$-\frac{5}{3}+\frac{10}{3}(5)$

~~Changes in the right hand side of Constraints ⇒~~

(II) Changes in the objective function coefficients ⇒

Example: Maximize $Z = 10X_1 + 15X_2 + 20X_3$

S/t

$$2X_1 + 4X_2 + 6X_3 \leq 24$$
$$3X_1 + 9X_2 + 6X_3 \leq 30.$$
$$X_1, X_2 \ \& \ X_3 \geq 0.$$

Solution: In equation form

$$Z - 10X_1 - 15X_2 - 20X_3 = 0$$

S/t

$$2X_1 + 4X_2 + 6X_3 + S_1 = 24$$
$$3X_1 + 9X_2 + 6X_3 + S_2 = 30$$
$$X_1, X_2, X_3, S_1, S_2 \geq 0.$$

The starting table is

CBi	Basic	X_1	X_2	↓ entering X_3	S_1	S_2	Solution	Ratios
	Z	-10	-15	-20	0	0	0	
0	leaving ← S_1	2	4	6	1	0	24	4
0	S_2	3	9	6	0	1	30	5

$3-6(\frac{1}{3})$ $\frac{1}{6} \times \frac{1}{3}$

$9-6(\frac{2}{3})$ $30-6(4)$

$\frac{1+3}{6}$ $\frac{?}{6}$

$-10+\frac{20}{3}$ $-15+20(\frac{2}{3})$

$4/(\frac{1}{3})$ $0+\frac{20}{6}=\frac{10}{3}$ $0+20(4)$

	Basic	X_1 (entering)	X_2	X_3	S_1	S_2	Solution	Ratios
C_{Bi}	Z	$-10/3$	$-5/3$	0	$10/3$	0	80	
20	X_3	$1/3$	$2/3$	1	$1/6$	0	4	12
0	S_2 (leaving)	1	5	0	-1	1	6	6

	Basic	X_1	X_2	X_3	S_1	S_2	Solution
C_{Bi}	Z	0	15	0	0	$10/3$	100
20	X_3	0	-1	1	$1/2$	$-1/3$	2
10	X_1	1	5	0	-1	1	6
	$C_j - Z_j$	0	-15	0	0	$-10/3$	

The last table is optimal as all of the coefficient in Z-row are non-negative.

(a) Find the range of the objective function coefficient C_1 of the variable X_1 such that the optimality is unaffected.

(b) Find the range of the objective function coefficient C_2 of the variable X_2 such that the optimality is unaffected.

(c) Check whether the optimality is affected, if the profit coefficients are changed from (10, 15, 20) to (7, 14, 15). If so, find the revised optimum solution.

<u>Solution:</u> After making some changes in the objective function coefficients, if the optimality is not affected, then the present set of basic variables will continue and in that case the $C_j - Z_j$ values of the basic variables will be equal to zero, however, the $C_j - Z_j$ values of the nonbasic variables will change. Hence, care should be taken in establishing the range of C_j values such that the corresponding $C_j - Z_j$ of non-basic variables

are limited to at most zero. **

(a) Since X_1 is a basic variable & C_1 is the coefficient of X_1, if the optimality is not affected, then the present ~~set~~ value of this basic variables will ~~change~~ continue and ~~C_1-Z_1~~ C_1-Z_1 of the basic variable X_1 will equal to zero.
However, $C_j - Z_j$ of X_2, S_1, S_2 will change.
These $C_j - Z_j$ values can be computed in terms of C_1 as follows:

$$C_2 - Z_2 = 15 - [20 \quad C_1]\begin{bmatrix} -1 \\ 5 \end{bmatrix} = 15 - (-20 + 5C_1)$$

$$\Rightarrow C_2 - Z_2 = 35 - 5C_1$$

Now $$C_4 - Z_4 = 0 - [20 \quad C_1]\begin{bmatrix} \frac{1}{2} \\ -1 \end{bmatrix} = -10 + C_1$$

& $$C_5 - Z_5 = 0 - [20 \quad C_1]\begin{bmatrix} -\frac{1}{3} \\ 1 \end{bmatrix} = \frac{20}{3} - C_1.$$

To maintain optimality, these $C_2 - Z_2$, $C_4 - Z_4$ & $C_5 - Z_5$ can be ~~taken to~~ restricted to at most zero. Thus

$$35 - 5C_1 \leq 0 \Rightarrow C_1 \geq 7$$

$$-10 + C_1 \leq 0 \Rightarrow C_1 \leq 10$$

$$\frac{20}{3} - C_1 \leq 0 \Rightarrow C_1 \geq \frac{20}{3}.$$ Hence

C_1 belongs to $[7, 10]$, i.e $7 \leq C_1 \leq 10$.
In this interval the optimality is not affected.

(b) Determination of the range of C_2 of the non-basic variable $X_2 \Rightarrow$

Since C_2 corresponds to one of the non-basic variable X_2, the range of C_2 can be obtained by just restricted $C_2 - Z_2$ to at most 0. Therefore,

$$C_2 - Z_2 = C_2 - \begin{bmatrix} 20 & 10 \end{bmatrix} \begin{bmatrix} -1 \\ 5 \end{bmatrix}$$

$$\Rightarrow C_2 - 30 \leq 0 \text{ or } C_2 \leq 30.$$

Hence, it is clear that the optimality will remain the same as long as the value of C_2 is less than or equal to 30.

(c) Checking the optimality ⇒

The new values of the objective function coefficients C_1, C_2 & C_3 of the variable X_1, X_2, X_3 are 7, 14, and 15, respectively. Thus

$$C_1 - Z_1 = 7 - \begin{bmatrix} 15 & 7 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0$$

$$C_2 - Z_2 = 14 - \begin{bmatrix} 15 & 7 \end{bmatrix} \begin{bmatrix} -1 \\ 5 \end{bmatrix} = -6$$

$$C_3 - Z_3 = 15 - \begin{bmatrix} 15 & 7 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0$$

$$C_4 - Z_4 = 0 - \begin{bmatrix} 15 & 7 \end{bmatrix} \begin{bmatrix} 1/2 \\ -1 \end{bmatrix} = -\frac{1}{2}$$

$$C_5 - Z_5 = 0 - \begin{bmatrix} 15 & 7 \end{bmatrix} \begin{bmatrix} -1/3 \\ 1 \end{bmatrix} = -2.$$

Since all $C_j - Z_j$ values are less than or equal to zero (all the new coefficients in Z-row of optimal table are non-negative), thus the optimality is unaffected.