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| Financial Engineering2 |
| PROJECT02 |
| -Valuation of Auto Callable Contingent Interest Notes - |

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| **Underlying Asset** | | Apple |
| **FV** | | 1000 |
| **Pricing date** | | 03/21/2019 |
| **Issue date** | | 03/26/2019 |
| **Maturity** | | 06/25/2020 |
| **Review date** | | 06/21/2019, 09/23/2019, 12/23/2019,  03/23/2020, 06/22/2020 |
| **Interest Payment dates** | | 06/26/2019, 09/26,/2019, 12/27/2019,  03/26/2020, 06/25/2020 |
| **Initial value** | | 195.09 |
| **Interest Barrier / Trigger rate** | | 78% \* S0  (=152.1702) |
| **Trigger event** | | At any day  (excluding pricing date and including maturity)  S(t) <0.78\*S0 |
| **Interest rate** | | 8.15% per annual  (Quarterly 2.0375%) |
| **Contingent Interest payment(CIP)** | **Does not auto call and**  **S(t) >= Barrier** | Payoff = 20.375 |
| **Automatically call** | **S(t) >= S0**  **(At any reference date excluding first)** | Payoff = 1000 +CIP |
| **Maturity**  **Payoff** | **[No auto call and S(T) >S0]or**  **No trigger event** | Payoff = 1000+CIP |
| **No auto call and**  **Barrier<S(T)<S0** | Payoff = 1000\*( CIP + stock return) |
| **No auto call and S(T)<S0 and Trigger event occurred,** | Payoff = 1000+(1000\*Stock return) |
| **Offered price** | | 958.9 |

1. **Terms of the Auto Callable Contingent Interest Notes**
2. **Parameters for pricing security**

We will estimate this product through Finite Difference Method (FDM). In addition to the information given above, the required parameters are interest rate, dividend rate, volatility, stock node and time node.

1. **Interest rate**

In the case of interest rate, there is no interest rate exactly matched from pricing date to maturity. After receiving the interest rate rata from Bloomberg, we calculated data for the period through interpolation and use it as constant. The interpolation formula is below.

1. **Dividend**

In the case of dividends, we obtain it from the dividend curve of Bloomberg and use as constant.

1. **Volatility**

We could not obtain the volatility corresponding barrier value from Bloomberg. Therefore, after obtaining the volatility data in Bloomberg, we extrapolated it to moneyness for getting the volatility corresponding to barrier. Extrapolation formula is below and interpolation method is the same with interest rate interpolation formula. We use it as constant.

1. **Stock Node**

Stock node is an important parameter in FDM. The higher the number, the more accurate the price but the longer it takes to calculate. We set 100 stock nodes and see how the estimated price changes as the stock node changes in the sensitivity analysis.

In addition, if we first set the number of time nodes, the stock node should be set to the following formula.

1. **Time Node**

Time node is also an important parameter like stock node. We set 780-time node because we set 100 stock node. To apply Explicit FDM, we have to satisfy below formula and the result is 780 when stock node is 100.

1. **Model selection**

To calculate this product price, we use Crank-Nicolson method. Since we want to see which method is most accurate, in sensitivity analysis we will how the price changes for each model as parameters change. CN method formula is below,

The below table summarizes the parameters.

|  |  |
| --- | --- |
| **Interest rate** | 2.5% |
| **Dividend rate** | 0.7% |
| **Volatility** | 25% |
| **Binomial model** | Crank-Nicolson method |
| **Number of stock nodes** | 100 |
| **Number of time nodes** | 780 |

1. **Pricing for the security**

We will briefly explain the process of pricing and if you want to know more detail, please see the attached code. Since we have estimated the parameters, we have to solve this matrix. To solve it, we have use two methods, one of them is LU decomposition and another is Thomas algorism.

To solve it, we have to set boundaries of a, b, c, d and value of the security.

We set lower boundary condition is zero and upper boundary condition is

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| **Time node** | **Condition** | | **Value of the security(** |
| Maturity | [ and No auto call]  Or  No trigger event | | 1000 + 20.375 |
| and No auto call] | |  |
| Otherwise | | 1000\* |
| From March 2019 to Jun 2020 | Auto callable date |  | 1000 + 20.375 |
| Coupon payment date | And no auto call | 20.375 + |
| Otherwise | | Matrix sovle |

Using boundaries condition and above table, we can obtain security price. The current price is located where time node is 0 and stock node is 50**. The price is 961.59,** which it is close to offered price (958.9).

