Optimizing Reserve Prices in Display Advertising Auctions

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Abstract

This paper examines how a publisher should set reserve prices for real-time bidding (RTB) auctions when selling display advertising impressions through ad exchanges, a $595 billion and growing market. Conducting a field experiment to induce exogenous variation in reserve prices at a major publisher, we find that setting reserve prices increases the publisher's revenues by 35% compared to using a zero reserve price. We also find empirical evidence that advertisers face a minimum impression constraint to ensure sufficient advertising reach.

Based on this insight, we develop a structural model of advertiser bidding behavior that incorporates impression constraints, allowing us to infer overall demand for advertising as a function of reserve prices. We then use this demand model to solve the publisher's pricing problem. Accounting for minimum impression constraints in setting reserve prices yields a profit increase of 9%% over a solution that does not incorporate the constraint. In a final field experiment, we validate our model's predictions by showing that ad revenues tend to be highest near the model-predicted optimal reserve prices.

Keywords: Display Advertising, Pricing, Auctions, Reserve Prices, Optimization, Real-Time Buying (RTB), Programmatic Buying, Real-Time Bidding, Impression Constraints, Field Experiments, Fluid Mean-Field Equilibrium

JEL Classification Codes: D4, L1, L2, M3

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1 Introduction

1.1 Overview

This paper considers how publishers (e.g., CNN, Wall Street Journal, Facebook, Youtube) should set reserve prices for auctions when selling display advertising impressions in real-time through ad exchanges (e.g., Google Ad Exchange, OpenX, Magnite). Global programmatic display ad spend was estimated $595 billion in 2024 and expected to grow considerably to $800 billion by 2028.[1] Drivers behind this growth include an upswing in mobile activities, a proliferation in online video ad formats, connected TV, and technological advancements in real-time buying (RTB) through ad exchange auctions.

While there is substantial research on advertisers' strategies in display advertising markets and the measurement of ad effectiveness, empirical study into publishers' strategies, especially regarding ad pricing, is significantly lacking. In the context of display ad auctions, the reserve prices set by the publisher can considerably influence their revenue outcomes. This paper seeks to bridge the notable gap in the literature by developing an approach that aids publishers in maximizing revenues through the optimization of reserve prices in display ad markets.

To achieve this aim, we first consider how advertisers' bidding behaviors would respond when the publisher changes reserve prices. More specifically, we develop a structural model of advertisers' bidding behaviors that can be affected by (i) own valuations of the ad impressions, (ii) competitors' valuations, (iii) the reserve prices, and (iv) the constraints advertisers might face. These constraints include a budget for the ad campaign or a set goal of impressions the campaign tries to reach. This model enables us to make predictions about how advertisers will respond, for example, when the constraint starts to bind tighter as the publisher increases reserve prices. Leveraging our advertiser model and the predicted responses, we develop a pricing model for the publisher that yields optimal reserve prices aimed at maximizing revenues.

We collect a novel dataset from a major publisher detailing its advertisers' bidding behaviors

Harvard University, HEC Paris, HKUST, the University of Illinois at Urbana-Champaign, Johns Hopkins University, Junior Faculty Development Forum at WashU, Lingnan University, the University of Miami, MSI State of Marketing Science Summit, Northwestern University, the University of Notre Dame, the University of Toronto, Rice University, the University of Rochester, Southern Methodist University, Stanford University, the University of Texas at Austin, Tilburg University, U.S. Department of Justice, the University of Vienna, and Yonsei University for comments and suggestions.

[1] https://www.statista.com/topics/2498/programmatic-advertising/#topicOverview

through an ad exchange and run a series of reserve pricing experiments in collaboration with the same publisher. Using this experimental variation in reserve prices, we examine advertiser responses and connect them to the theoretical predictions of our advertiser bidding model. This analysis offers insights into whether practical constraints - such as budget or impression/reach - are binding for advertisers, if any, and identifies the predominant constraint (the one that binds tighter) driving the behavioral patterns observed in our experiment. Subsequently, we derive the optimal reserve pricing policy on the part of the publisher in response to advertisers' demand and bidding behaviors that are generated from a model consistent with the experimental findings.

Several novel findings emerge. First, the implementation of reserve prices plays a crucial role in enhancing publisher's revenues. Experimental results suggest that setting reserve prices based on the simplistic assumption that advertisers face no practical constraints can increase publisher revenues by an average of 35% relative to setting no reserve prices. Second, the exogenous experimental variation enables us to test this naive assumption of no advertiser budget or impression constraints. Empirical analyses indicate that bids adjust to reserve prices in a manner consistent with advertisers predominantly facing minimum impression constraints. By incorporating these constraints into counterfactual exercises, we find that publishers could now attain 44% increase in revenues over the case without reserve prices. This additional 9 percentage points lift (from 35% to 44%) comes from accounting for the practical constraints advertisers face. Finally, an additional validation experiment affirms that revenues are generally optimized at the reserve price levels recommended by our model.

1.2 Relevant Research

Much of the display advertising literature centers on advertiser-side managerial challenges, such as measuring advertising effectiveness (e.g., Barajas et al. 2016, Gordon et al. 2019, Hoban and Bucklin 2015, Johnson et al. 2017, Lewis and Rao 2015, Sahni 2015, Waisman et al. 2025) and the resulting implications for ad buying and targeting decisions (e.g., Goldfarb and Tucker 2011, Rafieian and Yoganarasimhan 2021, Tucker 2014). Within display ad auctions, a substantial body of work focuses on advertisers' real-time bidding strategies and algorithms (e.g., Alcobendas and Zeithammer 2025, Balseiro and Gur 2019, Cai et al. 2017, Ghosh et al. 2009, Iyer et al. 2014, Johnson 2013, Lee et al. 2013, Sayedi 2018, Tunuguntla and Hoban 2021, Xu et al. 2015, Yuan et al. 2013). In contrast, our focus is on the supply side of the market, specifically examining publishers' pricing strategies in ad exchange markets.

On the supply side, an emerging body of research explores publishers' strategies including ad impression allocation mechanisms (e.g, Celis et al. 2014) and information provision policies (e.g., Ada et al. 2022). With respect to pricing strategies, Despotakis et al. 2021 solve for the optimal display ad reserve prices in a single-shot game framework. Balseiro et al. 2015 consider reserve prices where advertisers face budget constraints over the duration of their campaigns. We build on this literature in two ways; (i) we assume an empirical approach to set optimal reserve prices by developing a structural model to be used in practice and (ii) we consider the possibility of advertiser impression constraints (that is, the advertiser seeks to win at least a specific number of ad impressions over the duration of a campaign).[2]

The consideration of impression constraints is motivated by the various buying tools available to advertisers, which enable them to select advertising campaign goals as well as spending.[3] For example, Google's Ad Manager demand-side platform (DSP) allows advertisers to choose from a range of campaign objectives, such as brand awareness and reach that focus on obtaining impressions. Similar options are available on other major platforms including Meta, Amazon, TikTok, and LinkedIn. Figure 1, for instance, illustrates advertisers' ability to maximize impressions at Meta. Such branding goals help advertisers obtain better access to retailer shelf space and higher organic ranks at online retailers, increase the efficacy of their advertising by enhancing recall and recognition, induce more media coverage, enable them to charge higher prices, motivate employees, and expand word of mouth including via social media feed algorithms (Keller et al. 2010).[4]

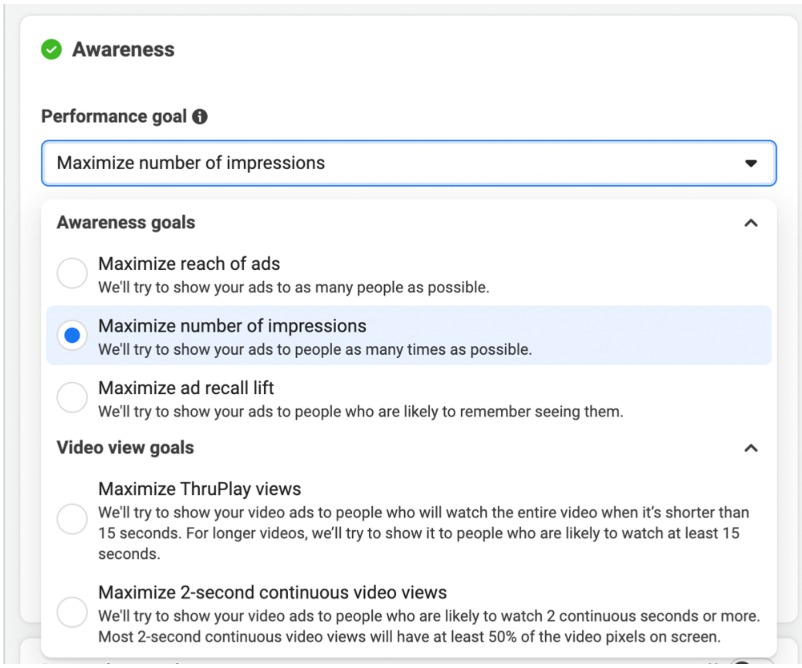
In recovering advertisers' valuations in this dynamic setting, we adopt the fluid mean-field equilibrium (FMFE) framework developed in Balseiro et al. 2015. Of note, the solution to the advertiser bidding game only depends on the steady-state distribution of rival bids, and not on the current single auction-specific state or the rivals' individual states. This equilibrium concept provides

[2] Coey et al. 2021 develop a scalable approach to compute optimal reserve prices without the need to recover bidder valuations and apply it to e-commerce auctions on eBay. Unlike our context, their setting assumes that bidders have one opportunity to bid and play the weakly dominant strategy of bidding their true valuations. In addition, our emphasis is display advertising.

[3] Please see Choi et al. 2020 for operational details of display ad markets.

[4] Other goals and constraints exist, such as clicks and conversions. However, we focus on impressions as the vast majority of ad inventory in our data (over 90%) is sold based on a CPM (cost-per-mille) basis, where advertisers pay per impression. Although clicks are relatively rare, they are conditioned on impressions and are often correlated with them. As such, evidence of an impression constraint in our context may, in part, reflect underlying click-related considerations. Notably, unlike this research, prior empirical work in display advertising has largely abstracted away from advertiser constraints altogether. Extending this literature to incorporate a broader set of goals and constraints would be a promising direction for future research.

Figure 1: Impression Goals at Meta's Ad Buying Dashboard



a computationally tractable way of modeling advertiser bidding behaviors and competition while capturing the dynamic nature of the advertiser decisions across auctions.[5] We extend this framework to accommodate a minimum impression constraint, and suggest estimation and identification strategies to link the FMFE theory to our empirical context.

Finally, in contrast with most of this prior research, we conduct field experiments to provide exogenous variation to test assumptions used in our model and demonstrate the potential of using theoretical insights to improve auction outcomes. On this dimension, our research is related to Ostrovsky and Schwarz 2023, who conduct a large field experiment in the context of search advertising. They find that setting appropriate reserve prices guided by theory leads to substantial increases in seller revenues. We similarly corroborate the importance of setting reserve prices, but in the context of display ad auctions (instead of sponsored search auctions). Different from Ostrovsky and Schwarz 2023, we additionally examine the causal effect of reserve prices on advertiser bidding behaviors and use a structural model to back out advertiser primitives.

While conceivable to conduct multiple experiments to evaluate a broader range of pricing levels,

[5] Backus and Lewis 2025 and Hendricks and Sorensen 2015 study bidders having a unit demand with multiple opportunities to bid in second-price sealed-bid auctions, and apply the framework to eBay's market. They also similarly use the belief formation and the mean-field equilibrium concept in Krusell and Smith 1998, Weintraub et al. 2008, and Iyer et al. 2014, but for a unit demand without a reach or budget constraint.

publishers often face significant constraints when setting reserve prices through ad exchanges, for example, limitations on the total number of pricing rules allowed.[6] Conducting such experiments are also costly in practice owing to engineering overhead and risks of losing revenue by selecting the wrong reserve prices. In the absence of a structural model, it is generally not feasible to extrapolate from a limited set of experimental cells to identify the optimal reserve prices, particularly when doing so requires tracing out the entire revenue curve along a continuum of reserve price levels. Further, structural models uncover market primitives that inform optimal reserve prices beyond experimental cells and can provide insights such as which constraints (e.g., budget or impression) matter and how frequently they bind in practice.

1.3 Organization

This paper is organized as follows. Section 2 first characterizes our data, demonstrating considerable heterogeneity in advertiser bidding behaviors. Next in Section 3, we present the advertiser bidding model and the publisher pricing model. Section 4 provides experimental evidence of constraints. Section 5 discusses the estimation method and the identification argument in inferring advertiser valuations, and Section 6 presents the estimation results. Finally in Section 7, we compute the optimal reserve prices and the revenue gains, and Section 8 presents a validation experiment of our model predictions.

2 Data

In this section, we describe the publisher and its data source to describe the research context and to show that the advertiser bids differ for different types of impressions. Heterogeneity in bids is instrumental in setting reserve prices; it suggests that advertisers differ in their valuations for advertising impressions and that they consider information about impressions such as the type of ad inventory in setting their bids.

2.1 The Publisher and its Data

The data for this study are collected from a large, premium publisher, ranked within the U.S. Top 10 by comScore.[7] The publisher has over 35 brands (sites). We focus our attention on display ads

[6] In our context, the ad exchange limited the publisher to just 150 pricing rules across all its dozens of websites and inventory types, defined by combinations of site, device, ad format, and ad type.

[7] https://www.comscore.com/Insights/Rankings

and exclude video ads in our analyses.[8]

The publisher sells its ad inventory through a large ad exchange and advertisers bid for this inventory using various demand side platforms (DSPs).[9] DSPs facilitate bidding on behalf of advertisers, often through automated systems. We assume that these systems reflect advertisers' stated objectives and are largely aligned with their campaign goals and budgets. For example, when an advertiser specifies brand-awareness or impression goals, DSPs' automated bid-optimization tools typically adjust bids to improve win rates (impressions won ÷ bids submitted). When win rates decline, the tools raise bids to help ensure that the campaign secures its targeted number of impressions.[10] In this way, automated bidding tools, algorithms, and bots play a central role in helping advertisers achieve their campaign objectives.

An advertiser bids on an impression based on i) data available from the third party (e.g., data management platforms or DSPs) potentially including demographics, past purchases, and advertising response and ii) publisher provided information. Information provided by the publisher to the advertiser typically includes users' article consumption (topic interests), data collected during user registration, and whether a given user or cookie has previously been exposed to the advertiser's ads (used for frequency capping). The publisher can choose to share this information with the ad exchange, subject to the privacy policy in place. The extent of the user (cookie) level information available to the advertiser also depends on the privacy regulation and has changed over time. In our context, the publisher does not release different information between treatment and control in the experiment and the information disclosed does not vary with reserve prices.

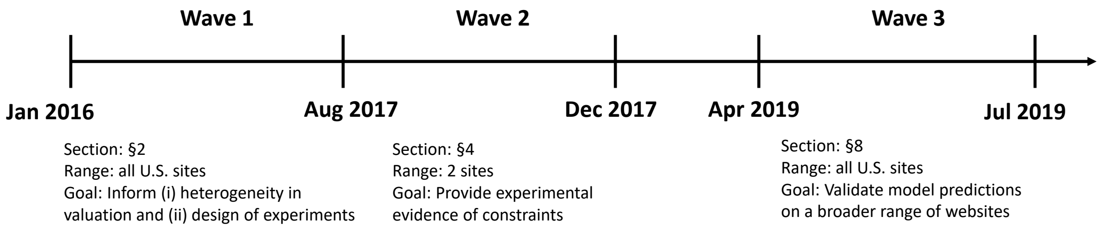
The publisher's ad exchange partner provides a report on auction outcomes related to this publisher's ad impressions and this report is generated at the daily level and includes advertiser ID, DSP ID, day, site where ad was placed, ad type (ad size, ad location on a page, device), number of bids submitted, number of impressions won, bidding amount, payment amount, and click responses. Thus the observational unit (i.e., dimension) is at the advertiser-DSP-day-site-ad type delivered, and the metrics provided for each observational unit are number of bids submitted, number of impressions won, bidding amount, payment amount, and number of clicks received. While number

[8] We adopt this sampling criterion due to the publisher's selling policy, where video ads are rarely available for sale in the ad exchange.

[9] Demand Side Platforms (e.g., DV 360, The Trade Desk, Yahoo! Ad Tech, MediaMath, Xandr) facilitate the bidding process for advertisers.

[10] https://support.google.com/displayvideo/answer/2997422?hl=en&ref_topic=3094964&sjid=14140632331992608595-NA#zippy=%2C

Figure 2: Timeline



of impressions won, payment amount, and number of click responses are available for all advertisers (we call this the "payment data"), number of bids submitted and the bidding amount are only available for a subset of advertisers who opt-in to share their bidding data with the publisher (we call this the "bidding data").

The data are collected in three waves as outlined in Figure 2. We collect observational data in Wave 1 to characterize heterogeneity in advertiser bidding behaviors and to inform the design of reserve price field experiment in Wave 2. The reserve price field experiment in Wave 2 is conducted at two sites and are used to distinguish between various theories regarding advertiser behavior, as will be described in later Section 4. These insights on advertiser behavior enable us to recommend an optimal reserve price policy for the publisher as described in Section 7. Wave 3 then conducts a validation experiment at a broader range of sites to test our reserve price policy.

2.2 Summary Statistics

In this section, we provide summary statistics from the observational data (Wave 1) to show there exists considerable variation in advertiser bidding behaviors which can inform the design of reserve price field experiments.

A total of 15,745 advertisers participate in the auctions from January 2016 to August 2017. Summary statistics of advertiser buying behaviors are presented in Table 1. All dollar metrics, including cost-per-mille (CPM) paid and bid CPM are scaled by a common, multiplicative constant throughout the paper for confidentiality. At the observational unit level (i.e., advertiser-DSP-day-site-ad type), advertisers on average win 14 impressions at \$0.93 CPM rate. The minimum CPM payment is close to zero, because the publisher imposed no reserve prices for most of its inventory during this period.

Bids are observed from the 81% of the advertisers who opted-in (default setting) to share their bidding information. Opt-in advertisers pay a higher CPM (\$1.18) than the full sample average

Table 1: Summary Statistics of Advertiser Behaviors

Per Observation Unit		Mean	Median	Std Dev	Min	Max
All	#Impressions Won	14.32	1	760.74	0	$2.99M$
	CPM paid ($)	0.93	0.59	1.94	0.00	4646.95
Opt-In	#Impressions Won	6.98	0	720.88	0	$2.29M$
	CPM paid ($)	1.18	0.71	2.34	0.00	1459.91
	# Bids Submitted	230.15	4	8306.13	1	$14.44M$
	Bid CPM ($)	1.48	0.66	12.25	0.00	37500.00

Note: CPMs paid are calculated conditioned on #impressions won > 0. CPMs are scaled by a common, multiplicative constant to preserve publisher confidentiality.

($0.93), but buy a much smaller number of impressions, constituting about 20% of the total revenues. Hence, there may be selection concerns arising from using the bid CPMs for inferring advertiser valuations in estimation. Fortunately, data are available on the CPM paid from all advertisers. Accordingly, we rely on the payment data in our estimation and only use the bidding data (number of bids submitted and the bid CPM) when reporting descriptive patterns (Subsection 2.3) and experimental findings (Subsection 4.2.2).

2.3 Heterogeneity in Valuations

In order to understand how to set reserve prices, it is necessary for the publisher to infer the distribution of advertiser valuations. The results below suggest substantial heterogeneity in bids with respect to the observed characteristics, and thus the potential to use reserve prices to price discriminate.

Using the bid data from January 2016, Figure 3 depicts the distribution of bid CPM z-scores.[11] The bid CPM z-score is calculated as (bid CPM - weighted mean) / (weighted stdev), where the number of bids for a given observational unit (advertiser-DSP-day-site-ad type) is used as the weight.[12] The x-axis represents these bid CPM z-scores, and the y-axis represents the percentage of number of bids. The figure evidences considerable heterogeneity in bidding with the minimum of -0.22 and the maximum value being 19506.13.

To explain the variation in the advertisers' bid CPMs, we estimate a weighted least square regression of bid CPM z-scores on the ad type (device, web vs. app, ad location, ad size), day, site, DSP, and advertiser. The weight used in the least square regression is the number of bids submitted for each observational unit.[13] Table 2 indicates that advertisers bid more for the desktop

[11]The results from other months are qualitatively similar.

[12]Bid CPM observations are daily averages (i.e., total bidding amount/#bids) for given observational units (advertiser-DSP-day-site-ad type) so we use the number of bids as the weight in constructing the bid CPM z-score.

