Sensitivity analysis ⇒ In many situations, the parameters and characteristics of a linear programming model may change over a period of time. Also the analyst may be interested to know the effect of changing the parameters and characteristics on the optimality. This kind of sensitivity analysis can be carried out in the following ways:

(i) Making changes in the right-hand side constants of the constraints

(ii) Making changes in the objective function coefficients

(iii) Adding a new constraint

(iv) Adding a new variable.

Changes in the right hand side constants of constraints ⇒

Example:

Maximize $Z = 6X_1 + 8X_2$

Subject to

$$5X_1 + 10X_2 \le 60$$
$$4X_1 + 4X_2 \le 40$$
$$X_1 \,\&\, X_2 \ge 0.$$

$-6+\frac{8}{2}$

~~48~~ 60

$0-4(1/10) = -2/5$

Sol 1.

Maximize: $z-6X_1-8X_2=0$ or $z-6X_1-8X_2+0S_1+0S_2=0$

s/t $5X_1+10X_2+S_1=60 \longrightarrow (1)$

$4X_1+4X_2+S_2=40 \longrightarrow (2)$

$X_1, X_2, S_1, S_2 \geq 0$.

The starting tableau is

CBi	Basic	X_1	X_2 (entering)	S_1	S_2	Solution	Ratios
	z	-6	-8	0	0	0	
0	S_1 (leaving)	5	10	1	0	60	6
0	S_2	4	4	0	1	40	10

P.C

CBi	Basic	X_1 (entering)	X_2	S_1	S_2	Solution	Ratios
	z	-2	0	$4/5$	0	48	
8	X_2	$1/2$	1	$1/10$	0	6	$\frac{6}{(1/2)} = 6\times2=12$
0	S_2 (leaving)	2	0	$-2/5$	1	16	$16/2 = 8$

P.C

$\frac{4}{5}+2(-\frac{1}{5})$

$\frac{1}{10}+\frac{1}{20}$

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$\frac{-(1/5)}{6}$ $\frac{1}{2}(-\frac{1}{5})$

CBi	Basic	X_1	X_2	S_1	S_2	Solution
	Z	0	0	$\frac{2}{5}$	1	64
8	X_2	0	1	$\frac{1}{5}$	$-\frac{1}{4}$	2
6	X_1	1	0	$-\frac{1}{5}$	$\frac{1}{2}$	8

*Table A

(a) If the right hand side constants of constraint (1) & constraint (2) are changed from 60 & 40 to 40 & 20, respectively. Determine the optimal solution associated with these changes.

Solution (a) The revised ~~right hand side constants~~ solution of the problem after incorporating the changes in the constraints can be obtained by using the formula

Basic variables in the optimal tableau = [Technological coefficient columns in the optimal tableau w.r.t basic variables in initial tableau] [New R.H.S constants]

Applying the formula we have

$$\begin{bmatrix} X_2 \\ X_1 \end{bmatrix} = \begin{bmatrix} 1/5 & -1/4 \\ -1/5 & 1/2 \end{bmatrix} \begin{bmatrix} 40 \\ 20 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Since $X_2 = 3$ & $X_1 = 2$ are non-negative, so the revised solution is ~~positive~~ feasible and optimal. The corresponding objective function value is $Z = 6(2) + 8(3) = 36$.

(b) If the right hand side constants of the constraint are changed from 60 & 40 to 20 & 40, respectively.

<u>Solution</u> ⇒ The revised solution of the basic variables in last tableau after incorporating the changes in the right hand side values of the constraints are obtained as

$$\begin{pmatrix} X_2 \\ X_1 \end{pmatrix} = \begin{pmatrix} 1/5 & -1/4 \\ -1/5 & 1/2 \end{pmatrix} \begin{pmatrix} 20 \\ 40 \end{pmatrix} = \begin{pmatrix} -6 \\ 16 \end{pmatrix}$$

$$Z = 6(-6) + 8(16)$$

$6(16)+8(-6)$

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Since the value of X_2 is negative, the solution is infeasible. This infeasibility can be removed from dual simplex method.

Based on tablea A, the tablea for the dual simplex method can be constructed as

CBi	Basic	X_1	X_2	S_1	S_2 ↓entering	Solution
	2	0	0	$2/5$	1	48
8	X_2 ← leaving	0	1	$1/5$	$-1/4$	~~2~~ -6 → pivot row
6	X_1	1	0	$-1/5$	$1/2$	~~8~~ 16
	Ratios	–	–	–	-4 pivot column	

$\Rightarrow |-4| = 4$.

CBi	Basic	X_1	X_2	S_1	S_2	Solution
	2	0	4	$6/5$	0	24
0	S_2	0	-4	$-4/5$	1	24
6	X_1	1	2	$1/5$	0	4

(64)

Since all the solution values are non-negative, so the solution is feasible & hence optimal. Therefore the optimum result is

$X_1 = 4, \quad X_2 = 0, \quad S_1 = 0, \quad S_2 = 24$

& $Z = 6(4) = 24$ (Optimum).