Financial Engineering2

PROJECT02

-Valuation of Auto Callable Contingent Interest Notes -

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1. Terms of the Auto Callable Contingent Interest Notes

Underlying Asset		Apple
FV		1000
Pricing date		03/21/2019
Issue date		03/26/2019
Maturity		06/25/2020
Review date		06/21/2019, 09/23/2019, 12/23/2019, 03/23/2020, 06/22/2020
Interest Payment dates		06/26/2019, 09/26,/2019, 12/27/2019, 03/26/2020, 06/25/2020
Initial value		195.09
Interest Barrier / Trigger rate		78% * S0 (=152.1702)
Trigger event		At any day (excluding pricing date and including maturity) S(t) <0.78*S0
Interest rate		8.15% per annual (Quarterly 2.0375%)
Contingent Interest payment(CIP)	**Does not auto call and S(t) >= Barrier**	Payoff = 20.375
Automatically call	**S(t) >= S0 (At any reference date excluding first)**	Payoff = 1000 +CIP
Maturity Payoff	**[No auto call and S(T) >S0]or No trigger event**	Payoff = 1000+CIP
	No auto call and Barrier<S(T)<S0	Payoff = 1000*(CIP + stock return)
	No auto call and S(T)<S0 and Trigger event occurred,	Payoff = 1000+(1000*Stock return)
Offered price		958.9

2. Parameters for pricing security

We will estimate this product through Finite Difference Method (FDM). In addition to the information given above, the required parameters are interest rate, dividend rate, volatility, stock node and time node.

1) Interest rate

In the case of interest rate, there is no interest rate exactly matched from pricing date to maturity. After receiving the interest rate rata from Bloomberg, we calculated data for the period through interpolation and use it as constant. The interpolation formula is below.

$$r^* = \frac{(t^* - t_1) * (r_2 - r_1)}{t_2 - t_1} + r_1$$

2) Dividend

In the case of dividends, we obtain it from the dividend curve of Bloomberg and use as constant.

3) Volatility

We could not obtain the volatility corresponding barrier value from Bloomberg. Therefore, after obtaining the volatility data in Bloomberg, we extrapolated it to moneyness for getting the volatility corresponding to barrier. Extrapolation formula is below and interpolation method is the same with interest rate interpolation formula. We use it as constant.

$$\sigma^* = \frac{(M^* - M_1) * (\sigma_2 - \sigma_1)}{M_2 - M_1} + \sigma_1, \qquad \mathrm{M: Moneyness}$$

4) Stock Node

Stock node is an important parameter in FDM. The higher the number, the more accurate the price but the longer it takes to calculate. We set 100 stock nodes and see how the estimated price changes as the stock node changes in the sensitivity analysis.

In addition, if we first set the number of time nodes, the stock node should be set to the following formula.

$$\text{Stock node} \leq \sqrt{\frac{time\ \ node}{sigma^2 * T}}$$

5) Time Node

Time node is also an important parameter like stock node. We set 780-time node because we set 100 stock node. To apply Explicit FDM, we have to satisfy below formula and the result is 780 when stock node is 100.

$$\text{time node} \geq \ stock\ \ node^2 * \ \ sigma^2 * T$$

6) Model selection

To calculate this product price, we use Crank-Nicolson method. Since we want to see which method is most accurate, in sensitivity analysis we will how the price changes for each model as parameters change. CN method formula is below,

$$\mathrm{a[j]} = 0.25 * [sigma^2 * j^2 - (r - div) * j]$$

$$\mathrm{b[j]} = -0.5 * [sigma^2 * j^2 + r + 2/\Delta t]$$

$$\mathrm{c[j]} = 0.25 * [sigma^2 * j^2 + (r - div) * j]$$

$$\mathrm{d[j]} = -a[j] * V_{j-1}{}^{i+1} - b[j] * V_j{}^{i+1} - c[j] * V_{j+1}{}^{i+1}$$

The below table summarizes the parameters.

