Lecture 6:- Ex 11 p 51. If p then q.

"If a person is a female ∧ and is a parent then this person is Sancaris mother"
p → q.
q.

x ∈ Person
y ∈ " .

∀x. f(x).
for all x, x is a person, x is a female
and x is a parent p(x) then, there exist ∃y. y is a person.
x is a mother of y.
m(x,y).

f(x) = x is a female.
p(x) = x is a parent
mother(x,y) = x is a mother of y.

$\forall x\,(f(x) \wedge p(x)) \rightarrow \exists y\, M(x,y)$.

Ex 12 p 52 "Everyone has exactly one best friend.".

for all x, x is a person. there exist y, y is a person,
if y is a friend of y, Then such that, there should not exist
z, x is a friend of z. ∧ y & z are different people.

$\forall x\, \exists y\; f(x,y) \rightarrow \exists_z f(x,z) \wedge (y \neq z)$.

x, y, z ∈ persons.
f(x,y) = x is a friend of y.

Ex 13:- p 52 there exist a women who has taken flight on every airline.

there exist x, x is a person, if x is women,
for all y, y is an airline, Then x has taken flight on y.

$\exists x\; W(x) \rightarrow \forall y f(x,y)$.

x ∈ person.
y ∈ Airlines.
W(x) = x is a women.
f(x,y) = x has taken flight on y.

p 54 Rules of Inference.

p1: p → q. "If you have a Correct password Then you can log onto Network."
p
q.

p2:- p. "you have a correct password".

∴ Then

c:- q. "you can log onto the network".

p	q	p → q
→ T	T	T →
T	F	F
→ F	T	T →
→ F	F	T →

Premises.

Conclus.

p1
p2
p3
:
pN
∴ c.

((p1 ∧ p2 ∧ p3 --- ∧ pN) → c) tautology.

((p → q) ∧ p) → q tautology.

p	q	p → q	(p → q) ∧ p	((p → q) ∧ p) → q.
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

RULES OF INFERENCE.

1- p, p → q, ∴ q. Modus Ponens.

2- ¬q, p → q, ∴ ¬p. Modus Tolens.

3- p → q, q → r, ∴ p → r. Hypothetical Syllogism.

4- p ∨ q, ¬p, ∴ q. Disjunctive Syllogism.

5- p, ∴ p ∨ q. Addition.

6- p ∧ q, ∴ p. p ∧ q, ∴ q. Simplification.

7- p, q, ∴ p ∧ q. Conjunction.

8- p ∨ q, ¬p ∨ r, ∴ q ∨ r. Resolution.

Ex3:- 62. "It is below freezing now"

therefore "It is below freezing or raining now".

Let p: It is below freezing.
q: It is raining.

$$\frac{p}{\therefore p \vee q}$$

It is Valid argument based on addition.

Ex6:- "It is not Sunny (p) this afternoon ($\wedge$) and it is Colder than yesterday" ($\neg p \wedge q$).

$r \rightarrow p$. "We will go Swimming (r) only if it is Sunny (p)".

$\neg r \rightarrow s$. "If we do not go Swimming ($\neg r$) then we will take a Canoe trip (s)".

$s \rightarrow t$. "If we take a Canoe trip (s) then we will be home by Sunset (t)".

$\therefore t$. leads to Conclusion. "We will be home by Sunset" (t).

P1:- $\neg p \wedge q$ ✓
P2:- $r \rightarrow p$. ✓
P3:- $\neg r \rightarrow s$. ✓
P4:- $s \rightarrow t$. ✓
C $\therefore t$.

(5) from (P1) $\neg p$ by Simplification. ✓
(6) from (5, P2) $\neg r$ by MT ✓
(7) from 6, P3 s by MP ✓

1-	p, $p \rightarrow q$, $\therefore q$	Modus Ponens.	5-	p, $\therefore p \vee q$	Addition.
2-	$\neg q$, $p \rightarrow q$, $\therefore \neg p$	Modus Tolens.	6-	$p \wedge q$, $\therefore p$ / $\therefore q$	Simplification.
3-	$p \rightarrow q$, $q \rightarrow r$, $\therefore p \rightarrow r$	Hypothetical Syllogism.	7-	p, q, $\therefore p \wedge q$	Conjunction.
4-	$p \vee q$, $\neg p$, $\therefore q$	Disjunctive Syllogism.	8-	$p \vee q$, $\neg p \vee r$, $\therefore q \vee r$	Resolution.

⑦ from 6, P3 s by MP ✓ 4. $p \wedge q$, $\neg p$, $\therefore q$ Disjunctive Syllogism. 8. $p \vee q$, $\neg p \vee r$, $\therefore q \vee r$. Resolution.

⑧ from ⑦, P4 t which Conclusion.