Publisher Strategies in Display Advertising Markets
Dissertation Proposal

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Abstract

This proposal considers how publishers' ad inventory pricing decisions, the allocation of ad inventory across distribution channels (direct/exchange channels), and the amount of customer information it shares with the advertisers affect its revenues in display advertising markets.

Focusing on the first question, and in particular setting the optimal reserve price in the exchange channel, a series of field experiments show that setting the reserve price can increase publisher's revenues substantially (32%). Further, we find that advertisers appear to behave as if they face minimum impressions constraint and deviate from truth-telling.

Based on this insight, an advertiser bidding model is constructed to incorporate the minimum impressions goal. The optimal reserve price with the minimum impressions constraint is computed and the magnitude of the profit loss in ignoring this constraint is assessed.

Subsequent iterations of this research will address the remaining allocation and information provision questions.

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1 Introduction

1.1 Overview

This paper considers how a publisher's pricing, allocation, and information sharing decisions affect its advertising revenues in display advertising markets, a question of considerable economic consequence. Display advertising (display ad herein) markets exist to match advertisers to publishers and are estimated to be \$39.4 billion in the US in 2017, growing considerably from \$31.2 billion in 2016.[1] Drivers behind this 26% double-digit growth rate include an upswing in mobile activities, a proliferation in online video ad formats, and technological advancements in ad targeting and measurement. Display ad spending is forecasted to surpass even search ad spending and continue its rapid revenue ascent.[2] In spite of this, empirical research into display markets is limited. With an emphasis on publisher marketing strategies, this is a void this paper seeks to fill.

A publisher in display ad markets (e.g., Facebook, Wall Street Journal, Youtube), sells its advertising inventory via guaranteed and non-guaranteed selling channels. The guaranteed selling channel (so-called direct sale, 'direct channel' herein) involves the advance sale of a bundle of publisher ad inventory (i.e., impressions) directly to the advertiser at a fixed price. The non-guaranteed sales channel (so-called Real-Time Buying, RTB, 'exchange channel' herein) occurs in real time via ad exchanges and focuses on selling single impressions through auctions. Because the sale is at the impression level, advertisers buying in the non-guaranteed channel can typically obtain viewer specific information for specific impressions. The exchange channel is estimated to be 35% ($\approx 82.5\% * 42\%$) of the total display ad revenue in 2018.[3] As such, the main characteristics distinguishing the exchange channel from the direct channel are the pricing mechanism (fixed price in direct vs. auction in exchange), the ability to buy a single, targeted impression, and the information available about the impressions (thus targeting ability).

This aforementioned sales channel structure motivates us to consider i) how the publisher's pricing of ad inventory should be set (i.e., fixed price in the direct channel and the reserve prices for auctions in exchange), ii) how to allocate ad inventory across two selling channels, and iii) how much information should be revealed to the advertisers about specific ad impressions (e.g., should past viewing information on the publisher site be shared with the

[1] https://www.iab.com/wp-content/uploads/2018/05/IAB-2017-Full-Year-Internet-Advertising-Revenue-Report.REV_.pdf. These revenue figures include display-related formats including banners (banners, rich media, and sponsorship) and video.

[2] Search advertising revenue totaled \$40.6 billion in 2017, little above display advertising, but the growth rate is abating (17%).

[3] https://www.emarketer.com/content/more-than-80-of-digital-display-ads-will-be-bought-programmatically-in-2018

advertisers).

The first step in answering these questions is understanding advertisers' underlying valuations. Therefore, we first focus on the exchange channel in this proposal, because bidding and payment data are highly informative about advertiser valuations and can be used to find the optimal pricing in auctions. Setting the reserve price to the optimal level will maximize the publisher's revenue in the exchange channel (conditioned on the decisions made in the direct channel).

As the dissertation develops, the subsequent objectives will be addressed. More specifically, pricing decisions in the dual selling channel structure will be considered using a two-stage game framework. In this framework, the publisher sells impressions via direct channel first, then sells the remaining unsold impressions via exchange in the second stage. Each stage is assumed to reflect the key characteristics of the corresponding sales channel. Alternative inventory allocation (i.e. apportioning ad inventory across direct and exchange channels), and information sharing strategies (i.e., how much viewer-level information about impressions should be shared withe advertisers) will be explored as counterfactual exercises based on the dual channel structure.

The data we use is furnished by a a large, premium publisher, ranked within U.S. top 10. Data are collected both for the direct (e.g., guaranteed contractual terms, ad delivery information) and the exchange channel (e.g., bids, payments). Importantly, we combine observational and experimental data in the exchange channel to validate our assumptions and test our advertiser bidding model, which becomes the foundation for recovering advertiser valuations.

1.2 Relevant Research

This paper builds on research in display advertising and dual-selling channels. To better frame our research contribution, each is discussed in turn.

1.2.1 Display Advertising

Most of the empirical display advertising literature focuses on measuring the effectiveness of ads and the attendant implications for advertisers' ad buying and targeting decisions conditioned on a single selling channel (Barajas et al. 2016, Hoban and Bucklin 2015, Johnson, Lewis, and Nubbemeyer 2016, Johnson, Lewis, and Reiley 2016, Lewis and Rao 2015, Sahni 2015, Sahni and Nair 2016, Tucker 2014, Goldfarb and Tucker 2011). In contrast, we consider i) both advertiser and publisher strategies and (ii) a dual channel structure of the market.

Focusing on the exchange channel, impressions are typically sold in second-price, sealed-bid auctions - hence there is a substantial literature on this auction mechanism (Krishna 2009, Laffont and Maskin 1980). The weakly dominant strategy for an advertiser in a private value, second-price auction of a single object is bidding truthfully, hence truth-telling strategy is commonly assumed in display markets research (e.g., Sayedi 2017, Celis et al. 2014). However, some empirical evidence exists that advertisers face practical constraints that might lead them to deviate from truth-telling when bidding in the exchange auctions: the advertiser (i) faces a budget constraint for a given campaign in repeated auctions (Balseiro et al. 2015; Balseiro and Gur 2017), (ii) learns own and/or other's true valuations over time (Iyer et al. 2014; Cai et al. 2017), (iii) sets a number of impressions to attain (Ghosh, Rubinstein, et al. 2009), and (iv) sets pacing options so that the budget is spent smoothly over a specified time period (Lee et al. 2013; Yuan et al. 2013; Xu et al. 2015). For example, Balseiro et al. 2015 show that the optimal bidding strategy for an advertiser facing a (binding) budget constraint is to shade values to account for the option value of future opportunities. This raises the empirical question of whether or not advertisers bid truthfully in display markets.

In the sponsored search context, Ostrovsky and Schwarz 2011 use a large field experiment to set reserve prices and find that setting appropriate reserve prices (guided by the auction theory) can lead to substantial increases in auction revenues. We conduct a similar test in display markets, and analyze the effect of reserve prices on advertiser bidding behaviors, in addition to the effect on publisher's revenues. The findings suggest that advertiser bidding behaviors are not consistent with truth-telling, but appear to reflect a minimum impressions goal. Accordingly, we incorporate this practical constraint into advertisers' bidding model in inferring their true valuations.

1.2.2 Dual-Selling Channels in Display Advertising Markets

Two theoretical papers consider dual channels and publishers in display advertising market. Athey et al. 2017 demonstrate that, with consumers' switching (multi-homing) across publishers and imperfect tracking technology, advertisers seeking broader reach would favor larger publishers to avoid inefficient duplication.[4] This favoring results in premium prices for high-reach sites and can potentially create an incentive for contracting guaranteed deals.

Second, Sayedi 2017 explores dynamic allocation of ad inventory across the two channels, wherein bids are requested for an impression in the exchange first, then are compared to the prices in the direct channel contracts when allocating impressions.[5] Under this setting, Sayedi 2017 shows that advertiser's profit is convex with respect to the amount bought through the direct channel and that symmetric advertisers exhibit asymmetric strategies where one

[4] Tracking may be imperfect, for example, when users traverse through multiple devices (desktop / mobile / tablet).

[5] https://support.google.com/dfp_premium/answer/3721872?hl=en

advertiser buys the maximum amount made available for the direct channel by the publisher. He also shows that leveraging both selling channels yields higher publisher profits than selling in the direct or exchange channel alone when using the dynamic allocation.

Our work complements this theoretical research by developing an empirical structural approach to infer the distribution (uncertainty) of advertiser valuation, and to quantify optimal pricing for the publisher within the dual selling channel structure. Using this dual channel model, the benefit/cost of using the alternative inventory allocation (i.e., dynamic allocation) can be analyzed as a counterfactual exercise.

