### Problem Set 2

Total 15 points

Due Monday, 4 February 2019 before 11:59pm

*Instructions*: This is a group assignment.

Handwritten solutions are acceptable here, please either photograph them to submit electronically or hand them to me in class on Monday. Of course, you can also insert the photos of your handwritten solutions into an electronic file (or embed in word/excel etc) and upload them electronically.

**Digital Options (Black-Scholes adaptation)**

**Digital options** have payoffs as follows: Calls:



Puts



with exercise price K, cash amount A and time to maturity T.

1. By considering the Black-Scholes equation on a dividend paying stock, write down the value of a Digital call at t = 0.
2. What portfolio of risk-free assets and digital puts replicates the payoff of a digital call?

Morgan Stanley are issuing **Dual Directional Trigger PLUS securities** contracts documented on Page 4 On March 5th, five-year zero coupon bonds on a principal of $100 were trading for $88.24969. European options on the S&P500 were available with five years to maturity with exercise prices of 1680 to 3920, the market prices are below, you also estimate N(d1) and N(d2).

1. Find a portfolio of zero-coupon bonds and European options (calls, puts, digital calls, digital puts) that can replicate the payoff of the note. Calculate the estimated value of the Dual Directional Trigger PLUS security (your answer should be between $940 and $980.
2. On the same day Barclay’s Bank issued a product **on the same index** but with a slightly different payoff and maturity see Page 6. Do you think the Barclay’s product had a (relative) estimated price higher or lower than the Morgan Stanley one above? Explain your answer and assumptions. **Assume here that the principal is $1000!**

**Link between binomial model and Black-Scholes PDE (Coming Soon!)**

**2.**  (3 points) Consider a simplified binomial model with time step of *dt* where , (note that this is the CRR model for very small time steps dt). The option value today is *V = V(S,t),* option value in the up-state is *V+ = V(u*×*S, t+dt*), and option value in the down-state is *V- = V(d*×*S, t+dt)* show that in the limit of *dt 🡪 0* we recover the Black-Scholes partial differential equation:



*Hint*. This question is asking you to carry out a small bit of analysis. Expand *V*+ and *V*- in Taylor series (your Taylor series needs to be second order in *S* and first order in *t*), substitute the expansions into the standard binomial pricing formula,

substitute in the definition of q and expand all of the exponential terms using *ex = 1 + x,* ignoring smaller terms. With any luck, you should obtain the partial differential equation above.

**Practice with the Binomial spreadsheet**

**3.** (6 points) Adapt the trees from ImpBinomial.xls to value a slightly simplified version of this product:

<https://www.sec.gov/Archives/edgar/data/1114446/000091412119000083/ub53184218-424b2.htm>

Microsoft (MSFT) product. I have tried to match the r, d, and  as closely as possible!

Face value: $1000

Payoff at maturity: if the stock price is greater than or equal to $90.46 then you receive the face value plus the final coupon payment. If the stock price is below $ you receive a cash amount equivalent to 1000/90.46 = 11.0546 stocks plus the final coupon payment.

Autocall feature: If the stock price is greater than or equal to the initial price on any of the observation dates t = 1/4, 1/2, 3/4 then the notes are immediately called for the face value + coupon.

Coupons: there is a monthly coupon of 9.05% (this is an annual figure) of the face value, payable at t = 1/12, 2/12 etc.

To set up your tree:

Choose N = 50 (i.e the same as in Impbinomial)

S0 = 102.80

T = 1.

r = 2.713%

 = 27.044%

Proportional Dividends: 1.84% (annual figure) quarterly at t = 1/12, 4/12, 7/12, 10/12, so assume 0.25 x 1.84% is paid each quarter.

Use the same u, d formulas as given in the Impbinomial spreadsheet (obviously adjusting for ).

