lecture 18:-

ER → EC → Partition.
" ← " ← ".

Ex 12 HW.
498

Ex 13 :- List the tuples in the Relation R produced by partition $A_1 = \{1, 2, 3\}$.
499
$A_2 = \{4, 5\}$
$A_3 = \{6\}$.

$S = \{1, 2, 3, 4, 5, 6\}$.

$R = \{ (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3),$
$(4,4), (4,5), (5,4), (5,5), (6,6)\}$. HW.

Ex 500 - 503.
1- 40 (HW.)

$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & - & - & - \\ 1 & 1 & 1 & - & - & - \\ 1 & 1 & 1 & - & - & - \\ - & - & - & 1 & 1 & - \\ - & - & - & 1 & 1 & - \\ - & - & - & - & - & 1 \end{bmatrix} \end{array}$$

Ex 35 :- [3] $R = \{(a,b) \mid a \equiv b \mod 5\}$.
501

$A = \mathbb{Z}$.

$[3] = \{3, 8, 13, 18, 23, \ldots\ldots$
$\underbrace{-2, -7, -12,}\ldots\ldots\}$. HW.

PARTIAL ORDER :- 1- Reflexive.

2- Anti Symmetric.
3- Transitive.

$(S, \preceq)$.

POSET.

Partial order. Set.

$(a,b) \in R$.
$a \preceq b$.

R, A.
(A, R).

$(a,b) \in R$.

Ex 6 : $R = \{(a,b) \mid a \le b\}$. $A = \mathbb{Z}$.
595

Reflexive $\forall a \in A$ $(a,a) \in R$.
$\forall a \in \mathbb{Z}$ $a \le a$ ✓.

Anti Symmetric $\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \to a = b$.
$\forall a,b \in \mathbb{Z}$ if $a \le b \wedge b \le a \to a = b$. ✓.

Transitive: $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \to (a,c) \in R$.
$\forall a,b,c \in \mathbb{Z}$ if $a \le b \wedge b \le c \to a \le c$. ✓.

$(S, \preceq) = (\mathbb{Z}, \le)$.

Ex 7: $R = \{(a,b) \mid a / b\}$. $A = \mathbb{Z}^+$.
505

Reflexive $\forall a \in A$ $(a,a) \in R$.
$\forall a \in \mathbb{Z}^+$ a / a ✓.

Anti Symmetric $\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \to a = b$.
$\forall a,b \in \mathbb{Z}^+$ if $a / b \wedge b / a \to a = b$. ✓.

Transitive :- $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

$\forall a,b,c \in \mathbb{Z}^+$ if $a/b \wedge b/c \rightarrow a/c$.. ✓.

$\leq, /, \underline{=, \geq}$ ✓.
HW.

$(<, >)$ Not PO.
HW.

$(S, \preceq) = (\mathbb{Z}, \leq)$.
$(\mathbb{Z}^+, /)$.
$(\mathbb{Z}^+, =)$.
$(\mathbb{Z}, \geq)$.

not PO.
$R = \{(a,b) \mid a$ & b has met $\}$.

Comparable :- $(S, \preceq)$. 'a' & 'b' $\in S$ are Comparable if either $a \preceq b$ or $b \preceq a$.
$(a,b) \in R$ or $(b,a) \in R$.

Exs :- $(\mathbb{Z}^+, /)$. 5, 7 are Comparable ?
3, 9 " " ?

Sol

$a \preceq b$ or $b \preceq a$

$5 \preceq 7$ or $7 \preceq 5$

$5/7$ or. $7/5$ 7, 5 are Not Comparable.

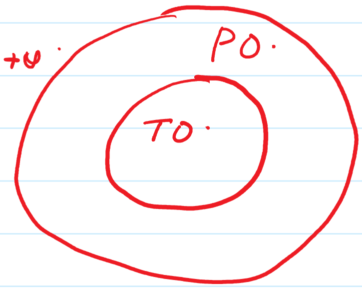
$3/9$ or $9/3$. 3, & 9 are Comparable.

Total Order Set :- $(S, \preceq)$.

if $\forall a,b \in S$ a and b are Comparable then $\preceq$ will be a total order Set.

Total order Examples. $\leq, \quad \geq$.

$-\infty \leq -\infty + 1 < \dots \; -1 \leq 0 \leq 1 \leq 2 \dots \leq +\infty$.

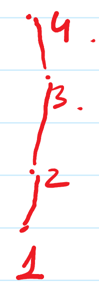


HASSE. DIAGRAM. 1898 – 1979.

$A = \{1, 2, 3, 4\}$.

$R = \{(a,b) \mid a \leq b\}$.

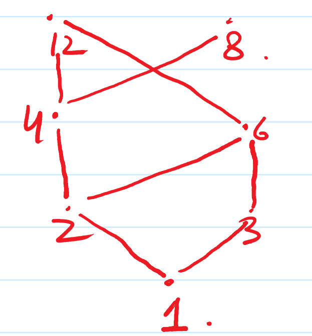
$= \{\cancel{(1,1)}, (1,2), (1,3), (1,4),$
$\cancel{(2,2)}, (2,3), (2,4),$
$\cancel{(3,3)}, (3,4),$
$\cancel{(4,4)}\}$.



Ex 12: P508

$R = \{(a,b) \mid a \text{ divides } b\}$. $A = \{1, 2, 3, 4, 6, 8, 12\}$.

$R = \{\cancel{(1,1)}, (1,2), (1,3), (1,4), (1,6), (1,8), (1,12),$
$\cancel{(2,2)}, (2,4), (2,6), (2,8), (2,12),$
$\cancel{(3,3)}, (3,6), (3,12),$
$\cancel{(4,4)}, (4,8), (4,12),$
$\cancel{(6,6)}, (6,12),$
$\cancel{(8,8)},$
$\cancel{(12,12)}\}$.

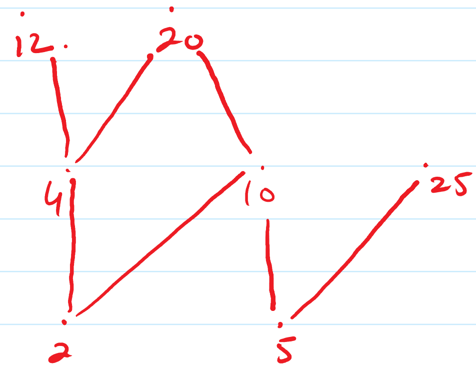


→ Same level. link.
→. two level. jump.

Ex 14 $(\{2, 4, 5, 10, 12, 20, 25\}, \mid)$.

Ex 14
509

$(\{\cancel{2}, 4, \cancel{5}, 10, \cancel{12}, \cancel{20}, \cancel{25}\}, \mid)$.



H.W.
find out original Relation from Hasse Diagram.