In the empirical context, inferring the distribution of advertiser valuation (uncertainty) is important, because advertisers have differential information states in the direct and in the exchange channels. Those advertisers with higher uncertainty have a higher option value of passing over the bundled impressions in the direct channel to buy in the exchange where more information about individual impressions are typically revealed (e.g., cookie information). We explore the role of advertiser uncertainty and information, including how coarsely should impressions be bundled in the direct channel and what information should be shared with the advertisers in the exchange channel, in our counterfactual.

1.3 Organization

This paper is organized as follows. Section 2 overviews the key characteristics of display advertising markets. Section 3 describes our data and highlights key features pertinent to display ad markets. Here we also provides supporting evidence for the minimum impressions constraint using experimental data. Next we present advertiser bidding model that incorporates the minimum impressions constraint. Section 5 discusses estimation method and identification argument in inferring advertiser valuations, and Section 6 presents the estimation results. In Section 7, the optimal reserve prices and the revenue gains are computed to address questions that are interests to practitioners. Subsequent versions of this research will also accommodate additional analyses of pricing and information sharing in the dual channel (direct and exchange) setting.

2 Display Advertising Markets

Given our primary interest is publisher pricing in the exchange channel, this section first overviews the advertisers and intermediaries in this market, and then introduces the pricing mechanism used in the exchange.[6]

Second, we address how the presence of the direct selling channel can influence pricing

[6] We will also briefly touch upon pricing in the direct channel as a prelude to future research.

and inventory decisions in the exchange channel. The presence of an alternative channel for purchase of advertising inventory creates an externality in exchange pricing - as more inventory is allocated to direct, the less is available for exchange, which affects prices in equilibrium.

Third, we consider the role of information asymmetries in display markets, which determines advertiser targeting capabilities, and therefore publisher revenues. Much of the analysis along the second and third dimensions will be completed in future drafts.

2.1 Players

The display ad market is a two-sided market: on one side, advertisers seek to find the sites that generate the best outcomes for their products (such as clicks, sales, or profits). On the other side of the market, publishers with consumers' impressions purvey their ad inventory to advertisers seeking to find advertisers with the highest valuations for those impressions. In between, intermediaries exist to facilitate the match between advertisers and publishers by pooling and managing information about who sees the advertisements and providing optimization tools and algorithms to facilitate the serving of ads.[7]

2.2 Distribution and Pricing

Two main channels exist for selling advertising, direct and exchange.

In the direct channel, advertisers buy blocks of inventory directly from the publisher. Advertisers and a publishers negotiate a fixed price pertaining to when, where, and how the ads will be displayed. These contractual arrangements guarantee the number of impressions to be delivered satisfying certain targeting criteria, at a negotiated fixed price (cost-per-mille, CPM), during a specified time period (e.g., 1M impressions to female users in SF greatly enjoy during July for $5000).

In the exchange channel, ad inventory is sold at the impression level in real-time rather than being bundled and bought in advance as is common in direct distribution. It is considered 'real-time' buying, because an advertiser's buying decision is made immediately after the user arrives on the website. It is non-guaranteed, because an auction takes place for the incoming impression and the advertiser's ad is served to the end user only if the advertiser wins the auction. The centralized marketplace, *ad exchanges* (e.g., DoubleClick Ad Exchange, OpenX), typically conduct a second-price, sealed-bid auction for each available impression in real time. As such, publishers can monetize beyond the advance sales (i.e., guaranteed contracted volume).

[7]Readers are referred to Choi et al. 2017 for details of the ecosystem and industry practices of the display markets.

Advertisers in display markets face the fundamental question of how to price advertising in each channel. In the direct channel, this involves a fixed price for a bundle, and in the exchange channel this involves setting reserve prices in the auctions. Importantly, because ads are partial substitutes across channels, the pricing decisions are likely interdependent. Our primary research goal is to consider these questions.

2.3 Inventory Allocation across Direct and Exchange

Inventory allocation across the direct and exchange channels is a challenging task for the publisher. The presence of inventory in one channel affects the demand and pricing for inventory in the other. Viewed from this lens, to the extent advertising on direct and exchange are perceived as substitutes by advertisers, the publisher faces a joint pricing and inventory problem.

Historically, publishers sought to sell all their inventory in advance via direct sales, and if unable to do so, passed the unsold (remnant) inventory to the ad exchanges (so-called "waterfalling"). In recent years, as the exchange channel has increased in its importance and size, and publishers have been adopting "dynamic allocation" in which bids are requested from the ad exchanges first so the winning bid can be compared to the option value of assigning the impression to the best matching guaranteed contract (Ghosh, McAfee, et al. 2009; Balseiro et al. 2014; Arnosti et al. 2016; Chen 2017; Sayedi 2017). These studies show that dynamic allocation can yield higher profits for the publishers than first sending impressions to the guaranteed contracts.

Nevertheless, many publishers (including the one we consider) still prefer selling premium high quality inventories (e.g., front page, leader-board) via guaranteed contracts, and limit the application of dynamic allocation to lower quality inventories with lower guaranteed contract prices. Waterfall strategies reduce the adverse effect of cherry picking by the advertisers in exchange. If high quality impressions (e.g., younger women in Lambrecht and Tucker 2017) are all cherry picked in exchange first under dynamic allocation, only the low quality impression will be left to serve the guaranteed contracts, which will in turn reduce the premium advertisers pay in the direct channel. For example, advertisers that are highly concerned with ensuring brand safety (i.e., ads appearing on guaranteed high-quality, reputable sites) typically pay more to attain it.

Given the ambiguity regarding the relative profitability of the waterfall and dynamic allocation strategies, a secondary goal of this paper is to contrast the waterfall and dynamic allocation strategies in an empirical context (when advertisers potentially value direct channel buying and pay premium).

2.4 Information

The information available to advertisers about the publisher's inventory differs markedly across channels, affecting how advertisers purchase ads. This has ramifications for how advertisers can target and how impressions are bundled and distributed. In this section, we outline the role of information in pricing and the attendant implications for our analysis.

Direct Channel In direct channel, impressions are bought and sold as bundles, thus targeting also occurs at the bundle level. Targeting is based on the publisher's knowledge of its content viewers (analogous to TV markets), for example by demographic (e.g., only female), contextual (e.g., only on certain pages with relevant contents), and/or behavioral (e.g., visits pattern to publisher's website) characteristics. As advertisers value more information publishers can price discriminate by charging a premium for an additional layer of such targeting.

Contracting targeted bundles of ads several months in advance via guaranteed contracts, however, is often difficult in practice. Because ads are sold in advance, it is not always possible to forecast traffic precisely. Hence, the publisher might not be able to generate a sufficient number of promised impressions for a certain segment if traffic for such finely targeted segment is too low. Thus, targeting criteria are typically coarse, and often at most based on demographic and contextual targeting. Further, when the publisher does not know advertisers' heterogeneous valuations, bundling can reduce information asymmetry. The diminution in this asymmetry can, in turn, lead to higher revenues for the publisher. A tertiary question this paper addresses is how coarse these aggregations should be in the context of the direct channel guaranteed contracts.

Exchange Advertisers have considerably more information and control over targeting in exchange. Each impression is sold separately in real time, meaning that the value of an impression can more readily be evaluated by matching the incoming user with the advertiser's own data on users' past behavior (e.g., purchase patterns on the advertiser's website). Advertisers further leverage additional data bought from third parties (e.g., income level, job history, home ownership, monthly car payment, visits to other relevant sites) and use *Data Management Platforms* (DMPs herein, e.g., Salesforce DMP, Adobe Audience Manager) to import data from multiple sources and to build audience segments from the integrated data. Thus, advertisers can benefit largely from behavioral targeting in exchange, in addition to the demographic and contextual targeting.

From the publisher's perspective, sharing information with the advertisers poses a similar trade-off as in the direct channel. Providing more (publisher's) information in exchange may yield higher advertiser bids conditional on participating in the auctions (De Corniere and De Nijs 2016; Hummel and McAfee 2016). At the same time, providing more information decreases the number of participating bidders and creates thin markets, as fewer advertisers are interested in a given impression with highly differentiated attributes (Levin and Milgrom 2010; Fu et al. 2012; Chen and Stallaert 2014). It is another goal of this paper to consider this trade-off as well.