[13]The ad size metric is calculated in (pixel^2/10000) unit. For example, we calculate the ad size for a (300×250) ad

Figure 3: Bid CPM Z-Scores: Heterogeneity

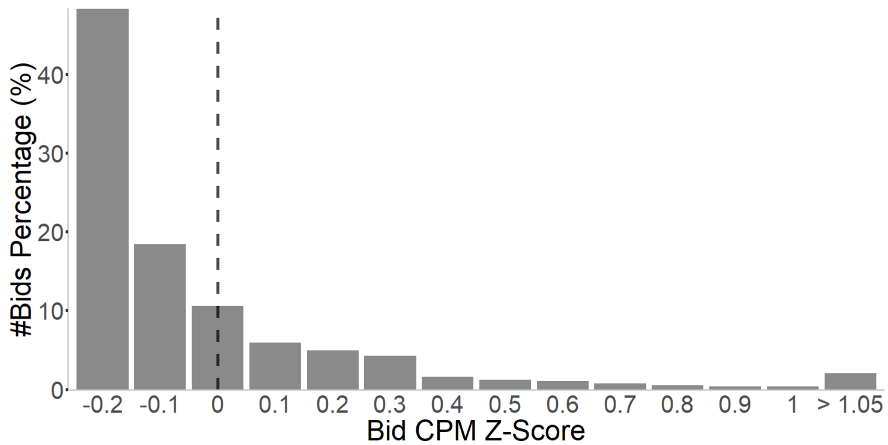


Table 2: Bid CPM Z-Score: Weighted Least Square Regression

DV: Bid CPM Z-Score	Estimates	SE
Desktop vs. Tablets	**0.11*****	(0.02)
Mobile vs. Tablets	**0.05*****	(0.02)
Web vs. App	0.02	(0.05)
Above the Fold vs. No Info	**0.04*****	(0.01)
Mid vs. No Info	0.01	(0.02)
Below the Fold vs. No Info	0.01	(0.02)
Size	**0.02*****	(0.00)
Day FE	Yes	
Site FE	Yes	
Advertiser FE	Yes	
DSP FE	Yes	
R-squared	0.14	

Note: An observational unit is (advertiser-DSP-day-site-ad type). Standard errors in parentheses are clustered at site level. ***denotes 1% significance. The number of observations is not reported to conceal the daily total number of ad impressions for the publisher firm.

ads (relative to mobile, tablets), above-the-fold ads (relative to mid, below-the-fold, or no info), and bigger size ads. These results imply that optimal reserve prices can be set differentially based on these ad types. Figure 4 plots the advertiser fixed effects and the site fixed effects estimated from the weighted least square regression of bid CPM z-scores. Both advertiser and site explain considerable variation in bid CPMs, suggesting they can also be used to price discriminate when setting reserve prices.[14]

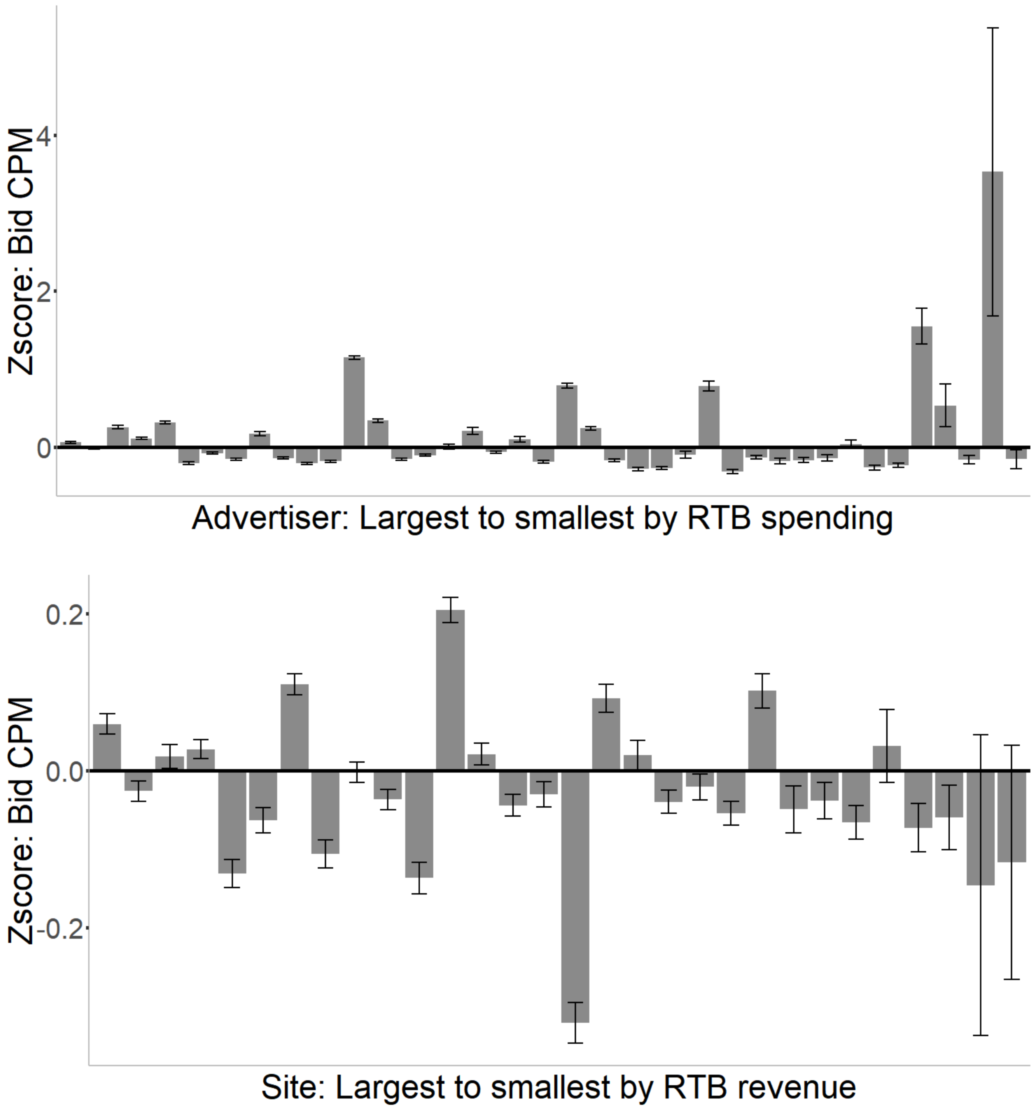
3 Model

This section develops a model of advertiser ad buying as a function of advertisers' valuations and reserve prices. Subsequently, the publisher's reserve price decision is modeled taking advertisers' ad

to be 7.5 ($= 300 \times 250 \div 10000$).

[14]The results from using bid CPMs ($) in lieu of bid CPM z-scores are qualitatively similar. We report bid CPM z-scores instead of levels to preserve the confidentiality of the bid CPM levels.

Figure 4: Advertiser Bid CPM: Advertiser and Site Fixed Effects



Note: Advertisers and sites are ordered by the spending (top figure) and revenues (bottom figure) in the real-time-buying (RTB) ad auctions, presented from the largest on the left and the smallest on the right.

buying decisions into account. Our approach builds upon the theoretical work of Balseiro et al. 2015, who use a fluid mean-field equilibrium (FMFE) to approximate the dynamic optimal bidding problem of budget-constrained advertisers buying multiple impressions for a campaign. In this section, we extend this modeling approach to the context of a minimum impression constraint. Analogous to a budget constraint, a minimum impression constraint induces inter-temporal, dynamic interactions over multiple second-price auctions that influence how reserve prices affect advertiser ad buying and publisher revenues. As we shall show, this constraint induces advertisers to bid higher than their true valuations (subject to a participation constraint). We conclude this section by contrasting the two types of constraints and how they affect advertiser bidding behaviors.

3.1 Model Overview

The ad exchange conducts a second-price, sealed-bid auction for each available impression. An impression is delivered to the winner who bids the highest above the reserve price (i.e., the winner's ad is displayed to the consumer). If no advertiser bids above the reserve price, the impression remains unsold and is perishable. The winner pays the second highest bid, or the reserve price if the second highest bid falls below the reserve price. Hence, we consider a second-price auction mechanism.[15]

The advertising game proceeds as follows:

1. Publisher: The publisher moves first and selects the reserve price, r, in the second-price auction. The publisher's objective is to maximize its expected revenues from the auctions. The publisher knows the distribution from which the advertiser's valuation is drawn, but does not know the advertiser's realized true value for each particular available impression.

2. Advertisers: The advertisers move in the second stage. Each advertiser decides how much to bid b, based on the realized valuation v for each given impression, the reserve price r, and the distribution of competing bids. The advertiser's objective is to maximize its expected utility (= valuation − cost) across auctions over the duration of the advertiser's ad campaign and given its minimum impression level, y, for that ad campaign.

[15] Generally speaking, retail media and walled gardens (e.g., Amazon or Facebook where the ad exchange and publisher are integrated) use second-price auctions (SPA) while independent ad exchanges use first-price auctions (FPA) (Despotakis et al. 2021, Alcobendas and Zeithammer 2025). Generalizing our approach across auction mechanisms is conceptually straightforward. When the estimation model uses a SPA, assessing the effect of a FPA requires a counterfactual analysis. In contexts where data are generated using FPAs, the estimation model would be adapted to bidder behaviors under FPAs, while the effect of a SPA could be conducted using a counterfactual analysis.

We will first introduce the equilibrium concept. Then using backward induction, we solve the advertiser's ad buying problem, and then characterize the publisher's reserve pricing problem.

3.2 Equilibrium Concept

Given the auction rules set by the ad exchange and the reserve price set by the publisher, advertisers participate in a game of incomplete information as they compete against other bidders. Furthermore, because the minimum impression constraints link advertisers' decisions across periods, the game becomes dynamic and cannot be simplified to a series of static auctions.

3.2.1 Fluid Mean-Field Equilibrium (FMFE)

We adopt the fluid mean-field equilibrium (FMFE) concept to address the challenges mentioned above. A FMFE is defined as "a set of bidding strategies such that (i) these strategies induce a competitive landscape as represented by the steady-state distribution of the maximum bid, and (ii) given this landscape, advertisers' optimal fluid-based bidding strategies coincide with the initial ones." (Balseiro et al. 2015). In other words, this equilibrium concept requires that the bidders' conjectured steady-state distribution of the maximum bid they face in competition is consistent with the fluid-based bidding strategies they themselves employ.

Applying this equilibrium concept to display advertising markets, an advertiser needs to form some belief about the distribution of competing bids when choosing an amount to bid. There can be various ways to model such an expectation depending on the information available to the advertiser and the level of sophistication expected. The first approximation FMFE considers is a *mean-field* approximation (Weintraub et al. 2008, Iyer et al. 2014) to relax the informational requirements of agents, requiring only that they know some aggregate and stationary representation of the competitors' bids and form beliefs based on these aggregate states, as opposed to tracking the specific action of each and every of the potentially 1000's of other bidders. In addition to the mean-field approximation, FMFE further incorporates a *stochastic fluid* approximation from the revenue management literature that reduces the computational burden in addressing the complex inter-temporal dynamics of the advertisers' bidding problem. We discuss these two approximations in detail next.

3.2.2 Mean-Field and Stochastic Fluid Approximation

Mean-Field Approximation The mean-field approximation assumes that the distribution of competitors' bids across auctions is stationary with the overall number of advertisers in the market being large. Moreover, it posits that an individual advertiser's bids do not affect this overall distribution. In a large market where any individual player has little effect on the subsequent play of others, a bidding strategy that relies on the aggregate and stationary representation of the competitors' bids well approximates the rational behavior of advertisers. This approximation is well suited for our display advertising context where thousands of advertisers compete over a duration of a campaign and where it is costly to advertisers to track and forecast the exact actions of all individual bidders. Institutionally, tracking the specific actions of all agents is impractical and implausible because advertisers generally do not have access to complete information about bid history. Rather, advertisers often rely on "bid landscape" information provided by the ad exchanges (Ghosh et al. 2009, Iyer et al. 2014, Balseiro et al. 2015). Bid landscapes are stationary representations of bidding distributions over a recent time horizon, which is exactly the relaxed information requirement in FMFE.

The mean-field approximation, while predicated on the overall number of advertisers in the market being large, allows a small fraction to participate in each auction, and the average number of bidders *per auction* need not be large. Roughly speaking, even when only a small fraction of competitors participate in any given auction, that set varies from auction to auction and the distribution of their bids "averages out" and appears stationary over many auctions an advertiser faces during its campaign (which often extends for weeks or months). For this reason, running auctions and setting reserve prices remain useful, as a small number of bidders with heterogenous valuations participate in each one of these auctions.

Stochastic Fluid Approximation Using the mean-field approximation outlined above, the advertiser's bidding behavior can be modeled as a stochastic dynamic programming problem where bidding decisions are inter-temporally linked through a minimum impression constraint. However, solving the associated Hamilton-Jacobi-Bellman equation is challenging, as it typically lacks a closed-form solution. To enhance computational tractability, FMFE introduces a second layer of approximation: the stochastic fluid approximation (Gallego and Van Ryzin 1994).

The stochastic fluid approximation assumes that minimum impression constraints need only be satisfied ex ante, in expectation, when advertisers solve for their optimal bidding strategies. The rationale is that advertisers typically face thousands of bidding opportunities daily over multi-week campaigns, allowing the minimum impression constraints to be readily met in expectation. In such a setting, the stochastic fluid model provides a good approximation of the advertiser's stochastic dynamic programming problem, and without loss of optimality, one can focus on stationary bidding strategies that depend solely on the actual realization of the bidder's valuation, ignoring the individual auction specific state. More specifically, adopting the stochastic fluid approximation yields a simple and behaviorally appealing characterization of the optimal bidding function: an advertiser needs only to adjust its own valuation by a *constant scale* (see Proposition 1). This constant factor guarantees that the advertiser's minimum impression constraint is met in expectation, thus it captures the dynamic nature of the advertiser decisions well while significantly reducing computational complexity.

3.2.3 Relation of FMFE to Other Equilibrium Concepts

Weak Perfect Bayesian Equilibrium (WPBE), is commonly used to solve dynamic games of incomplete information. WPBE, along with its refinements such as perfect Bayesian equilibrium and sequential equilibrium, specify players strategy profiles (a mapping from histories to bids) in conjunction with a set of beliefs regarding market dynamics. Specifically, WPBE assumes strong rationality as it entails tracking high-dimensional belief distributions and optimizing over history-dependent strategies for each and every player in the market. As the number of competitors grow, these requirements make the optimization problem high-dimensional and computationally infeasible. More importantly, the complexity and sophistication expected of agents is likely unrealistic in context.

As a result, we adopt the FMFE described above. However, the FMFE is not without its limits. FMFE requires two approximations; mean-field and stochastic fluid approximations. As noted by Balseiro et al. (2015, p. 870), the assumptions that underpin FMFE are more likely realized in "large markets where the number of bidding opportunities is large", such as in our display advertising market context. Hence, the FMFE concept may not apply as generally as a WPBE.

Further, the mean-field approximation for large-scale dynamic games has also appeared in other auction context and industrial organization applications. The equilibrium concept based on this type of approximation is known as Oblivious Equilibrium (Weintraub et al. 2008, Light and Weintraub

2022) or Mean Field Equilibrium (Iyer et al. 2014), in which each agent is assumed to make decisions based solely on its own state and knowledge of the long-run equilibrium distribution of states, disregarding information about rivals' current individual states. Oblivious Equilibrium and Mean Field Equilibrium omit the stochastic fluid approximation component.

3.3 Advertiser Problem

3.3.1 Advertiser Bidding Model

The goal of the advertiser bidding model is to determine the optimal bidding policy for advertisers. Subsequently, we will use the advertiser bidding model to infer the distribution of advertiser valuations by matching the optimality conditions derived in this section to the moments in the data (such as advertisers' payments). Advertiser valuations reflect all aspects of the impression that affect advertiser current and future profits, including current and future conversions Choi et al. 2020. Once these valuations are inferred, it becomes possible to explore the effect of reserve prices on advertiser bidding behaviors and the revenue implications when advertisers face practical constraints – the ultimate goal of this paper.

Following Balseiro et al. 2015, we consider a continuous-time infinite horizon setting in which users (ad impressions) arrive to the publisher's website according to a Poisson process with intensity η. Similarly, advertisers arrive to the ad exchange according to a Poisson process with intensity λ. Advertiser k is characterized by a type vector $\theta_k = (y_k, s_k)$; advertiser k has a minimum impression level y_k for the campaign length s_k (the duration over which an advertiser is using its bidding rule). Also, following the symmetric, independent private value assumption commonly adopted in prior work (e.g., Ostrovsky and Schwarz 2023, Balseiro et al. 2015), we assume that valuations for ad impressions are independent and identically distributed with a continuous cumulative distribution $F_V(\cdot|Z)$, where Z are auction specific observed characteristics. This specification allows for values that are correlated via the observed attributes Z. For example, Z may contain site dummies to capture that site1 is valued more than site2 among the advertisers. Z can also contain month dummies to control for seasonality.[16]

[16]Because publishers can only condition on observables such as site–ad type (e.g., they cannot observe whether an advertiser engages in retargeting and thus may have higher valuations than others), we adopt a conditional i.i.d. assumption aligned with the publisher's information set for price discrimination, and develop a model that predicts advertiser bidding behavior with respect to reserve prices.

Publishers can only condition reserve prices on their information set such as site–ad type, but not advertiser-side audience data. Hence, we assume conditional i.i.d. that is aligned with the publisher's information set for price

Advertisers are assumed to have a quasi-linear utility function, where utility is defined as the sum of the advertiser's valuations from the impressions won less the total payment.[17] The advertiser maximizes its expected utility (= valuation − cost) from the ad auctions, given the minimum impression constraint and the participation constraint.

In an auction, the advertiser competes against other bidders and also against the reserve price r. The reserve price is assumed to be known to all potential bidders (advertisers). In practice, the reserve price is generally not announced to the advertisers, but they can infer it from their repeated auction experience (for example, via experimentation, automated bidding, and machine learning algorithms). We let D be the steady-state maximum of the competitors' bids, where the publisher is also considered as one competitor that submits a bid equal to the reserve price r. The distribution of D will endogenously be determined in equilibrium, which we denote as F_D. We assume that both publisher and advertisers form rational expectations on F_D.

We focus on the bidding strategy $\beta_\theta^F(v|F_D)$ for an advertiser type $\theta = (y, s)$ given the distribution of D, to be a function of the advertiser's own valuation v. The advertiser faces the optimization problem given by

$$\begin{aligned} &\max_b \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{b(V) \geq D\right\}(V - D)\right] \\ &\text{s.t. } y_\theta \leq \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{b(V) \geq D\right\}\right] \\ &\quad 0 \leq \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{b(V) \geq D\right\}(V - D)\right] \end{aligned} \tag{1}$$

where the expectation is taken over both F_V (the distribution of valuations) and F_D (the distribution of the maximum competing bid).[18]

In the first line, ηs_θ (the expected arrival rate times the duration of the bidding interval) indicates the total number of impressions arriving during the campaign period. $\mathbf{1}\left\{b(V) \geq D\right\}$ within the expectation $E_{V,D}$ indicates the probability of winning the auction on a given arrival, where the advertiser's bid is higher than the maximum of the competitors' bids. Lastly, $(V - D)$ indicates

discrimination.

[17]In the face of declining marginal value of advertising impressions, one could presume that per-impression valuations decline as more impressions are purchased. Such a concave valuation function might arise if advertisers first target high-value impressions and later relax their criteria to meet impression goals, or if limited supply forces them to expand targeting to less desirable placements. However, in our setting targeting is typically fixed at the outset of a campaign, and with sell-through rates generally below 70%, advertisers rarely adjust targeting mid-course. We therefore adopt a quasi-linear, rather than concave, utility specification.

[18]If the advertiser is assumed to maximize its expected utility given a budget constraint (Balseiro et al. 2015), the constraint in the model will be specified as $B_\theta \geq \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{b(V) \geq D\right\} D\right]$ where B_θ is the maximum budget level.

valuation minus payment, where the payment is consistent with the second-price auction rule.

In the second line, the right hand side is the expected number of impressions won at the end of the campaign period. The inequality constraint assures that the expected number of impressions won is greater than the minimum impression level y_θ. This inequality can also be written as $\frac{y_\theta}{\eta s_\theta} \leq E_{V,D}\left[\mathbf{1}\left\{b(V) \geq D\right\}\right]$, implying that the advertiser tries to attain a minimum auction winning rate of $\frac{y_\theta}{\eta s_\theta}$ in expectation. To define a well-behaved optimization problem, we assume $y_k < \eta s_k$, that is the minimum impression level is lower than the total available impressions.

The third line captures the advertiser's participation constraint that its expected utility in bidding in the ad exchange is greater than the outside option value, which is normalized to be zero.[19]

Below we characterize advertisers' optimal bidding strategies, assuming that the participation constraints hold (do not bind) in equilibrium. In Subsection 3.4, we discuss how the participation constraint is imposed in calculating the optimal reserve price.

