lecture 12:- Relations.

$A \times B$.

$R \subseteq A \times B$.

$pow(A \times B) = 2^{|A \times B|} = 2^{|A| \times |B|}$

$|A| = 5 \quad |B| = 6$.

$2^{5 \times 6} = 2^{30}$.

reflexive:- $\underline{\forall a \in A}$ $\underline{(a,a) \in R}$. $A \times A$.

Ex:- $A = \{1,2,3,4\}$. $A \times A = \{(1,1), (1,2), \dots (4,4)\}$.

$R = \{(a,b) \mid a = b\} = \{(1,1), (2,2), (3,3), (4,4)\}$.

$\{ \underbrace{\quad}_{\text{syntax}} \mid \underbrace{\quad}_{\text{symentics}} \}$.

P462:- Is divides on $\underline{\text{set of +ve}}$ Integers. Reflexive?.
Ex9

$R = \{(a,b) \mid a \div b\}$. $Z^+ \times Z^+$.

$\forall a \in A \quad (a,a) \in R$.
$\forall a \in Z^+ \quad a$ divides a

✓

$\underset{a}{(6},\underset{b}{12)}$.

$(1,1)$
$(2,2)$
$(3,3)$
(∞,∞).

$x \in \{1,2, \cdots N\}$.
$\forall x P(x) = P(1) \land P(2) \land P(3) \land \dots \land P(N)$.

Symmetric:-
$\underline{\forall a,b \in A}$ if $(a,b) \in R \to (b,a) \in R$.

Ex11:- $A = \{1,2,3,4\}$.

$R_1 = \{\underset{a\ a}{(1,1)}, \underset{a\ b}{(1,2)}, \underset{a\ b}{(2,1)}, (2,2), \overset{X}{(3,4)}, (4,1), (4,4)\}$. X.

$R_2 = \{2$ ✓

$R_2 = \{\}$. ✓

$R_3 = \{(1,1)\}$. ✓

$R_4 = \{(3,4)\}$. X.

Anti Symmetric:- $\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a=b$.

$R_1 = \{(\overset{a}{1},\overset{b}{1}), (\overset{a}{1},\overset{b}{2}), (\overset{b}{2},\overset{a}{1}), (2,2), (3,4), (4,1), (4,4)\}$. X.

$R_2 = \{\}$ ✓

$R_3 = \{(1,1)\}$ ✓

$R_4 = \{(\underset{a}{3},\underset{b}{4})\}$. ✓

$R_5 = \{(1,1),(2,2)\}$.

$R_6 = \{(1,2),(3,1),(4,2)\}$. ✓

Ex12

463:- Is divides on Set of +ve Integers. Symmetric? AntiSymmetric.

Symmetric: $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.

$A = Z^+$

$\forall a,b \in Z^+$ if a divides b $\rightarrow$ b divides a.

6 divides 12 $\rightarrow$ 12 divides 6 X.

Antisymmetric:- $\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a=b$.

$\forall a,b \in Z^+$ if a divides b $\wedge$ b divides a $\rightarrow$ a=b.

✓

$R = \{(a,b) \mid a \geq b\}$. $A = Z^+$.

Symmetric: $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.

$\forall a,b \in Z^+$ if $a \geq b \rightarrow b \geq a$.

$\forall a,b \in \mathbb{Z}^+$ if $a \geq b \rightarrow b \geq a$.
$5 \geq 4 \rightarrow 4 \geq 5$. X.

Anti Symmetric: $\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$.

$\forall a,b \in \mathbb{Z}^+$ if $a \geq b \wedge b \geq a \rightarrow a = b$.
$5 \geq 5 \wedge 5 \geq 5 \rightarrow 5 = 5$.

Transitive: $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

Ex: $A = \{1,2,3,4\}$.

$R_1 = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)\}$. X.

a b . b c.

$R_2 = \{\}$. ✓
$R_3 = \{(1,2)\}$. ✓

$R_4 = \{(1,2), (3,4)\}$. ✓

Quiz #5 04-10-2022.

$A = \{1\}$.

How many Relations defined on the Set Above are

1- Reflexive. (find).
2- Symmetric. (find)
3- AntiSymmetric. (find).
4- Transitive (find).

$\forall a \in A \quad (a,a) \in R$.
$(1,1) \in R$.

$A \times A = \{(1,1)\}$.

X R ✓ R

$pow(A \times A) = \{\phi, \{(1,1)\}\}$.

S ✓ S ✓.

AS. AS.
✓ ✓
T T
✓ ✓

Quiz # 6. 04-10-2022.

→ find all reflexive from the power set of $A \times A$

$A = \{a, b\}$.

$A \times A = \{(a,a), (a,b), (b,a), (b,b)\}$.

$Pow(A \times A) = \{\phi, \{(a,a)\}, \{(a,b)\}, \{(b,a)\},$
$\{(b,b)\}, \{(a,a),(a,b)\},$
$\{(a,a),(b,a)\}, \{(a,a),(b,b)\},$
$\{(a,b),(b,a)\}, \{(a,b),(b,b)\},$
$\{(b,a),(b,b)\},$
$\{(a,a),(a,b),(b,a)\},$
$\{(a,a),(a,b),(b,b)\},$
$\{(a,b),(b,a),(b,b)\}.$
$\{(a,a),(b,a),(b,b)\}.$
$\{(a,a),(a,b),(b,a),(b,b)\}\}.$