2.5 Summary

In light of the foregoing discussion, we firstly consider (or will consider) three pricing questions: setting reserves in the exchange, setting prices in the direct channel, and setting prices jointly. We will secondarily consider how advertising inventory should be apportioned across the two channels, including the question of whether to allocate inventory first to exchange or direct. Thirdly, we will consider two questions pertaining to information asymmetries between the publisher and advertiser: which impression level information to avail the advertiser when bidding for an impression and how impressions should be bundled (by which observable impression characteristics) in the direct channel. With this in mind, we turn next to the data and approaches to be used to address these questions.

3 Data

Consistent with our objective (e.g., setting reserves in the exchange channel), this section overviews the data available to address the research goal and presents descriptive information regarding advertiser valuations and the attendant pricing implications. First, we describe the publisher and the data source. Second, we show that advertiser valuations differ for different types of impressions. This heterogeneity in valuations is instrumental in setting reserve prices and also suggests that advertisers consider information about impressions in their bids. Lastly, as a prelude to a bidding model, we examine the commonly adopted assumption of truth-telling bidding behaviors in display markets. Typically, the display advertising literature assumes bids are equivalent to valuations. Rather than take this assumption as given, we run a series of experiments that suggest otherwise. Of note, findings suggest that imposing the reserve prices led to a 32% increase in profits, providing concrete evidence there is substantial room to enhance pricing outcomes for the publisher.

3.1 The Publisher and Data Source

The data for this study are collected from a large, premium publisher, ranked within U.S. top 10 by comScore.[8] The publisher has over 30 brands (sites) internationally, among which

[8]https://www.comscore.com/Insights/Rankings

we focus on the top 20 U.S. based sites for our analyses. We further focus our attention to display ads, and exclude video in our analyses.[9] The data are collected from January 2016 to August 2017.

The data for the exchange channel are collected from the ad exchange used by the publisher. The ad exchange provides the data to the publisher at the daily level and the data provided on the various measures are available as daily averages. These data include advertiser id, Demand Side Platform (DSP herein) id,[10] day, site where ad was placed, ad type (ad size, ad location on a page, device), # bids submitted, # impressions won, bidding amount, payment amount, and click responses. Thus the observational unit (i.e., dimension) is at the advertiser-DSP-day-site-ad type delivered, and the metrics provided for each observational unit are # bids submitted, # impressions won, bidding amount, payment amount, and # click received. Focusing on the open auctions in the exchange channel, we have $1,466M$ observations. While the # impressions won, payment amount, and click responses are available for all advertisers, # bids submitted and the bidding amount are only available for a subset of advertisers who opt-in to reveal (share) their data with the publisher.

3.2 Summary Statistics

In this section we provide summary statistics of the data to show there exists considerable variation in advertiser bidding/payment and to explain the reason behind using the payment data in our estimation (as opposed to using the bidding data).

Summary statistics of advertiser buying behaviors are presented in Table 1. At the observational unit level (i.e., advertiser-DSP-day-site-ad type delivered), advertisers on average won 17 impressions at \$1.54 CPM rate. The minimum CPM payment is close to zero, because the publisher currently imposes no reserve price in the auctions.

During the sample period, $14,612$ advertisers participate in the auctions. Bids are observed from the 82% of the advertisers who opted-in (default setting) to share their bidding information. Opt-in advertisers pay a higher CPM (\$1.89) than the full sample average (\$1.54), but buy a much smaller number of impressions, constituting about 16% of the total revenues.

Hence, there are selection concerns arising from using the bid CPMs for inferring advertiser

[9] We adopt this sampling criterion due to the publisher's current waterfalling policy; video ads are mostly sold-out in advance via the direct sales, and are rarely available for sale in the ad exchange. Also the online video ads are often sold with the TV ads as a bundle, which hinders our analyses in the absence of the data on TV ads.

[10] Demand Side Platforms (e.g., MediaMath, DataXu, Turn, Rocket Fuel, Adobe Advertising Cloud) help advertisers by facilitating the real-time bidding process. Recognizing that bid optimization is a difficult managerial problem for the advertisers, DSPs specialize in calculating and submitting bids based on the users' behavioral data and on targeting criteria provided by the advertiser.

valuations in estimation. Fortunately, data are available on the CPM paid (i..e, the second highest bid in an auction). Unlike bid CPM, this CPM paid measure is available from all advertisers. Accordingly, we rely on the payment data in our estimation, and only use # bids submitted and bid CPM data to present evidence of heterogeneity in valuation in sub-section 3.3.

Table 1: Summary Statistics of exchange

	Per Observation Unit	Mean	Median	Std Dev	Min	Max
All	#Impressions Won	69.9	2	2345	0	587307
	CPM paid ($)	1.61	0.86	2.74	0.003	191.5
Opt-In	#Impressions Won	10.1	0	1312	0	777817
	CPM paid ($)	1.89	1.00	3.05	0.002	191.5
	# Bids Submitted	280	5	15042	1	$5.4M$
	Bid CPM	2.7	1.2	67.4	0.00003	50000

3.3 Heterogeneity in Valuations

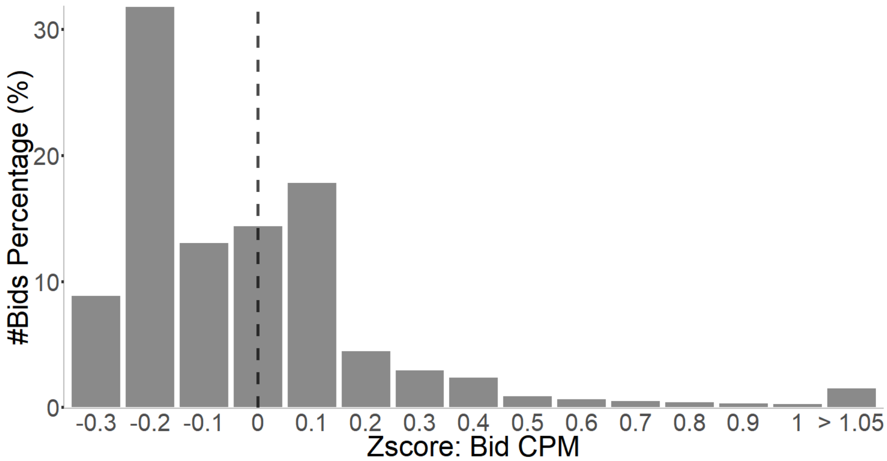
In order to understand how to set reserve prices, it is imperative for the publisher to determine the distribution of advertiser valuations. Accordingly, this sub-section explores advertisers' bidding data under the common assumption that bid CPM directly reveals advertiser valuation under the standard second-price, sealed-bid auctions (Krishna 2009), an assumption we shall explore subsequently. The results below suggest substantial heterogeneity in valuations with respect to the observed characteristics meaning that optimal reserve prices can potentially be set based on the observed key characteristics driving the heterogeneity for better price discrimination.

First in Figure 1, we explore the distribution of bid CPMs. The x-axis represents zscore of bid CPMs, and the y-axis represents the percentage of number of bids.[11] There exists considerable heterogeneity with the minimum of -0.28 and the maximum value being 14721.44.

To explain variation in the advertisers' bid CPMs, we estimate a weighted least square regression of bid CPM zscores on the ad type (ad location, device), site, advertiser, and days. The weight used is the number of bids submitted. Table 2 indicates that advertisers bid more for the desktop ads (relative to mobile, tablets, in-app) and ATF ads (relative MID, BTF). These results imply that optimal reserve prices can be set differentially based on these ad types. Figure 2 plots the advertiser fixed effects and the site fixed effects estimated from the weighted least square regression of bid CPM zscores. Both advertiser and site explain

[11]Bid CPM data are daily averages. Thus in calculating the zscore, we use the weighted mean and weighted standard deviation, where the number of bids for a given observational unit is used as the weight.

