lecture 15:- Checking Properties of Relations.

→ when relation is in Matrix form.

Structure
- Sets
- Matrices
- Graphs
- Trees.
  - Trie
  - Max Stack
  - Stack
  - Queue
  - Priority.
  - linklist.
  - Circular link
  - doubly link list
  - -
  - -
  - -
  - -

$R_1 \cap R_2$ $\quad$ $R_1 \cup R_2$.

→ How to Compute Intersection & Union.

Ex4 P478.

$$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$M_{R_1 \cap R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ By taking Conjunction.

$$M_{R_1 \cup R_2} = \begin{bmatrix} \text{HW} \end{bmatrix}$$ By taking disjunction.

Composite of a Relation:- Relations in Matrix.

R	(a,b)	$A \times B$
S	(b,c)	$B \times C$.

→ $(a,c) \in S \circ R$ if $\exists\, (a,\underline{b}) \in R \wedge (\underline{b},c) \in S$.

Note:- If — Then of logic → B diff from. If else of Code.

$M_R = [r_{ij}]$. $\quad M_S = [s_{ij}]$. $\quad M_{S \circ R} = [t_{ij}]$.

→ $t_{ij} = 1$. if $\exists_k$ $r_{ik} = s_{kj} = 1$. for Some k.

Ex5 :- 478

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \qquad M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Ex 3 :
478

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$M_{S \circ R} = \begin{array}{c c} & \begin{array}{ccc} 1 & 1 & 1 \end{array} \\ \begin{array}{c} \\ 0 \\ \\ \end{array} & \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix} \end{array}$$

H.W. Complete this.

t_{12} (i j)   if $r_{1K} = s_{K2} = 1$ $K \in \{1, 2, 3\}$.

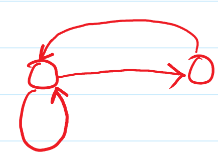
$T(r_{11} = s_{12} = 1) \vee$
$r_{12} = s_{22} = 1 \vee$
$r_{13} = s_{32} = 1.$

$$\left.\begin{array}{l} M_{R^2} = M_{R \circ R}. \\ M_{R^3} = M_{R^2 \circ R}. \\ M_{R \circ S} = \end{array}\right\} \text{H.W.}$$

Representing Relations Using Graphs.

1. Two Sets.   - Set of Vertices.
   - Set of Edges.

2) Vertices = ○
   Edges = —
   →.



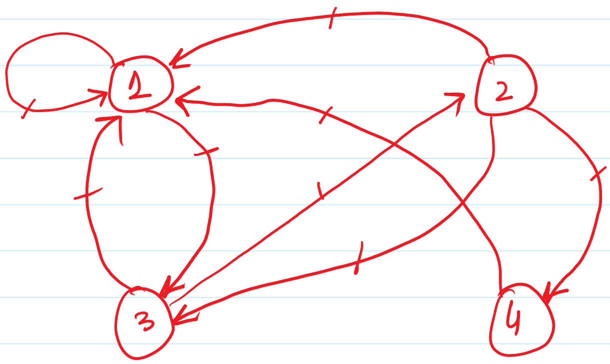
Vertices Set = A.   A is the based on which Relation is defined.

Set of Edges = R.

Ex8 :-
P480

$R = \{(1,1), (1,3), (2,1), (2,3), (2,4), (3,1), (3,2), (4,1)\}.$

$A = \{1,2,3,4\}.$



find R^{-1} Graph . H.W.
" $\overline{R}$ Graph H.W.

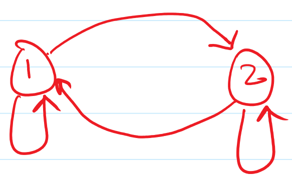
$R^{-1} = \{(b,a) \mid (a,b) \in R\}.$

$\overline{R} = \{(a,b) \mid (a,b) \notin R\} = A \times A - R.$

Properties of Relation in Graph.

1) Reflexive $\forall a \in A \quad (a,a) \in R.$

Ex:-



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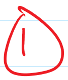

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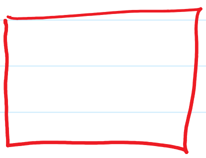