Interest rate	2.5%
Dividend rate	0.7%
Volatility	25%
Binomial model	Crank-Nicolson method
Number of stock nodes	100
Number of time nodes	780

3. Pricing for the security

We will briefly explain the process of pricing and if you want to know more detail, please see the attached code. Since we have estimated the parameters, we have to solve this matrix. To solve it, we have use two methods, one of them is LU decomposition and another is Thomas algorism.

$$\begin{pmatrix} b_0 & c_0 & 0 & \cdots & 0 \\ a_1 & b_1 & c_1 & \ddots & \vdots \\ 0 & a_2 & b_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & c_{jmax-1} \\ 0 & \cdots & 0 & a_{jmax} & b_{jmax} \end{pmatrix} \begin{pmatrix} V_0{}^i \\ V_1{}^i \\ V_2{}^i \\ \vdots \\ V_{jmax}{}^i \end{pmatrix} = \begin{pmatrix} d_0{}^i \\ d_1{}^i \\ d_2{}^i \\ \vdots \\ d_{jmax}{}^i \end{pmatrix}$$

To solve it, we have to set boundaries of a, b, c, d and value of the security.

$$b_0 = 1, c_0 = 0, a_{jmax} = 0, b_{jmax} = 1$$

$$d_0{}^i = lower\ \ boundaty\ \ condition, d_{jmax}{}^i = upper\ \ boundary\ \ condition$$

We set lower boundary condition is zero and upper boundary condition is

$$V_{jmax}{}^i = (1000 + 20.375) * \ \ e^{-(next\ \ call\ \ date - i*\Delta t)}$$

Time node	Condition		Value of the security($V_j{}^i$)
Maturity	[$S_T \geq S_0$ and No auto call] Or No trigger event		1000 + 20.375
	$B < S_T < S0$ and No auto call]		$1000 * (20.375 - \frac{S_j - S_0}{S_0})$
	Otherwise		1000*($\frac{S_j - S_0}{S_0}$)
From March 2019 to Jun 2020	Auto callable date	$S_j \geq S_0$	1000 + 20.375
	Coupon payment date	$S_j > B$ And no auto call	20.375 + $V_j{}^i$
	Otherwise		Matrix sovle

Using boundaries condition and above table, we can obtain security price. The current price is located where time node is 0 and stock node is 50**. The price is 961.59,** which it is close to offered price (958.9).

