lec 10 = Sessional.

lec 11 :- RELATIONS.

Binary Relations :- It is a Subset of $A \times B$.
A and B are two Sets.

Q:- How many Relations on $A \times B$ if $|A| = 4$ & $|B| = 3$.
Ans:- Possible Relations. $= pow(A \times B) = 2^{|A| \times |B|} = 2^{4 \times 3} = 2^{12}$.

Ex4 :- 462

$A = \{1,2,3,4\}$ $A = \{1,2,3,4\}$

$R = \{(a,b) \mid a \text{ divides } b\}$. $A = \{1,2,3,4\}$

$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$.

$A \times A = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)\}$.

Set builder notation = $\{\underbrace{(a,b)}\overbrace{\in A \times A \mid}^{\text{syntax.}} \underbrace{a \geq b}_{\text{Semantics.}}\}$.

$\{\underline{a} \in A \mid a \geq 2\}$.

$\{A \in pow(A) \mid \underline{\qquad}\}$.

Ex5 :- 462

$R_1 = \{(a,b) \mid a \leq b\}$. $A = \{1,2,3,4\}$.

$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)\}$.

$R_2 = \{(a,b) \mid a > b\}$.
$= \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)\}$.

$R_3 - R_6$ Do it at home.

$R_5 = \{(a,b) \mid a = b+1\}$.
$= \{(2,1), (3,2), (4,3)\}$.

Ex6:- How many relations on a Set with n elements.
Sol:- Let A be the Set.

$|A| = n$. $R \subseteq A \times A$.

$pow(A \times A) = 2^{|A \times A|}$
$= 2^{|A| \times |A|}$
$= 2^{n \times n}$
$= 2^{n^2}$

خصوصیت

$= 2^{|A| \times |A|}$
$= 2^{n \times n}$
$= 2^{n^2}$

خصوصيات

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Properties of Relations.

1- Reflexive :- $\forall a \in A, \quad (a,a) \in R.$

Ex7 :- $A = \{1,2,3,4\}.$

$T\ (1,1) \in R \wedge$
$T\ (2,2) \in R \wedge$
$T\ (3,3) \in R \wedge$
$T\ (4,4) \in R.$

$R_1 = \{(1,1)\}$ ✗
$R_2 = \{(1,1), (2,1), (2,2), (1,2), (3,3)\}.$ ✗
$R_3 = \{(1,2), (2,1)\}.$ ✗

$R_4 = \{(1,1), (1,2), (2,2), (2,3), (3,3), (4,4)\}.$ ✓

$A \qquad R \subseteq A \times A.$

$2^{|A| \times |A|}.$

$\underline{\underline{A = \phi}} \qquad R \subseteq A \times A \qquad 2^{|A| \times |A|} = 2^{0 \times 0} = 2^0 = 1.$

$R = \{\phi\}.$

$R = \{\ \}. \qquad \forall a \in A \quad (a,a) \in R.$

$A = \{1\}.$

$R = \{(1,1)\}.$

$A = \{1,2\}.$ How many Reflexive Relations.

Ans :-
$R_1 = \{(1,1), (2,2)\}.$
$R_2 = \{(1,1), (2,2), (1,2)\}.$
$R_3 = \{(1,1), (2,2), (2,1)\}.$
$R_4 = \{(1,1), (2,2), (1,2), (2,1)\}.$

Think at home.

Session 1 :- Solution. (3).

Q1 :- P $+1.$ $\quad$ C1 :- P $+1.$

$P \to q.$ $\quad$ C2 :- $\neg P \vee q.$

Q1:- p +1.

$p \to q$

$q \to \neg r$

$\therefore \neg r$.

C1:- p +1.

C2: $\neg p \vee q$

C3: $\neg q \vee \neg r$.

C4: r.

C5: q from C1 & C2. +1.

C6: $\neg r$ from C5 & C3.

C7: $\square$ from C4 & C6

Q2:- $\forall x \forall y \ (P(x,y) \to \neg Q(x,y))$.

$= \neg(\forall x \forall y (P(x,y) \to \neg Q(x,y))$.

$= \neg \forall x \ \forall y \ (P(x,y) \to \neg Q(x,y))$.

$= \exists x \neg \forall y (P(x,y) \to \neg Q(x,y))$.

$= \exists x \ \exists y \ \neg(P(x,y) \to \neg Q(x,y))$.

$= \exists x \exists y \neg(\neg P(x,y) \vee \neg Q(x,y))$. DeMorgan's.

$= \exists x \exists y \ P(x,y) \wedge Q(x,y)$.

(b) $\exists x \forall y \ (P(x,y) \vee \neg Q(x,y))$.

$= \neg \exists x \forall y (P(x,y) \vee \neg Q(x,y))$.

$= \forall x \ \neg \forall y \ (P(x,y) \vee \neg Q(x,y))$.

$= \forall x \ \exists y \ \neg(P(x,y) \vee \neg Q(x,y))$ DeMorgans.

$= \forall x \exists y \ \neg P(x,y) \wedge Q(x,y)$.

Q3:- (i) $\neg \forall x \forall y \ P(x,y)$ $x,y \in \{1,2\}$.

$= \exists x \exists y \neg P(x,y)$. ✓

$= \exists x (\neg P(x,1) \vee \neg P(x,2))$.

$= \neg P(1,1) \vee \neg P(1,2) \vee \neg P(2,1) \vee \neg P(2,2)$.

(ii) $\exists x \ \neg \forall y \ P(x,y)$. $= \exists x \ \exists y \neg P(x,y)$. ✓.

Q4 :-

$p =$ B is a Knight. $\neg p = ?$ Knight = lies.
$q =$ A is a Knight. $\neg q = ?$ Knaves = Truth.

A Says " " $\rightarrow$ $P \wedge Q$.
B " " " $\rightarrow$ $\neg Q$.

−6.

X.

CASE 1 :- A Knight, B Knight.

$p = T$ $\neg p = F$
$q = T$ $\neg q = F$.

−9
−8.

$P \wedge Q = F$
$\neg Q = F$.

−3.

CASE 2 :- A Knight, B Knave.

$P \wedge Q = F$
$\neg Q = T$

$p = F$ $\neg p = T$
$q = T$ $\neg q = F$.

CASE 3 :- A Knave, B Knight.

$P \wedge Q = T$
$\neg Q = F$.

$p = T$ $\neg p = F$.
$q = F$ $\neg q = T$

CASE 4 : A Knave, B Knave.

$P \wedge Q = T$
$\neg Q = T$

$p = F$ $\neg p = T$
$q = F$. $\neg q = T$.

Q5 :- "If $\overbrace{\text{it is Sunny}}^{p}$ than $\overbrace{\text{I will not go to beach}}^{\neg q}$"

Contrapositive.

$p \rightarrow q$
$\neg q \rightarrow \neg p$.

Implication :- $q \rightarrow \neg p$.

Converse :- $\neg p \rightarrow q$.

Inverse :- $\neg q \rightarrow p$.

Contrapositive :- $p \rightarrow \neg q$.

b