Proposition 1. *Suppose that $E[D] < \infty$. An optimal bidding strategy that solves (1) is given by*

$$\beta_\theta^F(v|F_D) = v + \mu_\theta^*$$

where μ_θ^* is the optimal solution of the dual problem

$$inf_{\mu \geq 0}\, \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V \geq D - \mu\right\}(V - D + \mu)\right] - \mu y_\theta$$

That is the advertiser bids higher than its own valuation by a constant factor μ_θ^*, which is the optimal dual (Lagrangian) multiplier of the minimum impression constraint. Intuitively, this means that the advertiser foregoes some utility to satisfy the constraint. This constant factor μ_θ^* guarantees that the advertiser meets in expectation the minimum impression constraint at the end of the campaign period. Of note, the dynamic nature of the repeated auctions is captured by this constant factor μ_θ^*, and the bidding strategy becomes static in the sense that μ_θ^* does not depend

[19] Advertiser valuation v in our model represents the advertiser's long-term expectation of the net present value of the impression. Thus, future purchases from the person viewing the exposure in the current period or customer-specific reputational effects at the impression level are subsumed in v. Our participation constraint ensures that advertisers participate and submit bids if it's profitable to do so, that is, this long-term expectation of the net present value v (including the potential value from building up reputations) is greater than zero (the normalized outside option value). To exemplify these constraints, consider a new firm seeking to enhance its reputation capital via advertising. If the reputational benefits manifest as future sales to the impressed customers (increasing v), this effect would relax the participation constraint. If reputation instead means sufficient market awareness facilitate greater channel access or attract investment capital, it would provide an incentive to set a higher impression constraint (thus increasing y). In our model, we allow both minimum impression constraint and the participation constraint to bind (or either one to bind), and we are agnostic as to which one of these two dominates the other.

on the current single auction-specific state.[20, 21]

Proposition 2. *If the participation constraints do not bind in equilibrium, the equilibrium can be characterized as follows:*

$$\beta_\theta^F\left(v|F_D\right) = v + \mu_\theta^*$$

where μ_θ^* *is*

$$\begin{cases} \mu_\theta^* = 0 & \text{if } y_\theta < \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V \geq D\right\}\right] \\ y_\theta - \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V + \mu_\theta^* \geq D\right\}\right] = 0 & \text{if } y_\theta \geq \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V \geq D\right\}\right] \end{cases} \tag{2}$$

The proposition states that if the minimum impression constraint is not binding, then in equilibrium advertisers will bid truthfully ($\mu_\theta^* = 0$). On the other hand, if the minimum impression constraint does bind, then advertisers will bid higher than the true valuations, where μ_θ^* solves the implicit function $y_\theta - \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V + \mu_\theta^* \geq D\right\}\right] = 0$. Of note, $y_\theta - \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V + \mu_\theta^* \geq D\right\}\right]$ is the number of expected impressions shy of the minimum impression level at the end of the campaign, when the optimal bid function is employed.

Based on this proposition, the cost of the minimum impression constraint, μ_θ^*, which we call "bid premium", increases with the minimum impression level (y_θ), and decreases with the number of impressions and length of campaign (ηs_θ). Perhaps more importantly for our purposes, an increase in the maximum competing bid D, lowers $E_{V,D}\left[\mathbf{1}\left\{V + \mu_\theta^* \geq D\right\}\right]$ and increases the bid premium μ_θ^*, thus increases the resulting bid $\beta_\theta^F = v + \mu_\theta^*$. Because an increase in reserve price can increase D (as the publisher is also considered as one competitor that submits a bid equal to the reserve price r), an increase in reserve price can lead to higher bids. The proofs for Proposition 1 and Proposition 2 are in online Appendix A.1.

3.4 Publisher Problem

The publisher's objective is to maximize the long-run expected revenues from the auctions given advertiser valuations, and the publisher can maximize its revenues by choosing the reserve price optimally.

[20] In the budget constraint case (Balseiro et al. 2015), advertisers will shade down theirs bids by a constant factor to account for the future bidding opportunities (future value). In our minimum impression constraint case, advertisers shade up by this constant factor μ_θ^* to satisfy the constraint at the end of the campaign period.

[21] Under FMFE, advertisers do not need to accelerate or decelerate their bids toward the end of a campaign. Instead, they adjust bids by the constant factor μ_θ^* throughout, ensuring that the minimum impression constraint is satisfied in expectation, almost surely, at the end of the campaign. In our empirical context, consistent with this logic, we find that bids are largely stationary over time, with no evidence of systematic acceleration or deceleration.

3.4.1 Publisher's Reserve Price Optimization Problem

We denote $G_\theta(\boldsymbol{\mu}, r) = E_{V,D}[\mathbf{1}\{V + \mu_\theta \geq D\} D]$ to be the expected payment of a θ-type advertiser when advertisers bid according to the profile $\boldsymbol{\mu}$, and the publisher sets a reserve price r (G_θ term is the product of the payment times the likelihood the impression is won). We define $I(\boldsymbol{\mu}, r) = F_D(r|\boldsymbol{\mu})$ as the probability that the impression is not won in the ad exchange.

The publisher's problem can then be written as

$$\max_r \eta \sum_\theta \{\lambda p_\theta s_\theta G_\theta(\boldsymbol{\mu}, r)\} + \eta c I(\boldsymbol{\mu}, r) \tag{3}$$

$$\text{s.t. } \mu_\theta \geq 0 \ \perp \ y_\theta \leq \eta s_\theta G_\theta(\boldsymbol{\mu}, r) \ \forall \theta \in \Theta$$

where p_θ is the probability that an arriving advertiser is of type θ and $c > 0$ is the publisher's valuation (i.e., the outside option value if the impression is not won by some advertiser in the ad exchange). Recall that η denotes the arrival rate of users (ad impressions) to the publisher's website, and λ denotes the arrival rate of advertisers. In this continuous-time, infinite-horizon setting, the first line represents the publisher's expected revenue rate aggregated across all advertiser types in the auctions at any given time. Advertisers with shorter campaign windows can be viewed as having a lower effective arrival rate at any given time, λs_θ, compared to advertisers with longer campaign durations. Given the large number of advertisers and bidding opportunities, the mix of advertiser types is stationary under our FMFE framework.

The first term in the first line of Equation (3) indicates the average expenditure of the advertisers (i.e., the second highest bids when the auctions are won by some advertisers). $F_D(r|\boldsymbol{\mu})$ in the second term indicates the probability that the impression is not won in the ad exchange. Thus, the second term in the first line indicates the publisher's outside option value when the impression is not won by any advertiser, as it reflects the product of the scrap value of the impression and the probability the impression is not sold. In sum, the first line can be interpreted as the publisher's value over selling and not selling an ad impression. The constraints in the second line reflect the conditions for the Lagrangian multipliers in ensuring the minimum impression constraints are met, when advertisers bid according to the profile $\boldsymbol{\mu}$ and the publisher sets a reserve price r.

3.4.2 Advertiser Participation Constraints

In the counterfactual, where we increase the reserve price to the optimal level, some advertisers will start to face binding participation constraints as the reserve price increases.[22] To incorporate the effect of the participation constraints in the policy simulation, we ascertain whether the bidding profile $\boldsymbol{\mu}$ satisfies the participation constraint $(0 \leq \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V + \mu_\theta \geq D\right\}(V - D)\right])$ at the considered reserve price level r for $\forall \theta \in \Theta$.

In the case the participation constraint does not hold for some advertiser types θ, we calculate the maximum μ_θ that satisfies the participation constraint such that

$$\bar{\mu}_\theta = \max_{\mu_\theta \geq 0}\left[0 \leq \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V + \mu_\theta \geq D\right\}(V - D)\right]\right]$$

This is, the advertiser increases its bid to $v + \bar{\mu}_\theta$ to bid as closely as possible to its minimum impression level while satisfying the participation constraint.[23]

3.5 The Effect of Constraints on Advertiser Bidding

The theoretical prediction under the standard second-price, sealed-bid auction is that bidders bid truthfully, meaning that it is a weakly dominant action for the advertiser to bid the true valuation for an ad impression (e.g., the value they place on displaying an ad to the consumer). The truth-telling strategy is tractable and often assumed in models of display markets (e.g., Celis et al. 2014, Sayedi 2018), because advertisers' unobserved valuations for the ad impressions can directly be inferred from the observed bids. However, there exists empirical evidence that advertisers face practical constraints when bidding in the ad exchange auctions (Balseiro et al. 2015; Balseiro and Gur 2019, Ghosh et al. 2009), in which case advertisers may deviate from bidding their true valuations. Therefore, we consider how advertiser facing these practical constraints would change their bidding behaviors when reserve prices are increased. Specifically, we consider the effect of increasing reserve prices when advertisers face (i) a maximum budget constraint (Balseiro et al. 2015; Balseiro and Gur 2019), (ii) a minimum impression constraint (Ghosh et al. 2009), and (iii) no binding constraints.

[22] In the extreme case where all advertisers face binding impression constraints, advertisers' bidding strategies will reach ∞ as the reserve price increases to ∞ without the participation constraints.

[23] Because of the participation constraints, the advertiser may not achieve the intended minimum impression level under the counterfactual. In this case, we are assuming that advertisers purchase as many impressions as possible toward the minimum impression level while satisfying the participation constraints. Alternatively, we can specify the advertisers to drop-out all together ex-ante from the ad exchange and purchase zero impression when the participation constraints are expected to bind at the onset of the campaign, but we think the former is more realistic in our context. Analogous to standard pricing models, when reserve prices exceed advertisers' expected utility, demand will decline as advertisers no longer participate.

Table 3: Theory Predictions

Mechanism	Reserve Level		Effect of Imposing Reserve Prices		
	$r = 0$	$r = r^*_{nc} > 0$	Bid CPM	#Impressions Won	Total Payment
Max Budget Constraint	Not Bind	Not Bind	No Change	$-$	$+$
	Not Bind	Bind	$-$	$-$	$+$
	Bind	Bind	$+/-$	$+/-$	No Change
Min Impression Constraint	Not Bind	Not Bind	No Change	$-$	$+$
	Not Bind	Bind	$+$	$-$	$+/-$
	Bind	Bind	$+$	No Change	$+$

Table 3 presents the theory predictions for bid CPM, the number of impressions won, and the total payments (publisher revenue), when the reserve price is changed from $r = 0$ (i.e., no reserve price) to $r^*_{nc} > 0$. r^*_{nc} is the optimal reserve calculated under the no constraint, single-shot standard (truth-telling) model. As the reserve price increases from ($r = 0$) to ($r^*_{nc} > 0$), advertisers face tighter constraints, and each row represents a possible scenario of the underlying state: (not bind, not bind), (not bind, bind), (bind, bind) where bind implies the respective constraint binds. Each constraint is considered in isolation. That is, when we consider the budget constraint, we assume that the minimum impression constraint does not bind in both ($r = 0$) and ($r^*_{nc} > 0$).

Note that the no constraint case is nested within the non-binding scenario (Not Bind, Not Bind). Here, neither impression nor budget constraints bind, and the advertiser bidding model collapses to a standard second-price auction without the constraint. In this case, advertisers bid their true valuations, and the distribution of bids will be invariant to the reserve price level. As the reserve price increases from ($r = 0$) to ($r < r^*_{nc}$), advertisers will pay more when winning impressions, since the effective second-highest bid rises. Overall impressions fall, because increasing reserve prices lead to fewer advertisers bids exceeding them. Total payments increase because r^*_{nc} represents the optimal reserve price that maximizes publisher revenue under the non-binding (no-constraint) case. This case corresponds to the first and fourth rows in the table.

When advertisers' constraints become binding in response to higher reserve prices (i.e., as the reserve price increases from ($r = 0$) to ($r^*_{nc} > 0$)), the (Not Bind, Bind) scenario arises, corresponding to the second and fifth rows of the table. In the budget constraint setting (row two), advertisers shade their bids below true valuations to conserve budget for future arrivals over the campaign horizon. This behavior reduces bid CPMs. At the same time, a higher reserve price lowers the probability of winning impressions (e.g., auctions fail to clear when all bids fall below r^*_{nc}), so the number of impressions sold declines. Total payments rise, because the budget constraint becomes binding, meaning advertisers spend more. In the minimum impression constraint case (row five), when the reserve price increases from ($r = 0$) to ($r^*_{nc} > 0$), total impressions decline because advertisers who previously purchased beyond their minimum impression level at $r = 0$ can no longer do so at $r = r^*_{nc}$; in other words, they shift from the Not Bind to the Bind state. As a result, the minimum impression constraints tighten, inducing advertisers to bid more aggressively to secure enough impressions to satisfy their campaign requirements. The net effect on total payments is ambiguous, depending on the relative magnitude of higher bids versus fewer impressions sold.

When advertisers' constraints bind both at ($r = 0$) and at ($r^*_{nc} > 0$), the (Bind, Bind) scenario manifests, corresponding to the third and sixth rows of the table. In the budget constraint case (row three), the effects on bids and winning rates are ambiguous, since optimal bid shading with respect to r is not monotonic when advertisers face binding budget. When budgets bind both at ($r = 0$) and ($r^*_{nc} > 0$), total payments will remain the same at the budget-constrained level. In the minimum impression constraint case (row six), a higher reserve price tightens the constraint, prompting advertisers to bid more aggressively to secure the required impressions. The number of impressions won remains unchanged, as advertisers purchase the minimum number of impressions both at ($r = 0$) and ($r^*_{nc} > 0$).[24] Accordingly, total payments rise with higher bids as the reserve price increases.

Online Appendix A.2 provides detailed rationale for each of the predictions presented in Table 3. Table 3 suggests that, by varying the reserve price and observing how the outcomes change, we can determine which of these explanations is more consistent with the advertiser behavior given the bids observed in our data. Such a comparison is necessary for developing a pricing model concordant with advertiser behavior and of theoretical interest in its own right. Hence, before estimating a model for the purpose of setting reserve prices, the next section describes an experiment intended to assess which of these constraints is predominant, if any.

4 Experimental Evidence of Constraints

As demonstrated in the preceding section, advertisers' bidding behavior is differentially influenced by the practical constraints they face - reach (minimum impressions to buy) or budget. Moreover, an exogenous increase in the reserve price changes the tightness of these constraints. To investigate

[24]The number of impressions can decrease if the participation constraint binds for some advertisers.

how such changes affect both advertiser bidding behavior and publisher revenue, a field experiment was designed to vary reserve prices in order to assess which constraint is predominantly binding, if any, in our setting.

In this section, we begin by characterizing the design of the experiment. Next, we detail the results. We find that i) advertisers' bidding behaviors are not consistent with truth-telling but appear to reflect a minimum impression constraint, and ii) our use of theory-based reserve prices leads to a 35% increase in revenues for the publisher, over the case of no reserve price, providing concrete evidence there is substantial room to enhance its pricing outcomes.

4.1 Experimental Setting

4.1.1 Pairwise Randomized Experiment

A pairwise randomized experiment (Athey and Imbens 2017) was conducted on two selected websites, comprising twelve matched pairs. The two sites and the twelve pairs were chosen to be closest in terms of ad characteristics, contents, user demographics, revenues, and number of impressions (user visits). Two experimental ad units within a pair were randomized into the treatment and the control group. For example, for the first pair, ad unit (site1, desktop, below-the-fold, 300x250) and ad unit (site1, desktop, above-the-fold, 300x250) were selected. Within the pair, the randomly chosen (site1, desktop, below-the-fold, 300x250) was assigned to the treatment group, whereas (site1, desktop, above-the-fold, 300x250) was assigned to the control group. Table 9 in the online appendix includes the details of the experimental pairs and the assignment of treatment/control groups. Each experimental pair is referred to by an Experiment Pair ID (i.e., Experiment Pair 1 to 12). The unit in a pair (i.e., a treatment or a control) is henceforth referred to as an experimental cell.

The sample of experimental observations collected between 10/18/2017 - 12/03/2017 constitute the post-treatment period when the reserve prices were changed for the treatment group. The same duration sample from 08/30/2017 to 10/15/2017 constitutes the pre-treatment period. Both the pre- and post- periods are used for the difference-in-difference (DiD) analyses. Our experimental manipulation is at the (site–ad type) level. At this level, there are 12 treatment units and 12 control units, each observed both pre and post, yielding 48 observations in total. However, the data are collected at a more granular level (i.e., advertiser-DSP-day-site-ad type level), where treated and control nest the sampling-level granularity. Overall, we observe $5,382,829$ payment data observations from all advertisers and $3,635,899$ bid data observations from the opt-in advertisers at the (advertiser-DSP-day-site-ad type) level.

The table of balance reported in online appendix (Table 10) shows that none of the observables are statistically different between the treatment and control groups in the pre-treatment period.[25] Nevertheless, to control for potential level differences between the treatment and control groups, our identification strategy is to compare the treatment and the control within a pair over time by conducting a DiD analysis. Online Appendix B.1.3 discusses the pre-trend assumption required for our identification.

4.1.2 Setting Reserve Prices

As advertisers vary bids across observed ad characteristics such as site, device, ad location, and size (Table 2), the publisher can price discriminate to enhance its revenues by setting different reserve prices across these observed ad characteristics. Hence, we design the experiment to set reserve prices at the site-ad type (device, ad location on a page, size) level.[26]

The experimental reserve prices in the treatment condition were derived using a naive theoretical model presuming that advertisers do not face any binding constraints and that they play a single-shot game for each available ad impression. Under this framework, advertisers follow a weakly dominant strategy of bidding their true valuations, and their valuation distribution can be directly inferred from observed bids.

The optimal reserve price depends on this inferred valuation distribution. Raising the reserve price reduces the probability of a sale, since impressions go unsold if no bids exceed the reserve price, but increases the revenue conditional on a sale, as the payment equals the higher of the reserve price and the second-highest bid. As such, setting reserve price involves a trade-off; higher reserve prices lower the likelihood of a sale but raise revenues when sales occur. The optimal reserve price balances this trade-off to maximize expected revenue. Because the valuation distribution is estimated non-parametrically, the optimal reserve price is computed numerically. Further details on

[25] Exact matching pairs of cells is infeasible. For example, matching (Site1 above-the-fold, Desktop, 300x600) with (Site2, above-the-fold, Desktop, 300x600) could yield different average bid CPM levels in the pre-treatment period, because Site 1 and Site 2, though similar, are not identical.

[26] Although possible to further vary reserve prices across days, advertisers, and consumers, we do not because of the publisher's concern for fairness across advertisers and its preference toward a simpler pricing scheme. This preference is shaped both by constraints imposed by the ad exchanges and the engineering costs associated with implementing and managing complex pricing rules. In this regard, the effect we report on the publisher's revenue when setting reserve prices at the (site-ad type) level can be considered as a lower bound to the gains possible in a more flexible system.

the calculation of these experimental reserve prices are provided in online Appendix B.1.4. During the experiment, the reserve prices were set to these calculated levels for the treatment group, while they were kept at the historical levels for the control group (i.e., no reserve prices).[27]

Testing the theoretical predictions of a binding budget or impression constraint requires ascertaining if and how bids change with reserve prices. That is, the level of the treatment reserve prices need not be at the optimal level for testing, merely that the reserve prices vary exogenously. The suggested experimental reserve prices based on the naive, standard model provide reasonable initial points and are likely to increase the publisher's revenues toward the global maximum relative to the condition of no reserve prices. Further improvements on the publisher's revenue are possible when practical constraints are considered in setting the reserve prices, as we shall discuss later.

4.2 Experimental Results

We begin by reporting experimental results regarding the publishers' revenues to gauge the effectiveness of setting reserve prices in auctions. Next, we discuss the effect of reserve prices on advertisers bidding behaviors to asses which of the constraints (reach or budget) is predominant, if any.

4.2.1 Effect on eCPM

The outcome measure considered is *eCPM* (effective CPM, industry vernacular), which yields a per supplied impression revenue.

$$eCPM = \frac{\text{Revenue}}{\text{\# Impressions Supplied to Ad Exchange (in thousand)}}$$

Table 4 shows the treatment effect on eCPM, where eCPM is multiplied by a common, multiplicative constant for confidentiality. The increase in revenues (holding the number of impressions supplied to the ad exchange the same) is 35%, thereby affirming the importance of setting reserve prices in running the auctions.

[27]For pairs 8 and 10, positive reserve prices were in place in the pre-period. We provide additional discussions on these two cells in online Appendix C.2.