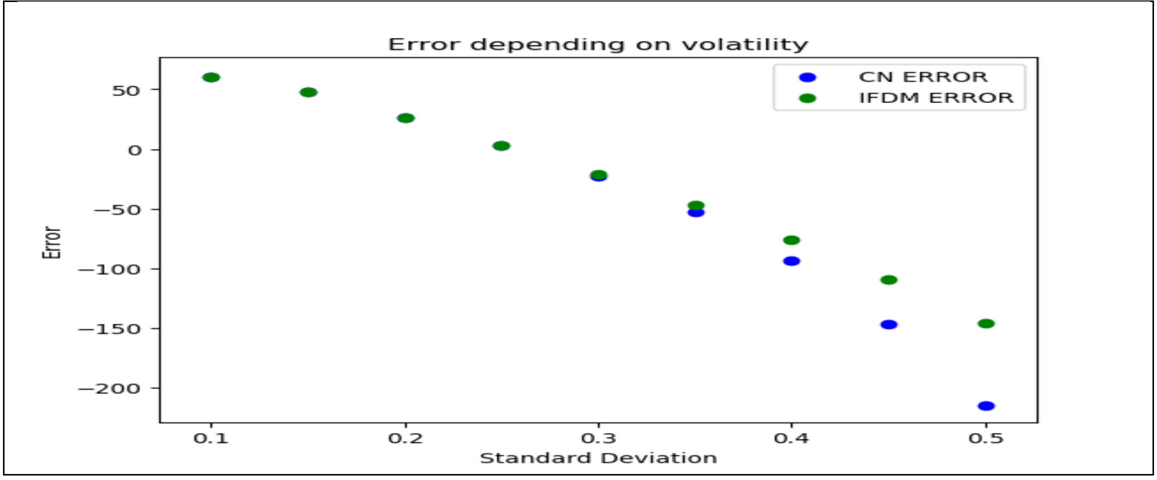
4. Sensitivity Analysis

Although we estimated the price close to the published price, we will analyze various scenarios to identify parameters that affect price. We will conduct a sensitivity analysis on all models (EFDM/IFDM/CN). We will look at which parameters affect each model. Sensitivity analysis is performed only for time node, stock node and volatility because interest rate and dividend rate is well estimated.

1) Volatility analysis

To analyze how the volatility change affects the price, other variables are fixed to the existing values. As a result of the scenario analysis, it can be seen that the volatility has a big influence on the price. We can see that in all model volatility has big effect on price. Below table shows price based on volatility. As you can see under EFDM model, the price changes greatly depending on the volatility and under IFDM and CN model as the volatility increases, the price decreases. Below figure shows error based on volatility and we get conclusion that CN model more accurate than IFDM and EFDM.

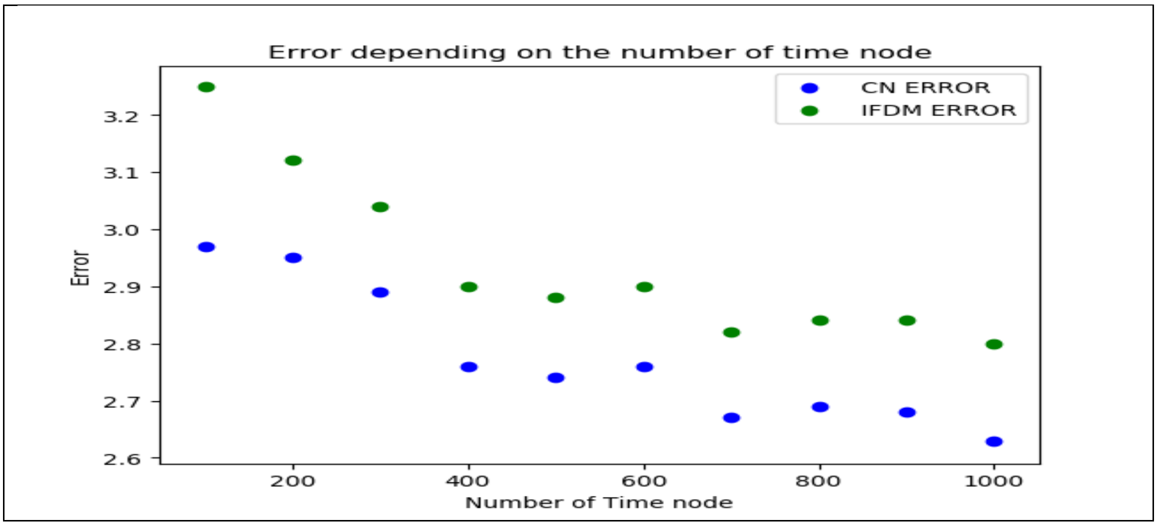
Volatility	EFDM	IFDM	CN
10%	1019.39	1019.38	1019.38
15%	1007.11	1007.12	1007.11
20%	985.41	985.46	985.43
25%	961.79	961.74	961.59
30%	-2.6e+27	937.56	936.28
35%	-6.6e+90	911.6	905.69
40%	-3.3e+213	882.31	865.17
45%	Nan	849.23	811.63
50%	Nan	812.82	743.81



2) Number of Time node analysis

To analyze how the number of time node affects the price, other variables are fixed to the existing values including the number of stock node. As a result of the sensitivity analysis, the number of time node does not greatly effect on the price under IFDM and CN model. However, under EFDM model we cannot calculate the price because we fixed stock node, which means that we did not followed time and stock node formula. To calculate price, we need minimum 780-time node under 100 stock node. Below table shows the price based on the number of time node and below figure shows error based on the number of time node and we get conclusion that CN model more accurate than IFDM and EFDM.

