# Project 1: Linear Programming

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## Introduction

Marketing budgets now comprise **11** percent of total company budgets based on the CMO survey, sponsored by the Fuqua School of Business at Duke University, Deloitte LLP, and the American Marketing Association. However, the effectiveness of marketing varies significantly: on the one hand, P&G cut more than $100 million in digital marketing spending because their digital ads were largely ineffective; on the other hand, Netflix plans a 54% boost in ad spending because they got very positive feedback in international markets.

One potential reason for such variation is the way of making marketing budget allocations. Namely, how much to invest in each advertisement platform. As stated in the Handbook of Marketing Analytics:

> *... budget decisions are often based on gut feelings or on the negotiations skills of individual managers. Consequently, politics and individual opinions tend to shape the decision process instead of fact-based discussions. Obviously, these rules and practices bear the risk of results far away from the optimal, profit-maximizing budget.*

Indeed, the marketing strategy of Netflix seems to be steered by data[1]. In this project, we try to use linear programming to build a simple marketing budget allocation strategy.

## The Linear Program

The marketing department from a consumer packaged goods company needs to allocate **$10 million** budget across platforms to maximize return.

Based on historical data, one analytic group estimates the ROI (Return of Investment) from each campaign is

| Platform | Print | TV | SEO | AdWords | Facebook | LinkedIn | Instagram | Snapchat | Twitter | Email |
|---|---|---|---|---|---|---|---|---|---|---|
| ROI | 3.1% | 4.9% | 2.4% | 3.9% | 1.6% | 2.4% | 4.6% | 2.6% | 3.3% | 4.4% |

The following are the constraints based on the guidance of the CMO:

a) The amount invested in print and TV should be no more than the amount spent on Facebook and Email. Surprisingly, email seems to be a great channel for reaching real people.
b) The total amount used in social media (Facebook, LinkedIn, Instagram, Snapchat, and Twitter) should be at least twice of SEO and AdWords.
c) For each platform, the amount invested should be no more than $\$3M$.

Follow the steps below to build the linear programming.

1. Formulate the budget allocation as a linear programming problem. List and describe the decision variables, the objective, and all the constraints.
2. Use the following test case and solve the LP in R.
    - Solve the problem and we call its solution **alc1** which should be

[1] See the previous Netflix news article.

| Platform | Print | TV | SEO | AdWords | Facebook | LinkedIn | Instagram | Snapchat | Twitter | Email |
|---|---|---|---|---|---|---|---|---|---|---|
| Amount($) | 0 | 3M | 0 | 1M | 0 | 0 | 3M | 0 | 0 | 3M |

3. Next, we will write a function in R that can construct an allocation for any ROI vector, upper bound ($3M in the previous case), and budget ($10M in the last example).
   - Name the function as allocation() and save it as **allocation_gZ.R** where Z is your group number. For example, group 3's filename will be **allocation_g3.R**.
   - Then, from the R code used in the previous step, construct an R function that takes three inputs ROI_vec, upper_bound, and budget, and outputs the optimal objective value and optimal solution. For example, if you assign result = allocation(ROI_vec, upper_bound, budget), result$objval and result$sol will return the optimal objective value and the optimal solution respectively.
   - To test your function, use the test case from the previous step.
4. Use allocation() function to calculate the optimal objective value without the 3rd constraint. Report both the optimal objective value and optimal solution, which we call **alc2**, and compare them with the $3M counterparts.

## The Optimizers' Curse

The CMO is happy to see the promising results presented by the marketing department. However, she is also very concerned: because she recalls being somewhat disappointed after following such recommendations in the past. To be cautious about the decision, she asks the allocation being tested with ROI (shown below) estimated independently by another analytic group.

| Platform | Print | TV | SEO | AdWords | Facebook | LinkedIn | Instagram | Snapchat | Twitter | Email |
|---|---|---|---|---|---|---|---|---|---|---|
| ROI | 4.9% | 2.3% | 2.4% | 3.9% | 4.4% | 4.6% | 2.6% | 1.9% | 3.7% | 2.6% |

1. Get the optimal objective value (with $3M upper bound) and the corresponding solution, **alc3**, using the new ROI vector. Are they the same as their counterparts using the previous ROI vector?
2. The disappointment of an allocation is defined as the difference between the objective values using the old and new ROI vector.
   - Calculate the disappointment for alc1 and alc2 in the previous section.
   - Do you think the 3rd constraint based on CMO's experience is valuable?
3. Try to find an allocation that dominates alc1, alc2, and alc3 regarding the average objective values using both the old and new ROI vector. There are at least two possible ways to achieve this allocation.
   - Hint one: you might want to tweak ROI vector. This is a little cheating because it might be impossible to hire an independent analytic group.
   - Hint two: you might want to tweak the upper bound. This is a more practical solution than the previous one.

## Multi-Period Allocation

The CMO gained permission to reinvest **half** of the return. For example, if the marketing obtains a 4% return in January, the budget of February will be $\$10M + \$10M \times 4\% \times 50\% = \$10.2M$.

The monthly ROI for next year[2] is given in **Project1.Rdata**. The three constraints[3] given by CMO are still

[2]Notice that the first row is the average of the previous two ROI vectors
[3]See page 1, constraint a), b), and c).

in place for each month.

1. What's the optimal allocation for the whole year?
   - Hint: you can use c(ROI_mat) to convert the matrix to a vector.
   - Hint: **ROI_mat are in percentages**.
   - Hint: you might want to use *index_mat* given in the Rdata file.
2. What's the connection between the multi-period and previous single-period problem?
3. To obtain a stable relationship with each platform, the monthly change for each should be **no more than 1M**. Does the previous connection still hold? **You do not need to solve the new optimization**

## Submission Instructions

Please submit your report and allocation_gZ.R file to Canvas.