$$\frac{3}{5}S_1 - \frac{9}{4}S_2 + S_3 = -6. \longrightarrow \text{(iii)}$$

Now by including the above constraint in optimal table we can write

C_B	Basic	X_1	X_2	S_1	S_2 (entering)	S_3	Solution
	Z	0	0	$2/5$	1	0	64
8	X_2	0	1	$1/5$	$-1/4$	0	2
6	X_1	1	0	$-1/5$	$1/2$	0	8
0	leaving $\leftarrow S_3$	0	0	$3/5$	$-9/4$	1	-6
ratios		-	-		$\lvert -4/9 \rvert = 4/9$		

One can see that the solution is infeasible b/c S_3-row contains negative right hand side constant. This infeasibility can be removed through dual simplex method.

$-\frac{1}{5}-\frac{1}{2}\left(\frac{-4}{15}\right)$ $\quad -3+\frac{2}{15}$ $\quad -\frac{1}{2}\left(\frac{-4}{9}\right)$ $\quad 64-(1)\left(\frac{8}{3}\right)$ $\quad \frac{1}{4}\left(-\frac{4}{9}\right)$

$\frac{1}{5}+\frac{1}{4}\left(-\frac{4}{15}\right)$ $\quad 8-\frac{1}{2}\left(\frac{8}{3}\right) = 8-\frac{4}{3} = \frac{24-4}{3}$

C_{B_i}	Basic	X_1	X_2	S_1	S_2	S_3	Solution
	Z	0	0	$10/15$	0	$4/9$	$184/3$
8	X_2	0	1	$2/15$	0	$-1/9$	$8/3$
6	X_1	1	0	$-1/15$	0	$2/9$	$20/3$
0	S_2	0	0	$-4/15$	1	$-4/9$	$8/3$

Since the infeasibility is removed and also the solution is optimal, hence,

$X_1 = \frac{20}{3}, X_2 = 8/3, S_2 = \frac{8}{3}, S_1 = 0, S_3 = 0,$

& $Z(\text{optimum}) = \frac{184}{3}$.

(IV) Adding a new variable ⇒

In a problem the product mix problem, over a period of time, a new product may be added to the existing product mix. Under such situation one will ~~find~~ be interested in finding the revised optimal solution from the optimal table of the original problem.

In this analysis, the following items are to be determined after incorporating the data of the new variable (new product).

The $C_j - Z_j$ value ⇒

$$C_j - Z_j = C_j - [CB]_{1\times m} \begin{bmatrix} \text{Technological coefficients of} \\ \text{optimal table w.r.t basic} \\ \text{Variables of the initial table} \end{bmatrix}_{m\times m} \times \begin{bmatrix} \text{Constraint coefficients} \\ \text{of new variable} \end{bmatrix}_{m\times 1},$$

where m is the number of constraints in the problem. If the $C_j - Z_j$ ~~values~~ of the new variable indicates the optimality as per nature of the optimization (maximization or minimization), the optimality of the problem after including the new variable is not affected. Otherwise, the constraint coefficients (technological coefficients) of the new variable are to be computed.

The constraint coefficients (technological coefficients) of the column corresponding to the new variable can be given by formula:

$$\begin{bmatrix} \text{Revised constraint} \\ \text{coefficients of the} \\ \text{new variable} \end{bmatrix}_{m\times 1} = \begin{bmatrix} \text{Technological coefficients of the} \\ \text{optimal table w.r.t basic} \\ \text{variables of the initial table} \end{bmatrix}_{m\times m}$$

$$\times \begin{bmatrix} \text{Constraint coefficients of} \\ \text{the new variable} \end{bmatrix}_{m\times 1}. \rightarrow (A2)$$

Note: These coefficients are incorporated in the current optimal table and the necessary number of iterations is carried out from the current table till the optimality has been reached.

Example: Maximize $Z = 6X_1 + 8X_2$,

Subject to
$$5X_1 + 10X_2 \le 60$$
$$4X_1 + 4X_2 \le 40$$
$$X_1 \ \& \ X_2 \ge 0.$$

The optimal table is

C_{B_i}	Basic	X_1	X_2	S_1	S_2	Solution
	Z	0	0	$2/5$	1	64
8	X_2	0	1	$1/5$	$-1/4$	2
6	X_1	1	0	$-1/5$	$1/2$	8

* A new product P_3 is included in the existing product mix. The profit per unit of the new product is Rs. 20. The processing requirements of the new product on lathe & milling machines are 6 hrs/unit & 5 hrs/unit, respectively.

(a) Check whether the inclusion of the product P_3 changes optimality.

(b) If it changes the optimality, find the revised optimal solution.

Solution:- The LP problem after incorporating the data of the new product P_3 is shown below:

Maximize $Z = 6X_1 + 8X_2 + 20X_3$

S/t $5X_1 + 10X_2 + 6X_3 \leq 60$

$4X_1 + 4X_2 + 5X_3 \leq 40$

$X_1, X_2, X_3 \geq 0.$

(a) $C_j - Z_j = C_j - [CB]$ [Technological coefficients of the optimal table w.r.t basic variables of the initial table] [Constraint coefficients of the new variable]

$$\Rightarrow C_3 - Z_3 = 20 - \begin{bmatrix} 8 & 6 \end{bmatrix} \begin{bmatrix} 1/5 & -1/4 \\ -1/5 & 1/2 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$\Rightarrow c_3 - z_3 = \frac{63}{5}$.

This shows that coefficient of X_3 in z-row is $-\frac{63}{5}$. Hence, the resulting table is not optimal b/c X_3 plays the role of entering variable. Hence, the inclusion of new product (new variable) changes the optimality.

(b) $\begin{bmatrix} \text{Revised constraint} \\ \text{Coefficients of} \\ \text{new Variable} \end{bmatrix} = \begin{bmatrix} \text{Technological coefficients of} \\ \text{the optimal table w.r.t} \\ \text{basic variables of initial table} \end{bmatrix} \begin{bmatrix} \text{constraint coefficients} \\ \text{of new variable} \end{bmatrix}$

$$= \begin{bmatrix} 1/5 & -1/4 \\ -1/5 & 1/2 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \end{bmatrix} = \begin{bmatrix} -1/20 \\ 13/10 \end{bmatrix}$$

Hence, using (*) & the value of $C_3 - Z_3$ in the optimal table, we obtain the following modified table

C_B	Basic	X_1	X_2	X_3	S_1	S_2	Solution
	Z	0	0	$-63/5$	$12/5$	1	64
8	X_2	0	1	$-1/20$	$1/5$	$-1/4$	2
6	X_1	1	0	$13/10$	$-1/5$	$1/2$	8

The above table is not optimal, & hence after performing two or more iterations, one can get the following optimal result

$X_1 = 0, \quad X_2 = 0, \quad X_3 = 8, \quad S_1 = 12,$

$S_2 = 0, \quad Z(\text{optimum}) = 160.$