Figure 1: Advertiser Bid CPM: Heterogeneity



explain

Table 2: Advertiser Bid CPM: Weighted Least Square Regression

DV: Bid CPM ZScore	Estimates
Desktop	**0.059** (0.007)
Mobile	−**0.062** (0.006)
Tablets	−**0.057** (0.006)
Display vs. App	**0.149** (0.004)
Above the Fold vs. No info	0.005 (0.004)
Mid vs. No info	−**0.012** (0.004)
Below the Fold vs. No info	−**0.013** (0.005)
Site	Yes
Advertiser	Yes
Days (control for supply, competition)	Yes

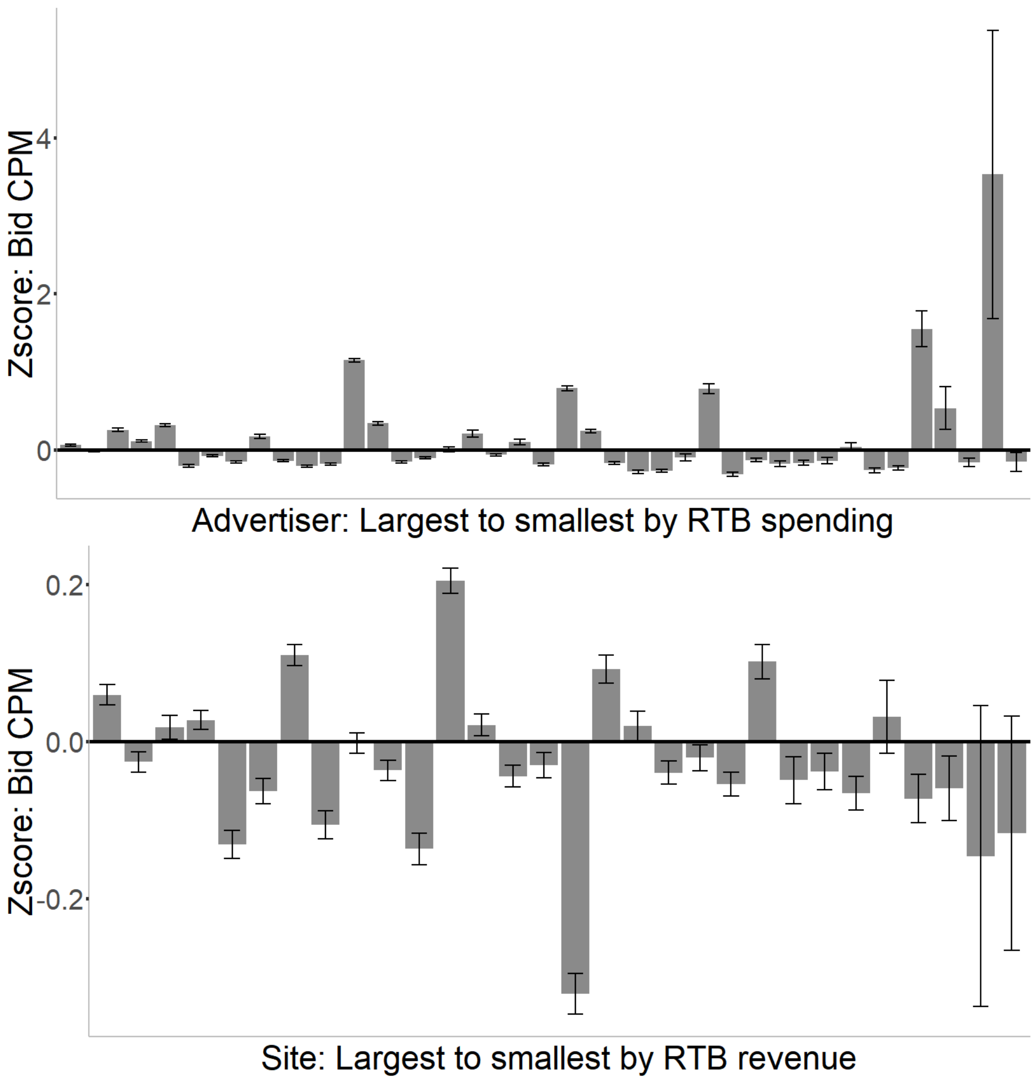
considerable variation in bid CPMs, suggesting they can also be used to price discriminate via differences in reserve prices.[12]

3.4 Truth-Telling Assumption

Theoretical prediction under the standard second-price, sealed-bid auction format is that bidders bid truthfully, meaning that advertisers' underlying valuations can directly be inferred from the observed bids. Though the assumption of truth-telling strategy is tractable and often assumed in models of display markets (e.g., Sayedi 2017, Celis et al. 2014), there exists some empirical evidence that advertisers face practical constraints when bidding in the exchange auctions. We begin by outlining the practical constraints advertisers might face when bidding in the exchange. Then we provide evidence that advertisers' bidding behaviors

[12] In our empirical analysis, we examine reserve prices based on different ad types and sites, while keeping the reserve price the same across advertisers. This is due to the publisher's concern for fairness across advertisers and the preference toward a simpler pricing scheme.

Figure 2: Advertiser Bid CPM: Advertiser and Site Fixed Effects



are not consistent with truth-telling, but appear to reflect a minimum impressions goal.

3.4.1 Practical Constraints in Bidding

Practical constraints discussed in existing literature include (i) budget constraint (Balseiro et al. 2015; Balseiro and Gur 2017), (ii) imperfect information about own and/or other's true valuations (Iyer et al. 2014; Cai et al. 2017), (iii) minimum impressions goal (Ghosh, Rubinstein, et al. 2009), and (iv) pacing options where the budget is spent smoothly over a specified time period (Lee et al. 2013; Yuan et al. 2013; Xu et al. 2015). Advertisers' optimal bidding strategies may differ from the truth-telling when faced with these practical constraints. If one or more of these constraints seem to bind advertisers' bidding behaviors, those constraints will need to be considered in building the advertiser bidding model. Otherwise, predictions of advertisers' bidding behaviors under the counterfactual (e.g., when increasing the reserve prices) will be incorrect and will lead to wrong inference on the optimal reserve prices.

Table 3: Theory Predictions

Mechanism	Reserve Level		Effect of Imposing Reserve		
	$r = 0$	$r = r^*_{nc} > 0$	Bid CPM	#Impressions Won	Total Payment
Max Budget Constraint	Not Bind	Not Bind	No Change	−	+
	Not Bind	Bind	−	−	+
	Bind	Bind	+/−	+/−	No Change
Min Impression Constraint	Not Bind	Not Bind	No Change	−	+
	Not Bind	Bind	+	−	+/−
	Bind	Bind	+	No Change	+

Our goal then is to change reserve prices and see how advertisers respond in order to determine which mechanism stands out as the main driver behind their behaviors. Table 3 presents the theory predictions on bid CPM, the number of impressions won, and the total payment, when the reserve price is changed from $r = 0$ (i.e. no reserve) to $r^*_{nc} > 0$. r^*_{nc} is the optimal reserve calculated under the no constraint (truth-telling) model.[13] As the reserve price increases from ($r = 0$) to ($r^*_{nc} > 0$), advertisers face tighter constraints, and each row represents a possible scenario of the underlying state; (not bind, not bind), (not bind, bind), (bind, bind). Each mechanism is considered in isolation. That is when we consider the budget constraint, we assume that the minimum impression constraint does not bind in both ($r = 0$) and ($r^*_{nc} > 0$).

In sum, by varying the reserve and observing how the metrics change, we can determine which of these explanations are most consistent with the data, and ensure our advertiser

[13] Online Appendix B.2 outlines the rationale for these predictions.

bidding model specification is consistent with the bidding outcomes.

3.4.2 Experimental Setting

A series of experiments were developed to vary the reserve prices in a set of display auctions. For purposes of the experiment, advertiser valuations are estimated assuming advertisers use truth-telling strategies, and the optimal reserve prices are calculated conditioned on this assumption (this assumption is not necessary to conduct a test of truth-telling but does provide a first order approximation of the optimal reserve prices).

Reserve prices were set to the optimal level for the treatment group, while they were kept at the historical level for the control group (i.e., no reserve prices).

Data were collected for the period 07/01/2017 - 02/11/2018, where 07/01/2017 - 10/15/2018 constitute 'Prior' period and 10/19/2017 - 02/11/2018 constitute 'Post' period where the change in reserve took place for the treatment group. The details of the experiment design and calculating the optimal reserve prices can be found in online Appendix A.

3.4.3 Experimental Results on eCPM

We begin by reporting experimental results regarding the publishers' revenues to gauge into the effectiveness of setting reserve prices in auctions. We then assess whether the changes in publisher's revenues are consistent with the magnitude predicted from the truth-telling model.

Effect on eCPM The outcome measure considered is *eCPM* (effective CPM, industry vernacular), which yields a per supplied impression revenue.

$$eCPM = \frac{\text{Revenue}}{\text{\# Impression Supplied to Exchange (in thousand)}}$$

Table 4 shows the treatment effect on eCPM, where eCPM is multiplied by a common, multiplicative constant for confidentiality. The increase in revenues (holding the impressions supplied to exchange the same) is huge, 32%, thereby affirming the importance of setting the optimal reserve prices in running auctions.

$$\begin{aligned}\text{Increase in revenue} &= \frac{(0.49 - 0.38) - (0.33 - 0.34)}{0.38} \\ &= \frac{0.12}{0.38} \simeq 32\%\end{aligned}$$

Figure 3 plots the treatment effect by the experimental groups, and the dotted horizontal line represents the overall percentage change in eCPM, 32%.[14] The red bars indicate the

[14]Details of the experimental groups are included in Table 9 in online Appendix.

