lecture 5 :-

Representing English Statements Using Quantifiers.

E20/p39 :- there is an honest politician. subject.
$\downarrow$
$P(x) \leftarrow x \textcircled{>} 3$.
$\uparrow$
there exist x, x is a politician, x is honest. predicate

$\exists_x P(x)$.

let $P(x) = x$ is honest.

$x \in \{$ politicians $\}$. ✓
$x \in$ set of politicians.

Ex20

39. All Americans eat cheese burger.

$-x \in$ Set of Americans.

for all x, x is an American x eat cheese burger.

$P(x) = x$ eat cheese burger.

$\forall_x P(x)$.

HW 4:00 O clock.

Ex21 :- find Negation of $\forall_x (x^2 > x)$ & $\boxed{\exists_x (x^2 = 2)}$.

let $P(x) = x^2 > x$.

$\neg(\forall_x P(x))$

$\neg \forall \neg P(x) = \exists_x \neg P(x)$.

$= \exists_x \neg P(x)$.

$= \exists_x \neg (x^2 > x)$

$\neg(>) = \leq$.

$= \exists_x (x^2 \leq x)$.

$\boxed{p \rightarrow q \equiv \neg p \vee q}$

Ex23:-   "Every Student in This Class has Studied Calculus"

for all x, x is a Student in this class, x has studied Calculus

$\forall x \, P(x)$. $P(x) =$ x has studied Calculus.

x ∈ Set of Students in This class.

Ex24:- $\underline{Part \, a}$ HW 4:30 pm. today.

Every Student in this Class has either Visited Canada or Mexico.

for all x, x is a Student in this class, x has Visited Canada or x has Visited Mexico.

let x ∈ Student in This Class.
$P(x) =$ x has Visited Canada
$Q(x) =$ " " " Mexico.

$\forall x \, (P(x) \vee Q(x)). = \forall x \, P(x) \vee \forall x \, Q(x).$

X → $\boxed{\forall x \, P(x) \vee Q(x)}$.

Exercise 7:-   Translate into English.
$\overline{43}$

$C(x) =$ x is a Comedian.   x ∈ Set of all people.
$F(x) =$ x is Funny.

$\forall x \, (C(x) \rightarrow F(x))$.

for all x, x is a person. If x is a Comedian then x is funny.

$\underline{Q \, 7:-}$ (a) $\exists x \, P(x)$.   Using only ∧, ∨, ⌐.   x ∈ {0, 1, 2, 3, 4}.
$\overline{44}$.

    $= P(0) \vee P(1) \vee P(2) \vee - - \vee P(4)$.

x ∈ {1, --- N}.
$\exists x \, P(x) = P(1) \vee P(2) \vee P(3)$
    $- - \vee P(N)$.

(c) $\exists x \neg P(x) = \neg P(0) \vee \neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4)$.

(f) $\neg \forall x \, P(x) = \exists x \neg P(x) =$

P	$P \wedge P$
T	T
F	F

$\neg \forall x (P(x) \wedge P(x)) = \neg \forall x \, P(x)$.

$P \wedge P = P$.

$\underline{Ex \ 25}$:- (a) No one is perfect
$\underline{91}$

$\underline{\text{for all } x,}$ x is person.   x is $\underline{not}$ Perfect.

$\forall x \neg P(x)$.

Let $x \in$ persons.
$P(x) = x$ is Perfect.

(d)   At least one of your friend is perfect.

$x \in$ Set of my friends.
$P(x) = x$ is perfect.

there exist x. x is my friend, x is perfect.

$\exists x \, P(x)$.

"Every one likes Some One".

$x, y \in$ persons.
$P(x, y) = x$ likes y.

for all x, there exist y, x & y are person,
$\qquad\qquad\qquad$ x likes y.

$\forall x \exists y . P(x, y)$.

Nested Quantifiers:-

$x, y \in \{1, 2, 3, \cdots M\}$.

$\forall x \forall y \, P(x, y)$.

$\forall x P(x) = P(1) \wedge P(2) \cdots$

$= \forall x \big(P(x, 1) \wedge P(x, 2) \wedge P(x, 3) \cdots \wedge P(x, N) \big)$.

$= \underline{\forall x \, P(x, 1)} \; \underline{\wedge} \; \underline{\forall x \, P(x, 2)} \wedge \cdots \wedge \underline{\forall x \, P(x, N)}$.

$\big(P(1, 2) \wedge P(2, 1) \wedge P(3, 1) \wedge \cdots \wedge P(N, 1) \big) \wedge$.

$$(P(1,2) \land P(2,2) \land P(3,2) \land \cdots \land P(N,2)) \land.$$

$$\overline{=} \ \overline{-} \ \overline{-}$$

$$(P(1,N) \land P(2,N) \land P(3,N) \land \cdots \land P(N,N)).$$

$$\forall x \exists y \ P(x,y).$$

$$\forall x \ [\ P(x,1) \lor P(x,2) \lor P(x,3) \lor \cdots \lor P(x,N)].$$

$$= \ \underline{\forall x \ P(x,1)} \ \underline{\lor} \ \forall x \ P(x,2) \lor \cdots \ \lor \forall x \ P(x,N).$$

$$= \ (P(1,1) \land P(2,1) \land \cdots \land P(N,1)) \times$$

$$\overline{-} \ \overline{-} \ \overline{\vdots} \qquad \qquad \vdots$$

$$[P(1,N) \land P(2,N) \land \cdots \land P(N,N)]$$

$$\exists x \forall y \ P(x,y). \quad \Big\} \ 4:30 \ pm.$$

$$\exists x \exists y \ P(x,y). \quad \Big\}$$

$$\neg \forall x \forall y \ P(x,y) = \ ? \ \Big\} \ 4:30 pm.$$

$$\neg \exists x \forall y \ \neg P(x,y) = \ ? \ \Big\}$$

Ex1:

$$\forall x \forall y \quad (x+y = y+x) \qquad x,y \in \mathbb{Z}.$$

$$\forall x \exists y \quad (x+y = 0). \qquad \qquad '' \quad \in \mathbb{R}.$$

$$\overset{\textstyle\bigcirc}{-\infty}$$

$$\vdots \ -1$$

$$\vdots$$

$$+\infty.$$

Ex 4:- $Q(x,y) = x+y = 0.$

$\exists y \, \forall x \, Q(x,y) = ?$ F

$\forall x \, \exists y \, Q(x,y) = ?$ T. $+\infty.$

$-\infty$

Ex 5:- $Q(x,y,z) = x+y = z.$

$\forall x \forall y \, \exists z \, Q(x,y,z) = ?$ T

$\exists z \, \forall x \forall y \, Q(x,y,z) = ?$ F.

Quiz 2:- 09-Sep-2022.

① $\forall y \, P(1,y)$ $y \in \{0, 1, 2\}.$

marks 5 expand this using $\wedge, \vee, \neg$ & Predicate only.

$= \underline{\hspace{6cm}}.$

marks 5.

② Every one Speaks hindi. Express using Quantifiers.