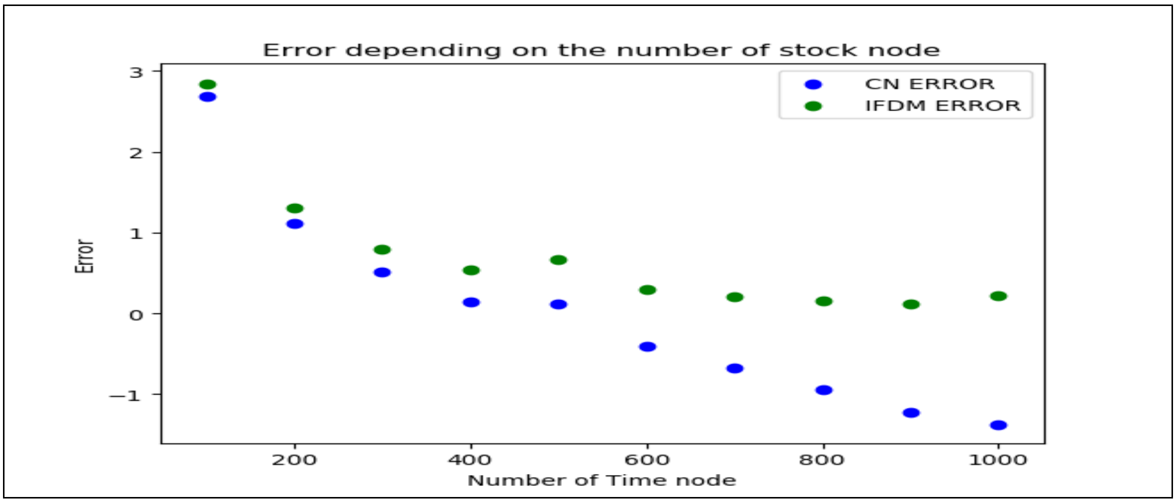
Number of Time node	EFDM	IFDM	CN
100	-2.12e+41	962.15	961.87
200	-5.5e+30	962.02	961.85
300	-8.65e+35	961.94	961.79
400	4.06e+31	961.8	961.66
500	1.05e+18	961.78	961.64
600	3.02e+22	961.8	961.66
700	2.3e+15	961.72	961.57
800	961.8	961.74	961.59
900	961.81	961.74	961.58
1000	961.77	961.7	961.53



3) Stock node analysis

To analyze how the number of stock node affects the price, other variables are fixed to the existing values including the number of time node. As a result of the sensitivity analysis, the number of stock node does not greatly effect on the price under IFDM and CN model. Rather the higher the number of nodes, the more accurate the price. However, under EFDM model, it cannot calculate the price because we fixed time node, which means that we did not followed time and stock node formula. Below table shows the price based on the number of stock node and below figure shows error based on the number of stock node and we get conclusion that CN model more accurate than IFDM and EFDM.

Number of stock node	EFDM	IFDM	CN
100	961.79	961.74	961.59
200	Nan	960.21	960.01
300	Nan	959.69	959.41
400	Nan	959.44	959.04
500	Nan	959.56	959.02
600	Nan	959.19	958.49
700	Nan	959.11	958.23
800	Nan	959.06	957.96
900	Nan	959.02	957.68
1000	Nan	959.12	957.52



4) Time node set using formula

Above two analysis, we know that if we does not satisfy node formula, we cannot calculate the price under EFDM. Therefore, we analyze the price calculating necessary time node according to the stock node. The below table shows the result and all models have reasonable price. below figure shows the error based on the number of error. In here, CN model has the most accurate price.

Number of	EFDM	IFDM	CN

Stock node and Time node			
(50, 196)	961.82	961.83	961.69
(100,780)	961.79	961.74	961.59
(150,1758)	959.76	959.69	959.45
(200,3125)	960.21	960.13	959.75