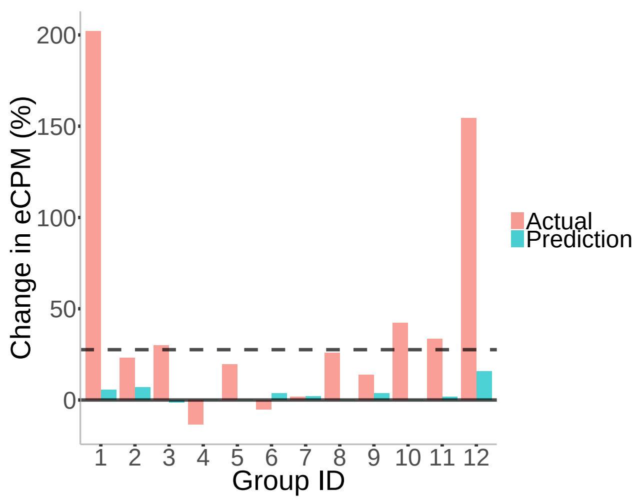
Table 4: Treatment Effect on eCPM

Group	Reserve	Prior (17/08/24 -17/10/15)	Post (17/10/19 - 17/12/03)
Treatment	Optimal	0.38	0.49
Control	No	0.34	0.33

Note: eCPM is scaled by multiplicative constant for confidentiality.

actual change in eCPM, whereas the blue bars indicate the predicted eCPM based on the truth-telling advertiser bidding model. The truth-telling model predicts 5.0% increase in publisher's revenue, which is much lower than the actual 32% increase.[15]

Figure 3: Treatment Effect on eCPM by Group



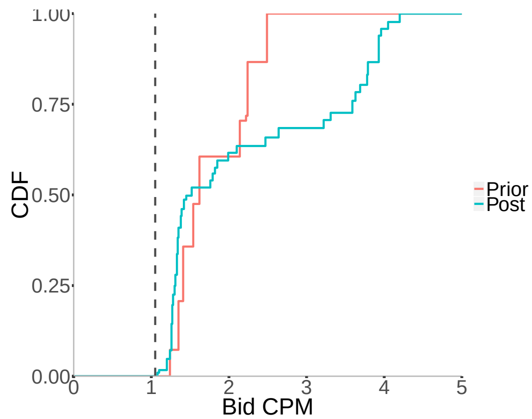
3.4.4 Experimental Results on Bidding Behaviors

To further investigate how advertisers' bidding behaviors are affected by the (exogenous) increase in reserve prices, we consider their bid CPM data. If advertisers bid truthfully, the distribution of bids would be identical pre- and post-, accounting for the truncation due to changes in reserve prices. Figure 4 plots the distribution of bids for one chosen combination (advertiser-DSP-site-ad type), before and after the change in the reserve price.[16] The (truncated at the reserve) cdf shifts to the right after the increase in reserve price, indicating that bid CPM increases with the increase in the reserve price.

[15]The results from a formal difference-in-difference analysis on eCPM is reported in Table 10 in online Appendix.

[16]Because the bid CPM might not be observed below the reserve price, the (truncated) cumulative density function is drawn for the bid CPM data that are above the reserve price set for the experiment (dotted line). If the cost to participate in the auction is high, advertisers will not submit bids knowing that the reserve price is above the valuation.

Figure 4: Treatment Effect on Bid CPM Distribution



More generally, the first column in Table 5 presents the results from difference-in-difference (DiD) analysis, which controls for the time trend before and after the experiment, advertiser fixed effects, and other observables.[17] The observations are at (advertiser-DSP-site-ad type) and weighted by the number of bids submitted in running the weighted least square regression.[18] Bid distributions change with respect to the (exogeneous) increase in reserve prices, which strongly suggests that advertisers do not bid truthfully.[19] [20]

3.4.5 Theoretical Rationale for Experimental Results

In addition to the bid CPM analysis reported in Table 5, the DiD analyses are also conducted on the number of impressions won and the total payment. The second and third columns in Table 5 indicate that the number of impressions won decreases while the total payment increases. Combining these results with the observation that bid CPM increases upon raising the reserve, the findings are suggestive that advertisers set a number of impressions to attain when bidding in the exchange auctions (Table 3). These findings are also consistent with the industry practice of setting and monitoring the 'win rate' (= #impressions won/#bids submitted) as one of the KPIs.[21], and our discussions with industry participants.

[17]From the experiment, we have 4,113,816 bid-metrics at the advertiser-DSP-day-site-ad type level from the opt-in advertisers. For this analysis, both the prior and post data are truncated at the reserve price set in the post period, which leaves us $N = 24,239$ observations.

[18]The post-experimental period indicator is not estimated since the day fixed effect is added.

[19]The coefficient for "Treated" is significant. Although control conditions included sites-ad types that are closest in revenues and ad requests, randomization was not perfect. Thus we prefer the DiD specification over the simple comparison of (Treat, Post) - (Control, Post).

[20]The same analysis as in the first column of Table 5 was conducted using the payment data (second highest bids) from all advertisers to avoid the selection issue discussed in sub-section 3.2. Results are robust inasmuchas second highest bids also increase with respect to the increase in the reserve prices, conditioned on payments exceeding the reserve price set for the experiment.

[21]https://www.adtaxi.com/2018/04/24/exchange-win-rates-bigger-always-better/

Table 5: DiD Regression Results

DV	Bid CPM		# Impressions Won		Total Payment	
	Estimate	SE	Estimate	SE	Estimate	SE
Treated× Post	0.17**	0.07	-2.46**	0.69	0.008**	0.001
Treated	-0.11*	0.06	3.69**	0.49	0.006**	0.001
Post	–		–		–	
Day fixed effect	yes		yes		yes	
Group fixed effect	yes		yes		yes	
Site-Adtype controls	yes		yes		yes	
Advertiser fixed effect	yes		yes		yes	
DSP fixed effect	yes		yes		yes	
Adjusted R^2	0.53		0.001		0.002	
N	$24,239$		$5,937,287$		$5,937,287$	

Note: $^*p < 0.1$ $^{**}p < 0.05$

Based on these findings from the experiment, the advertiser bidding model is constructed to incorporate the minimum impression goal and departs from the commonly adopted truth-telling strategy.

3.5 Summary

Below we summarize this section's findings on pricing in the exchange market, as that is our key goal. While the focus of this proposal is pricing in the exchange, as a prelude to future work, we also outline preliminary descriptive insights into our other objectives: inventory allocation and the effect of information on targeting.

3.5.1 Exchange Pricing

This section first outlined the data and showed considerable variation in advertiser valuations for impressions across sites, devices, and ad formats. This suggests that advertisers value inventory heterogeneously both within and across advertisers, meaning that there is value to knowing the advertiser's underlying valuation in pricing, and that price discrimination via mechanisms such as auctions are important in this market,

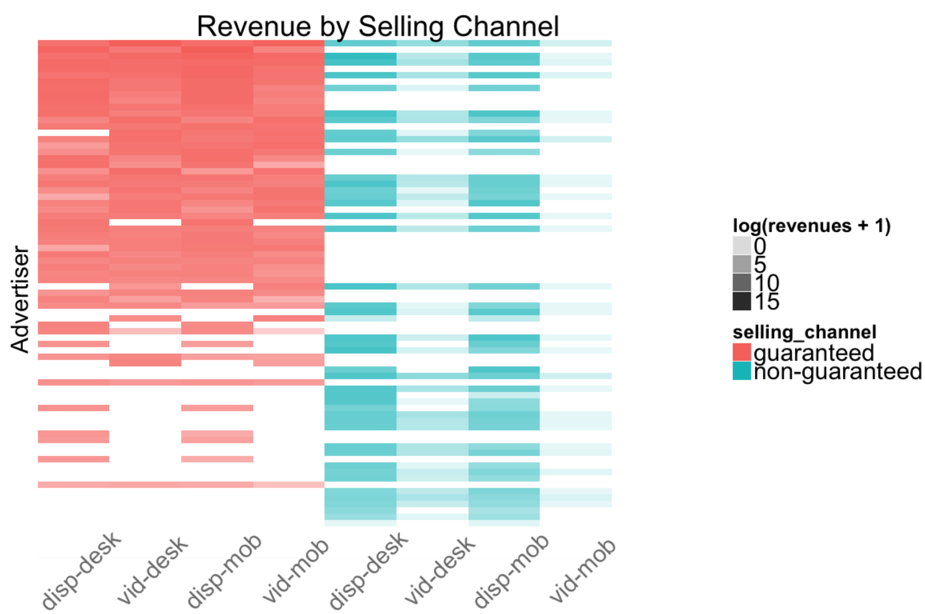
Given the importance of imputing advertiser valuations, we next considered the role of reserve prices in advertiser bidding strategies and (via a series of experiments) found advertiser behaviors are not consistent with truth telling, but rather a minimum impressions constraint. In the process, we showed that varying reserves has a material consequence on auction revenue. We use this information next, as we turn to a model of advertiser bidding decisions.

