lecture 17:- Sessional 2 Course "Relations". Lecture 19. Included.

Equivalence Relations.

1- Reflexive.
2- Symmetric.
3- Transitive.

Ex1 : P494 $R = \{(a,b) \mid a=b \text{ or } a=-b\}$ $A = \mathbb{Z}$. ✓.

Reflexive. $\forall a \in A$ $(a,a) \in R$.
$\forall a \in \mathbb{Z}$ $(a=a \text{ or } a=-a)$. ✓

Symmetric: $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.
$\forall a,b \in \mathbb{Z}$ if $a=b \text{ or } a=-b \rightarrow b=a \text{ or } b=-a$.

Transitive: $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

$\forall a,b,c \in \mathbb{Z}$ if $a=b \text{ or } a=-b \wedge b=c \text{ or } b=-c \rightarrow a=c \text{ or } a=-c$.

Ex2 P494 $R = \{(a,b) \mid (a-b) \in \mathbb{Z}\}$ $A = \mathbb{R}$.

Reflexive. $\forall a \in \mathbb{R}$ $(a,a) \in R$.
$\forall a \in \mathbb{R}$ $(a-a) \in \mathbb{Z}$. ✓

Symmetric: $\forall a,b \in \mathbb{R}$. if $(a,b) \in R \rightarrow (b,a) \in R$.
$\forall a,b \in \mathbb{R}$ if $(a-b) \in \mathbb{Z} \rightarrow (b-a) \in \mathbb{Z}$ ✓

Transitive: $\forall a,b,c \in \mathbb{R}$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

$\forall a,b,c \in \mathbb{R}$. if $(a-b) \in \mathbb{Z} \wedge (b-c) \in \mathbb{Z} \rightarrow (a-c) \in \mathbb{Z}$ ✓.

Hence Equivalence Relation.

Hence Equivalence Relation.

Ex 3:- $R = \{(a,b) \mid a \equiv b \mod m\}$ $m > 1$, $m \in Z^+$. $A = Z$.

Reflexive. $\forall a \in A$ $(a,a) \in R$.
$\forall a \in Z$ $a \equiv a \mod m$. ✓.

Symmetric:- $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$
$\forall a,b \in Z$ if $a \equiv b \mod m \rightarrow b \equiv a \mod m$ ✓.

Transitive:- $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

$\forall a,b,c \in Z$ if $a \equiv b \mod m \wedge b \equiv c \mod m$
$\rightarrow a \equiv c \mod m$.

$4 \equiv 8 \mod 2 \wedge 8 \equiv 64 \mod 2$.
$\rightarrow 4 \equiv 64 \mod 2$. ✓

✓ Rough work.
$(5,11) \in R$ $m = 3$.

$5 \equiv 11 \mod 3$.

$5 \div 3 = 1$ remainder 2; $11 \div 3 = 3$ remainder 2.

21-OCT-2022. 10:pm. EX4 & EX5 494. HW.

Ex 6:- 495 $R = \{(a,b) \mid a \text{ divides } b\}$. $A = Z^+$

Reflexive. $\forall a \in A$ $(a,a) \in R$.
$\forall a \in Z^+$ a divides a ✓

Symmetric:- $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.
$\forall a,b \in Z^+$ if a divides $b \rightarrow b$ divides a.
$(2,4) \in R \rightarrow (4,2) \notin R$. X.

$(2,4) \in R \rightarrow (4,2) \notin R$. $\wedge$.

Transitive : $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

$\forall a,b,c \in \mathbb{Z}^+$ if

Not Equivalence Relation.

Ex 7. $R = \{(a,b) \mid a \geq b\}$. $A = \mathbb{Z}$. Not Equivalence.

"	"	$a \neq b$	"	"	".
"	"	$a < b$	"	"	"
"	"	$a \leq b$	"	"	".

21-OCT-2022. H.W.

Ex 7 : 495 :- $R = \{(a,b) \mid |a-b| < 1\}$. $A = \mathbb{R}$.

Reflexive. $\forall a \in A$ $(a,a) \in R$.
$\forall a \in \mathbb{R}$ $|a-a| < 1$.

Symmetric : $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.
$\forall a,b \in \mathbb{R}$ if $|a-b| < 1 \rightarrow |b-a| < 1$. ✓

Transitive : $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

$\forall a,b,c \in \mathbb{R}$. if $|a-b| < 1 \wedge |b-c| < 1 \rightarrow |a-c| < 1$.
$|1-0.5| < 1 \wedge |0.5-0| < 1 \rightarrow |1-0| \not< 1$
X.

Not Equivalence Relation.

Equivalence Classes :-

$[a] = \{s \mid (a,s) \in R\}$. set builder Notation. $\{\_\_\_ \mid \_\_\_\}$.

$$[a] = \underbrace{\{ s \mid (a,s) \in R \}}_{7 \;=\; \{ \quad \}}.$$

$$\{ \underbrace{\quad\quad}_{\text{Syntax. type.}} \mid \underbrace{\quad\quad\quad}_{\text{Semantics. Condition.}} \}.$$

Ex8:- 496.

$R = \{ (a,b) \mid a = b \text{ or } a = -b \}.$ $A = \mathbb{Z}.$

$[7] = \{ 7, -7 \quad \}.$

Ex9:- $[0] = ?$ $[1] = ?$ $R = \{ (a,b) \mid a \equiv b \bmod 4 \}.$

$A = \mathbb{Z}.$

$[0] = \{ 0, 4, 8, 12, \ldots$
$-4, -8, -12, \ldots \}.$

$[1] = ?$

Partition Let $A_1, A_2, \ldots A_n$ be the family of sets belonging to $P.$ it will create a partition of set $P.$

(i) $\forall_i \; A_i \neq \Phi.$ ✓

(ii) $\forall_{i,j} \; A_i \cap A_j = \Phi.$

(iii) $\bigcup_{i=1}^{n} A_i = P.$

Ex13:- 499

$A_1 = \{1, 2, 3\}.$
$A_2 = \{4, 5\}.$
$A_3 = \{6\}.$

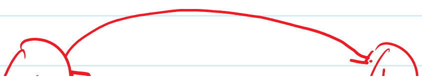
$S = \{1, 2, 3, 4, 5, 6\}.$

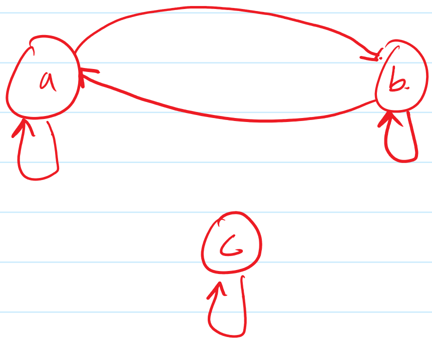
A_1, A_2, A_3 creates a partition?

$A_1 \cap A_2 = \Phi.$
$A_1 \cap A_3 = \Phi.$ ✓.
$A_2 \cap A_3 = \Phi.$

$A_1 \cup A_2 \cup A_3 = S.$ ✓.

The Equivalence classes always. Creates a partition.





$$\begin{array}{ccc} a & b & c \\ \hline \end{array}$$
$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

P 500 - 503. HW.

Ex. 1 - 30.