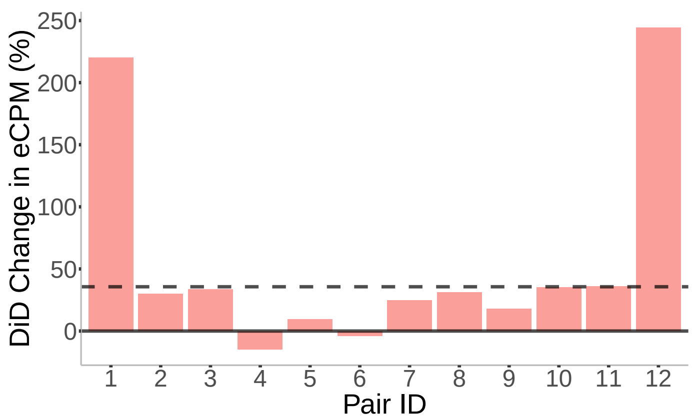
$$\text{Increase in revenue} = \frac{(0.49 - 0.37) - (0.36 - 0.37)}{0.37}$$
$$= \frac{0.13}{0.37} \simeq 35\%$$

Table 4: Treatment Effect on eCPM ($)

Group	Reserve	Pre (08/30/17 - 10/15/17)	Post (10/18/17 - 12/03/17)
Treatment	Yes	0.37	0.49
Control	No	0.37	0.36

Figure 5 uses data at the experimental manipulation level (site–ad type) to compute the percentage DiD change in eCPM and plots this treatment effect by the experimental pair. The dotted horizontal line shows the overall percentage DiD change in eCPM across pairs, 35%.[28],

Figure 5: Treatment Effect on eCPM by Experimental Pair



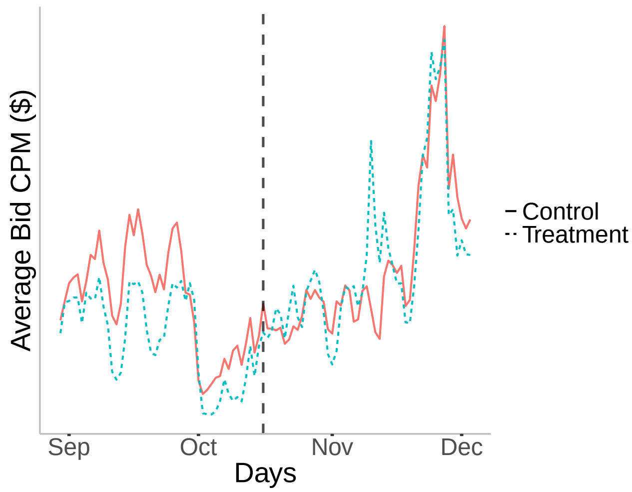
4.2.2 Effect on Bidding Behaviors

To assess how advertisers' bidding behaviors are affected by the exogenous increase in reserve prices, we consider their bid CPM data. If advertisers bid truthfully, the distribution of bids will remain invariant to the experimental manipulation of the reserve price. However, this is not the case. Figure 6 plots the average bid CPM trend line, where the vertical dotted line indicates the start date of the experiment. In the pre-period, the control group mean level is higher than the treatment group. But the mean level for the treatment group becomes higher as soon as reserve prices are imposed. Even though reserve prices are not published, advertisers respond to the changes in reserve prices

[28]The treatment effects on eCPM from alternative DiD specifications (e.g., using more granular units of analysis and adding control variables) are qualitatively similar and are reported in online Appendix B.2.1.

within days by adjusting their bidding strategies, suggesting they can quickly infer the reserve price levels based on the auction outcomes.

Figure 6: Bid CPM Trend



Note: Y-axis levels are not displayed for confidentiality.

The first column in Table 5 presents the result from a DiD regression of bid CPM. Day fixed effects are included to control for the time trend such as seasonality. Further, we include advertiser fixed effects as our emphasis is on within advertiser bid changes over time and the number of ad impressions to control for inventory. Each observation in the DiD regression is weighted by the number of bids submitted.[29] The increase in (scaled) bid CPM with respect to the exogenous increase in reserve prices is \$0.039. This represents a 19% increase and suggests that advertisers do not bid their true valuations.[30]

4.3 Theoretical Rationale for Experimental Results

In addition to the bid CPM analysis reported in Table 5, the DiD analyses are also conducted on the number of impressions won and the total payment. The second and third columns in Table 5 respectively indicate that the change in impressions bought is not statistically significant, while the (scaled) total payments significantly increase by \$0.005, or about 29%. Combining these two results with the observation that bid CPM increases upon raising the reserve prices suggests that

[29] Recall that a bid CPM observation is a daily average (i.e., total bidding amount/#bids) for a given observational unit (advertiser-DSP-day-site-ad type). Thus the weighted DiD regression uses the number of bids for a given observational unit as the weight.

[30] The analyses presented in Table 5 are conducted at the most granular observational unit in our data, advertiser-DSP-day-site-ad type level. This most granular observational unit differs from the level of experimental manipulation (at the site-ad type level). Our granular analysis can be construed as a repeated measure approach, where the granularity of observations is nested within the treatment and control units.

Table 5: Treatment Effect on Bidding Behaviors

DV (Scaled)	Bid CPM ($)	# Impressions Won	Total Payment ($)
Treated × Post	**0.039*****	0.351	**0.005*****
	(0.013)	(0.419)	(0.000)
# Impressions Supplied (in thousand)	Y	Y	Y
Treated	Y	Y	Y
Day	Y	Y	Y
Advertiser	Y	Y	Y
R-squared	0.212	0.197	0.188
Observations	3,635,899	5,382,829	5,382,829

Note: The dependent variables are all multiplied by a common, multiplicative constant for confidentiality. The unit of observation for the analysis is (advertiser-DSP-day-site-ad type). Bid CPM analysis uses 3,635,899 bid data observations from the opt-in advertisers. Ad impression and total payment analyses use 5,382,829 payment data observations from all advertisers. Standard errors are clustered by advertiser. ***denotes 1%.

advertisers' bids are affected by practical constraints. Specifically, the results are most consistent with the theory predictions presented in the last row in Table 3 in which the minimum impression constraint binds for at least some advertisers facing a change from zero to positive reserve prices. Intuitively, when the reserve price increases, the probability of winning an impression decreases for an advertiser. The advertiser therefore increases bid CPMs in order to achieve at least the minimum impression level set for an advertising campaign.[31, 32] This bidding behavior increases the total advertiser payment and keeps the number of winning impressions the same. Were advertisers to consider both the minimum impression level (reach) and the budget in setting their ad campaign and bids, one or the other constraint would be more binding. Our data are consistent with the minimum impression constraint affecting advertisers' bids more tightly than the budget constraint.

Based on these findings from the experiment, the advertiser bidding model is estimated with the minimum impression constraint to allow advertisers to depart from the commonly adopted truth-telling strategies. In the next section, we outline a non-parametric approach for estimating advertiser valuations with the minimum impression constraint and discuss model identification.

[31]As bid CPMs incorporate advertisers' underlying valuations for the ad impressions, an alternative interpretation for our results could be that valuations change with reserve prices. This might occur were an increase in reserve prices lead to fewer impressions being sold and therefore less ad clutter. Less clutter might increase users' attention or click-through-rate (CTR) for the displayed ads and therefore the advertiser valuations. We don't think this chain of reasoning is likely in our setting for two reasons. First, the publisher shows house ads (e.g., cross-promoting publisher's other brands/sites) in the unsold ad slots, thus keeping clutter relatively constant. Second, the DiD analysis of CTR shows no evidence of CTR change due to the increase in reserve prices. Less clutter would imply more clicks.

[32]When advertisers select the automated bidding strategy with the goal of Maximizing Brand, Google Display & Video, one of the largest DSPs, explains their algorithm as follows: 'your line item's bid will be raised until your budget can be spent', and further notes that 'average CPM may need to be adjusted to improve win rate'. These descriptions align with our findings: as reserve prices increase, advertisers win fewer impressions, resulting in lower win rates. When more auctions are lost and campaign budgets remain unspent, the algorithm responds by increasing bids to improve the win rate and achieve the campaign's brand or reach objectives.

Source: https://support.google.com/displayvideo/answer/2997422?hl=en&ref_topic=3094964&sjid=14140632331992608595-NA#zippy

5 Identification and Estimation

This subsection discusses the identification and estimation strategies for the inference of advertiser valuations.

5.1 Identification

We consider the identification of the model primitives, μ^* and F_V, from the observable data, F_D. Specifically, we consider a sequence of steps: i) the distribution of second highest bids, F_D ii) the distribution of shifted values, F_W, iii) the Lagrangian multipliers or bid premiums, μ^*, and iv) the distribution of valuations, F_V.

The Distribution of Second Highest Bids, F_D Because only the payment data are used in estimation (see Subsection 2.2), the distribution of the observed payments needs to be linked to the distribution of valuations. Denoting D to be the winning payments, F_D^0 is defined as the distribution of the second highest bids.[33]

The Distribution of Shifted Valuations, F_W The observed, non-parametric distribution of second highest bids, F_D, is functionally linked and identifies the distribution of valautions up to a location constant μ^*. Based on the optimal solution to the advertiser's problem and the resulting optimal bidding strategy in Proposition 1, we define the shifted valuation as $w = v + \mu^*$, where v is the true valuation and μ^* is the optimal Lagrangian multiplier (2). Then, the distribution of order statistics implies that

$$\begin{aligned} F_D(w) &= n(n-1)\int_0^{F_W(w)} u^{n-2}(1-u)du \\ &\equiv \varphi\left(F_W(w)|n\right) \\ f_D(w) &= nf_W(w)(n-1)F_W(w)^{n-2}\left(1-F_W(w)\right) \end{aligned}$$

In the last line, $nf_W(w)$ indicates that one of the n advertisers (i.e., $\begin{pmatrix} n \\ 1 \end{pmatrix} = n$) draws w exactly, and $(n-1)F_W(w)^{n-2}\left(1-F_W(w)\right)$ indicates that $(n-2)$ out of the remaining $(n-1)$ advertisers (i.e., $\begin{pmatrix} n-1 \\ n-2 \end{pmatrix} = n-1$) draw shifted valuations lower than w, $\left(F_W^0(w)^{n-2}\right)$, and 1 advertiser

[33] As the publisher currently imposes no reserve price, the identification strategy is described conditioned on no reserve price. When the reserve prices exist, the identification strategy will be similar, but the distribution of bids will represent a truncated distribution of valuations, because the impressions are not sold and payment data do not exist below the reserve price to infer valuations.

draws shifted valuation higher than w, $(1 - F_W(w))$.

Since $\varphi(\cdot|n)$ is a strictly monotonic function given n, F_W is identified from the distribution of observed winning payments F_D when the number of potential bidders n is known (Paarsch, Hong, et al. 2006). Thus, F_W is recovered by inverting the order statistic function such that $F_W(w|n) = \varphi^{-1}(F_D(w))$. In this way, F_W is non-parametrically identified from the distribution of F_D and the use of order statistics.

Lagrangian Multipliers μ^* Conditioned on the advertiser type $\theta = (s, y)$, identification of μ^* follows from knowledge of F_W and the conditions in Proposition 2. Specifically, μ^* satisfies the equation $y_\theta - \eta s_\theta E_{W,D}\left[\mathbf{1}\left\{W \geq D\right\}\right] = 0$ when $y_\theta \geq \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V \geq D\right\}\right] = \eta s_\theta E_{W,D}\left[\mathbf{1}\left\{W - \mu^* \geq D\right\}\right]$. Thus, conditioned on observing θ, F_W, and F_D, μ^* is functionally identified from the Karush–Kuhn–Tucker (KKT) conditions of the advertiser problem.

The Distribution of Valuations, F_V Once F_W and μ^* are identified, the distribution F_V can be recovered from the relationship $w = v + \mu^*$. Thus, F_V is identified from the structure of w, which itself arises from the solution to the first-order condition of the advertiser's problem - that is, from the functional form of the optimal bidding strategy. This first-order condition implies that v and μ^* enter additively and are separable.

In sum, F_D and F_W are identified non-parametrically from the observed prices paid, and $\mu^*$and F_V are identified from F_W,the observed types, and the functional forms arising from the solution to the first order conditions of the advertiser problem (see Equation 2).

5.2 Estimation

The estimation strategy is detailed in this subsection. The model primitives to be estimated are F_V and μ^*.

Under the truth-telling scenario, when the constraint does not bind ($\mu^* = 0$), F_D^0 can be estimated by substituting the sample analogue for the population quantity as

$$\begin{aligned} \hat{F}_D(v) &= \frac{1}{T}\sum_{t=1}^{T}\mathbf{1}\left[d_t \leq v\right] \qquad (4) \\ &= n(n-1)\int_0^{\hat{F}_V(v)} u^{n-2}(1-u)du \end{aligned}$$

The asymptotic properties of the estimator $\hat{F}_D(v)$ (and the resulting $\hat{F}_V$) are discussed in Paarsch, Hong, et al. 2006.

The strategy to incorporate the minimum impression constraint is estimating $F_{V|Z}$ together with μ^* for a given z. The estimation is done in two stages. In the first stage, we estimate location shifted distribution $F_V^0(w)$ where $w = v + \mu$. In the second stage, conditioned on the recovered distribution $F_V^0(w)$, μ is estimated using the condition in Proposition 2.

Incorporating (discrete) covariates is possible as follows. For a given characteristic $z \in \boldsymbol{Z}$, the estimator of $F_{V|Z}^0(v|z)$ is specified as

$$\hat{F}_{D|Z}(v|z) = n(n-1)\int_0^{\hat{F}_{V|Z}(v|z)} u^{n-2}(1-u)du \tag{5}$$

Because the number of participants vary across auctions, n-specific non-parametric empirical cumulative distribution functions (ECDFs) are estimated first for a particular combination of the z (by n-specific, we mean an ECDF is inferred separately for each auction with n bidders). Then $\hat{F}_{V|Z}(v|z)$ is obtained by kernel smoothing across the n-specific ECDFs to obtain a single ECDF. The exceptionally large number of payments observed given a particular combination of ad characteristics facilitates inference in this context of display ad markets.[34]

One challenge in estimating valuations when the impression constraint binds is that D, the distribution of maximum of competing bids, is a function of others' bids. As such, advertisers need to form beliefs about the bids of other advertisers. In estimation, this maximum is observed and equivalent to the rational expectation of the advertisers' regarding the maximum of the competitors' bids. The estimation procedure is outlined in the online Appendix D.1.

5.3 Institutional Details

This subsection discusses four institutional aspects of the data that warrant additional attention: (i) metrics (e.g., payments, number of impressions won) are only available as daily averages instead of at the auction (impression) level, (ii) the number of potential bidders, n, are not directly observed, (iii) operationalizing the minimum impression levels and campaign lengths, and (iv) which data points are included in Z.

Daily Aggregate Data The metrics (e.g., payments, # impressions won) provided by the ad exchange are aggregated to day level, and are not available to this or other publishers at more granular

[34]The observed characteristics affecting the valuation are discrete in our context, such as site, ad type (ad location, ad size, device), and time (month). Although the dimension of $\boldsymbol{Z}$ considered is large, we also have many observations per given particular combination of Z, enabling non-parametric estimation. If the covariates are continuous, semi-parametric approaches such as single-index models (e.g., the density-weighted derivative estimator in Powell et al. 1989, the maximum rank-correlation estimator in Han 1987) can instead be used to reduce the curse of dimensionality.

levels. More specifically, an observational unit in the ad exchange data represents total payments, number of impressions won, and number of clicks attained for each given advertiser-DSP-day-site-ad type. For each observational unit, the average (daily) CPM paid is calculated as (total payments / number of impressions won). This average (daily) CPM paid is used as d_t in Equation (4) in forming the estimator for $\hat{F}_D(v)$.

As each observational unit represents a different number of impressions won, we weigh the average CPM paid by the number of impressions won when estimating the distribution of valuations. That is, a data point (y average CPM paid, x number of impressions won) is treated as if there are x number of observations with y CPM payment.

Number of Potential Bidders The identification strategy discussed above requires that the number of potential bidders n is known. The number of potential bidders for a given observational unit (advertiser-DSP-day-site-ad type) will be

$$\begin{aligned} n = & \, n_1(\text{\# advertisers with CPM payment, i.e., positive \# impressions won}) \\ & + n_2(\text{\# advertisers with bids submitted, but zero impressions won}) \end{aligned}$$

n_1 is observed in the data, while n_2 is observed only for the opt-in advertisers. Thus, to operationalize n, we use the following proxy:[35]

$$\begin{aligned} \hat{n} = & \, n_1(\text{\# advertisers with CPM payment, i.e., positive \# impressions won}) \\ & + n_{2,opt-in}(\text{\# opt-in advertisers with bids submitted, but zero impressions won}) \end{aligned}$$

Minimum Impression Level and Campaign Length When advertisers do not face binding minimum impression constraints, advertisers bid truthfully and the distribution of valuations can be recovered without the knowledge of the minimum impression level or the campaign length. In the case the minimum impression constraints bind for some advertisers, the identification strategy discussed in Subsection 5.1 requires information pertaining to the observed levels of the constraint. More specifically, the estimator in Equation (14) in the online appendix, requires the term $\tau_\theta = \left(\frac{y_\theta}{\eta s_\theta}\right)$ as an input. This term reflects the minimum winning rate the advertiser aims to attain for a campaign. Thus, τ_θ is advertiser-campaign specific. In the estimation (and the counterfactual), we allow τ to

[35] $n_{2,opt-out}$ will be small for two reasons. First, 82% of advertisers opted-in to share their bidding information (default setting). Second, the opt-out advertisers (18%) constitute about 84% of the total revenues, meaning opt-out advertisers are highly likely to be included in n_1, which is observed. We provide robustness checks for this operationalization in online Appendix C.4.2.

vary across advertisers, but hold constant the minimum winning rate within an advertiser across campaigns. Denoting k to be the advertiser, τ_k is proxied by

$$\hat{\tau}_k = min\left(I_{k1}, ...I_{km}, ...I_{kM}\right),\ I_{km} > 0\,\forall m$$
$$I_{km} = \frac{1}{\sum_{day\in m}\mathbf{1}\left[i_{k,day} > 0\right]u_{day}}\sum_{day\in m} i_{k,day}$$

where $i_{k,day}$ represents the impressions won by advertiser k and u_{day} represents the impressions available for sale on a given day. I_{km} in the second line constructs the winning rate for a given month m, conditional on participating in the ad exchange. In other words, $\hat{\tau}_k$ is computed as the minimum of the monthly winning rates observed in the data, conditioned on the advertiser participating in the ad exchange. Accordingly, the optimal bidding strategy profile $\boldsymbol{\mu}|\boldsymbol{Z}$, is recovered at the advertiser level, instead of advertiser-campaign level.[36]

The solution to the publisher's optimal reserve price in Equation (3) also involves the term $\delta_\theta = p_\theta s_\theta$ which represents the share of each advertiser's type present in the auction (where type θ is uniquely defined by a campaign length and a minimum impression level). This share of advertiser types is determined by the product of the advertiser type's arrival probability, p_θ, and the campaign duration, s_θ; this product represents the fraction of advertisers in an auction that play bidding strategy profile $\boldsymbol{\mu}_\theta$. We do not observe p_θ and s_θ, but can nonetheless compute δ_θ at the advertiser-type level (that is, setting $\theta = k$). We do this by approximating the weight for each advertiser by its observed share of participation. Thus, we use below proxy for $\delta_{\theta=k}$

$$\hat{\delta}_k|Z = \frac{\sum_{day}\mathbf{1}\left[i_{k,day} > 0\right]}{\sum_k\sum_{day}\mathbf{1}\left[i_{k,day} > 0\right]}$$

where $i_{k,day}$ represents the impressions won by advertiser k on a given day. $\mathbf{1}\left[i_{k,day} > 0\right]$ takes value 1 if the advertiser k wins a positive amount of impressions on a given day. We include 'month' as the observable characteristics in Z, so the summation is done over days within a month. This $\hat{\delta}_k|Z$ is the weight advertiser k (with the bidding strategy profile μ_k) plays toward the platform's revenue given Z.

[36]The identification strategy discussed in Subsection 5.1 relies solely upon the *observational data* to compute minimum impression levels. An alternative identification strategy could leverage the *experimental data* (i.e., the causal change in bid CPMs) to estimate the minimum impression levels. Specifically, the optimality condition in Proposition 2 implies a one-to-one mapping between i) the minimum impression level and ii) the causal change in bid CPM observed in the experimental data. Hence, the operationalization of minimum impression levels in the observational data can be validated using the experimental data. Doing so, we find a strong positive correlation between the minimum impression constraints imputed from the observational data and those estimated from the experimental data (see Online Appendix C.4.1)

Observed Covariates The distribution of valuations is estimated separately for each combination of Zs to control for heterogeneity. The observed covariates considered in Z are

- Site: 20 U.S. based sites are considered.
- Ad location: these include above-the-fold (ATF), MID, below-the-fold (BTF), and no information available.
- Ad size: are represented by 300x250, (728x90, 970x66), 320x50, and 300x600. These are the sizes conforming to Interactive Advertising Bureau (IAB) standard guideline and are most commonly used by the advertisers.[37]
- Device: the various devices include desktop, mobile, and tablet.
- Month: controls are included for seasonality.

In sum, we estimate the distribution of valuations for each $11,520$ (20x4x4x3x12) combination of Zs.[38]

6 Results

This section outlines the advertiser bidding model results used to infer the advertiser valuation distribution. Recall, estimation recovers i) the distribution of the underlying advertiser valuations F_V, and ii) the vector of optimal Lagrangian multipliers $\boldsymbol{\mu}^* = (\mu_1^*, .., \mu_\theta^*, ..\mu_\Theta^*)$ which reflects the tightness of the advertisers' minimum impression constraints. The pair $(F_V, \boldsymbol{\mu}^*)$ is obtained for each combination of observables $\boldsymbol{Z}$ (discrete space of site-ad location-ad size-device-month). Below we report the estimates for the treatment group in the experimental data in the month prior to the experiment (09/2017).

6.1 The Valuation Distribution, F_V

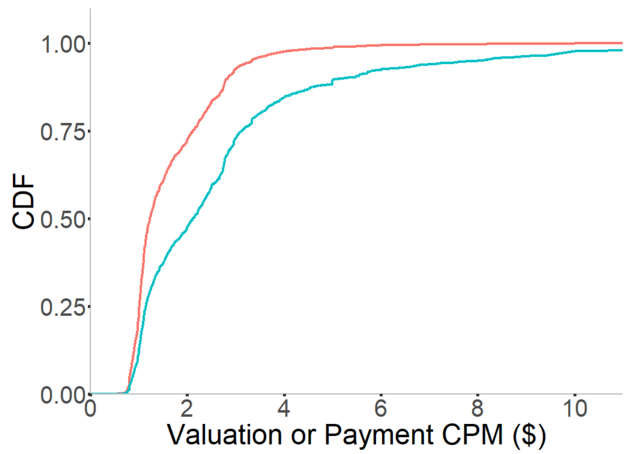
The cumulative density function of the advertiser valuations, F_v, is plotted in Figure 7 for Experimental Cell 1. The cdf of the advertiser valuations (F_V, blue line) is recovered from the observed payments (F_D, red line).[39] This figure shows how valuations exceed payments. For

[37] https://www.iab.com/guidelines/

[38] Advertiser (or DSP)-specific distribution of valuations will be explored in the future to control for the heterogeneity across advertisers (or DSPs).

