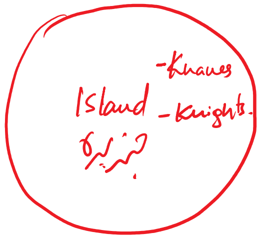
lecture 3:-

Exercise 49-54.
P20.

Applications of propositional logic.

1. System Consistency.
2. Logical puzzles.

P-15
Ex18.



A, B.

→ A says "B is a Knight"
→ B says "the two of us are of opposite type".

A = ? B = ?

Let p = A is a Knight.
$\neg p$ = A is a Knave.

q = B is a Knight
$\neg q$ = B is a Knave.

q

$(p \wedge \neg q) \vee (\neg p \wedge q)$

$\wedge$ and (اور)

A نائٹ ہے اور B نیو ہے

$\vee$ OR (یا)

A نیو ہے اور B نائٹ ہے

and $\wedge$.

Case 1:- A Knight, B Knight

1 → $q = T$

2 → $(p \wedge \neg q) \vee (\neg p \wedge q) = T$

$p = T$ $\neg p = F$
$q = T$ $\neg q = F$.

1 → $T = T$ ✓

2 → $(T \wedge F) \vee (F \wedge T) = T$
$F \vee F \neq T$
$F \neq T$.

This does not hold.

CASE 2:- A Knight, B Knave.

1 → $q = T$

$p = T$ $\neg p = F$
$q = F$ $\neg q = T$

2 → $(p \wedge \neg q) \vee (\neg p \wedge q) = F$

(1) → $F \neq T$ This case also does not hold.

p = A is a Knight.
$\neg p$ = A is a Knave.

q = B is a Knight
$\neg q$ = B is a Knave.

CASE 3

A Knave, B Knight

A Knave, B Knight

① → $q = F$ $\quad P = F \quad \neg P = T$ $\quad q = T \quad \neg q = F$.

② → $(P \wedge \neg q) \vee (\neg P \wedge q) = T$

① → $T \neq F$ does not hold.

CASE 4:- A Knave. B Knave.

① $q = F$ $\quad p = F \quad \neg p = T$ $\quad q = F \quad \neg q = T$

② $(P \wedge \neg q) \vee (\neg P \wedge q) = F$

① → $F = F$ ✓

② → $(F \wedge T) \vee (T \wedge F) = F$ this holds.

$F \vee F = F$

$F = F$

A is a Knave

B is a Knave.

Knight, Knight, Knight
Knight, Knight, Knave.
Knight, Knave Knight
Knight, Knave, Knave.
Knave, Knight Knight
Knave, Knight, Knave.
Knave, Knave Knight
Knave, Knave, Knave.

p = A is a Knight $\neg p =$
q = B is a Knight $\neg q =$
γ = C is a Knight. $\neg \gamma =$

A Says " b is a Knight or C is a Knight. $q \vee \gamma$

B Says " I am a Knave!! $\neg q$

C Says Nothing.

A Knight B Knight C Knight.

$p = T \quad \neg p = F$
$q = T \quad \neg q = F$
$\gamma = T \quad \neg \gamma = F$.

① $q \vee \gamma = T$
② $\neg q = T$

Ex :- 55-59.

Sessional Qs:-
1- System Consistency.
Ex 44-54.

2- Knave Knight.
Ex 55-59.

Ex: 55/p20.

A Says "Atleast one of us is a Knave".

B Says "".

p = A is a Knight ¬p = —
q = B is a Knight ¬q = —

(A Knave ∧ B Knight) ∨
(A Knight ∧ B Knave).

A Knight B Knight

$(\neg p \wedge q) \vee (p \wedge \neg q) = T$

$p = T \quad \neg p = F$
$q = T \quad \neg q = F$

$(F \wedge T) \vee (T \wedge F) = T$
$F \vee F \neq T.$

Does not hold.

A Knight B Knave.

$(\neg p \wedge q) \vee (p \wedge \neg q) = T$

$p = T \quad \neg p = F$
$q = F \quad \neg q = T$

$(F \wedge F) \vee (T \wedge T) = T$
$F \vee T = T$
$T = T$

Holds.

A Knave B Knight

$(\neg p \wedge q) \vee (p \wedge \neg q) = F$

$p = F \quad \neg p = T$
$q = T \quad \neg q = F.$

$(T \wedge T) \vee (F \wedge F) = F$
$T \vee F = F$
$T \neq F.$

Does Not hold.

Knave Knave.

$(p \wedge \neg q) \vee (\neg p \wedge q) = F$

$p = F \quad \neg p = T$
$q = F \quad \neg q = T$

$(F \wedge T) \vee (T \wedge F) = F$
$F \vee F = F$
$F = F$

Holds.

A	B
Knight Knave	Knave. Knave.

B is a knave, A is Unknown.

Propositional Equivalence :-

1) Tautology :- T
2) Contradiction :- F
3) Contingency :- T, F.

P	¬P	P ∨ ¬P	P ∧ ¬P
T	F	T	F
F	T	T	F
	Contingency.	Tautology.	Contradicting.

$P \wedge q$ = ?

$(P \wedge q) \wedge \neg(P \wedge q)$ = Contradiction.

$(P \vee q) \vee \neg(P \vee q)$ = Tautology.

Two Expressions are equivalent if you take the bi conditional of both & it results in tautology.

Ex2/22 :- $\neg(P \vee q)$, $\neg P \wedge \neg q$.

Proof They are equivalent.

P	q	P∨q	¬(P∨q)	¬P	¬q	¬P∧¬q.	¬(P∨q) ↔ ¬P∧¬q.
T	T	T	F	F	F	F	T
T	F	T	F	F	T	F	T
F	T	T	F	T	F	F	T
F	F	F	T	T	T	T.	T

Homework :- 1.5 Speed. for listening to lecture.
Exercise. 49-54 ?
55-59 ?. on Teachable.