

# Homework 7

Han Nguyen - TXN200004

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## Question 1

```
# Data: Cola can fill amounts (ounces)
fill <- c(15.997, 16.005, 15.981, 15.954, 15.986, 16.021, 15.985, 16.001, 16.018, 16.056)
```

### Step 1: State the null and alternative hypotheses

- $H_0: \mu = 16.00$  (The mean fill of cola cans is 16.00 ounces)
- $H_a: \mu < 16.00$  (The mean fill of cola cans is less than 16.00 ounces)

This is a one-sided (left-tailed) test.

### Step 2: Choose the significance level

$\alpha = 0.05$

### Step 3: Determine the appropriate test statistic

Since we have a small sample ( $n = 10$ ) and we don't know the population standard deviation, we use a one-sample t-test with the test statistic:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

where  $\bar{x}$  is the sample mean,  $\mu_0 = 16.00$  is the hypothesized mean,  $s$  is the sample standard deviation, and  $n = 10$  is the sample size.

### Step 4: Formulate the decision rule

With  $\alpha = 0.05$  and degrees of freedom  $df = n - 1 = 9$ , we will reject  $H_0$  if:

- $t < -t_{\alpha, df}$  (critical value approach), or
- p-value  $< \alpha$  (p-value approach)

```
# Critical value for one-sided test
alpha <- 0.05
df <- length(fill) - 1
t_critical <- qt(alpha, df)
```

The critical value is  $t_{0.05,9} = -1.8331$ .

### Step 5: Collect data and calculate the test statistic

```
# Sample statistics
n <- length(fill)
x_bar <- mean(fill)
s <- sd(fill)
mu_0 <- 16.00
```

```

# Calculate test statistic
t_stat <- (x_bar - mu_0) / (s / sqrt(n))

# Calculate p-value (one-sided, left-tailed)
p_value <- pt(t_stat, df)

```

Sample mean:  $\bar{x} = 16.0004$  ounces

Sample standard deviation:  $s = 0.0276$  ounces

Test statistic:  $t = 0.0459$

#### Step 6: Make a decision

```

# Decision based on critical value
reject_H0_critical <- t_stat < t_critical

# Decision based on p-value
reject_H0_pvalue <- p_value < alpha

```

P-value = 0.5178

Since  $t = 0.0459$  is greater than the critical value of -1.8331, and the p-value (0.5178) is greater than  $\alpha = 0.05$ , we fail to reject  $H_0$ .

#### Step 7: Draw a conclusion

Based on the sample of 10 cola cans, we fail to reject the null hypothesis at the 0.05 significance level. There is not sufficient statistical evidence to conclude that the mean fill of cola cans is less than 16.00 ounces. Mark's data does not support the claim that the cans contain less than 16 ounces.

```

# Verification using t.test function in R
t_test_result <- t.test(fill, mu = 16.00, alternative = "less")
print(t_test_result)

```

```

##
##  One Sample t-test
##
## data:  fill
## t = 0.045909, df = 9, p-value = 0.5178
## alternative hypothesis: true mean is less than 16
## 95 percent confidence interval:
##      -Inf 16.01637
## sample estimates:
## mean of x
## 16.0004

```

## Question 2

```
# If I wrote code that would be used in all following letters, it would go here
```

## Question 3

```
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```

## Question 4

```
# If I wrote code that would be used in all following letters, it would go here
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## Question 5

```
# This is where my code for this question goes
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## Question 6

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# This is where my code for this question goes
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## Question 7

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# This is where my code for this question goes
```

## Question 8

```
# This is where my code for this question goes
```