[39] The recovered cdf of the advertiser valuations for all experimental cells are included in Figure 12 in online appendix.

Figure 7: Advertiser Valuations Distribution



Note: The blue line is the recovered advertiser valuations distribution (F_V) and the red line is the observed payments (F_D).

example, about 40% of advertisers have a valuation of less than $2.00, and 60% of advertisers pay less than $2.00.

Table 6: Advertiser Valuations F_V: Mean and Standard Deviation

Experimental Cell	Mean	SD
1	3.08	5.72
2	2.58	3.84
3	2.96	3.69
4	4.07	4.84
5	2.71	3.27
6	2.74	3.79
7	3.19	3.48
8	15.0	6.76
9	4.77	5.26
10	8.11	4.25
11	2.48	2.98
12	3.04	4.28
(Weighted) Mean	3.94	4.17
(Weighted) SD	2.84	0.98

Note: The weight used is the number of impressions won for each experiment. The mean and the standard deviation are computed across the 12 experimental cells.

In Table 6, the first and the second columns respectively represent the mean and the standard deviation of the advertiser valuations for each treatment group in the experiment. The standard deviation of the means for the distributions is large (2.84). In other words, the valuation distributions appear to vary by observables across cells such as (site-ad location-ad size-device-month), implying that different reserve prices should be set for different auctions.

6.2 The Impression Constraint, $\boldsymbol{\mu}^*$

Recall, μ^* is the premium advertisers pay to meet the minimum impression constraint (over the solution where there exists no such constraint). The first column in Table 7 reports the percentage of advertisers bound by the minimum impression constraint (i.e., those with positive Lagrangian multipliers $\mu_k^* > 0$) at $r = 0$. On average, about 19% of the advertisers face binding minimum impression constraints during the prior experimental period when $r = 0$. Moreover, as the average advertiser valuation for an impression is about \$3.94, a "back of the envelope" calculation suggests that advertisers bid around 20% (0.78/3.94) higher than their true valuations because of the binding minimum impression constraint when there is no reserve, $r = 0$. Also of note, the standard deviations of $\boldsymbol{\mu}$ are high (see fifth column in Table 7), meaning the cost of the constraint varies significantly across advertisers.

The percentage of advertisers bound by the constraint increases from 19 to 32 as the publisher increases the reserve prices. As a result, the cost of the constraint increases from \$0.78 to \$0.99 and the overall cost of the constraint across advertisers grows substantially, from 20% to 25%.

Table 7: The Impression Constraint, $\boldsymbol{\mu}^*$

Experimental Cell	% Advertisers Constrained		$\boldsymbol{\mu}$: Mean		$\boldsymbol{\mu}$: SD	
	$r = 0$	r^*	$r = 0$	r^*	$r = 0$	r^*
1	19.0	33.5	0.66	0.77	4.68	3.33
2	15.3	36.1	0.54	0.96	2.54	3.51
3	17.1	35.8	0.62	1.08	2.27	3.88
4	17.4	22.0	0.93	0.97	3.45	3.42
5	27.8	38.4	0.72	0.78	15.3	15.3
6	26.1	56.4	0.43	0.52	5.53	5.39
7	19.4	26.5	0.88	0.90	5.51	3.85
8	9.19	9.2	1.26	1.35	5.87	5.80
9	21.4	45.9	1.34	1.85	5.63	5.59
10	18.2	17.7	1.27	1.43	3.06	3.92
11	16.7	37.5	0.62	1.08	2.70	3.55
12	26.1	55.1	0.79	0.91	11.1	10.9
(Weighted) Mean	18.7	31.7	0.78	0.99	4.94	5.07
(Weighted) SD	4.35	10.7	0.24	0.22	3.91	3.66

6.3 Sensitivity and Robustness

Our approach requires a number of assumptions pertaining to the experimental conclusions, identification, and estimation. We discuss the implications of these assumptions in depth in online Appendix B.2, but summarize them here.

6.3.1 Model

FMFE relies on two key approximations: the mean-field approximation and the stochastic fluid approximation. These assumptions are more likely to hold in "large markets where the number of bidding opportunities is large". We provide empirical support for these assumptions in our context in online Appendix C.1.1. We also discuss the level of minimum impression constraints in online Appendix C.1.2.

6.3.2 Experimental Analysis

Several aspects of the experimental analysis warrant more discussion. First, bid truncation might arise if advertisers cease to bid when reserves increase. This could happen if bids are costly (bid automation however suggests costs are negligible) and short of the increased reserve gains do not offset the cost. Online Appendix C.2.1 reports no decrease in bids in the treatment group, implying truncation does not occur. A second concern is SUTVA inasmuch as advertisers in the treatment group can redirect bids to the control group as prices rise in the treatment condition relative to control. Online Appendix C.2.2 finds no evidence of decreased (increased) bids in the treatment (control) group. Third, experimental pairs 8 and 10 used identical, non-zero reserves in the pre-experimental period. As detailed in online Appendix C.2.3, the difference in difference analysis controls for identical non-zero pre-treatment reserves. Fourth, online Appendix C.2.4 considers heterogeneous treatment effects, specifically the thinness of markets and how far a cell is below optimal reserves in the pre-treatment period. The appendix finds i) directional support that thinness of markets amplifies treatment effects, because changing reserves do not bind when markets are thick, and ii) directional support that cells far below optimal evidence greater gains from using the optimal reserve. Finally, online Appendix C.2.5 discusses the potential for bidding bots to impute higher advertiser values from higher reserves, noting that any such imputation is likely to be brief in the presence of frequent feedback and monetary rewards.

6.3.3 Identification

Parameter recovery can be affected by i) the exchange's use of daily aggregate rather then auction level bid level reporting, and ii) the imputation of impression constraints from the data (see Section 5.3). Online Appendix C.3.1 addresses the first issue by simulation and estimation, showing that modestly higher levels of aggregation have little effect on the parameter estimates or recommended reserve prices. Online Appendix C.4.1 addresses the second issue by using the experimental data period to estimate impression constraints and showing that these estimated constraints align with those imputed using the observational data period.

6.3.4 Estimation

We consider two key assumptions pertaining to the information used in estimation. The first major assumption involves the calculation of the minimum impression constraint and the second major assumption involves the calculation of the number of potential bidders. Regarding the former, our main analysis uses monthly durations to compute minimum impression constraints (see Section 5.3). Online Appendix C.4.1 explores different levels of aggregation for computing the minimum impression constraint and finds little effect of these changes on the results. Regarding the latter, whether a bidder loses or does not participate is not observed for bidders that do not opt in to the exchange, making it hard to ascertain the total number of bidders in an auction. Online Appendix C.4.2 consider a range of assumptions regarding participation and reports our insights and findings are robust within this range.

7 Setting Reserve Prices

In this section, we compute the optimal reserve price to maximize the publisher's revenues in the ad exchange auctions. Based on the recovered advertiser distribution F_V, the optimal reserve can be obtained by solving the publisher's optimization problem prescribed in Equation (3).

7.1 Solving for the Optimal Reserve Price

This subsection discusses the numerical approach used to solve for the optimal reserve price. For each experimental group, we compute two reserve prices. The first reserve price, r^*_{nc}, is calculated under the assumption that advertisers bid truthfully, that is, they face no impression constraint ($\boldsymbol{\mu}^* = 0$),. This can be done by solving the implicit function in Equation (15) in online appendix.

The second reserve price, r^*, is calculated with the minimum impression constraint in place by solving the publisher's optimization problem in Equation (3). The second reserve price r^* takes into account advertisers' best responses with the minimum impression constraint in FMFE.

The computation of the second reserve price requires updating advertisers' beliefs on D to ensure that advertisers' beliefs are consistent with the bidding profile $\boldsymbol{\mu}$ at the new reserve price r considered. Thus solving the optimal reserve price r^* with the minimum impression constraint involves embedding the iterative best-response algorithm to find FMFE under the new reserve price. The detailed procedure is included in online Appendix D.2.2.

7.2 Optimal Reserves

Table 8: Policy Simulation Results

Experimental Cell	Reserve Price Bias $\left(\frac{r^*_{nc}-r^*}{r^*}\right)$	Profit Loss
1	-18.2	4.55
2	-59.3	29.3
3	-48.1	12.7
4	-28.6	5.90
5	-33.2	14.3
6	-5.71	5.50
7	-7.41	1.04
8	0.0	0.0
9	-40.0	17.7
10	-5.56	0.09
11	0.0	0.0
12	-4.98	4.98
(Weighted) Average	-27.2	9.09

Table 8 details the predicted reserve price bias and the profit loss arising from ignoring the minimum impression constraint across all the experimental cells. The reserve price bias is calculated as the (optimal reserve without the constraint - optimal reserve with the constraint)÷(optimal reserve with the constraint). The profit loss is calculated as the difference in profits when setting the optimal reserve with the constraint and without. On average, the optimal reserve price calculated with no constraint is about 27% lower than the optimal reserve calculated with the minimum impression constraint, but can be as high as 59%. Further, the profit loss in ignoring this constraint is calculated to be around 9% across experimental cells, but can be as high as 29%. From this, we conclude that the minimum impression constraint can lead to even more substantial gains in publisher revenues than a naive approach that assumes no constraint.

8 Model Validation

The experiment in Section 4 showed that setting reserve prices under the naive assumption of a one-shot auction with no constraints led to a 29% increase in auction revenues relative to the context of no constraint. While substantial, this lift raises the question of how much higher profit lift could have been if reserve prices were instead set using the minimum impression constraint model.

Based on the advertiser distribution F_V recovered in the previous section, it is possible to compute what the optimal reserve price should have been by solving the publisher's optimization problem prescribed in Equation (3). We show in Subsection 7.2 that the use of the minimum impression constraint model would have led to an additional 9 percentage points of profit over the naive model (i.e., the lift would have increased from 29% to 38%).

While this forecast of what might have occurred had we used our optimal reserve pricing model is informative it remains a forecast. To obtain concrete empirical evidence that the minimum impression approach can increase revenues over a naive model with no constraint or a budget constraint model, this section reports the result of a validation exercise where we randomized reserve prices over a much larger range of experimental cells (than just 12 pairs), thereby generating an experimentally induced demand curve of revenues by reserve prices.

8.1 Holdout Experiment Design

8.1.1 Pairwise Randomized Experiment

The goal of the holdout experiment was to gather new data and assess how well our model predicts and if our reserve prices with minimum impression constraint is indeed optimal. A corollary benefit of the holdout exercise is to assess the robustness of our model with respect to its various assumptions. In general, to the degree the model assumptions differ from the true data generating mechanism, our predictions of the reserve prices where revenues are expected to be maximized will not be correct; that is, the actual optimal reserve price will differ from the predicted optimal reserve price.

The holdout experimental data in Wave 3 (Figure 2) were collected for the period 04/01/2019 - 07/31/2019, where 04/01/2019 - 06/20/2019 constitute the 'Pre' period and 06/21/2019 - 07/31/2019 constitute the 'Post' period where the changes in reserve prices took place for the treatment group. Similar to the design of the initial experiment in Wave 2, a pairwise randomized experiment was conducted, but this time across all publishers' websites comprising 31 pairs. These pairs were chosen to be closest in terms of contents, user demographics, revenues, and number of impressions (user visits). Within a pair, a randomly chosen ad unit was assigned to the treatment group, and the other was assigned to the control group. For the treatment cells, the reserve prices were manipulated as described next. The control cells did not change the ad-hoc reserve prices used by the publisher.[40, 41]

[40]Our results from the first experiment motivated the publisher to raise their the reserve prices to non-zero levels for many of their ad units.

[41]The balance table shows that none of the observable characteristics differ significantly between the treatment and control groups in the pre-treatment period of the validation experiment. The pre-trend assumption required for our identification is also consistent with the discussion in online Appendix 10.

8.1.2 Experimental Reserve Prices

The validation experiment randomized reserve prices across experimental cells for the treatment group in the post-period, independently of the model or analysis. However, the experimental reserve prices were not fully randomized. Instead, they were randomly perturbed around the pre-period (ad-hoc) reserve prices for the treatment group, $r_{T,pre}$, to remain aligned with the publisher's existing business practices and to mitigate risk and operational disruptions, while still enabling experimentation across a broad range of sites. More specifically, the reserve price in the post-period, $r_{T,post}$ was randomly drawn from $r_{T,post} \sim U\left[0.7 \times r_{T,pre}, 1.3 \times r_{T,pre}\right]$. This broad span of reserve prices enables us to trace a revenue curve and ascertain whether or not the highest revenue aligns with our prediction of where the highest revenue should be, that is r^*.[42, 43]

8.2 Holdout Experiment Results

8.2.1 Results

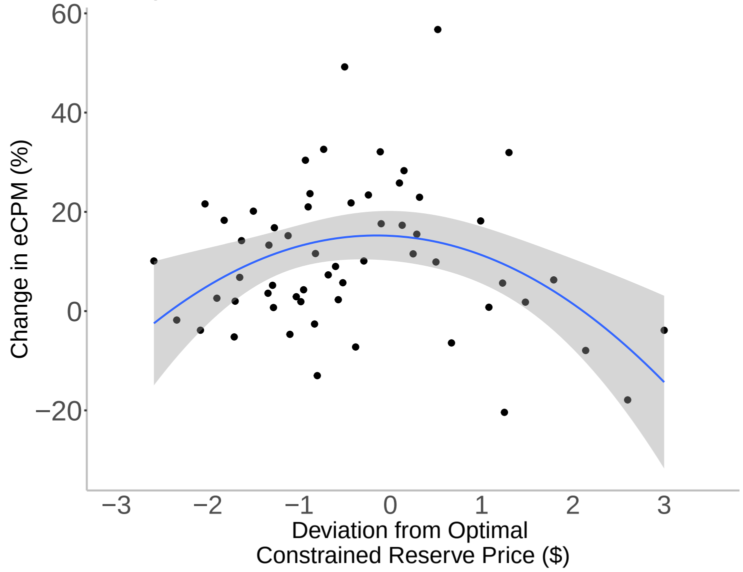
Figure 8 presents the results from the holdout experiment. To evaluate the outcomes, we first compute the DiD percent change in eCPM (relative to the pre-experimental period eCPM) for each of those paired cells. Next, we plot these DiD percent changes as a function of the deviation between the randomized reserve price and the optimal reserve price predicted by the model. For example, suppose the experimental reserve prices are randomly assigned as \$1, \$2, and \$3 across cells, while the minimum impression constraint model predicts the optimal reserve price to be \$2 in each case. The deviation between experimental and optimal reserve prices would thus be -\$1, \$0, and \$1. If the minimum impression constraint model is correct, one would expect to see DiD gains in eCPMs, on average, to be largest in the cell where the deviation in reserve prices is \$0 (that is, the randomly

[42] Across the experimental cells, the randomized reserve price levels in our validation experiments span the minimum of r^*_{nc} values and the maximum of r^* values. This range provides sufficient variation in the experimental manipulation to determine whether the publisher's maximum ad revenue aligns with r^* or with some other value.

[43] An alternative experimental design would have involved setting, for each of the 31 experimental cells: (i) the naive optimal reserve price r^*_{nc}, (ii) the optimal reserve price predicted by our minimum impression constraint model r^*, and (iii) a zero reserve price $r = 0$. This design would allow for a direct test of whether the model-predicted optimal reserve price r^* yields higher revenue than either the naive benchmark or no reserve. However, we did not pursue this approach for three main reasons. First, the design would require 63 distinct reserve pricing rules (31 for r^*, 31 for r^*_{nc}, and one for $r = 0$), exceeding the number the publisher was willing (and able) to implement due to the constraints (total number of pricing rules) and operational costs. Second, it was unlikely that the publisher would have agreed to implement $r = 0$ condition, as the initial experiment suggested that setting a zero reserve would result in substantial revenue loss. Third, by introducing broader variation in reserve price levels, specifically in their deviation from the model-predicted optimum, we are able to empirically trace out the revenue curve that maps reserve prices to revenue outcomes. This richer variation allows us to identify the peak of the revenue function and assess whether it aligns with r^*. Such analysis would be difficult with only three discrete reserve price levels.

set reserve price matches the optimal reserve). Figure 8 portrays this logic and outcome.

Figure 8: Validation Experiment Result



Note: The x-axis plots the deviation from the optimal reserve price: (randomly chosen experimental reserve price − optimal reserve price predicted by the minimum impression constraint model). The y-axis plots the percentage DiD change in eCPM. If the model predicts well, then we would expect the DiD change in eCPM to be the highest on the y-axis around zero on the x-axis, and become lower as we move further away from zero on the x-axis.

The data imply that the minimum impression constraint solution is best because the peak of the curve is close to zero deviation. Conditioned on the relatively small sample available, this pattern (peak at zero deviation) is what we would hope to observe if the minimum impression constraint approach increases revenues over a naive model with no constraint.[44, 45, 46]

[44] Each experimental cell consists of two inventory types; branded and anonymous. In the branded case, advertisers were informed of the specific publisher site on which the impression would appear. In the anonymous case, they were informed only of the publisher, but not the specific site. Two types exist for both treatment and control ad units. As a result, advertisers bid different amounts depending on the level of information available. This, in turn, leads to different optimal reserve prices for branded and anonymous types within each experimental cell. Consequently, although our experimental design manipulates 31 reserve price levels - since the publisher's existing reserve pricing rule $r_{T,\text{pre}}$ did not distinguish between branded and anonymous inventory - we observe 62 difference-in-differences (DiD) percentage changes in eCPM, one for branded and one for anonymous within each experimental cell. These 62 estimates are visualized in Figure 8.

[45] Ideally, one would want to test if the change in eCPM with the minimum impression constraint approach is statistically different from the naive model. A formal test will involve calculating the loss in prediction error for each of these cells and conducting a statistical test such as KS test. Given we have 31 experimental pairs and given these experiments are highly costly for the publisher to run (e.g., the revenue risk they face), we have very low power and the difference will be statistically insignificant. Nevertheless, the peak of the curve is where we predict it would be, subject to all these caveats discussed.

[46] Another way to utilize the validation experiment data would be to use the structural estimates to get predicted eCPM and comparing those to the actual eCPM. We find that the correlation between the predicted eCPM and actual eCPM is 0.67, which suggests that our structural model performs reasonably well.

8.2.2 Remarks

It is worth noting that our model makes a number of assumptions as discussed in Section 6.3. A key reason for using a holdout task and endeavoring to validate our model predictions is that such assumptions tend to work against finding that the predicted and observed optimal align, and that the predicted optimal reserve performs better than the naive reserve. Moreover, the validation task implies that publishers can use our approach to optimize reserves; which is ultimately the key goal of the research.

9 Conclusion

With the continued rapid growth in display advertising markets, there is an increasing value in characterizing the advertisers' valuations for ad impressions, and how advertiser strategies are affected by practical constraints (such as reach or budget) or by the reserve price in advertising exchange markets. Taking the perspective of the publisher, we consider how reserve prices should be set for auctions when selling display advertising impressions through these ad exchanges to advertisers play a repeated ad buying game and face practical constraints such as budget or impression counts.

In a field experiment, setting the reserve price under the assumption advertisers play a one-shot game without any constraints is shown to increase publisher's revenue substantially, by 35% (notably, at no additional cost to the publisher). By manipulating the reserve in this fashion, we test the extent to which reach or budget constraints (across multiple auctions) affect advertisers' bidding behaviors. Experimental findings indicate that increasing the reserve price increases advertisers' bid CPMs and total payments, while it does not have an impact on the total number of impressions won by the advertisers. We show these patterns are most consistent with advertisers playing repeated auctions with minimum impression constraint.

Subsequently, we construct an advertiser bidding model that incorporates the minimum impression constraint. The model builds on the notion of a fluid mean-field equilibrium developed in Balseiro et al. 2015, which well approximates the rational behavior of thousands of advertisers competing in repeated auctions with some constraints. We extend this theoretical framework to incorporate the minimum impression constraint, and suggest estimation and identification strategies in applying it to our empirical context. Our counterfactual results show that the reserve price solved without imposing the minimum impression constraint is 27% lower than the optimal reserve level with the minimum impression constraint. Because ignoring the minimum impression constraint biases the solution downward, the profit loss in ignoring this constraint is found to be about 9%. We then conduct an additional experiment to validate this pricing policy and find that the prices set most closely to the predicted optimal price yield the highest revenues on average.

