

# Lecture 29:-

## HASSE DIAGRAM.

1898-1979.

Ex:-  $R = \{(a,b) \mid A \subseteq B\}$ .

$P(S) \times P(S)$ .

$A \subseteq B$   
 $B \supseteq A$ .

$S = \{a, b, c\}$ .

$\subseteq$  Prove in Hw.

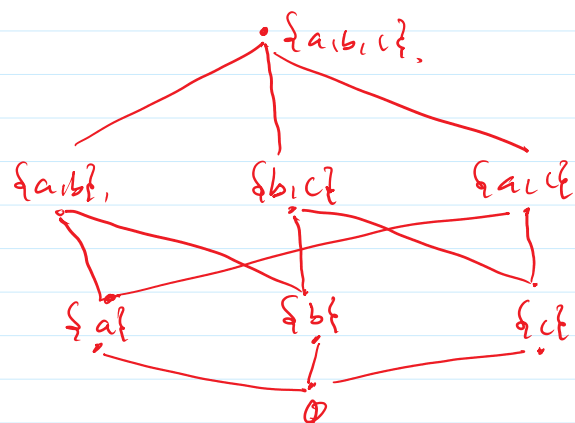
$P(S) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\}\}$ .

$P(S) \times P(S) = \{(\emptyset, \emptyset), (\emptyset, \{a\}), (\emptyset, \{b\}), \dots$   
 $(\{a\}, \emptyset), (\{a\}, \{a\}), (\dots)$

$\}$

$R = \{(\emptyset, \emptyset), (\emptyset, \{a\}), (\emptyset, \{b\}), (\emptyset, \{c\}), (\dots)$   
 $(\{a\}, \{a\}), (\{a\}, \{a,b\}), (\{a\}, \{a,c\}), (\{a\}, \{a,b,c\}),$   
 $(\{b\}, \{b\}), (\{b\}, \{a,b\}), (\{b\}, \{b,c\}), (\{b\}, \{a,b,c\}),$   
 $(\{c\}, \{c\}), (\{c\}, \{a,c\}), (\{c\}, \{b,c\}), (\{c\}, \{a,b,c\}),$   
 $(\{a,b\}, \{a,b\}), (\{a,b\}, \{a,b,c\}),$   
 $(\{a,c\}, \{a,c\}), (\{a,c\}, \{a,b,c\}),$   
 $(\{b,c\}, \{b,c\}), (\{b,c\}, \{a,b,c\}),$   
 $(\{a,b,c\}, \{a,b,c\})\}.$

$(P(\{a,b,c\}), \subseteq) \Rightarrow$



$P(S) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\}\}$

Hw.

Ex:-

$R = \{(a,b) \mid \frac{|a \cup b|}{|a|} \geq 1\}$ .

$P(S)$   
 $S = \{1, 4, 8\}$ .

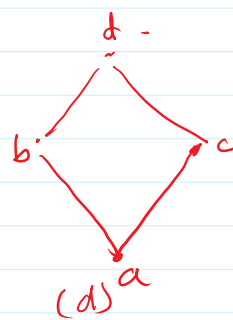
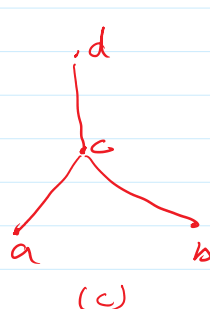
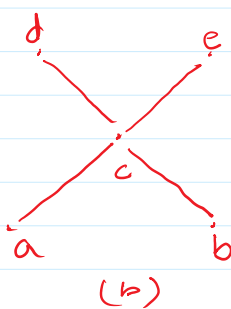
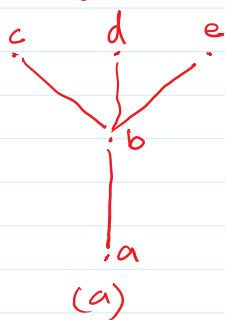
Ex 14.  $(\{2, 4, 5, 10, 12, 20, 25\}, |)$

HASSE DIAGRAM.

### SOME BASIC DEFINITIONS.

- PS09.
- 1- Greatest  $\checkmark$  :-  $a \in S$  is the greatest in  $(S, \leq)$   
if  $b \leq a \quad \forall b \in S.$
  - 2- Least :-  $a \in S$  is the least in  $(S, \leq).$   
if  $a \leq b \quad \forall b \in S.$
  - 3- Maximal :-  $a \in S$  is maximal in  $(S, \leq),$   
if  $\neg \exists b \in S. a < b.$
  - 4- Minimal :-  $a \in S$  is minimal in  $(S, \leq).$   
if  $\neg \exists b \in S. b < a.$

Ex 17./PS10.



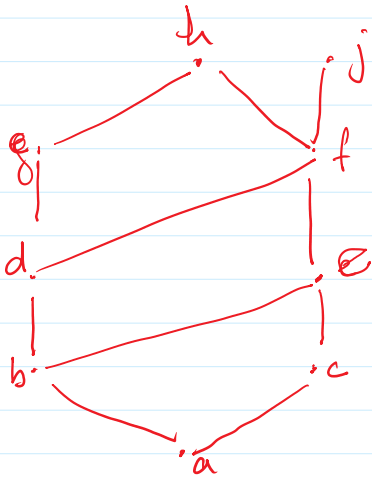
- Observation :-
- if  $\checkmark$  Greatest exist  $\rightarrow$  that element is maximal.
  - if  $\checkmark$  Least  $\checkmark$   $\rightarrow$   $u$  is minimal.
  - if  $\checkmark$  Greatest does not exist  $\rightarrow$  we may or may not have maximal.
  - if  $\checkmark$  Least does not exist  $\rightarrow$  we may or may not have minimal.

