1 i) $3.5 - 1.5 = 2 = h$

$$h_0 = \frac{2}{2^0}$$
$$h_1 = \frac{2}{2^1}$$
$$h_2 = \frac{2}{2^2}$$
$$\vdots$$
$$h_n = \frac{2}{2^n} = \frac{1}{2^{n-1}}$$

ii) $(1.5 + h_n) - (1.5 + h_n) = 0$

root midpoint

2 $c_0 = b_0 - a_0$ $x \in [a_0, b_0)$ is root

$$c_n = \frac{1}{2^n}(b_0 - a_0).$$
$$\frac{|x - c_n|}{x} \le \frac{\frac{1}{2}c_n}{x}$$
$$\le \frac{\frac{1}{2}c_n}{a_0}$$
$$= \frac{b_0 - a_0}{2^{n+1}a_0}$$
$$\Rightarrow \frac{b_0 - a_0}{2^{n+1}a_0} \le \varepsilon$$
$$2^n \ge \frac{b_0 - a_0}{2a_0\varepsilon}$$
$$n\log 2 \ge \log(b_0 - a_0) - \log\varepsilon - \log a_0 - \log 2$$
$$n \ge \frac{\log(b_0 - a_0) - \log\varepsilon - \log a_0}{\log 2} - 1$$

3 $p(x) = 4x^3 - 2x^2 + 3 = 0$

$p'(x) = 12x^2 - 4x$

由 $x_{n+1} = x_n - \frac{p(x_n)}{p'(x_n)}$

$p(-1) = -3$

$p'(-1) = 16$

当 $n = 0$, $x_1 = -1 + \frac{3}{16} = -0.8125$

$p(-0.8125) = -0.4658$

$p'(-0.8125) = 11.1719$

当 $n = 1$, $x_2 = -0.7708$

$p(-0.7708) = -0.0201$

$p'(-0.7708) = 10.2128$

当 $n = 2$, $x_3 = -0.7688$

$p(-0.7688) = 0.000286$ ← $p(x) < \varepsilon$, postconditions

$p'(-0.7688) = 10.1678$

4 令 x 为根，$e_n = x_n - x$

由 $f(x) = f(x_n) + (x - x_n) f'(\xi)$

$0 = f(x_n) + (x - x_n) f'(\xi)$

$f(x_n) = (x_n - x) f'(\xi)$

$= e_n f'(\xi)$

又 $e_{n+1} = x_{n+1} - x$

$e_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - x$

$= e_n - \frac{f(x_n)}{f'(x_n)}$

则 $e_{n+1} = e_n - \frac{e_n f'(\xi)}{f'(x_n)}$

$= e_n \left(1 - \frac{f'(\xi)}{f'(x_n)}\right)$

由 $e_{n+1} = C e_n^s$

$C = 1 - \frac{f'(\xi)}{f'(x_n)}$

5 当 $x_0 = 0$，收敛

当 $0 < x_0 < \frac{\pi}{2}$

$0 < \tan^{-1} x < x$

$0 < \tan^{-1} x_n < x_n$

收敛

当 $-\frac{\pi}{2} < x_0' < 0$

$0 < x_0 < \frac{\pi}{2}$

收敛

∴ 循环收敛

6 当 $x_1 = \frac{1}{p}$，$x_n = \frac{1}{p + x_{n-1}}$

∵ $p > 1$，$x \in (0, 1)$

令 $f(x) = \frac{1}{p+x}$，$f(x) \in (0, 1)$

$\lambda = \max_{x \in (0,1)} |f'(x)|$

$= \max_{x \in (0,1)} \left| -\frac{1}{(p+x)^2} \right|$

$< \frac{1}{p^2}$

< 1

$f(\alpha) = \alpha$

$\frac{1}{p+\alpha} = \alpha$

$\alpha^2 + p\alpha - 1 = 0$

$\alpha = \frac{-p \pm \sqrt{p^2+4}}{2}$ ∴ 收敛

No.: Date:

7 $x \in [a_0, b_0]$ is root

$c_0 = b_0 - a_0$

$c_n = \frac{1}{2^n}(b_0 - a_0)$

$$\frac{|x - c_n|}{|x|} \le \frac{(b_0 - a_0)}{2^{n+1}|x|}$$

$$\frac{b_0 - a_0}{2^{n+1}|x|} \le \varepsilon$$

$$2^n \ge \frac{b_0 - a_0}{2\varepsilon |x|}$$

$$n \log 2 \ge \log(b_0 - a_0) - \log \varepsilon - \log|x| - \log 2$$

$$n \ge \frac{\log(b_0 - a_0) - \log \varepsilon - \log|x|}{\log 2} - 1$$

$x \to 0$, $n \ge +\infty$

Relative error can't be appropriate measure.