While this paper addresses a question of growing economic importance with a novel dataset, a number of additional extensions are possible. In particular, publishers often sell advertising inventory via direct sales as well as RTB. Direct selling involves advance sale of a bundle of impressions directly to the advertiser at a fixed price. Extending our research to consider optimal joint pricing (e.g., fixed price in the direct, reserve price in the ad exchange), and whether inventory should first be made available to one channel or another, are interesting directions for future research.[47] A second question of interest is motivated by noting that advertiser valuations are incumbent upon the information available to the advertisers about the impression. This raises the question of whether and how much information a publisher should share with an exchange. Another interesting extension would be to extend our analysis to the context of a first-price auction (FPA). To explore how the optimal reserve would change under a FPA, one could conduct a counterfactual analysis using a FPA. Another promising direction for future research is to incorporate both types of constraints, minimum impression constraints and budget constraints, into a unified framework, and to investigate heterogeneity in constraint types across advertisers. This would become feasible with richer datasets that directly observe either budgets or minimum impression levels. Finally, in search advertising ads are sold via position auctions, and extending our analysis to optimize reserve prices in that context would be a valuable contribution.

To the best of our knowledge, this paper is among the first to empirically consider the issues of pricing in display advertising markets. Given our initial results and the growth in these markets, we hope this and future research will continue to yield economically meaningful implications for publishers in these rapidly growing markets and lead to more research in this area.

[47] We consider reserve prices, taking direct sales decisions made as given (e.g., price, number of impressions bought and sold in direct). Taking the direct channel as given is an assumption that mirrors the structure of the market we consider, where the impressions are sold in the ad exchange after they do not sell in the direct sales channel. Thus, our solution to the optimal reserve prices can be viewed as the sub-game perfect equilibrium solution to the second-stage, taking the first-stage decisions as given.

Funding and Competing Interests

The data provider has a right to review the manuscript for the following conditions:

- The name of the data provider or its affiliates is not used;
- No data or any information from which the identity of the data provider or its affiliates could be inferred is used.
- No information relating to visitation frequencies, traffic levels or sales levels on the data provider affiliates' sites is disclosed, except for anonymous references to small portions or samples of the data provider affiliates, and from which overall traffic and sales patterns of the data provider affiliates' sites cannot be determined;
- No personal data is presented that identifies registrants or users of the data provider sites;

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Online Appendix

A Model

A.1 Proofs

A.1.1 Proposition 1

The proof for Proposition 1 follows the steps established in Balseiro et al. 2015 Section A.1. First, the dual of the primal problem in Equation (1) is introduced using a Lagrange multiplier for the minimum impression constraint. Second, the first-order conditions are derived to determine the solution for the dual problem.

Step 1: The Lagrangian for type θ is denoted as

$$\begin{aligned}\mathcal{L}_\theta(b,\mu) &= \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{b(V)\geq D\right\}(V-D)\right] + \mu\left[\eta s_\theta E_{V,D}\left[\mathbf{1}\left\{b(V)\geq D\right\}\right] - y_\theta\right] \\ &= \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{b(V)\geq D\right\}(V-D+\mu)\right] - \mu y_\theta\end{aligned} \tag{6}$$

where a Lagrange multiplier for the minimum impression constraint is $\mu \geq 0$. The dual problem (converting from maximizing the advertiser's objective function given its minimum impression constraint in Equation (1) to minimizing the Lagrangian multipliers while maximizing the objective function) is given by

$$\begin{aligned}\Psi_\theta(\mu) =& \inf_{\mu\geq 0}\sup_{b(\cdot)}\mathcal{L}_\theta(b,\mu) \\ =& \inf_{\mu\geq 0}\left\{\eta s_\theta \sup_{b(\cdot)}\left\{E_{V,D}\left[\mathbf{1}\left\{b(V)\geq D\right\}(V+\mu-D)\right]\right\} - \mu y_\theta\right\} \\ =& \inf_{\mu\geq 0}\left\{\eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V+\mu\geq D\right\}((V+\mu)-D)\right] - \mu y_\theta\right\} \\ =& \inf_{\mu\geq 0}\left\{\eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V\geq D-\mu\right\}(V-(D-\mu))\right] - \mu y_\theta\right\}\end{aligned} \tag{7}$$

the inf is the Lagrangian minimization step and the sup is the goal maximization step.[48]

In a standard second-price auction without constraints, an advertiser with valuation W solves the following problem:

$$\sup_{b(\cdot)}\left\{E_{W,D}\left[\mathbf{1}\left\{b(W)\geq D\right\}(W-D)\right]\right\} \tag{8}$$

which is maximized by bidding truthfully, $b^*(W) = W$. The maximized expected utility then becomes:

$$\begin{aligned}&E_{W,D}\left[\mathbf{1}\left\{b^*(W)\geq D\right\}(W-D)\right] \\ =&E_{W,D}\left[\mathbf{1}\left\{W\geq D\right\}(W-D)\right]\end{aligned} \tag{9}$$

In our setting, advertiser's inner optimization problem in Equation (7) is expressed as:

$$\sup_{b(\cdot)}\left\{E_{V,D}\left[\mathbf{1}\left\{b(V)\geq D\right\}(V+\mu-D)\right]\right\} \tag{10}$$

Our goal is to determine the function $b(V)$ that maximizes this objective. Recognizing that

[48] Intuitively, the dual problem can be thought of as i) choosing the bid to maximize the objective function given the shadow cost of the constraint and then ii) minimizing the cost of that constraint, while the Lagrangian from the primal problem can be thought of as i) minimizing the shadow cost of the constraint and then ii) choosing the bid to maximize the objective function.

this objective function resembles the standard second-price auction problem in Equation (8) and also recognizing the solution to this problem in Equation (9), we can conclude that the objective in Equation (10) will be maximized when setting the function $b^*(V)$ as $V + \mu$. In other words, the function $b^*(V) = V + \mu$ is the solution to the problem in Equation (10) that maximizes the objective.

In Equation (7), the equality in the third line comes from the fact that the inner optimization problem is similar to an advertiser's problem who faces value $v + \mu$ and seeks to maximize its expected utility in the second-price auction so that bidding truthfully becomes optimal (consider Equation 1 without the constraints). That is for any given multiplier $\mu \geq 0$, the inner expectation term is maximized with the policy $b(V) = V + \mu$. Further, the term within the expectation in the last line is convex in μ, and the expectation preserves convexity, leading to a convex dual problem.

Step 2: The first order condition of $\Psi_\theta(\mu)$ with respect to μ (that is, the FOC for the $\inf_{\mu\geq 0}\{\cdot\}$) is given by

$$(d/d\mu)\Psi_\theta(\mu) = \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V \geq D - \mu\right\}\right] - y_\theta = 0 \tag{11}$$

To explain the solution to this FOC, we begin by noting that $\Psi_\theta(\mu)$ is convex in μ. If the constraint does not bind (i.e., $\eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V \geq D\right\}\right] \geq y_\theta$), then $(d/d\mu)\Psi_\theta \geq 0$ at $\mu = 0$. This condition implies the function $\Psi_\theta(\mu)$ is increasing in μ for all $\mu \geq 0$, such that the function is minimized at $\mu^* = 0$ (has a corner solution). Intuitively, the Lagrangian multiplier can be interpreted as the cost of the impressions constraint; If the constraint does not bind, the constraint is costless.

On the other hand, when the constraint binds (i.e., the optimal unconstrained number of impressions is less than the minimum impression level, that is $\eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V \geq D\right\}\right] < y_\theta$), then $(d/d\mu)\Psi_\theta(\mu)$ takes a negative value at $\mu = 0$. As $\mu \to \infty$, $(d/d\mu)\Psi_\theta(\mu)$ converges to a positive value $\eta s_\theta - y_\theta > 0$, because $lim_{\mu\to\infty} E_{V,D}\left[\mathbf{1}\left\{V \geq D - \mu\right\}\right] = 1$.[49] Thus, there exists a unique interior solution $\mu^* > 0$ for $(d/d\mu)\Psi_\theta(\mu^*) = 0$ as Ψ_θ is a convex function in μ.

Moreover, the complementary slackness conditions hold with the bidding function $b^*(V) = \beta_\theta^F = v + \mu^*$ and the optimal multiplier μ^* such that:

$$\mu^*\left[\eta s_\theta E_{V,D}\left[\mathbf{1}\left\{\beta_\theta^F(V) \geq D\right\}\right] - y_\theta\right] = 0,$$

That is, either i) the minimum impression constraint binds or ii) the Lagrangian multiplier μ^* is 0.

Lastly, there is no duality gap. That is, there is no difference between the primal (1) and dual (7) values. The bid function β_θ^F is primal feasible from the first-order conditions of the dual, and

[49] $(\eta s_\theta - y_\theta)$ is a finite positive number as we constrain $\eta s_\theta > y_\theta$ when defining the optimization problem in Subsection 3.3.1 (i.e., the minimum impression level is lower than the total available impressions).

the primal objective value calculated at the proposed bid function ($b(V) = \beta_\theta^F$) is then

$$
\begin{aligned}
&\eta s_\theta E_{V,D}\left[\mathbf{1}\left\{\beta_\theta^F \geq D\right\}(V-D)\right] \\
=&\left(\eta s_\theta E_{V,D}\left[\mathbf{1}\left\{\beta_\theta^F \geq D\right\}(V-D)\right] + \mu^*\left[\eta s_\theta E_{V,D}\left[\mathbf{1}\left\{\beta_\theta^F \geq D\right\}\right] - y_\theta\right]\right) - \mu^*\left[\eta s_\theta E_{V,D}\left[\mathbf{1}\left\{\beta_\theta^F \geq D\right\}\right] - y_\theta\right] \\
=&\mathcal{L}_\theta(\beta_\theta^F, \mu^*) - \mu^*\left[\eta s_\theta E_{V,D}\left[\mathbf{1}\left\{\beta_\theta^F(V) \geq D\right\}\right] - y_\theta\right] \\
=&\mathcal{L}_\theta(\beta_\theta^F, \mu^*) \\
=&\Psi_\theta(\mu^*)
\end{aligned}
$$

where the third equality follows from the complementary slackness conditions, as $\mu = \mu^*$ is the value for which $\mu^*\left[\eta s_\theta E_{V,D}\left[\mathbf{1}\left\{\beta_\theta^F(V) \geq D\right\}\right] - y_\theta\right] = 0$. Finally, the last equality follows from the fact that $\Psi_\theta(\mu^*) = \sup_{b(\cdot)}\mathcal{L}_\theta(b, \mu^*)$ and the optimal bid function β_θ^F solves the latter problem.

A.1.2 Proposition 2

Once we establish the optimal bidding function and the optimal multiplier as above, Proposition 2 follows from Proposition 4.1 in Balseiro et al. 2015, in which they characterize the equilibrium.

A.1.3 Existence and Uniqueness of the Equilibrium

Balseiro et al. 2015 Theorem 4.1 (p. 871) prove the existence of FMFE. The existence of an equilibrium with the minimum impression constraint follows the similar logic and steps laid out in their supplemental material in (http://dx .doi.org/10.1287/mnsc.2014.2022). The characterization of equilibrium and the proof of existence is provided for the case without participation constraints (i.e., for the case where the participation constraints do not bind in equilibrium). In our model, participation constraints are additionally considered and they can bind. In our empirical estimation, we didn't face issues with convergence, suggesting that the equilibrium exists with the participation constraints considered.

Uniqueness is established when bidders are homogenous and there is single advertiser type θ (Balseiro et al. 2015 Theorem 4.2). The supplementary appendix (http://dx.doi.org/10.1287/mnsc.2014.2022) provides sufficient conditions under which uniqueness holds with two types; namely that they have a common value distribution with positively homogeneous failure rate, such as exponential, Weibull, and Rayleigh distributions. It is extremely rare to find results regarding the uniqueness of equilibria in dynamic games (Doraszelski and Pakes 2007), and providing theoretical conditions for uniqueness is challenging for more than two types of bidders. In numerical experiments Balseiro et al. find that the myopic best-response algorithm, like the iterative best response we use in our estimation and counterfactuals, yields the same FMFE for a given model instance with two or more types, even when starting from different initial points. In our own empirical setting, we also generally found the same equilibrium when perturbing the initial points, which was reassuring.

If multiple equilibria do exist in our setting, we interpret and assume that the counterfactual simulation results we find is the closest equilibrium to the one observed in data as the status quo equilibrium. Note that our validation experiment results would likely not hold if another equilibrium were being played.

A.2 Theoretical Predictions

In this appendix, we outline the intuition for the predictions in Table 3 in Section 3. These predictions indicate how advertiser behaviors change as the reserve prices increase from $r = 0$ (i.e., not reserve price) to a reserve price level $r^*_{nc} > 0$, that is the optimal reserve price under the assumption that advertisers do not face binding constraints.

In the subsequent analysis, we consider several cases: i) when the constraint binds neither at 0 nor at r^*_{nc} (not bind, not bind), ii) when the constraint does not bind at 0 but does bind at r^*_{nc} (not bind, bind), and iii) when the constraint binds at both the 0 and r^*_{nc} reserve price levels (bind, bind). We consider two types of constraints: a maximum budget constraint and a minimum impression constraint. The minimum impression constraint and the maximum budget constraint are each considered in isolation. That is when we consider the budget constraint, we assume that the minimum impression constraint does not bind in both ($r = 0$) and ($r^*_{nc} > 0$).

A.2.1 No Binding Impression or Budget Constraint (Not Bind, Not Bind)

When the underlying state is (not bind, not bind), the advertiser bidding model collapses to the standard second-price auction without the constraint. In this case, advertisers bid their true valuations, and the distribution of bids will be invariant regardless of the reserve price level. The probability of winning an impression decreases at higher r^*_{nc}, but the total payment increases as r^*_{nc} maximizes publisher's revenues.

A.2.2 When Impression Constraints Bind ((Not Bind, Bind) or (Bind, Bind))

Impression Constraints: The Effect of the Reserve Price on the Optimal Bid The equilibrium Lagrangian multiplier μ^* increases monotonically with the increase in r (until the participation constraint binds). When the minimum impression constraint binds, the optimal $\mu^* > 0$ satisfies $y_\theta - \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V + \mu^* \geq D\right\}\right] = 0$ (see Equation 11). An increase in the reserve price will increase D (the steady-state maximum of the competitors' bids), and to offset this effect, μ^* needs to increase as well to satisfy the equality constraint. As the optimal bidding strategy is derived as $\beta^F_\theta = v + \mu^*$, an increase in r increases μ^*, which in turn increases bids for advertisers participating both at $r = 0$ and r^*_{nc}.

Impression Constraints: The Effect of the Reserve Price on Number of Impressions Won and the Total Payment When at least some advertisers face the case of (not bind, bind), the number of impressions won by advertisers will decrease as the reserve prices increase. This is because some advertisers who were able to buy in excess of the minimum impression constraint at $r = 0$ cannot buy as many impressions as they used to at r^*_{nc}.

The direction of the total payment is ambiguous when reserve prices increase in the (not binding, binding) condition because there are opposing effects. Although the number of impressions won decreases, advertisers increase their bids with the increase in the reserve price leading to a higher payment CPM per impression sold.

Finally, when the advertiser faces the (bind, bind) condition, the number of impressions won does not change, as the advertiser buys the minimum number of impressions when the constraint binds.[50] Accordingly, the total payment will increase with the increase in advertisers' bids as the reserve price increases.

A.2.3 When Budget Constraints Bind ((Not Bind, Bind) or (Bind, Bind))

Budget Constraints: Optimal Bidding Strategy Balseiro et al. 2015 establish that the optimal bidding strategy when advertisers face the maximum budget constraint is

$$\beta_\theta^F\left(v|F_D\right) = \frac{v}{1+\mu^*} \tag{12}$$

where μ^* is

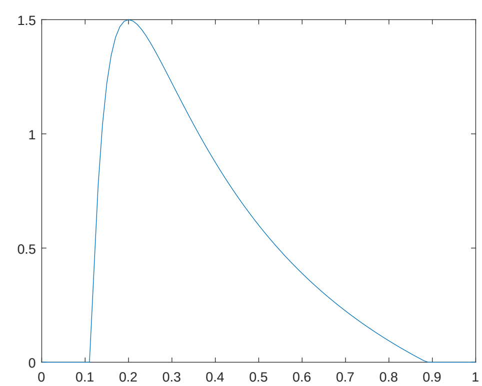
$$\begin{cases} \mu^* = 0 & \text{if } b_\theta > \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V \geq D\right\} D\right] \\ \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V/(1+\mu^*) \geq D\right\} D\right] - b_\theta = 0 & \text{if } b_\theta \leq \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V \geq D\right\} D\right] \end{cases}$$

and b_θ is the maximum (constrained) budget.

Budget Constraints: The Effect of Reserve Price on Optimal Bid When the advertiser faces the case of a (not bind, bind) budget constraint as the reserve price increases from 0 to r^*_{nc}, the bidding strategy will change from v to $\frac{v}{1+\mu^*}$ where $\mu^* > 0$. Thus, the bid will decrease in this case.

When the advertiser faces (bind, bind) constraint as the reserve price increases from 0 to r^*_{nc}, the effect on the bid is ambiguous. This is because $\frac{\partial \mu^*}{\partial r}$ is not monotonic. For example, Figure 9 shows the change in μ^* (y-axis) with respect to the change in r (x-axis), when $v \sim U[0,1]$ and $b = 0.1$, when there is one advertiser. The plot shows that μ^* first increases then decreases in the range $[0, r^*_{nc}] = [0, 0.5]$.

Figure 9: Optimal Bidding Strategy with a Maximum Budget Constraint



Note: The figure shows the change in μ^* (y-axis) with respect to the change in r (x-axis).

[50] The number of impressions can decrease if the participation constraint binds for some advertisers.

Budget Constraints: The Effect of Reserve Price on Number of Impressions Won and Total Payment When the advertiser faces the case of a (not bind, bind) budget constraint as the reserve price increases from 0 to r^*_{nc}, the number of impressions won will decrease as this advertiser now shades bids and the reserve price increases (see Equation 12). The total payment across advertisers increases, because advertisers spend more and the budget constraint becomes binding.

When the advertiser faces the case of a (binding, binding) budget constraint, the total payment will stay the same (at the binding budget level), but the effect on the number of impressions won is ambiguous because bid CPM may or may not increase with respect to the change in the reserve price.

B Experimental Evidence of a Constraint

This appendix section contains two components. First, it details the design of the experiment used in Section 4 to explore the effect of reserve prices on advertiser bidding. Second, it summarizes a number of robustness checks pertaining to the analysis of the experimental outcomes.

B.1 Experiment Design

B.1.1 Treatment and Control Groups

The results from Subsection 2.3 show that advertisers' bid CPMs vary by site, device, ad location, and size. Therefore, the experiment was designed to run across different ad types including (i) site (site1 and site2), (i) device (desktop, mobile, tablet), (iii) ad location (above-the-fold/ATF, MID, below-the-fold/BTF, no info, front door), and (iv) size (300x250, 728x90, 970x66, 320x50, 300x600, 970x250). An experiment was conducted over twelve pairs. The two units in a pair were closest in ad characteristics and were randomized either into the treatment or the control group. For example, (site1, desktop, BTF, 300x250) and (site1, desktop, ATF, 300x250) were paired for the first experiment, and the randomly chosen (site1, desktop, BTF, 300x250) was assigned to the treatment group, whereas (site1, desktop, ATF, 300x250) was assigned to the control group. Table 9 shows the full list of the pairs and the corresponding treatment and control group characteristics.

Table 9: Treatment and Control Groups

Pair ID	Device	Site and Position		Inventory Size	
		Treatment	Control	Treatment	Control
1	Desktop	Site1 BTF	Site1 ATF	300x250	300x250
2	Desktop	Site2 MID	Site2 ATF, BTF	300x250	300x250
3	Mobile	Site1 No Info	Site2 MID	300x250	300x250
4	Mobile	Site2 ATF	Site2 BTF	300x250	300x250
5	Desktop	Site1 ATF	Site1 BTF	728x90, 970x66	728x90, 970x66
6	Tablet	Site1 ATF, BTF	Site1 No Info	728x90, 970x66	728x90, 970x66
7	Mobile	Site2 MID, BTF	Site1 No Info, Site2 ATF	320x50	320x50
8	Desktop	Site1 ATF	Site2 ATF	300x600	300x600
9	All	Site1 Front Door BTF	Site1 Front Door BTF	300x250	728x90
10	Desktop	Site1 ATF	Site1 BTF	970x250	970x250
11	Tablet	Site1 No Info	Site2 ATF, MID, BTF	300x250	300x250
12	Desktop	Site2 BTF	Site2 ATF	728x90, 970x66	728x90, 970x66

B.1.2 Balance of Observables

Table 10 reports the balance of observables between the treated and control groups in the pre-period. The observable metrics are calculated for each experimental cell (12 treatment cells and 12 control cells) and the corresponding coefficients of variation are reported. The p-values are obtained from two-sided paired t-tests of differences in treatment vs. control groups. The observables are not statistically significantly different between the treatment and control groups. There is little power with 12 observations, nonetheless all but one of the p-values exceed 50%.

Table 10: Table of Balance

Variable	Coefficient of Variation		P-value for Difference in Means
	Treatment	Control	
Revenue	0.99	1.26	0.96
# Ad Requests	1.20	1.21	0.91
# Impressions Sold	0.91	1.15	0.68
# Advertisers	4.11	5.28	0.92
CPM paid	0.91	0.98	0.65
Click Through Rate	0.91	0.96	0.40
Bid CPM	1.09	1.12	0.56
eCPM	1.40	1.41	0.91
Sell Through Rate	1.53	1.41	0.91
#Bids / # Ad Requests	6.65	5.67	0.92

Note: Variables are calculated at the experimental cell level (12 treatment cells and 12 control cells). P-values are from two-sided paired t-tests of differences in treatment vs. control groups ($N = 12$).

