

Lecture 20:-

- 1- Greatest.
- 2- least
- 3- Maximal
- 4- Minimal

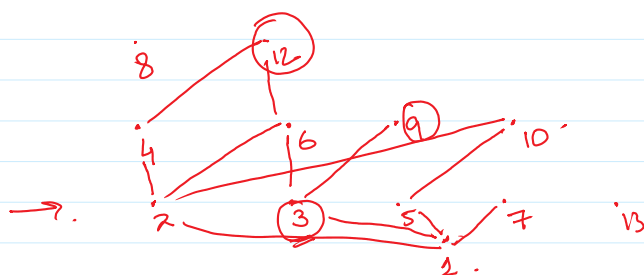
- 5- Upper Bound.
- 6- lower Bound.
- 7- least Upper.
- 8- Greatest lower.

Ex 20:-
511.

Greatest
least
Lower
Upper
Bound.

$\{3, 9, 12\}$ ✓
 $\{1, 2, 4, 5, 10\}$ HW.

$(\mathbb{Z}^+, 1)$.



$LB = \{3, 1\}$.

$GLB = \{3\}$

GCD.

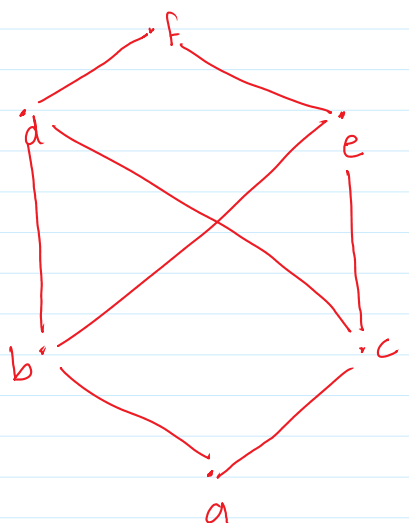
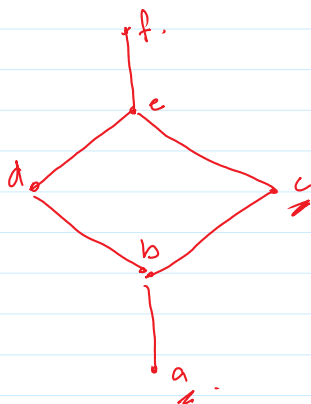
Upper Bound $\{12, 3, 12\}$.

UB $\{12, 3, 12\}$.

$= \{36, 72, 108, 144, \dots\}$.

LUB $= \{36\}$ LCM.

Lattice:- A Partial order in which every pair of elements has GLB & LUB.



UB $\{b, c\}$.

$= \{d, e, f\}$.

LUB $= \emptyset$.

Total order.

Lexicographic order:-

$$(A, \leq).$$

$$(A \times A, \leq).$$

$$A = \{1, 2\}.$$

$$A \times A \times A \times A = \{ \text{---, ---, ---, ---} \} \quad \text{H.W.}$$

$$|A| \times |A| \times |A| \times |A|.$$

$$= 2 \times 2 \times 2 \times 2 = 16.$$

$$\{0, \{(1, 1, 1, 1)\} \text{---} \}.$$

$$(A_1, \leq_1) \quad (A_2, \leq_2).$$

Lexicographic order. \leq , $A_1 \times A_2$.

$$a \leq b$$

$$(a, b) \in R.$$

$$(a_1, a_2) \leq (b_1, b_2).$$

$$\text{if } a_1 < b_1 \text{ or } a_1 = b_1 \wedge a_2 < b_2.$$

Ex 9:-
Prob.

$$(\mathbb{Z} \times \mathbb{Z}, \leq).$$

$$\leq = (< =).$$

$$(3, 5) < (4, 8) \quad \checkmark$$

$$\downarrow \downarrow \quad \downarrow \downarrow$$

$$a_1 \ a_2 \quad b_1 \ b_2.$$

$$(4, 9) < (4, 11).$$

$$(3, 8) < (4, 5).$$

$$(5, 3) < (4, 11).$$

$$(A, \leq), \quad (A, 1).$$

$$(3, 9) < (6, 12) \quad \text{H.W.}$$

$$(A_1 \times A_2 \times \dots \times A_n, \leq).$$

$$A_1 \times A_2 \times \dots \times A_n \times$$

$$A_1 \times A_2 \times \dots \times A_n.$$

$$(A_1, \leq_1), (A_2, \leq_2), \dots, (A_n, \leq_n),$$

$$A_1 \times A_2 \times A_3 \times \dots \times A_n.$$

$$(a_1, a_2, \dots, a_n) \leq (b_1, b_2, \dots, b_n).$$

$$\text{if } a_1 < b_1. \quad \text{or } \text{if } \exists i > 0 \quad a_1 = b_1, \dots, a_i = b_i \wedge a_{i+1} < b_{i+1}$$

$$i \geq 1.$$

Ex 10:-
p 506

$$(1, 2, 3, 5) \leq (1, 2, 4, 3).$$

$$\begin{array}{cccc} \downarrow & \downarrow & \downarrow & \downarrow \\ a_1 & a_2 & a_3 & a_4 \end{array} \quad \begin{array}{cccc} \downarrow & \downarrow & \downarrow & \downarrow \\ b_1 & b_2 & b_3 & b_4. \end{array}$$

$$a_1 = b_1 \quad a_2 = b_2 \quad a_3 < b_3.$$

Dictionary \rightarrow arrangement in lexicographic Order.

$$(A_1, \dots, A_n, \leq).$$

$$\underline{\text{dis}}\underline{\text{c}}\underline{\text{r}}\underline{\text{e}}\underline{\text{c}}\underline{\text{t}} \leq \underline{\text{dis}}\underline{\text{c}}\underline{\text{r}}\underline{\text{e}}\underline{\text{t}}.$$

z
c
b
a

W