**Information for Question 1**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| |  |  | | --- | --- | | **FINAL TERMS** | | | **Issuer:** | Morgan Stanley Finance LLC | | **Guarantor:** | Morgan Stanley | | **Maturity date:** | March 5, 2023 | | **Valuation date:** | March 5, 2023 | | **Underlying index:** | S&P 500® Index | | **Aggregate principal amount:** | $1,565,000 | | **Payment at maturity:** | If the final index value is *greater than* the initial index value:  $1,000 + leveraged upside payment  If the final index value is *less than or equal to*the initial index value but is *greater than or equal to* the trigger level:  $1,000 + ($1,000 x absolute index return)  *In this scenario, you will receive a 1% positive return on the Trigger PLUS for each 1% negative return on the underlying index. In no event will this amount exceed the stated principal amount plus $300.*  If the final index value is *less than* the trigger level:  $1,000 × index performance factor  *Under these circumstances, the payment at maturity will be less than the stated principal amount of $1,000, and will represent a loss of more than 30%, and possibly all, of your investment.* | | **Leveraged upside payment:** | $1,000 x leverage factor x index percent change | | **Leverage factor:** | 113% | | **Index percent change:** | (final index value – initial index value) / initial index value | | **Absolute index return:** | The absolute value of the index percent change.  For example, a –5% index percent change will result in a +5% absolute index return. | | **Index performance factor:** | final index value / initial index value | | **Initial index value:** | 2800.00, which is the index closing value on the pricing date | | **Final index value:** | The index closing value on the valuation date | | **Trigger level:** | 1960.00, which is 70% of the initial index value | | **Stated principal amount / Issue price:** | $1,000 per Trigger PLUS (see “Commissions and issue price” below) | | **Pricing date:** | March 5, 2018 | | **Original issue date:** | March 5, 2018 |   https://www.sec.gov/Archives/edgar/data/895421/000095010318002859/image_001.jpg |  |

**Option Price Data**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *K* | Call | Put | N(d1) | N(d2) |
| *1680* | 1005.21 | 77.82 | 0.89131 | 0.77084 |
| *1960* | 827.66 | 147.37 | 0.82126 | 0.66577 |
| *2240* | 676.04 | 242.86 | 0.74175 | 0.56231 |
| *2520* | 549.08 | 362.99 | 0.65885 | 0.46708 |
| *2800* | 444.26 | 505.27 | 0.57736 | 0.38331 |
| *3080* | 358.63 | 666.73 | 0.50056 | 0.31188 |
| *3360* | 289.16 | 844.37 | 0.43035 | 0.25226 |
| *3640* | 233.09 | 1035.39 | 0.36761 | 0.20324 |
| *3920* | 187.97 | 1237.37 | 0.31248 | 0.16335 |

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| |  |  | | --- | --- | | **Issuer:** | Barclays Bank PLC | | **Reference asset\*:** | S&P 500® Index (Bloomberg ticker symbol “SPX<Index>”) (the “underlier”) | | **Aggregate principal amount:** | $10,849,110 | | **Initial issue price:** | $1000 per Trigger PLUS (see “Commissions and initial issue price” below) | | **Pricing date:** | March 5, 2018 | | **Original issue date:** | March 5, 2018 | | **Valuation date†:** | March 5, 2024 | | **Maturity date†:** | March 5, 2024 | | **Interest:** | None | | **Payment at maturity:** | You will receive on the maturity date a cash payment per Trigger PLUS determined as follows:     If the final underlier value is *greater than* the initial underlier value:  $10 + leveraged upside payment     If the final underlier value is *less than or equal to* the initial underlier value but greater than or equal to the trigger value:  $10     If the final underlier value is *less than* the trigger value:  $10 × underlier performance factor | |  |  | | **Leveraged upside payment:** | $10 × leverage factor × underlier return | | **Leverage factor:** | 135% | | **Trigger value:** | 1820.00, which is 65% of the initial underlier value (rounded to two decimal places) | | **Underlier return:** | (final underlier value – initial underlier value) / initial underlier value | | **Underlier performance factor:** | final underlier value / initial underlier value | |  |
| |  |  | | --- | --- | | **Initial underlier value:** | 2800, which is the closing level of the underlier on the pricing date | | **Final underlier value:** | The closing level of the underlier on the valuation date | |  |
|  |  |
| https://www.sec.gov/Archives/edgar/data/312070/000095010318002915/image_005.jpg |  |
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