B.1.3 Pre-Trend

The DiD analysis is predicated upon parallel pre-trends. In Figure 10, we plot pre-trends for the metrics used in the DiD analysis; eCPM, average bid CPM, number of ad impressions won, and total payment. Although control and treatment groups have different mean levels for some metrics, in general the trends appear to be parallel. To formally test whether the two groups exhibit parallel trends during the pre-treatment period, we estimate a regression that includes a group indicator, a time variable, and their interaction. In Table 11, we report the coefficient and standard error of the interaction term from this regression for each outcome metric used in the DiD analysis.

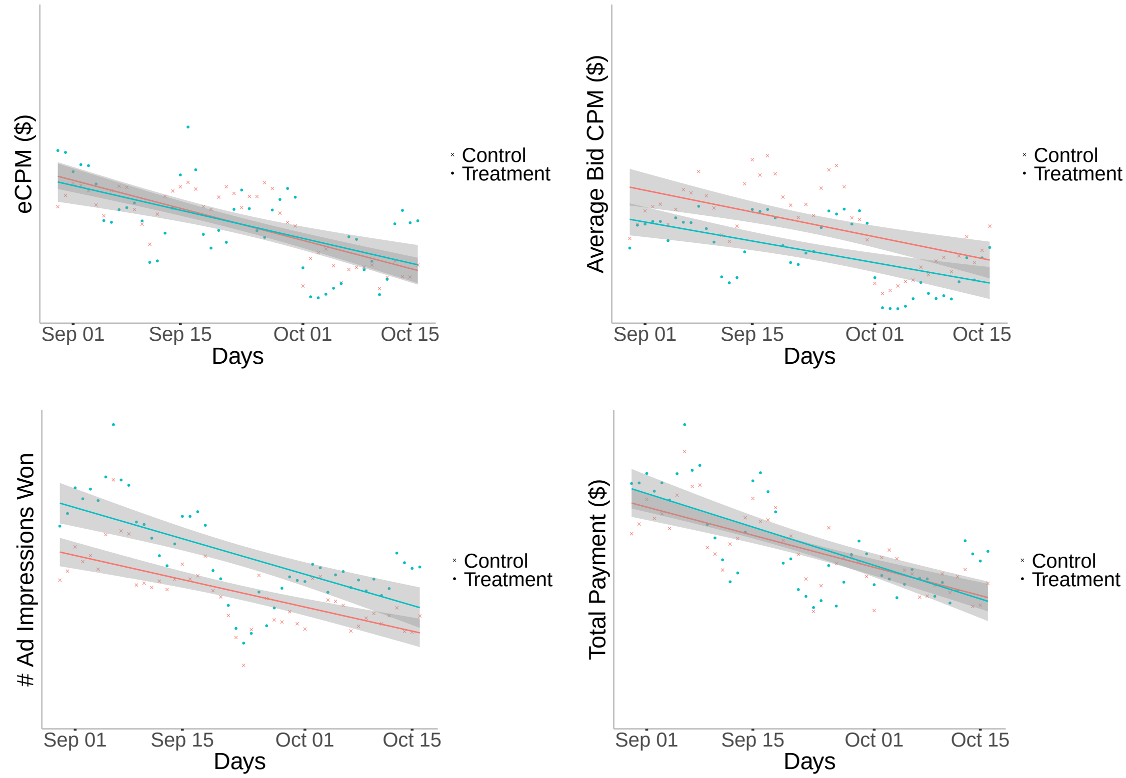
Table 11: Test of Parallel Trends: Interaction Term (Time $\times$ Treatment)

Variable	Estimate	SE
eCPM	0.0003	0.0009
Average Bid CPM	0.0004	0.0010
# Impressions Won	−47.28	249.79
Total Payment	−1.24	1.15

B.1.4 Calculating Reserve Prices

When inducing exogenous variation in the reserve pricing experiment, we selected price levels predicated on the assumption of no binding constraints in the hope of finding a level that would increase the publisher's revenues (had we set reserve prices far too high, revenues would have fallen compared to the historical zero reserve price levels). That is, the reserve prices set in the

Figure 10: Pre Trends



Note: Y-axis levels are not displayed for confidentiality.

experiment were calculated presuming advertisers bid truthfully and that the observed bids reflect their underlying valuations.

Under this setting, the publisher can choose the reserve price r to maximize the publisher's revenue by solving

$$\max_r r\left\{1 - F_V(r)\right\} \tag{13}$$

where F_V is the cdf of advertiser valuation distribution (Riley and Samuelson 1981). $\{1 - F_V(r)\}$ is the probability that the ad impression will be sold at the reserve price r.[51]

Let b_i be the average bid CPM and L_i be the number of bids submitted for a given observational unit i (advertiser-DSP-day-site-ad type). We estimate $\{1 - F_V(r)\}$ in Equation (13) by the empirical sample analogue

$$\{1 - \hat{F}_V(r)\} = \frac{\sum_{i=1}^{n} \mathbf{I}(b_i > r) L_i}{\sum_{i=1}^{n} L_i}$$

using the pre-period data. Equation (13) is optimized with respect to r to find the optimal reserve prices for this unconstrained, naive model. We use these reserve price levels for the treatment group

[51]Under the symmetric independent private value paradigm, the optimal reserve price is independent of the number of bidders (Riley and Samuelson 1981). Considering the case where the publisher faces a single bidder; Equation (13) represents the expected revenue from selling the ad impression at the reserve price r.

in our experiment (Section 4).

B.2 Experimental Results: Robustness Checks

B.2.1 The Effect of Reserve Prices on eCPM

Next, various DiD specification results are reported with various control variables. The outcome measure considered is *eCPM* (multiplied by a common, multiplicative constant for confidentiality). First, we present the analysis using the experimental cell as the unit of analysis, which has the virtue of transparency. In estimating the DiD regression, we weigh each observation by its ad impression requests (= number of ad impressions supplied to ad exchange) to account for the substantial amount of heterogeneity in experimental cell sizes. The result in Table 12 indicates that the causal increase in eCPM from setting reserve prices is about \$0.12 and not particularly sensitive to specification.

Table 12: Treatment Effect on eCPM (\$): Cluster Level Analyses

	(1)		(2)		(3)	
DV= eCPM (\$)	Estimate	SE	Estimate	SE	Estimate	SE
Treated × Post	0.13	(0.16)	0.11	(0.11)	**0.12***	(0.06)
Treated	−0.00	(0.12)	0.00	(0.15)	–	
Post	−0.01	(0.12)	0.00	(0.10)	−0.005	(0.04)
Experimental Pair	–		y		y	
Treated × Experimental Pair	–		–		y	
R-squared	0.03		0.39		0.94	
Observations	48		48		48	

Note: There are 12 treatment and 12 control cells, each with pre- and post- observations, giving us a total of 48 observations for this analysis. *Denotes 10 % significance.

One can conduct similar DiD analyses using more granular-level data to further control for ad characteristics. Table 13 considers data whose unit of observation is (day-site-ad type) in contrast to the prior analysis where the unit of observation is site-ad type. The effect of implementing reserve prices on eCPM is again estimated to be positive and significant, \$0.11 $\sim$ \$0.13, yielding 30% $\sim$ 35% increase in revenue across specifications from the baseline eCPM for the treatment group in the pre-period, \$0.37.

Using this unit of observation at the (day–site ad type) level, we also estimate heterogeneous treatment effects by experimental pair, as shown in Figure 11. We use the same regression specification as in Table 13, but now interact each experimental pair with the indicator (Treated × Post). The coefficients on this interaction term for each pair, along with their 95% confidence intervals, are plotted in the figure.

B.2.2 The Effect of Reserve Prices on Bidding Behaviors

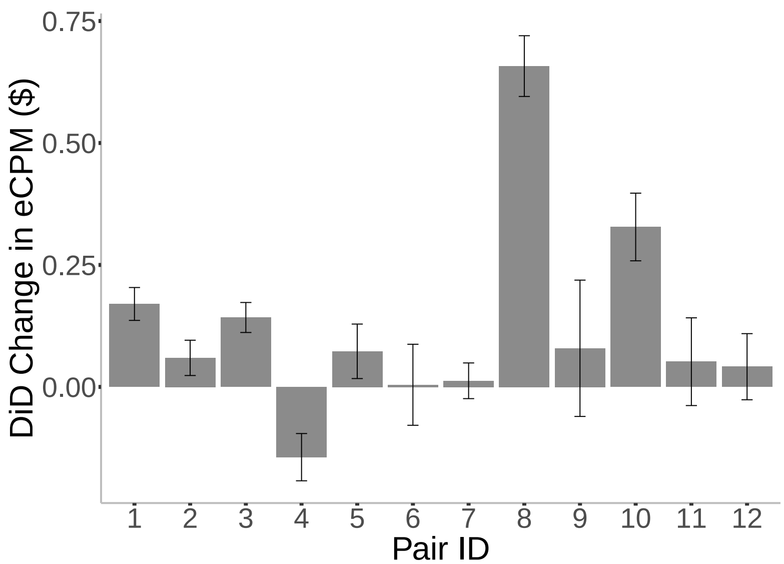
Table 14 reports the effect of reserve prices on advertiser bidding behaviors and its sensitivity to the inclusion of additional control variables. Column (1) is the specification reported in the main body of the paper (Table 5). Overall, the finding that the bid CPM and total payment increase while the number of impressions bought stays the same is qualitatively robust to various model specifications.

Table 13: Treatment Effect on eCPM ($): Unit Level Analyses

DV= eCPM ($)	(1)	(2)	(3)	(4)	(5)
Treated × Post	**0.13****	**0.13****	**0.11****	**0.12*****	**0.11*****
P-Value (Randomization Inference)	0.04	0.04	0.04	0.00	0.01
Treated	y	y	y	–	–
Post	y	–	–	–	–
Day	–	y	y	y	y
Experimental Pair	–	–	y	y	y
Treated × Experimental Pair	–	–	–	y	y
Site-Ad Type	–	–	–	–	y
R-squared	0.02	0.05	0.28	0.64	0.82
Observations	3,442	3,442	3,442	3,442	3,442

Note: There are 5,382,829 observations in the experiment at the (advertiser-DSP-day-site-ad type) level. Aggregation is necessary for the eCPM DiD analyses, because the denominator in the eCPM (i.e., impressions supplied to ad exchange) can only be defined at (day-site-ad type) level. Thus, the data are aggregated across advertisers and DSPs, leaving 3,442 observations. The p-value for testing the null hypothesis that the treatment has no effect is calculated using randomization inference, randomizing treatment assignment at the experimental pair-level. *** denotes 1% and ** 5 % significance.

Figure 11: Treatment Effect on eCPM by Experimental Pair



C Robustness Assessments

In this appendix section, we provide robustness assessments for assumptions made regarding the model, experimental analysis, identification assumptions, and estimation choices.

C.1 Model

This subsection outlines the assumptions underlying our model and their relevance to our empirical setting: specifically (i) assumptions required for the approximations to hold, and (ii) the level at which minimum impression constraints are imposed.

Table 14: Treatment Effect on Bidding Behaviors: Robustness Checks

DV= Bid CPM ($)	(1)	(2)	(3)	(4)	(5)
Treated × Post	**0.039****	**0.050****	**0.049****	**0.049****	**0.049****
P-Value (Randomization Inference)	0.037	0.026	0.019	0.033	0.035
# Impressions Supplied (in thousand)	y	y	y	y	y
Treated	y	y	–	–	–
Day	y	y	y	y	y
Advertiser	y	y	y	y	y
Pair	–	y	y	y	y
Treated × Pair	–	–	y	y	y
Site-Ad Type	–	–	–	y	–
DSP	–	–	–	–	y
R-squared	0.212	0.223	0.226	0.229	0.230
Observations	3, 635, 899	3, 635, 899	3, 635, 899	3, 635, 899	3, 635, 899

DV= # Impressions Won	(1)	(2)	(3)	(4)	(5)
Treated × Post	0.351	0.220	0.321	0.314	0.282
P-Value (Randomization Inference)	0.397	0.437	0.422	0.447	0.436
# Impressions Supplied (in thousand)	y	y	y	y	y
Treated	y	y	–	–	–
Day	y	y	y	y	y
Advertiser	y	y	y	y	y
Pair	–	y	y	y	y
Treated × Pair	–	–	y	y	y
Site-Ad Type	–	–	–	y	–
DSP	–	–	–	–	y
R-squared	0.197	0.197	0.198	0.198	0.199
Observations	5, 382, 829	5, 382, 829	5, 382, 829	5, 382, 829	5, 382, 829

DV= Total Payment ($)	(1)	(2)	(3)	(4)	(5)
Treated × Post	**0.005****	**0.005****	**0.005****	**0.005****	**0.005****
P-Value (Randomization Inference)	0.048	0.045	0.022	0.020	0.020
# Impressions Supplied (in thousand)	y	y	y	y	y
Treated	y	y	–	–	–
Day	y	y	y	y	y
Advertiser	y	y	y	y	y
Pair	–	y	y	y	y
Treated × Pair	–	–	y	y	y
Site-Ad Type	–	–	–	y	–
DSP	–	–	–	–	y
R-squared	0.188	0.189	0.189	0.189	0.194
Observations	5, 382, 829	5, 382, 829	5, 382, 829	5, 382, 829	5, 382, 829

Note: The dependent variables are all multiplied by a common, multiplicative constant for confidentiality. The unit of observation for the analysis is (advertiser-DSP-day-site-ad type). Bid CPM analysis uses 3, 635, 899 bid data observations from the opt-in advertisers. Ad impression and total payment analyses use 5, 382, 829 payment data observations from all advertisers. The P-value for testing the null hypothesis that the treatment has no effect is calculated using randomization inference (Athey and Imbens 2017). For randomization inference, we randomize treatment assignment at the experimental pair-level, the same way that the assignment was done in the experiment). ***denotes 1% and **5% significance.

C.1.1 FMFE Approximations

FMFE relies on two key approximations: the mean-field approximation and the stochastic fluid approximation. The mean-field approximation assumes that the distribution of competitors' bids across auctions is stationary in large markets and that a single advertiser's bids do not affect this distribution. The stochastic fluid approximation assumes that minimum impression constraints need only be satisfied in expectation, since advertisers face ample opportunities to bid. As Balseiro et al. (2015, p. 870) note, these assumptions are more likely to hold in "large markets where the number of bidding opportunities is large" such as our display advertising context. Below, we provide empirical support for these assumptions.

Large Markets In real-world markets, some advertisers may compete repeatedly in auctions targeting similar user groups, raising concerns about the assumption that a single advertiser's bids do not influence the overall distribution or market state. To assess the plausibility of this assumption in our empirical context, we compute the set overlap of auctions in which advertisers participated (over days) and then compute the cosine similarity matrix across advertisers. The resulting matrix shows low average pairwise similarity of 0.23, where 0 indicates no overlap and 1 indicates complete overlap. This low similarity suggests that advertisers' bidding activities are sufficiently orthogonal to support the required assumption.

Many Bidding Opportunities For each ad unit (site–ad type), we calculate the number of ad requests (i.e., the number of available impressions) and confirm that bidding opportunities are very large in our setting. Advertisers typically have thousands of daily bidding opportunities across multi-week campaigns.

C.1.2 Level of Minimum Impression Constraints

In our model, minimum impression constraints are assumed to be set at the level of (site-ad type). This assumption will more likely to hold when advertisers seek to diffuse impressions across ad units to broaden audience reach.

For purposes of setting the reserve prices, we would ideally want these constraints to be counterfactually invariant to changes in the reserve prices. If advertisers were to adjust the level of their constraints in response to higher reserve prices (or if such constraints were set at broader levels than we assume), this would manifest as SUTVA violation in our empirical context, as advertisers shift their ad buying from the expensive inventory with higher reserve prices to lower reserve prices. In our robustness checks in the subsequent Subsection C.2.2, we find that the concern for SUTVA violation is minimal, across various analyses.

C.2 Experiment

In this subsection, we address four concerns regarding our experimental design, data, and analysis: (i) bid truncation, (ii) SUTVA violation, (iii) experimental cells with non-zero reserve prices, (iv) heterogenous treatment effect, and (v) bidding bots inferring quality from the reserve prices.

C.2.1 Bid Truncation

When the experimental reserve prices exceed an advertiser's valuation, treated advertisers might not submit a bid (if, for example, the advertiser perceives submitting bids to be costly, though in reality bids are automated so there is little reason not to bid). In this instance, all bids will be accounted for the control group with zero reserve prices, but only the bids exceeding the reserve prices will be observed for the treatment group in the post-period. The average bid CPMs would therefore appear to increase for the treatment group due simply to truncation.

We explore this truncation issue using a DiD regression of the number of bids. Were advertisers to cease submitting bids below the reserve prices, one would expect to see a decrease in the number of bids in the post-period for the treatment group. The first two columns in Table 15 show the results from DiD regressions of the number of bids. The model (1) includes advertiser fixed effects to illustrate the within-advertiser change. The model (2), on the other hand, considers the extensive margin. In both cases, there is no significant decrease in the number of advertiser bids; in fact the coefficients are positive. Thus, there is no significant evidence of truncation in our context, presumably because the cost of submitting bids is so small as they are typically submitted by algorithms (and submitting bids below the reserve prices does not decrease advertisers' utility).

Table 15: Treatment Effect on Number of Bids and # Impressions Won

DV (Scaled)	# Bids		# Impressions Won	
	(1)	(2)	(1)	(2)
Treated × Post	2.86	0.74	0.351	0.033
	(2.88)	(2.48)	(0.419)	(0.545)
# Impressions Supplied (in thousand)	Y	Y	Y	Y
Treated	Y	Y	Y	Y
Day	Y	Y	Y	Y
Advertiser	Y	–	Y	–
R-squared	0.080	0.003	0.197	0.001
Observations	3,635,899	3,635,899	5,382,829	5,382,829

Note: The dependent variables are multiplied by a common, multiplicative constant for confidentiality. Bids analysis uses 3,635,899 bid data observations from the opt-in advertisers. Ad impression analyses use 5,382,829 payment data observations from all advertisers. ***denotes 1% and**5% significance.

C.2.2 SUTVA Assumption

In this subsection, we discuss the stable unit treatment value assumption (SUTVA). The assumption of no spillovers between different units implies that potential outcomes are invariant to random treatment assignment of others (Angrist et al. 1996).

Violation of this assumption may occur if, for example, treatment group advertisers strategically substitute the ad inventory (purchase more ad impressions) to the control group due to the increase in reserve prices in the treatment group ads. In this case, one should observe more (fewer) bids in the control (treatment) group after treatment. Revisiting Tables 15, we find no evidence the number of submitted bids and the number of ad impressions change with an increase in reserve prices. The first (last) two columns show the results from DiD regressions of the number of bids (the number of ad impressions won). The model (1) looks at the within-advertiser changes, whereas the model (2) excludes advertiser fixed effects to look at the extensive margin. As their is no negative interaction between the treatment and post, strategic substitution does not seem to be a major factor in our setting.To further explore this concern, we also assess whether advertisers arrivals in the control group post-treatment are disproportionately greater than in the treatment group. Were SUTVA violated, fewer new advertisers would arrive in the treatment condition where there is an larger increase in reserve prices, as they would substitute into the control group. Table 16 reports no statistically significant change in the number of advertisers between the treatment and control groups, again suggesting that SUTVA violation may not be a major consideration in our setting.

Table 16: Treatment Effect on Number of Advertisers

DV (Scaled)	# Advertisers
Treated × Post	11.02
	(34.45)
R-squared	0.068
Observations	3442

Note: The number of advertiser analyses use $3,442$ data observations at (day-site-ad type) levels.

C.2.3 Non Zero Reserve Prices in Pairs 8 and 10

In most pairs, the control group reserve prices were zero. However, for experimental pairs 8 and 10, the publisher set surprisingly high reserve prices in the pre-period that were the same for the treatment and control groups. As a result, in the post-period, the reserve prices were experimentally decreased to the unconstrained, naive optimal levels for the treatment group and decreased even more, to zero, for the control group.

As such, one would still expect increases in the eCPM in pairs 8 and 10 as moving from high reserve prices to the unconstrained, naive solution levels (experimental treatment setting) should yield a bigger lift in *revenues* than moving to the zero reserve prices (experimental control setting). Considering the last row in Table 3 in which the minimum impression constraint binds, one would again expect increases in *bid CPM* and *total payments* but no change in the *number of impressions* bought for pairs 8 and 10. Moving from extremely high reserve prices to zero would relax the minimum impression constraint to a larger extent so advertisers would decrease their bid CPM to a larger extent, compared to moving to the naive solution level with smaller decrease in their bid CPM. Hence, the difference in difference in bid CPMs should remain positive for pairs 8 and 10. Similarly for the total payments, the number of impressions won would be similar in the (bind, bind) condition, but the treatment group would be bidding higher amounts under higher reserve prices, which raises the total payment (as it is the product of impressions won and the second highest bids).