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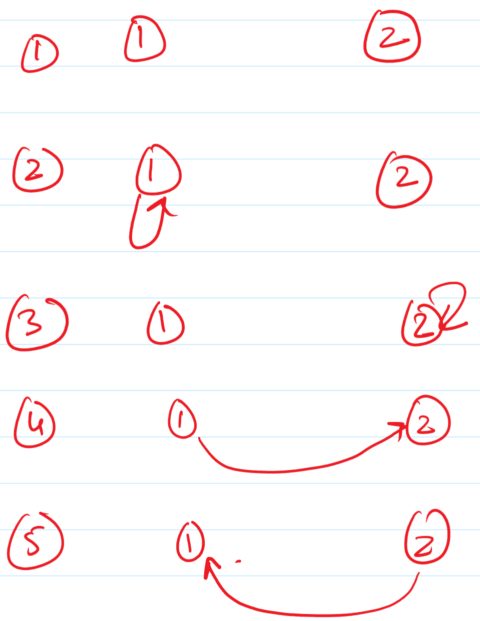
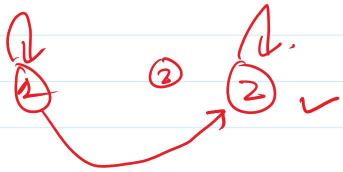


② 

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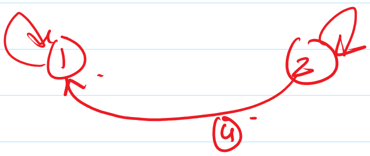
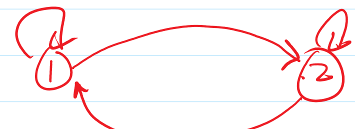
④ 

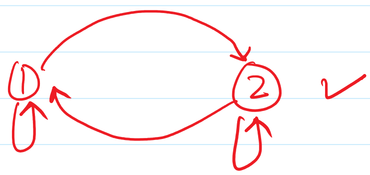
⋮

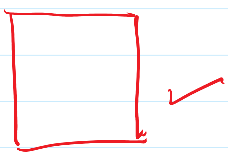
7 more. HW. find.

16 

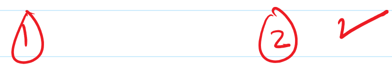
①

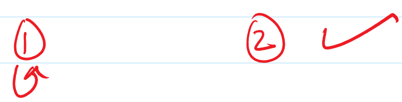
Symmetric: $\forall a, b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.

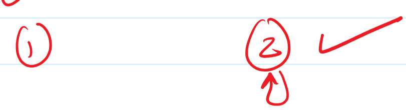
Ex. 

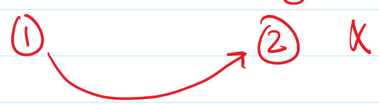


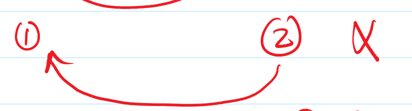
HW.

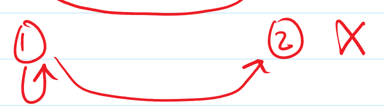
1) 

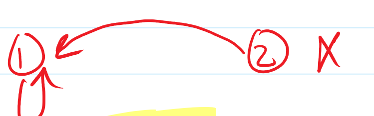
2) 

3) 

4) 

5) 

6) 

7) 

HW.

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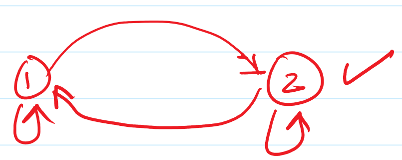
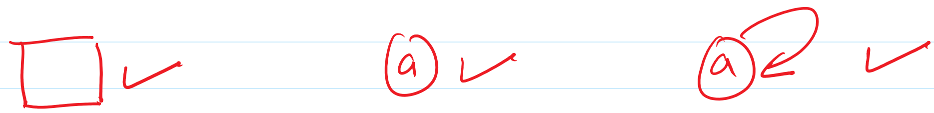
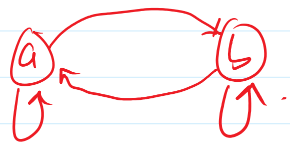
16) 

Anti Symmetric: $\forall a, b \in A$ if $\underline{(a,b) \in R} \wedge \underline{(b,a) \in R} \rightarrow \underline{a = b}$.

Ex





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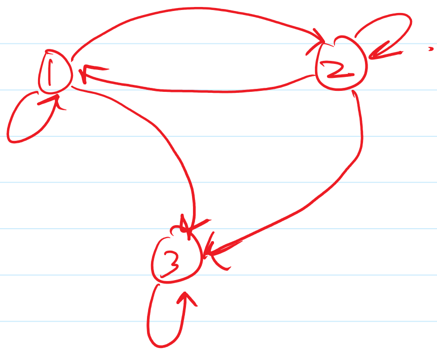

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Quiz #8 14-OCT-2022.

$A = \{ \ \}$?

Find Graphs of all Relations on A which are.

1- Reflexive.
2- Symmetric.
3- Anti Symmetric.