Lecture 13: "Operations on Relations".

Ex 17: p465

$A = \{1,2,3\}$ $B = \{1,2,3,4\}$.

$R_1 = \{(1,1),(2,2),(3,3)\}$ $R_2 = \{(1,1),(1,2),(1,3),(1,4)\}$.

$R_1 \cup R_2 = \{(1,1),(2,2),(3,3),(1,2),(1,3),(1,4)\}$.

$R_1 \cap R_2 = \{(1,1)\}$.

$R_1 - R_2 = \{(2,2),(3,3)\}$.

$R_2 - R_1 = \{(1,2),(1,3),(1,4)\}$.

Ex 466–468 (Ex 1–30).

Ex 18 HW.

$\neg(=) = \neq$.
$\neg(\leq) = >$.
$\neg(\geq) = <$.
$\neg(<) = \geq$
$\neg(>) = \leq$

$A = \mathbb{R}$
$\mathbb{R} \times \mathbb{R}$.

Ex 19: $R_1 = \{(a,b) \mid a < b\}$. ✓

$R_2 = \{(a,b) \mid a > b\}$. ✓

$R_1 \cup R_2 = \{(a,b) \mid a<b \vee a>b\} = \{(a,b) \mid a \neq b\}$.

$R_1 \cap R_2 = \{(a,b) \mid a<b \wedge a>b\} = \emptyset$.

$R_1 - R_2 = \{(a,b) \mid a<b \wedge \neg(a>b)\}$.

$= \{(a,b) \mid a<b \wedge a \leq b\} = \{(a,b) \mid a \leq b\}$.

$R_2 - R_1 = \{(a,b) \mid a>b \wedge \neg(a<b)\} = \{(a,b) \mid a>b \wedge a \geq b\}$.
$= \{(a,b) \mid a \geq b\}$.

$R_1 \oplus R_2 = R_1 \cup R_2 - R_1 \cap R_2$.
$= \{(a,b) \mid a \neq b\} - \emptyset = R_1 \cup R_2 = \{(a,b) \mid a \neq b\}$.

Composite of a Relation.

A, B, C. sets.

R $(a,b) \in A \times B$.

S $(b,c) \in B \times C$.

R $\quad (a,b) \in A \times B$.

S $\quad (b,c) \in B \times C$

$S \circ R$.

$(a,c) \in S \circ R$ if $\exists\ (a,b) \in R \wedge (b,c) \in S$. Then $(a,c) \in S \circ R$.

Ex 20 / 465 : $A = \{1,2,3\}$. $B = \{1,2,3,4\}$ $C = \{0,1,2\}$.

$R = \{(1,1), (1,4), (2,3), (3,1), (3,4)\}$ $A \times B$.

$S = \{(1,0), (2,0), (3,1), (3,2), (4,1)\}$. $B \times C$.

$S \circ R = \{(1,0), (1,1), (2,1), (2,2), (3,0), (3,1)\}$. $S \circ R \neq R \circ S$.

$S \circ S = ?$ $\quad S^2$

$R^2 = R \circ R$. $\quad R^3 = R^2 \circ R$.

$R^4 = R^3 \circ R$.

$R^n = R^{n-1} \circ R$.

Theorem : A relation R on A, is transitive iff. $R^n \subseteq R$. $n = 1,2,3,\ldots$.

Special types of Relations.

$\bar{R} = \{(a,b) \mid (a,b) \notin R\}$. $= \{(a,b) \in A \times A \mid (a,b) \notin R\}$.

Ex : $A = \{1,2,3\}$, $A \times A = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$.

$R = \{(a,b) \mid a = b\} = \{(1,1), (2,2), (3,3)\}$.

$\bar{R} = \{(1,2), (1,3), (2,1), (2,3), (3,1), (3,2)\}$.

Ex : $\bar{R} = A \times A - R$.

$= |A| \times |A| - 5$

If $|A| = 100$.

$|R| = 5$.

Ex:- $K = AKA - K$.

$= |A| \times |A| - 5$

$= 100 \times 100 - 5 = 9995$.

If $|A| = 200$.

$|R| = 5$.

$|\bar{R}| = ?$

$A = \{1, 2, \ldots\ldots 200\}$.

$R = \{(a,b) \mid a > b\}$.

$\bar{R} = ?$

$R^{-1} = \{\overline{(a,b)} \mid \overline{(b,a) \in R}\}$. $= \{(b,a) \mid (a,b) \in R\}$.

$A = \{1,2,3,4\}$. 2^{16}.

$R = \{(1,2), (3,4), (2,2)\}$. $|R| = |R^{-1}|$.

$R^{-1} = \{(2,1), (4,3), (2,2)\}$.

$R = \{(a,b) \mid a > b\}$ $R^{-1} = ?$ $A = \{1,2,3,4\}$.

$R = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)\}$.

$R^{-1} = \{(1,2), (1,3), (2,3), (1,4), (2,4), (3,4)\}$.

$R^{-1} = \{(b,a) \mid (a,b) \in R\}$.

$= \{(b,a) \mid a > b\}$.

$a \quad b$.

$R^{-1} = \{(a,b) \mid b > a\}$. ✓

Quiz #7 7-10-2022.

$A = \{1,2\}$ $B = \{1,2\}$ $C = \{1,2\}$.

$R = \{(a,b) \mid a \div b\}$

$R = \{(a,b) \mid a \div b\}$.

$S = \{(a,b) \mid a = b\}$.

find S.R.