3.5.2 Inventory Allocation

Pricing across channels might be interdependent because of demand externalities (each channel is a partial substitute). To the extend demand across channels is interdependent, pricing and inventory policies should be set jointly.

The data suggest that advertisers and publishers jointly consider the two selling channels when making the display ad decisions. Figure 5 plots ad type on the x-axis and each tile on the y-axis represents an advertiser. The red tiles represent the direct channel, whereas the blue tiles represent the exchange. The shade of tile color represents the level of the ad spending. We see some advertisers engage only in the direct channel and some advertisers engage only in the exchange, but in general advertisers buy ad impressions from both direct and exchange selling channels.[22]

Figure 5: Ad Spending by Selling Channel

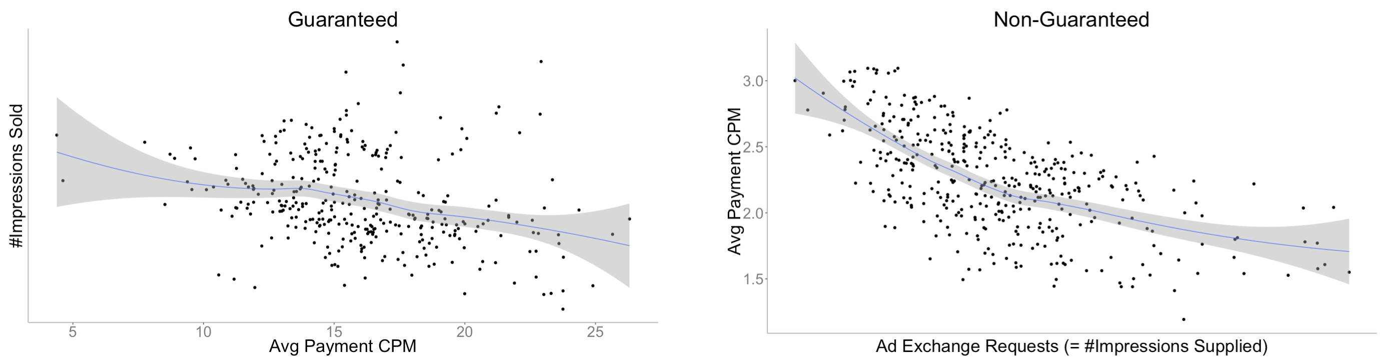


Because of this joint demand, the price and volume guaranteed in advance likely affect the supply, demand, and equilibrium outcome in exchange, and vice versa. In Figure 6, the left side shows a downward sloping demand curve in the direct channel; the plot shows the CPM paid (price) on the x-axis and the impressions sold on the y-axis. On the right side, we similarly plot the demand curve in the exchange channel, where the x-axis represents the

[22]Advertisers focusing on the direct channel include companies selling durable goods and/or luxury goods. Advertisers focusing on the exchange include tech companies and those who can readily link consumers' online behaviors to sales such as e-commerce sites.

number of impressions supplied to the exchange channel (i.e. unsold inventory from the direct channel) and the corresponding CPM payments on the y-axis. When the CPM price in the direct channel is set too high (low), more (less) impressions will be available in the exchange and lower (increase) the CPM paid in the exchange. The CPM paid in the exchange channel is affected, because the number of advertisers competing per impression is affected given advertisers' total demand for ad impressions. Thus, the fixed price in the direct channel should be set by jointly considering the two selling channels.

Figure 6: Interrelation of Direct and exchange Channels



3.5.3 The Role of Information in Targeting

In the context of the exchange market, we have already discussed how advertiser valuations can differ with respect to different ad inventory types (see sub-section 3.2). Using data from the direct channel, we present an analogous descriptive analysis of the heterogeneity in advertiser valuations and provide supporting evidence that the publisher can price discriminate based on how coarsely the impressions are bundled and the information shared with the advertisers.

Data Source The direct channel data include the contractual terms and the impressions' delivery (ad serving) information. Contractual terms include advertiser id, agency id, price, site, ad inventory type (ad size, ad location on a page, device), campaign start and end dates, number of impressions contracted, and user targeting (none, geographic or demographic). The delivery information is at the impression level, and includes user id, site, ad inventory type, delivery time, corresponding contract id (which links to the contractual terms), and click response (binary 0/1).[23] The direct channel consists of 220GB of compressed, impression-level data. When linking the direct channel data to the exchange data, the direct channel data

[23]Ad inventory type delivered is often granular than the ad inventory type contracted. For example, an ad would be shown on site1 in above-the-fold area on a page (ad type actually delivered) for an advertiser who contracts impressions for site1 (ad type contracted)

are aggregated to (advertiser-agency-day-site-ad type delivered-targeting) level, yielding us $280M$ observations. For each of the aggregated observational unit, available metrics are #impressions delivered, payment amount, and # clicks generated.

Summary Statistics In the direct channel, the publisher sells its impressions on a first-come-first-contracted basis. During the sample period, $152,868$ guaranteed contracts are drawn with 593 advertisers. Summary statistics of these contracts are reported in Table 6. The average campaign length is about a month. On average, $439,000$ impressions are contracted at a \$22.30 CPM rate. The large standard deviation in CPMs imply that advertisers differ in their willingness to pay for different contractual terms (e.g., ad types, campaign period).

Table 6: Summary Statistics of Guaranteed Contracts

Per Contract	Mean	Median	Std Dev	Min	Max
Campaign Duration (in days)	31.7	16	47.3	1	365
#Impressions Contracted (in thousand)	427	125	2594	0.1	170000
CPM	22.3	16.5	76.4	0.0(\$1.89)1	9450
CPM When Targeting Is Used	57.2	16.9	227.5	0.05	3329

Table 7: Targeting Used in Guaranteed Contracts

User/Time Targeting Used	4132
Geography	1418
Gender	36
Topic (Contextual)	2483
Time of Day	263
No User/Time Targeting Used	$148,736$
Total Contracts	$152,868$

Note: Multiple targeting criteria can be used in a contract

The last row in Table 6 indicates the CPM more than doubles when user targeting (either demographic, contextual, behavior targeting) is stipulated in the contract, suggesting the potential value of price discrimination and the value of targeting. Nevertheless, user targeting in direct channel is not common, and only in 3% ($= 4132/152,868$) of all the contracts we see user targeting criteria negotiated (Table 7). The average CPM rate in the direct channel is much higher than the CPM observed in the exchange channel, in part because i) the high quality inventory (e.g., front page, leaderboard) is often sold in advance via the direct channel first, and ii) advertisers pay a premium (above the expected value from the users' direct responses) when contracting guaranteed deals.

Heterogeneity in Valuations The goal of this sub-section is to suggest there exists considerable variation in advertiser-inventory valuations. This variation, in turn, suggests

Table 8: ANOVA: Factors Driving Direct Channel Ad Buying

DV: Ad Spending	% Explained
Site	44.8
Site * Device	27.8
Site * Industry	22.0
Device	3.9
Others	1.5
(Device * Industry, Industry, Month)	

the potential to price discriminate by bundling ad inventory based on site, device, or other observables.

Although user-specific targeting is not common in direct channels, advertisers appear to strategically select site and ad inventory types (i.e., ad location, size, and device). Ad location is categorized into above-the-fold (ATF), mid (MID), below-the-fold (BTF), or no info (i.e., anywhere on the page). Device includes desktop, mobile, or tablets. To apportion variation in advertisers' valuations in the direct channel, we estimate an ANOVA with ad spending as the dependent variable for the advertisers in the top three industries (telecommunications, auto, banking).[24] Results indicate that advertisers have heterogeneous valuations across sites and devices, suggesting the potential to price discriminate by bundling ad inventory (e.g., mobile + desktop at a bundle price). Moreover, industries differ in the sites they buy, suggesting strong match value between sites and industry (for example, auto makers are more likely to buy ads on auto sites).

