(III) Adding a new constraint ⇒

Sometimes a new constraint may be added to an existing linear programming model as per changing realities. Under such situations each of the basic variables in the new constraint is substituted with the corresponding expression based on the current optimal table. This will yield a modified version of a new constraint in terms of only the current non-basic variables.

* If the new constraint is satisfied by the values of the current basic variables, the constraint is said to be redundant one. Hence, the optimality of the original problem will not be affected even after the inclusion of a new constraint in the existing model.

** If the new constraint is not satisfied by the current values of the basic variables the optimality of the original problem will be affected. So the modified version of the new constraint is to be augmented into the optimal table of the original problem and iterated till the optimality is reached.

Examples

Maximize : $Z = 6X_1 + 8X_2$

S/t

$$5X_1 + 10X_2 \le 80$$

$$4X_1 + 4X_2 \le 40$$

$$X_1, X_2 \ge 0.$$

The optimal table of the aforementioned example is produced as follows:

CBi	Basic	X_1	X_2	S_1	S_2	Solution
	Z	0	0	2/5	1	64
8	X_2	0	1	1/5	-1/4	2
6	X_1	1	0	-1/5	1/2	8

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(a) Check whether the addition of the constraint $7X_1+2X_2 \leq 65$ affects the optimality. If it does, find the new optimum solution.

(b) Check whether the addition of the constraint $6X_1+3X_2 \leq 48$ affects the optimality. If it does, find the new optimum solution.

Solution: (a) The new constraint is

$$7X_1+2X_2 \leq 65.$$

The current optimum solution is $Z=64$, $X_2=2$, $X_1=8$.

Now $7(8)+2(2) = 60 \leq 65$, i.e., X_2, X_1 satisfy the new constraint. Hence new constraint is redundant and the optimality will not be affected even after including the new constraint into existing model.

(5) The new constraint is $6X_1+3X_2 \leq 48$

Now $6(8)+3(2) = 54 > 48$. Thus, this constraint is not ~~affected~~ satisfied by the values of current basic variables, i.e., $X_2=2, X_1=8$.

So the modified form of the new constraint $6X_1+3X_2 \leq 48$ in terms of non-basic variables should be obtained.

Inclusion of slack variable S_3 in $6X_1+3X_2 \leq 48$ yields

$$6X_1+3X_2+S_3 = 48. \longrightarrow (i)$$

From the optimal table we can write the expressions corresponding to X_1 & X_2 rows as

$$X_2+\frac{1}{5}S_1-\frac{1}{4}S_2 = 2 \rightarrow (ii)$$

$$X_1-\frac{1}{5}S_1+\frac{1}{2}S_2 = 8 \rightarrow (iii)$$

Putting (ii) & (iii) in (i) we have

$$\frac{3}{5}S_1 - \frac{9}{4}S_2 + S_3 = -6. \longrightarrow \text{(iii)}$$

Now by including the above constraint in optimal table we can write

C_{Bi}	Basic	X_1	X_2	S_1	S_2 (entering)	S_3	Solution
	Z	0	0	$2/5$	1	0	64
8	X_2	0	1	$1/5$	$-1/4$	0	2
6	X_1	1	0	$-1/5$	$1/2$	0	8
0	leaving ← S_3	0	0	$3/5$	$-9/4$	1	-6
ratios		-	-		$\lvert -4/9 \rvert = 4/9$.		

One can see that the solution is infeasible b/c S_3-row contains negative right hand side constant. This infeasibility can be removed through dual simplex method.

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C_{B_i}	Basic	X_1	X_2	S_1	S_2	S_3	Solution
	Z	0	0	$10/15$	0	$4/9$	$184/3$
8	X_2	0	1	$2/15$	0	$-1/9$	$8/3$
6	X_1	1	0	$-1/15$	0	$2/9$	$20/3$
0	S_2	0	0	$-4/15$	1	$-4/9$	$8/3$

Since the infeasibility is removed and also the solution is optimal, hence,

$X_1 = \frac{20}{3}, X_2 = 8/3, S_2 = 8/3, S_1 = 0, S_3 = 0,$

& $Z(\text{optimum}) = \frac{184}{3}.$

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(IV) Adding a new Varible ⇒