Lecture 4:-

1- $p \wedge T \equiv p$. Identity law.
$p \vee F \equiv p$

2- $p \vee T \equiv T$ Domination laws.
$p \wedge F \equiv F$

3- $p \vee p \equiv p$ Idempotent laws.
$p \wedge p \equiv p$.

4- $\neg(\neg p) \equiv p$ Double Negation

5- $p \wedge q \equiv q \wedge p$ Commutative.
$p \vee q \equiv q \vee p$.

6- $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$ Associative.
$p \vee (q \vee r) \equiv (p \vee q) \vee r$. "

7- $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$. Distributive.
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$.

8. $\neg(p \wedge q) \equiv \neg p \vee \neg q$ De Morgans.
$\neg(p \vee q) \equiv \neg p \wedge \neg q$.

Ex 6. Hw.

Predicates:- $p(x) = x + 4 = 8$. Predicate. $x \in \{1, 2, 3, 4\}$.

Subject

$\Rightarrow 1 + 4 = 8$ (F)
$\Rightarrow 2 + 4 = 8$ (F)
$\Rightarrow 3 + 4 = 8$ (F)
$\Rightarrow 4 + 4 = 8$ (T).

$p(x) = x + 4 > 10$.

subject predicate.

subject 'predicate.

Ex1 :- P31

let $p(x) = x > 3$. what are $p(4)$ & $p(3)$.

$p(4) = 4 > 3$ T

$p(3) = 3 > 3$ F.

Ex3 :- P31

$Q(x,y) = x = y+3$ $Q(1,2)$ & $Q(3,0)$.

$Q(1,2) = 1 = 2+3 = F$

$Q(3,0) =$ HW.

Ex2 :- P31 HW.

Ex4 :- let $A(c,n) =$ "Computer c is connected to network n"

$c \in$ { Computers on Campus }.

$n \in$ { Networks on Campus }.

→ Math1 is connected to CAMPUS2.

→ " is not " " " 1.

A(MATH1, CAMPUS1) = ? F

A(MATH1, CAMPUS2) = ? T.

Example 5 HW.

QUANTIFIERS :-

→ Universal :- $\forall$, for all for each, for every

$x \in \{1,2,3,4 \dots N\}$.

$$\forall_x P(x) = P(1) \wedge P(2) \wedge P(3) \wedge P(4) \wedge \dots \wedge P(N).$$

$\forall x P(x) = P(1) \wedge P(2) \wedge P(3) \wedge P(4) \wedge \cdots \wedge P(N).$

$= T$

→ Existential :- $\exists$ There exist. for some. atleast one.

$\exists x P(x) = P(1) \vee P(2) \vee P(3) \vee P(4) \vee \cdots \vee P(N).$

Ex8 :-
P33

$P(x) = x+1 > x \qquad x \in R.$

$\forall x P(x) = ?$ True.

Ex9
P34

$P(x) = x < 2 \qquad x \in R.$

$\forall x P(x) = 5 < 2$ (F).

F.

Ex10
P34

$P(x) = x^2 > 0 \qquad x \in Z.$

$x = 0$ False.

Ex11 :-
P34

$P(x) = x^2 < 10. \qquad x \in \{1, 2, 3, 4\}.$

$\forall x P(x) = ?$

$\forall x P(x) = P(1) \wedge P(2) \wedge P(3) \wedge P(4)$

$= (1^2 < 10) \wedge (2^2 < 10) \wedge (3^2 < 10) \wedge (4^2 < 10)$

$= T \wedge T \wedge T \wedge F$

$= F.$

Ex12 HW.

Ex13 :-
P34.

$\forall x (x^2 \geq x). \qquad x \in R.$

Let $P(x) = x^2 \geq x.$

$\forall x P(x) = \forall x (x^2 \geq x) \qquad (0.5)^2 \geq 0.5.$

$0.25 \geq 0.5$ (F).

False. $x = 0.5.$

Ex 15 :- $\exists x\, P(x)$ $P(x) = x + 1 = x$.
P34

false. $x \in \mathbb{R}$.

Ex 16–18 HW.

Negating Quantifiers. $\neg(p \land q) = \neg p \lor \neg q$.

$\neg(p_1 \land p_2 \land \cdots \land p_N) = \neg p_1 \lor \neg p_2 \lor \neg p_3 \cdots \lor \neg p_N$.

$\neg(p_1 \lor p_2 \lor \cdots \lor p_N) = \neg p_1 \land \neg p_2 \land \neg p_3 \cdots \land \neg p_N$.

$\forall x\, P(x) = P(1) \land P(2) \land P(3) \land \cdots \land P(N)$. $x \in \{1, 2, 3 \cdots N\}$.

Take Negation of both side.

$\neg(\forall x\, P(x)) = \neg(P(1) \land P(2) \land P(3) \land \cdots \land P(N))$.

$= \underbrace{\neg P(1)}_{P(1)} \lor \underbrace{\neg P(2)}_{P(2)} \lor \underbrace{\neg P(3)}_{P(3)} \lor \cdots \lor \underbrace{\neg P(N)}_{P(N)}$.

$= \exists x\, \neg P(x)$.

$\neg \forall x\, P(x) = \exists x\, \neg P(x)$.

$\exists x\, P(x) = P(1) \lor P(2) \lor P(3) \lor \cdots \lor P(N)$

Taking Negation on both Sides.

$\neg(\exists x\, P(x)) = \neg(P(1) \lor P(2) \lor P(3) \lor \cdots \lor P(N))$

$= \neg P(1) \land \neg P(2) \land \neg P(3) \land \cdots \land \neg P(N)$.

$= \forall x\, \neg P(x)$.

$\neg \exists x\, P(x) = \forall x\, \neg P(x)$.

Ex:- $\neg \forall x \overbrace{\exists y\, \forall z\, P(x, y, z)}^{P(x)} = \neg \forall x\, P(x)$.

$= \exists x\, \neg \exists y\, \underbrace{\forall z\, P(x, y, z)}_{P(y)}$. $= \neg \exists y\, P(y)$.

$$= \exists x \neg \exists y \underbrace{\forall z\, P(x,y,z)}_{P(y)}. \qquad = \neg \exists y\, P(y).$$

$$= \exists x\ \forall y\ \neg \forall z\ P(x,y,z).$$

$$= \exists x\ \forall y\ \exists z\ \neg P(x,y,z).$$

Ex $\neg \forall y \neg \exists z (\neg P(y,z)).$ $\qquad \neg \exists z\, P(z).$

$$= \neg \forall y\ \forall z\ \neg(\neg P(y,z)). \qquad = \forall z \neg P(z).$$

$$= \neg \forall y\ (\forall z\ P(y,z)).$$

$$= \exists y\ \neg \forall z\ P(y,z).$$

$$= \exists y\ \exists z\ \neg P(y,z).$$

Quiz . 1 :-

A Says " B is a Knight "

B Says " " .

C Says " I am a Knave".

p = C is a Knight $\qquad \neg p$ =

q = B is a Knight $\qquad \neg q$ =

r = A is a Knight $\qquad \neg r$. =

Knights . always Speak Truth.

" " lies.

s

we know that A is a Knight .

Find B, C .