Dual Simplex Method ⇒

Dual simplex method is a specialized form of simplex method in which optimality is ~~obtained~~ maintained in all iterations. Initially the solution may not be feasible. Successive iterations will remove the infeasibility. If the problem is feasible in an iteration then the procedure will be stopped b/c the solution obtained is feasible and optimal at that stage.

This method is demonstrated through the following numerical problem.

Example: Solve the following problem using dual simplex method.

Minimize $Z = 2X_1 + 4X_2$

S/t

$$2X_1 + X_2 \geq 4$$
$$X_1 + 2X_2 \geq 3$$
$$2X_1 + 2X_2 \leq 12$$
$$X_1, X_2 \geq 0.$$

Solution first, we will convert the constraint into ($\leq$) type wherever necessary, as ~~shown below~~ follows:

Minimize $Z = 2X_1 + 4X_2$

S/t

$$-2X_1 - X_2 \leq -4$$

$$-X_1 - 2X_2 \leq -3$$

$$2X_1 + 2X_2 \leq 12$$

$$X_1, X_2 \geq 0.$$

The canonical form can be obtained as follows:

Minimize $Z = 2X_1 + 4X_2$

$$-2X_1 - X_2 + S_1 = -4$$

$$-X_1 - 2X_2 + S_2 = -3$$

$$2X_1 + 2X_2 + S_3 = 12$$

$$X_1, X_2, S_1, S_2, S_3 \geq 0.$$

The starting tableau is

C_{Bi}	Basic	X_1 ↓entering	X_2	S_1	S_2	S_3	Solution
	z	-2	-4	0	0	0	0
0	$\boxed{S_1}$ leaving	-2 Pivot entry	-1	1	0	0	-4 most negative
0	S_2	-1	-2	0	1	0	-3
0	S_3	2	2	0	0	1	12

The above tablea clearly shows that the problem is optimal as no coefficient in the z-row is positive. But some of the values under the solution column are negative. These negative values are retaining the infeasibility of the solution. The infeasibility can be removed from the usage of following conditions.

<u>Feasibility condition</u> ⇒ The leaving variable is the variable which is having the most negative value (break ties arbitrarily). If all the basic variable have positive or zero values, then the feasible optimal solution is reached. Hence the procedure ends here.

Optimality condition ⇒ The entering variable can be selected from among the non basic variables as follows:

① Find the ratios of the coefficient in Z-row to the corresponding left side coefficients of the row associated with the leaving variable. Ignore the ratios associated with positive or zero denominators.

② The entering variable is the one with the smallest ratio if the problem is minimization, or the smallest absolute value of the ratios if the problem pertains to maximization. (break ties arbitrarily). If all the denominators are positive or zero, then the problem has no feasible solution.

Since in the previous table, S_1 has most negative value on the right side, thus S_1 is leaving variable. The entering variable can be obtained as

Variables	X_1	X_2	S_1	S_2	S_3
z	-2	-4	0	0	0
S_1	-2	-1	1	0	0
Ratio	1	4	—	—	—

As the problem is of minimization type, entering variable is the one which has smallest ratio, and it corresponds to X_1. So X_1 is entering.

C_{Bi}	Basic	X_1	X_2	S_1 ↓entering	S_2	S_3	Solution
	z	0	-3	-1	0	0	4
2	X_1	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	2
0	S_2 ← leaving	0	$-\frac{3}{2}$	$-\frac{1}{2}$	1	0	-1
0	S_3	0	1	1	0	1	8
	Ratios	—	$\frac{-3}{\left(-\frac{3}{2}\right)} = 2$	2	—	—	

Pivot column.

Since the problem is concerned with minimization, the entering variable is the one with a smallest positive ratio. The smallest ratio is 2 corresponding to X_2 & S_1. By breaking ties randomly, $\underline{S_1}$ is selected as entering variable. Therefore

C_{Bi}	Basic	X_1	X_2	S_1	S_2	S_3	Solution
	Z	0	0	0	-2	0	6
2	X_1	1	2	0	-1	0	3
0	S_1	0	3	1	-2	0	2
0	S_3	0	-2	0	2	1	6

Since all the solution values are non-negative, so they are feasible and hence optimal as well.

Therefore the results are

$$X_1 = 3, \quad X_2 = 0, \quad S_1 = 2 \ \& \ S_2 = 6.$$

& $Z = 2(3) + 4(0) = 6$. (Optimum value of objective function).