Lecture # 16 :-

Page 481 - 482 Ex.

Closures :-

1. Reflexive :-

Ex :- $R = \{(1,1), (1,2), (2,1), (3,2)\}$ $A = \{1,2,3\}$.

$R = \{(1,1), (1,2), (2,1), (3,2)\} \cup \{(2,2), (3,3)\}$.

$\Delta = \{(a,a) \mid a \in A\}$.

$R \cup \Delta =$ Reflexive.

$= \{(1,1), (2,2), (3,3)\}$.

Ex1 :- 483.

$R = \{(a,b) \mid a < b\}$ $A = \mathbb{Z}$.

find reflexive closure.

$\Delta = \{(a,a) \mid a \in \mathbb{Z}\}. = \{(-\infty,-\infty) \dots (0,0) \dots (+\infty,+\infty)\}$.

$R \cup \Delta = \{(a,b) \mid a < b \vee a = b\}$.

$= \{(a,b) \mid a \leq b\}$. $\leq$ less than or equal to.

Closure wrt Symmetric.

$R^{-1} = \{(b,a) \mid (a,b) \in R\}$.

$R \cup R^{-1}$

$R = \{(1,2)\}. \cup \{(2,1)\}$.

$R \cup R^{-1}$ $\{(1,2), (2,1)\}$

Ex2 :-
P483

Ex :- 484

Closure.

A = {1, 2, 3, 4}.

$\cancel{b}$ $\cancel{c}$.

b c

a b.

(3,4) ?
iteration

PATH :-

(T U G)

Semantic
Set of Vertices.
" " Edges.

_ _ _ _ _ _

Ex3 :-
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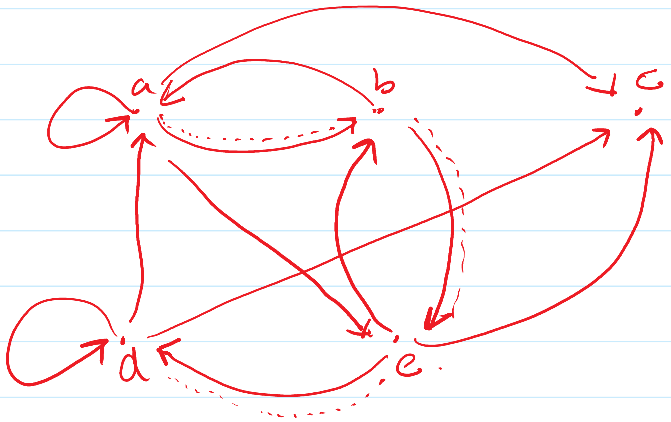
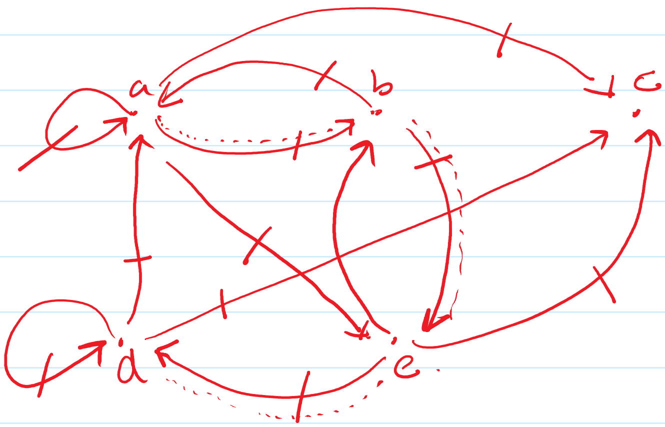


a to d. length of a path.
(a,b) (b,e), (e,d). 3
a, b, e, d. 3

Definition:- R is a relation on A.
$\exists$ a path of length n $n \in \mathbb{Z}^+$.
iff $(a,b) \in R^n$.



R = {(a,a), (a,b), (b,a)
(a,c), (a,e), (b,e),
(d,d), (d,a), (d,c),
(e,c), (e,d) (e,b)}.

$R \circ R = R^2$. HW.

Definition: R^* Connectivity Relation.

Definition:- R^* Connectivity Relation.

the Connectivity Relation Contains a tuple (a,b).
if $\exists$ atleast one path from a to b in R.

$$R^* = \bigcup_{i=1}^{\infty} R^i = R^1 \cup R^2 \cup R^3 \cdots \cup R^{\infty}.$$

Exh:- 485. $R = \{(a,b) \mid a \text{ has met } b\}$. A = Set of all People.
(Amjad, Kamran).
(— , —).

what is R^n.
" " R^*.

Solution:- $R^2 = R \circ R$.
$(a,b) \in R^2$ iff $\exists\, x_1$
Such That $(a, x_1) \in R \wedge$
$(x_1, b) \in R$.

R (a,b) $A \times B$
S (b,c). $B \times C$.
$(a,c) \in S \circ R$. $(a,b) \in R \wedge$
$(b,c) \in S$.

$R^3 = R^2 . R$.
$(a,b) \in R^3$ iff
$\exists\, x_1, x_2,$ $(a, x_1) \in R$
$(x_1, x_2) \in R$.
$(x_2, b) \in R$.
Then $(a,b) \in R^3$.

R (a, x_1) $A \times A$
R. (x_1, b). $A \times A$.
$(a,b) \in R \circ R$. $(a, x_1) \in R \wedge$
$(x_1, b) \in S$.

R^2 (a,b) $A \times B$
R. (b,c). $B \times C$.
$(a,c) \in S \circ R$. $(a,b) \in R \wedge$
$(b,c) \in S$.

$R^n = R^{n-1} . R$.
$(a,b) \in R^n$ iff
$\exists\, x_1, x_2, \ldots x_{n-1}$ Such That

a has met x_1 $(a, x_1) \in R$.
x_1 " " x_2 — —
x_2 " x_3 — —
— — .

- - .

x_{n-1} has b. - - - -

R^*. will contain (a,b) if a has met b.
or $\exists$ any number of people who met each other
in sequence with a being the first person in sequence
& b be the last.

Ex 6 :- $R = \{(a,b) \mid$ a and state b has common border $\}$ A = Set of all states in US.
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R^n

R^*.