm 6 \\ \hline 2 \end{array} \qquad \begin{array}{r} 0 \\ 3\overline{)2} \\ \pm 0 \\ \hline 2 \end{array}$$

$(-1, 9, 5) \in R$?

$(14, 0, 7) \in R$?

Relational Database.

Ex4:- $A \times N \times S \times D \times T$.

{ (PIA, PK744, PEW, KHI, 17:00)

—

—

—

⋮

}

A is the Set of Airlines.

N is the Set of flights.

S " " " Starting points.

D " " " Destinations.

T " " " Time.

— S " " " Starting Points.

— D " " " Destinations.

! T " " Time.

;

—.

}

Airlines Flights. Starting Destination Time.

→.

→.

Representing Relations:-

Using Matrices.

$T = t_{ij}$

{ }.

Unique elements.

[ ]

row, columns.

$A = \{a_1, a_2, ----a_m\}$ $B = \{b_1, b_2, ---b_n\}$.

$R \subseteq A \times B$.

m_{12}

$$m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R \\ 0 & \text{if } (a_i, b_j) \notin R. \end{cases}$$

$M \Rightarrow$ rows = $|A|$

Col = $|B|$.

Ex1:- $A = \{\underset{a_1}{1}, \underset{a_2}{2}, \underset{a_3}{3}\}$. $B = \{\underset{b_1}{1}, \underset{b_2}{2}\}$.

$R = \{(2,1), (3,1), (3,2)\}$.

Find M_R which Represents Relation R.

$$M_R = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & 0 & 0 \\ 2 & 1 & 0 \\ \end{array}$$

$$M_R = \begin{array}{c|cc} 2 & 1 & 0 \\ 3 & 1 & 1 \end{array}$$

Ex:- $A = \{5, 4, 3\}$. $A \times A$.

$R = \{(a, b) \mid a < b\}$.

$$\begin{array}{c|ccc} 5 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 \\ \hline & 5 & 4 & 3 \end{array}$$

How to check Properties Inside a matrix.

1- Reflexive. $\forall a \in A \quad (a, a) \in R$.

$\forall i \quad m_{ii} = 1$.

Ex:- $[\;]$ ✓ $[0]$ X. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

$\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ X $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ ✓

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ - - - - -

- - - - - - -

$|B|$

$|A| \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$2^{3 \times 3} = 2^9 = 512$.

Co... l...:-

Symmetric :-

$\forall a,b \in A \quad$ if $(a,b) \in R \rightarrow (b,a) \in R$.

$\forall i,j \quad$ if $(a_i, b_j) \in R \rightarrow (b_j, a_j) \in R$.

$\forall i,j \quad m_{ij} = 1 \longrightarrow m_{ji} = 1$.

$[\,]$ ✓ $\quad [0]$ ✓ $\quad [1]$ ✓

$m_{11} = 1 \rightarrow m_{11} = 1$.

$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. ✓ $\quad \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ ✓ $\quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. X.

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓ $\quad \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ ✓

$$M_R = [M_R]^T$$

Find all Symmetric matrices of size 2x2.

Anti Symmetric.

$\forall a,b \in A \quad$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$.

$\forall i,j \quad$ if $m_{ij} = 1 \wedge m_{ji} = 1 \rightarrow i = j$.

$[\,]$. ✓ $\quad [2]$ ✓ $\quad \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. ✓

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Next lecture Composite.