4 Model

With an eye towards linking reserve pricing to publisher revenues in the exchange channel, this section develops a model of advertiser ad buying as a function of advertisers' valuations and reserve prices. We extend the framework in Balseiro et al. 2015 where the rational behavior of advertisers in display markets is approximated using a fluid mean-field equilibrium (FMFE). In their framework, advertisers are assumed to participate in multiple second-price auctions over the length of a campaign with a prespecified budget. In light of our finding that bid CPMs are positively correlated with the reserve prices, we instead consider a FMFE where advertisers are assumed to have a minimum impressions goal.

Like a budget constraint, the minimum impressions constraint can induce advertiser interactions that influence how reserve prices affect publisher revenues. Specifically, with the minimum impressions constraint, advertisers bid higher than their true valuations to attain

[24] These three industries constitute about 25% of the total direct channel revenues.

their minimum impressions goal when faced with the higher reserve prices.

Below, we begin by describing the publisher-advertiser game and conclude by solving it. Using backward induction, we first solve the advertisers' ad buying decisions, and then characterize the publisher's reserve pricing problem.

4.1 Model Overview

The advertising game proceeds as follows:

1. Publisher: The publisher is the leader and selects the reserve price in the second-price auction r, for the unsold inventory from the direct channel. The publisher's objective is to maximize the long-run expected profit from the auctions. The publisher knows the distribution from which advertisers' valuations are drawn, but does not know the realized true value for each particular impression.

2. Advertisers: The advertisers are the followers and decides how much to bid b, based on the realized valuation v for each given impression and the reserve price r. The advertisers' objective is to maximize expected utility (= valuation − cost) from the auctions given the minimum impressions goal constraint, y.

4.2 Assumptions

This sub-section outlines the model assumptions used to solve the game.

4.2.1 Auction Rule

Reflecting the industry practice, the auction format for each impression is assumed to be second-price, sealed-bid and set exogenously by the ad exchange. The impression is delivered to the winner who bids the highest above the reserve price (i.e., winner's ad is displayed to the consumer). If no advertiser bids above the reserve price, the impression is unsold. The winner pays the second highest bid, or the reserve price if the second highest bid falls below the reserve price.

4.2.2 Independent Private Value (IPV)

Following the independent private value assumption adopted in (Edelman et al. 2007, Ostrovsky and Schwarz 2011, Balseiro et al. 2015) advertiser valuations are assumed to be drawn independently from the conditional distribution $F_{V|Z}(v|z)$, where Z are observed characteristics. Z controls for the factors that make the impressions (auctions) heterogeneous. After controlling for these observed covariates, advertiser valuations are assumed to be independent.[25] [26]

4.2.3 Reserve Price

The reserve price is assumed to be known to all potential bidders (advertisers). In practice, the reserve price is not announced to the advertisers, but they can infer it from repeated auction experience (e.g., using machine learning algorithm).[27] By assuming that advertisers know the reserve price, our prediction on publisher's profits on setting the optimal reserve price can be viewed as the expected profits after advertisers go through the reserve learning phase.

4.2.4 Utility Function

Advertisers are assumed to have a quasi-linear utility function, where utility is defined as the sum of the advertiser's valuations from the impressions won less the total payment for the impressions.

4.2.5 Fluid Mean-Field Equilibrium (FMFE)

The equilibrium concept we adopt is the fluid mean-field equilibrium (FMFE) as defined in Balseiro et al. 2015. This equilibrium considers a mean-field approximation (Weintraub et al. 2008, Iyer et al. 2014) to relax the informational requirements of agents. This FMFE equilibrium concept considers a stochastic fluid approximation from the revenue management literature that is suitable when the number of bidding opportunities is large (Gallego and Van Ryzin 1994). In a setting where a large number of advertisers compete, the rational expectation of competitors' bids can be formulated (approximated) based on the aggregate and stationary distribution of others' bids, instead of tracking each individual competitor's bids. This has two benefits. First, it lightens the assumed burden on part of the decision maker, who needs only to know the distribution of others' bids as opposed to all others' bids. In large markets, tracking all others' bids would be challenging, if not impossible, for bidders. Second, the approach reduces the computational burden of solving for a market equilibrium.[28]

[25] This specification allows values that are common (affiliated) via the covariate Z. For example, Z may contain site dummy, to capture that site1 is valued more than site2 among the advertisers. Z can also contain advertiser dummy to control for the heterogeneity across advertisers.

[26] Athey and Haile 2002 note that the conditional independence assumption (i.e., whether there is unobserved auction-specific heterogeneity after controlling for Z) can be tested if more than one bid is observed in each auction or the transaction price is observed in auctions with exogenously varying numbers of bidders (Theorem 3). Using the payment data alone, unfortunately, this conditional independence assumption cannot be tested.

[27] https://www.aarki.com/blog/understanding-hard-and-soft-price-floors-in-programmatic-media-buying

[28] Readers are referred to Balseiro et al. 2015 for more detail and a formal definition of the FMFE. They also provide a theoretical justification for using the FMFE as an approximation of advertisers' behaviors in display markets.

To employ the FMFE concept, therefore, several assumptions are required.

First, the distribution of competitor's bids is assumed to be stationary, conditional on the observed characteristics Z (which includes time dummy) and the reserve price r. Second, we assume that the bids of a single advertiser do not affect this distribution. Given that a large number of advertisers compete in the exchange channel and the marginal impact of any player is small, neither of these assumptions appear restrictive.[29]

Third, we assume the bidders' minimum impressions constraint need only be satisfied in expectation when bidders solve for their optimal bidding strategies.[30] With this third assumption, it can be shown that bidding strategies that do not condition on the individual state of other advertisers' valuations (instead depending only on the bidder's own valuation, the *distribution* of others' valuations, and the reserve) closely approximate the solution to the optimal bidding strategy that specifically conditions on the individual states of others. This third assumption is motivated by the fact that an advertiser has a large number of bidding opportunities over the campaign length.

4.3 Advertiser Bidding Model

The goal of the advertiser bidding model is to determine the optimal bidding policy faced by advertisers (which we can match to the data in order to back out the distribution of advertiser valuations in order to explore the effect of reserves).

The amount of advertising inventory varies over time. Following Balseiro et al. 2015, we assume that the arrival process of available advertising impressions ($\simeq$ users) at any given point in time follows a Poisson distribution with intensity η. As suggested in sub-section 3.4.5, we allow for advertiser k to have a minimum impressions goal y_k for the campaign length s_k (i.e., the duration over which an advertiser is using its bidding rule). Then, ηs_k (the expected arrivals times the duration of the bidding interval) indicates the total number of impressions arriving during the campaign period.

For a given impression i that arrives in this bidding interval (campaign length), denote advertiser k's value as v_{ik}, which is assumed to be drawn independently and identically from a continuous cumulative distribution $F_V(\cdot|Z)$. Z are observed characteristics, which are common across advertisers (e.g., ad type). Further, let D be the steady-state maximum of the competitors' bids, where the publisher is also considered as one competitor that submits a bid equal to r. The distribution of D will endogenously be determined in equilibrium, which we denote as F_D.

[29] This assumption doesn't require that the average number of bidders *per auction* to be large.

[30] That is, when solving the bidding strategy to maximize its expected utility, the advertiser chooses a bidding function that satisfies the minimum impressions constraint ex ante, in expectation.

The advertiser maximizes its expected utility (= valuation − cost) from the ad auctions, given the minimum impressions goal constraint and the participation constraint. With the assumptions made in sub-section 4.2, we focus on the bidding strategy $\beta_\theta^F(v_i|F_D,\theta)$ for an advertiser type $\theta=(s,y)$ (where advertise type is defined by the campaign length and the impression goal) to be a function of the advertiser's own valuation v_i. The advertiser faces the optimization problem given by

$$\begin{aligned} J_\theta^F(F_D) &= \max{}_b \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{b(V)\geq D\right\}(V-D)\right] \\ \text{s.t. } y_\theta &\leq \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{b(V)\geq D\right\}\right] \\ 0 &\leq \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{b(V)\geq D\right\}(V-D)\right] \end{aligned} \tag{1}$$

where the expectation is taken over both F_V and F_D. To define a well-behaved optimization problem, we assume $y_k<\eta s_k$, that is the minimum impressions goal is lower than the total available impressions.

In the first line, ηs_θ indicates the total number of impressions arriving during the campaign period. $\mathbf{1}\left\{b(V)\geq D\right\}$ indicates the probability of winning the auction on a given arrival, where the advertiser's bid is higher than the maximum of the competitors' bids. Lastly, $(V-D)$ indicates valuation minus payment, where the payment is consistent with the second-price rule.