C.2.4 Heterogenous Treatment Effect

Figure 5 shows that the eCPM of Pairs 1 and 12 increase by more than 200%, which are much higher than other pairs. It warrants additional attention as to what are unique about these two experimental pairs.

With only 12 pairs, the potential to test the underlying mechanism driving the heterogeneity in treatment effects across cells is limited. Auction theory suggests that the gains from using reserve prices would be largest when auctions are thin, because that is when reserve prices often bind (otherwise, the second highest bid determines the payment). To test whether reserve price effects are greater when markets are thin, we operationalize thinness as the ratio of bids recorded to impressions supplied and regress that on the gains observed by increasing reserve prices. Specifically, regressing (DiD % change in eCPM by pair) on (ebids by pair) where (ebids = #bids / # impressions supplied to ad exchange) yields a negative coefficient consistent with theory, but the effect is not statistically significant, possibly due to the small sample.

Another factor that can explain larger treatment effects of using reserve prices is the distance of the reserve price from the optimal level. That is, the revenue gains would be higher when the pre-experiment reserve prices are far from optimal. This is indeed what we observe in our setting. Running the regression of (DiD % change in eCPM by pair) on (increase in reserve price by pair) yields a positive coefficient, though it is not statistically significant, again possibly due to the small sample.

C.2.5 Bidding Bots

One possible explanation for the increase in bid CPMs alongside higher reserve prices is that bidding bots may interpret reserve prices as signals of impression quality (a form of price-quality inference). Were bots incorrectly infer valuations from reserve prices, this effect on advertiser bids would likely be short-lived. Over time, advertisers will better learn the true value of impressions arriving by observing performance metrics such as click-through and conversion rates. The bidding bots will then adjust bids accordingly to ensure advertisers' utilities - that is, valuation minus payment - are maximized.

Were incorrect valuations by bots to temporarily increase bids in the short run, advertiser auction participation would also increase. Participation rates would rise because bots think ROI is higher. In our experimental data, while there is an increase in bid CPMs, there is no accompanying change in auction participation. Although it's possible that the lack of change in participation reflects offsetting effects, where higher perceived valuations encourage participation while higher reserve prices discourage it, we find no compelling evidence that bots are inferring higher valuations.

In addition, we also explored whether higher reserve prices are associated with higher CTRs in our experiment. If there is no strong positive association, then bots would have less reason to infer higher quality from higher reserve prices. As noted in Footnote 31, the DiD analysis of CTR shows no evidence of CTR change with the increase in reserve prices.

C.3 Identification

In this subsection of the appendix, we address the implication of temporal data aggregation on identification as data are only available from the ad exchange for any publisher as daily aggregates.

C.3.1 Daily Data Aggregation

As the publisher's ad exchange partner only reports auction outcomes at the daily level, the available metrics are daily averages for the given observational units (advertiser-DSP-day-site-ad type). Model identification presumes the auction-level data, and our model can be construed as agents acting "as if" daily aggregates are informative about more granular information. After all, the daily aggregate information is all advertisers have available to form beliefs about the distribution of competing advertiser bids and all the publisher have available to make pricing decisions. That said, this "as if" assumption raises concerns about the potential inference bias in our estimation due to not having access to the auction-level data. We explore this concern several ways.

First, we assess the potential aggregation bias in computing reserve prices, our key prescription for publishers. To explore potential bias, we fit a normal distribution to the observed bid distribution with mean μ_k and variance σ_k^2 for each of the ad units k.[52] Using this distribution, we simulate auction-level data. Then we aggregate the simulated auction-level metrics to daily averages to construct an aggregated dataset. We then compare the distribution of valuations, bid premiums, auction-level versus daily-level, and also the resulting reserve prices across the disaggregated and aggregated data sets. We find that the mean of the valuation is not biased, but that the variance of the valuation distribution is biased downward when using the aggregate data. These findings are to be expected given the sample mean is an unbiased estimate (like the daily average we observe), but the variance of the sample mean distribution is smaller than that of the population distribution. However, we find that the impact of this reduced variance on the optimal reserve price level itself to be minimal (within $\$0.03 - \$0.07$ range). We repeat the same simulation exercise by increasing the variance of the simulated normal distribution ($1.5 \times \sigma_k^2$) and again find that the impact on the optimal reserve price level to be still small (within $\$0.05 - \$0.11$ range). We hypothesize that this small effect of aggregation on reserve prices occurs because the key determinant in calculating the reserve price is the cumulative probability of the advertiser's valuation distribution (not the variance per se). The cumulative probability reflects the likelihood of a sale at a given reserve price. If the cumulative probability around the optimal reserve price is similar between the granular auction-level data and the aggregated daily-level data, then the bias in the optimal reserve price itself will be minimal. We believe this is generally the case in our data, as the optimal reserve prices are not located at the extreme ends of the distribution.

[52] We recognize that the observed distribution and associated moments are aggregated, but using them provides an opportunity to simulate the effect of aggregation in the approximate neighborhood of the advertiser bids in the data.

At a more primitive level, aggregating auction-level data to daily-level relates to how the temporal aggregation in the second highest bid distribution, $f(d)$, affects our objects of inference, v and μ. The *mean* of $f(d)$ will not be affected, but as the results from the simulation exercise exhibit, the *variance* of $f(d)$ will be smaller when computed with increased aggregation. With the reduced variance of $f(d)$, it's not immediately clear whether there will be a systematic, large directional bias in v and μ, and/or whether aggregation will notably decrease the efficiency of the estimates of v and μ. We explore these questions by re-computing $f(d)$ at the two day aggregation and comparing it to the one day aggregation. In other words, we increase the level of aggregation to ascertain how increasing aggregation affects inferences of v and μ. Results are qualitatively similar for the one day and two day aggregations and reported in Table 17. In sum, we observe modest levels of temporal aggregation having little impact on inference for our model primitives.

Table 17: $f(d)$: One-Day Aggregation versus Two-Day Aggregation

$f(d)$		One Day Aggregation		Two Day Aggregation	
		(Weighted)Mean	(Weighted) SD	(Weighted)Mean	(Weighted) SD
F_V	Mean	3.94	2.84	4.01	2.73
	SD	4.17	0.98	3.49	0.92
% Advertisers Constrained at $r = r^*$		31.7	10.7	29.2	10.8
$\boldsymbol{\mu}$ at $r = r^*$	Mean	0.99	0.22	0.95	0.20
	SD	5.07	3.66	4.73	3.69
Reserve Price Bias $\left(\frac{r^*_{nc} - r^*}{r^*}\right)$		-27.2		-26.5	
Profit Loss (%)		9.09		9.00	

Note: The weight used is the number of impressions won for each experimental cell.

C.4 Estimation

In this subsection, we focus on the robustness of our findings to variable operationalizations, specifically the minimum impression constraint and the number of bidders.

C.4.1 Operationalization of Minimum Impression Constraint

Validation Using Experimental Data Our advertiser model and the resulting first order conditions provide a one-to-one mapping between minimum impression constraint and the bid premium ($\boldsymbol{\mu}$). In our main specification, we operationalize minimum impression constraint from the observational data and back out the bid premium ($\boldsymbol{\mu}$). This approach enables us to (i) recommend optimal pricing schemes for those sites upon which we have not conducted experiments and to (ii) retain the experimental data for use in providing an independent validation test for our model.

Regarding the second point above, experimental data can be used to identify (rather than impute) the minimum impression constraint, as we can map the observed changes in bid CPM with respect to exogenous variation in reserve prices to their underlying minimum impression constraint via the first order conditions in our model. This identification is only possible for the sites and unit units with experimental variation, and we won't be able to recommend reserve prices for other sites without having to run experiments. Nevertheless, this approach enables us to compare the estimated minimum impression constraints in the experimental data period to the operationalized impression constraints in the observational data period. This comparison helps to validate the operationalization of minimum impression constraints used in our main specification.

We isolate advertisers who have participated both in the observational and experimental data periods. For the observational data, we impute the minimum impression constraints as in our main specification. For the experimental data period, we estimate and infer the minimum impression constraint based on the observed changes in bid CPM with respect to exogenous variation in reserve price and mapping them to the optimality conditions in our model. Although the imputed and estimated minimum impression constraints are from different sample periods with different seasonality trends (4 months apart), we still find a strong positive correlation between the two minimum impression constraints (0.74). Hence, we conclude that the imputation approach used in our estimation is a reasonable operationalization of the minimum impression constraint.

Sensitivity to Periodicity in τ The estimator in Equation 14 requires the term $\tau = \left(\frac{y_\theta}{\eta s_\theta}\right)$, the minimum winning rate the advertiser aims to attain for a campaign. In our implementation, we compute the minimum of the winning rates over the months observed in the data, conditioned on the advertiser participating in the ad exchange.

Since advertisers set the minimum impression constraint at the campaign level, which corresponds to the minimum winning rate τ for the campaign, excessive granularity (e.g., calculating the minimum across weekly or daily winning rates) will not be ideal if the duration used to compute minimum impression constraints is shorter than the campaigns. In FMFE, it is acceptable for winning rates to fluctuate, being low on some days and high on others, as long as advertisers achieve the minimum winning rate for the campaign as a whole. Operationalizing τ at a more granular level would result in low τ values due to sampling variation, which in turn would underestimate the inferred bid premium μ. As the minimum impression constraints and the campaign-level minimum winning rates are not directly observed in our data, we select a monthly frequency. This decision is informed by discussions with industry experts and reviewing the direct sales data which indicate that ad campaigns typically run for about a month or longer.

Nevertheless, to further explore the implications of the periodicity used to compute the constraint, we consider the alternative of defining minimum winning rate as the minimum across two-weeks and across weekly levels. We find that the bid premiums are 13% and 17% lower than in our main specification, and the optimal reserve prices to be 9% and 12% lower. These declines are modest and in the expected direction, because the implied bid premiums are lower. Since i) the implied difference in reserve price setting policy using shorter intervals is modest, ii) the imputed and estimated minimum impressions show high correlation, and iii) the validation exercise in Section 8 suggests that our recommended reserve prices correspond to the optimal level with the monthly window (meaning that window validates well), our main specification retains the minimum across months.

C.4.2 Operationalization of the Number of Potential

As noted in Section 2.1, the set of total advertisers is greater than the set of opt-in advertisers, and we only observe bids for the opt-in advertisers. If advertisers do not opt-in, we do not know whether they bid and lost (a potential bidder) or did not bid at all (a non-potential bidder). Yet inference requires the number of potential bidders in the market, so the distinction between the two cases is material. We consider the following form of sensitivity analysis around the number of potential bidders by computing the upper bound and lower bound of the number of potential bidders and estimating the model for these two cases.

The number of potential bidders for a given observational unit (advertiser-DSP-day-site-ad type) will be

$$\begin{aligned} n = & \, n_1(\text{\# advertisers with CPM payment, i.e., positive \# impressions won}) \\ & + n_2(\text{\# advertisers with bids submitted, but zero impressions won}) \end{aligned}$$

n_1 is observed in the data, while n_2 is observed only for the opt-in advertisers.

Thus, to operationalize n, we use the following proxy in our main specification, which is the lower bound:

$$\begin{aligned} \hat{n} = & \, n_1(\text{\# advertisers with CPM payment, i.e., positive \# impressions won}) \\ & + n_{2,opt-in}(\text{\# opt-in advertisers with bids submitted, but zero impressions won}) \end{aligned}$$

As a robustness check, we also consider the upper bound as follows. We assume all advertisers observed across an entire week for the ad unit (i.e., winning at least one impression for the site-ad type), not just the one day, to be potential bidder set. The valuation distributions inferred across the two cases are not statistically different (p-value = 0.44 for KS test). Further, we believe that the true potential bidders on a given day will be closer to the lower bound, because most advertisers opt-in and those that do not tend to have positive impressions won on most days (i.e. are included in n_1) because they are large revenue advertisers (see footnote 35).

D Estimation and Policy Simulation

This section overviews the computational algorithms used in estimation and policy simulations.

D.1 Computational Steps in Estimation

The estimation proceeds as follows:

Stage 1 Denoting $w = v + \mu$, $\hat{F}_W(w)$ and $\hat{F}_D(d)$ are non-parametrically estimated as described in Equations (4) and (5).

Stage 2 Using the optimality condition in Proposition 2, $\boldsymbol{\mu}^*$ is solved for the FMFE using the following algorithm.

1. Start with an arbitrary vector of multipliers $\boldsymbol{\mu}$. That is $\mu_\theta^0 = \mu_\theta, \forall \theta \in \Theta$
2. Repeat

(a) Using the estimates $\left(\hat{F}_D(d),\ \hat{F}_W(w)\right)$ obtained in Stage 1, $N_{sim} = 100$ simulated values are drawn from the estimated distributions $v \sim \hat{F}_V = \hat{F}_W(v + \mu_\theta^i)$ and $d \sim \hat{F}_D(d)$ to construct

$$\hat{h}(\mu_\theta; \boldsymbol{\mu}^i) = \frac{y_\theta}{\eta s_\theta} - \frac{1}{N_{sim}} \sum \left[\mathbf{1}\left\{v_{sim} + \mu_\theta \geq d_{sim}\right\}\right] \tag{14}$$

This equation follows from the advertiser's optimal bidding strategy condition characterized in Proposition 2. Note that the first term is the advertiser's minimum auction winning rate for a given impression and a campaign as computed in Section 5.3, and the second term is the predicted winning rate conditional on μ_θ.

(b) μ_θ^{i+1} solves

$$\arg\min_{\mu_\theta \geq 0} \hat{h}(\mu_\theta; \boldsymbol{\mu}^i), \quad \forall \theta \in \Theta$$

where minimizing $\hat{h}(\mu_\theta; \boldsymbol{\mu}^i)$ over $\mu_\theta \geq 0$ ensures that the conditions in Proposition 2 are satisfied in equilibrium

(c) Compute the difference $\triangle = \left\|\boldsymbol{\mu}^{i+1} - \boldsymbol{\mu}^i\right\|$ and update $i = i + 1$

3. Until $\triangle < \epsilon$ where ϵ is a convergence threshold. We set $\epsilon = 10^{-4}$ in estimation.

D.2 Computational Approach to Computing Optimal Reserve Prices

This subsection discusses the numerical approach used to solve for the optimal reserve price. We discuss the case when the constraint does not bind ($\mu^* = 0$) first, then incorporate the case of the binding constraint ($\mu^* > 0$).

D.2.1 Case1: $\mu^* = 0$

For purposes of the first experiment (Waive 2), advertiser valuations are estimated assuming advertisers use truth-telling strategies, and the optimal reserve prices are calculated conditioned on this assumption. Under the standard second-price, sealed-bid auction, where advertisers bid truthfully, their valuation distribution can be identified and estimated from the observed payment data for each given ad characteristics.

The publisher can maximize the revenue from the ad exchange by choosing the reserve price optimally. The optimal reserve price r^* can be expressed as

$$r^* = c + \frac{[1 - F_V(r^*)]}{f_V(r^*)} \tag{15}$$

where c is publisher's valuation (Riley and Samuelson 1981). F_V and f_V are cdf and pdf of advertiser valuation distribution. For each of the twelve pairs, we use the estimated distribution $\hat{F}_{V|Z}$ and $\hat{f}_{V|Z}$ to calculate $r^*_{nc}(Z)$.

D.2.2 Case2: $\mu^* > 0$

In the policy simulation, we need to find the new FMFE under the considered reserve price level. To do so, the advertisers' beliefs on D (which reflects the bids of competing advertisers) need to be updated to ensure that the advertisers' new beliefs are consistent with the bidding profile $\boldsymbol{\mu}(D, r)$ at the new reserve price. Thus solving the optimal reserve price with the minimum impression constraint involves embedding the iterative best-response algorithm. That is, given the recovered F_v,

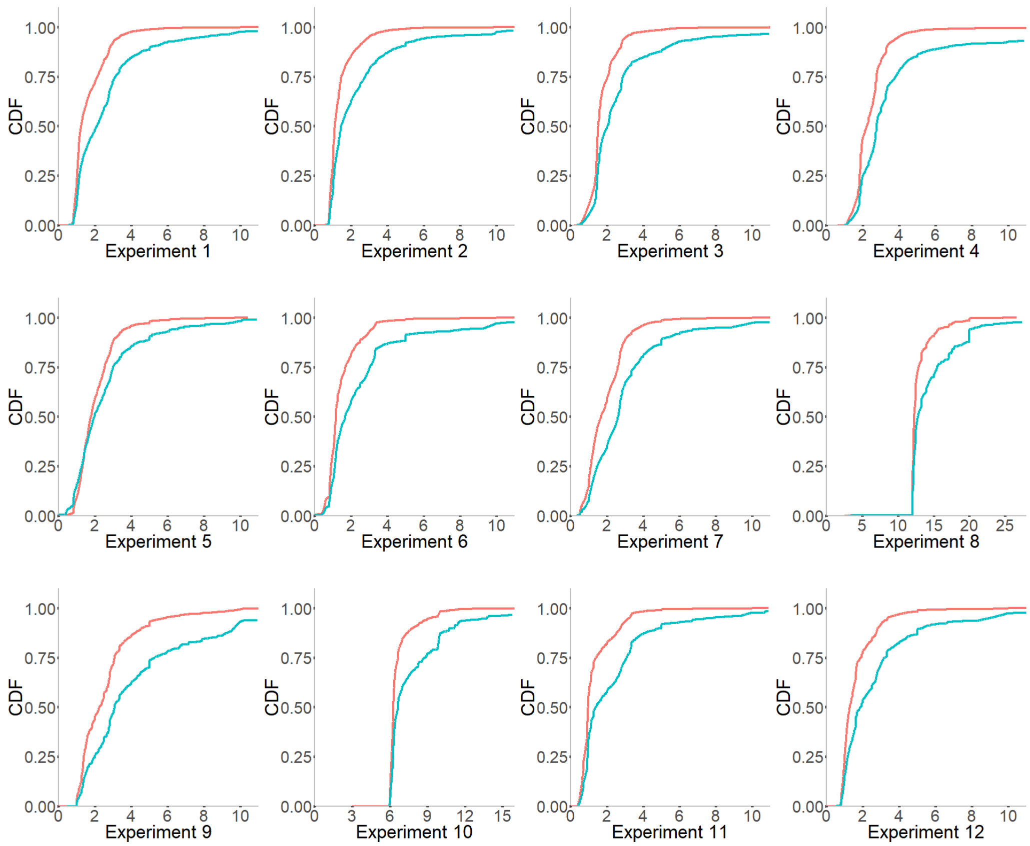
1. Start with an arbitrary r^j for $j = 0$ (we start with $r^0 = r^*_{nc}$, the optimal reserve when advertisers bid truthfully)
2. Repeat: r-step
   (a) Start with an arbitrary vector of multipliers $\boldsymbol{\mu}$. That is $\mu^0_\theta = \mu_\theta, \forall \theta \in \Theta$
   (b) Repeat: $\boldsymbol{\mu}$-step
      i. Obtain $F_D\left(\cdot|\boldsymbol{\mu^i}\right)$ using Equation (4).
      ii. μ^{i+1}_θ solves
      $$\begin{aligned} &\arg\min_{\mu_\theta \geq 0} h(\mu_\theta; \boldsymbol{\mu}^i), \quad \forall \theta \in \Theta \\ h(\mu_\theta; \boldsymbol{\mu}^i) = &\frac{y_\theta}{\eta s_\theta} - E_{V,D}\left[\mathbf{1}\left\{V + \mu_\theta \geq D\right\}\right] \end{aligned}$$
      which finds the bidding strategy profile, $\boldsymbol{\mu}_\theta$, that minimizes the difference between the minimum winning rate the advertiser aims to attain for a campaign (as computed in Subsection 5.3) and the predicted winning rate, $E_{V,D}\left[\mathbf{1}\left\{V + \mu_\theta \geq D\right\}\right]$, given $\boldsymbol{\mu}_\theta$.
      iii. Check the advertiser's participation constraint
      For θ with $0 > E_{V,D}\left[\mathbf{1}\left\{V + \mu_\theta \geq D\right\}(V - D)\right]$, update $\mu^{i+1}_\theta = \bar{\mu}_\theta$ where
      $$\bar{\mu}_\theta = \text{argmax}_{\mu_\theta \geq 0}\left[0 \leq \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V + \mu_\theta \geq D\right\}(V - D)\right]\right]$$
      In other words, if the expected utility to the advertiser is less than 0 (the first equation), the bids are reduced to the point where the advertiser surplus becomes zero (the second line).
      iv. Compute the difference $\triangle = \left\|\boldsymbol{\mu}^{i+1} - \boldsymbol{\mu}^i\right\|$ and update $i = i + 1$
   (c) Until $\triangle < \epsilon$
   (d) Optimize Equation (3) to compute the new reserve price r^{j+1}
3. Until the global maximum is found

E Results

E.1 The Valuation Distribution, F_V

The cumulative density function of the advertiser valuations, F_v, is plotted in Figure 12. The cdf of advertiser valuations (F_V, blue line) is recovered from the observed payments (F_D, red line). The shape of the distributions vary substantially. In other words, the valuation distributions appear to vary by observables such as (site-ad location-ad size-device-month), implying that different reserve prices should be set for different auctions.

Figure 12: Advertiser Valuation Distributions



Note: The blue line is the recovered advertiser valuations distribution (F_V) and the red line is the observed payments (F_D).