Definitions:-  
PS10.

Upper Bound :-  $u \in UB. (u \in S)$  if  $a \leq u$   
 $\forall a \in A.$  Then  
 $u \in UB$  for the Set  $A \subseteq S.$

Lower Bound :-  $l \in LB (l \in S).$  if  $l \leq a$   
 $\forall a \in A$  then  
 $l \in LB$  for the Set  $A \subseteq S.$

lower bound.  $a \geq LB$  ( $a \geq 0$ ).  $\forall a \in A$  then  $\downarrow \in LB$  for the Set  $A \subseteq S$ .



$\{a, b, c\}$  find Upper bound.

UB =  $\{e, f, j, h\}$ .

LB =  $\{a\}$ .

$\{a, b, c\}$ .

Least Upper Bound:-  $\{e\}$ .

PS 22.

Definitions:-

$\{a, b, c\}$ .

Greatest Lower Bound:-  $\{a\}$ .

Set =  $\{b, e\}$ .

UB =  $\{f, j, h, e\}$ .

LUB =  $\{e\}$ .

LB =  $\{a, b\}$ .

GLB =  $\{b\}$ .

Ex:-

3 elements.

(a) \* ..

4 elements.

ChW.

(b)



(c)

GCD / LCM.  
 $\downarrow$

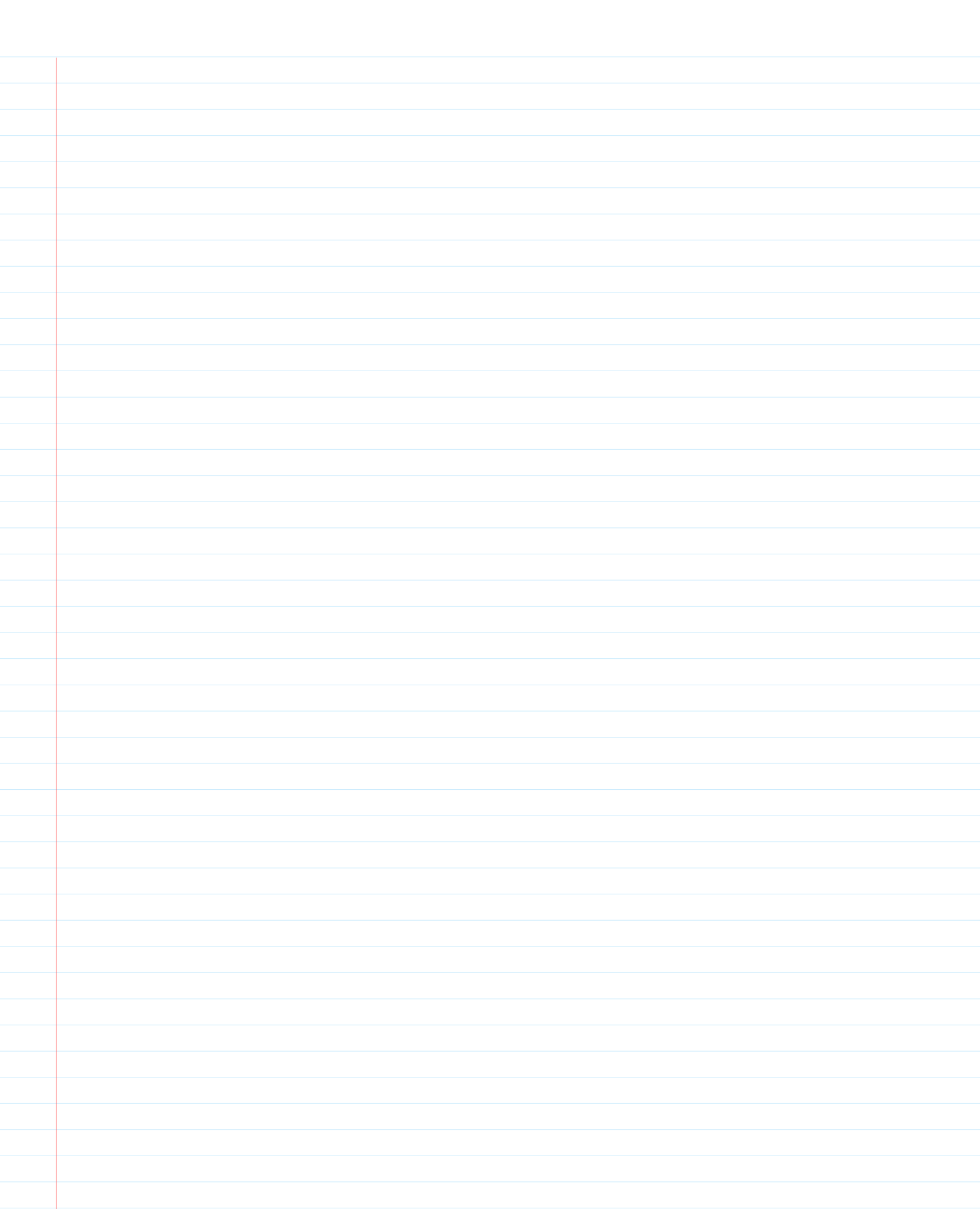
Next Lecture.

(d)



(d)





aaa

M

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