1. **Sensitivity Analysis**

Although we estimated the price close to the published price, we will analyze various scenarios to identify parameters that affect price. We will conduct a sensitivity analysis on all models (EFDM/IFDM/CN). We will look at which parameters affect each model. Sensitivity analysis is performed only for time node, stock node and volatility because interest rate and dividend rate is well estimated.

1. **Volatility analysis**

To analyze how the volatility change affects the price, other variables are fixed to the existing values. As a result of the scenario analysis, it can be seen that the volatility has a big influence on the price. We can see that in all model volatility has big effect on price. Below table shows price based on volatility. As you can see under EFDM model, the price changes greatly depending on the volatility and under IFDM and CN model as the volatility increases, the price decreases. Below figure shows error based on volatility and we get conclusion that CN model more accurate than IFDM and EFDM.

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| --- | --- | --- | --- |
| **Volatility** | **EFDM** | **IFDM** | **CN** |
| **10%** | 1019.39 | 1019.38 | 1019.38 |
| **15%** | 1007.11 | 1007.12 | 1007.11 |
| **20%** | 985.41 | 985.46 | 985.43 |
| **25%** | 961.79 | 961.74 | 961.59 |
| **30%** | -2.6e+27 | 937.56 | 936.28 |
| **35%** | -6.6e+90 | 911.6 | 905.69 |
| **40%** | -3.3e+213 | 882.31 | 865.17 |
| **45%** | Nan | 849.23 | 811.63 |
| **50%** | Nan | 812.82 | 743.81 |

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1. **Number of Time node analysis**

To analyze how the number of time node affects the price, other variables are fixed to the existing values including the number of stock node. As a result of the sensitivity analysis, the number of time node does not greatly effect on the price under IFDM and CN model. However, under EFDM model we cannot calculate the price because we fixed stock node, which means that we did not followed time and stock node formula. To calculate price, we need minimum 780-time node under 100 stock node. Below table shows the price based on the number of time node and below figure shows error based on the number of time node and we get conclusion that CN model more accurate than IFDM and EFDM.

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| --- | --- | --- | --- |
| **Number of Time node** | **EFDM** | **IFDM** | **CN** |
| **100** | -2.12e+41 | 962.15 | 961.87 |
| **200** | -5.5e+30 | 962.02 | 961.85 |
| **300** | -8.65e+35 | 961.94 | 961.79 |
| **400** | 4.06e+31 | 961.8 | 961.66 |
| **500** | 1.05e+18 | 961.78 | 961.64 |
| **600** | 3.02e+22 | 961.8 | 961.66 |
| **700** | 2.3e+15 | 961.72 | 961.57 |
| **800** | 961.8 | 961.74 | 961.59 |
| **900** | 961.81 | 961.74 | 961.58 |
| **1000** | 961.77 | 961.7 | 961.53 |

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1. **Stock node analysis**

To analyze how the number of stock node affects the price, other variables are fixed to the existing values including the number of time node. As a result of the sensitivity analysis, the number of stock node does not greatly effect on the price under IFDM and CN model. Rather the higher the number of nodes, the more accurate the price. However, under EFDM model, it cannot calculate the price because we fixed time node, which means that we did not followed time and stock node formula. Below table shows the price based on the number of stock node and below figure shows error based on the number of stock node and we get conclusion that CN model more accurate than IFDM and EFDM.

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| **Number of stock node** | **EFDM** | **IFDM** | **CN** |
| **100** | 961.79 | 961.74 | 961.59 |
| **200** | Nan | 960.21 | 960.01 |
| **300** | Nan | 959.69 | 959.41 |
| **400** | Nan | 959.44 | 959.04 |
| **500** | Nan | 959.56 | 959.02 |
| **600** | Nan | 959.19 | 958.49 |
| **700** | Nan | 959.11 | 958.23 |
| **800** | Nan | 959.06 | 957.96 |
| **900** | Nan | 959.02 | 957.68 |
| **1000** | Nan | 959.12 | 957.52 |

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1. **Time node set using formula**

Above two analysis, we know that if we does not satisfy node formula, we cannot calculate the price under EFDM. Therefore, we analyze the price calculating necessary time node according to the stock node. The below table shows the result and all models have reasonable price. below figure shows the error based on the number of error. In here, CN model has the most accurate price.

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| **Number of**  **Stock node and Time node** | **EFDM** | **IFDM** | **CN** |
| **(50, 196)** | 961.82 | 961.83 | 961.69 |
| **(100,780)** | 961.79 | 961.74 | 961.59 |
| **(150,1758)** | 959.76 | 959.69 | 959.45 |
| **(200,3125)** | 960.21 | 960.13 | 959.75 |

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