In the second line, the right hand side is the expected number of impressions won at the end of the campaign period. The inequality constraint assures that the expected number of impressions won is greater than the minimum impressions goal y_θ. This inequality can also be written as $\frac{y_\theta}{\eta s_\theta}\leq E_{V,D}\left[\mathbf{1}\left\{b(V)\geq D\right\}\right]$, implying that the advertiser tries to attain a minimum auction winning rate of $\frac{y_\theta}{\eta s_\theta}$.

The third line captures the advertiser's participation constraint that the its expected utility in bidding in the exchange channel is greater than zero. Below we characterize advertisers' optimal bidding strategies, assuming that the participation constraints hold (do not bind) in equilibrium. In sub-section 4.4, we discuss how the participation constraint is imposed in calculating the optimal reserve price.[31]

Proposition 1. *Suppose that $E[D]<\infty$. Assuming the participation constraint does not*

[31] In estimation, where advertiser valuations are inferred conditional on observing the data, this assumption implies that the participation constraints do not bind for those observed advertisers at their observed (optimal) bidding strategies given $r=0$. This assumption that the participation constraint does not bind in estimation is not restrictive, as it implies advertisers who participated in the exchange channel expected to gain positive utilities at $r=0$. In the counterfactual, where we calculate the optimal reserve price, advertisers might face binding participation constraints as the reserve price increases, thereby potentially dropping out of the auction.

bind, an optimal bidding strategy that solves (1) is given by

$$\beta_{\theta}^{F}\left(v|F_D\right)=v+\mu^{*}$$

where μ^* is the optimal solution of the dual problem

$$inf_{\mu\geq 0}\,\eta s_{\theta}E_{V,D}\left[\mathbf{1}\left\{V\geq D-\mu\right\}\left(V-D+\mu\right)\right]-\mu y_{\theta}$$

That is the advertiser bids higher than her own valuation by a constant factor μ^*, which is the optimal dual (Lagrangian) multiplier of the minimum impressions constraint. Intuitively, this means that the advertiser foregoes the expected utility to satisfy the constraint. This constant factor μ^* guarantees that the advertiser meets the minimum impressions goal at the end of the campaign period.

Proposition 2. *If the participation constraints do not bind in equilibrium, the equilibrium can be characterized as follows:*

$$\beta_{\theta}^{F}\left(v|F_D\right)=v+\mu^{*}$$

where μ^* is

$$\begin{cases}\mu^{*}=0 & \text{if } y_{\theta}<\eta s_{\theta}E_{V,D}\left[\mathbf{1}\left\{V\geq D\right\}\right]\\ y_{\theta}-\eta s_{\theta}E_{V,D}\left[\mathbf{1}\left\{V+\mu^{*}\geq D\right\}\right]=0 & \text{if } y_{\theta}\geq\eta s_{\theta}E_{V,D}\left[\mathbf{1}\left\{V\geq D\right\}\right]\end{cases}$$

The proposition states that if the minimum impressions goal is not binding, then in equilibrium advertisers will bid truthfully ($\mu^* = 0$, $\lambda^* = 0$). On the other hand, if the minimum impressions goal does bind, then advertisers will bid higher than the true valuation, where μ^* solves the implicit function $y_\theta - \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V+\mu^* \geq D\right\}\right] = 0$. Of note, $y_\theta - \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V+\mu^* \geq D\right\}\right]$ equals to the expected number of impressions short from the minimum impressions goal at the end of the campaign when the optimal bid function is employed.

Based on the proposition, the cost of the minimum impressions constraint, μ^*, increases with the goal (y_θ), and decreases with the number of impressions and length of campaign (ηs_θ). Perhaps more importantly for our purposes, an increase in the second highest payment, D, lowers $E_{V,D}\left[\mathbf{1}\left\{V+\mu^* \geq D\right\}\right]$ and thus increases μ^*, the bid premium. Because an increase in reserves can increase D when the reserve binds (the second highest valuation is lower than the reserve), reserves can lead to higher bids, consistent with our experimental data. The proofs for Proposition 1 and Proposition 2 are in online Appendix B.1.

The primitives to be estimated are (F_V, F_D, $\boldsymbol{\mu}$) given the reserve price r.

4.4 Optimal Publisher Ad Auction Reserve Price

The publisher's objective is to maximize the long-run expected profit from the auctions given advertiser valuations. Within the independent private value (IPV) paradigm, the publisher can maximize the revenue from the RTB channel by choosing the reserve price optimally.

4.4.1 Publisher's Optimization Problem

We denote $G_\theta(\boldsymbol{\mu}, r) = E_{V,D}[\mathbf{1}\{V + \mu_\theta \geq D\} D]$ to be the expected payment of a θ-type advertiser when advertisers bid according to the profile $\boldsymbol{\mu}$, and the publisher sets a reserve price r. Similarly, $I(\boldsymbol{\mu}, r) = F_D(r|\boldsymbol{\mu})$ is defined as the probability that the impression is not sold in the exchange, when advertisers bid according to the profile $\boldsymbol{\mu}$ and the publisher sets a reserve price r.

The publisher's problem can then be written as

$$\max_r \eta \left[\sum_\theta \{p_\theta s_\theta G_\theta(\boldsymbol{\mu}, r)\} + cI(\boldsymbol{\mu}, r) \right] \tag{2}$$

$$\text{s.t. } \mu_\theta \geq 0 \ \perp \ y_\theta \leq \eta s_\theta E_{V,D}[\mathbf{1}\{V + \mu_\theta \geq D\}] \ \forall \theta \in \Theta$$

where p_θ is the probability that an arriving advertiser is of type θ and $c > 0$ is the publisher's valuation (i.e., outside option value if the impression is not won by some advertiser in the exchange).[32]

The first part in the first line indicates the average expenditure of the advertisers (i.e., the second highest bids when the auctions are won by some advertisers). The second term in the first line indicates the publisher's outside option value when the impression is not won by any advertiser. The constraints in the second line present the conditions for the multipliers in ensuring the minimum impressions goals.

4.4.2 Advertiser Participation Constraints

In the counterfactual, where we increase the reserve price to the optimal level, some advertisers will start to face binding participation constraints as the reserve price increases.[33] To incorporate the effect of the participation constraints in the policy simulation, we ascertain whether the bidding profile $\boldsymbol{\mu}$ satisfies the participation constraints $(0 \leq \eta s_\theta E_{V,D}[\mathbf{1}\{V + \mu_\theta \geq D\}(V - D)])$ at the considered reserve price level r for $\forall \theta \in \Theta$.

In the case the participation constraint does not hold for some advertiser types θ, we calculate maximum μ_θ that satisfies the participation constraint such that

$$\bar{\mu}_\theta = \max_{\mu_\theta \geq 0} [0 \leq \eta s_\theta E_{V,D}[\mathbf{1}\{V + \mu_\theta \geq D\}(V - D)]]$$

This is, the advertiser increases its bid to $v + \bar{\mu}_\theta$ to bid as closely as possible to its minimum impressions goal while satisfying the participation constraint.[34]

[32]Note that, by definition, $I(\boldsymbol{\mu}, r) = F_D(r|\boldsymbol{\mu}) = 1 - \sum_\theta \{p_\theta s_\theta E_{V,D}[\mathbf{1}\{V + \mu_\theta \geq D\}]\}$. Thus, equation 2 can be interpreted as the publishers value expectation over selling and not selling an impression.

[33]In the extreme case where all advertisers face binding impressions goals, advertisers' bidding strategies will reach ∞ as the reserve price increases to ∞ without the participation constraints.

[34]Because of the participation constraint, the advertiser may not achieve the minimum impression goal

Given the publisher's outside option value c, the primitives to be recovered from the policy simulation are the optimal reserve price r^* and the corresponding outcomes (F_V, F_D, $\boldsymbol{\mu}$) at r^*.

4.5 Dual Channel Model

Before proceeding to estimation, we will briefly discuss the dual channel model which incorporates both direct and exchange. To capture the sequential nature of the market in which our data are collected, we intend to extend the two-stage game theory framework in Sayedi 2017 and develop an empirically tractable advertiser buying model; i) the first stage represents the direct channel, and (ii) the second stage represents the exchange.

In the direct channel, the publisher first sets the fixed price p for a bundle of impressions. Then the advertisers decide how many impressions to buy at the fixed price in the direct and how impressions to buy in the exchange based on the expectation about the competition landscape and the reserve price in the exchange. A theoretical simplified model encompassing both the direct and exchange channels and its implications are available in online Appendix B.3.

5 Identification and Estimation

This sub-section discusses the identification and estimation strategies for the inference of advertiser valuations.

5.1 Identification

Since the publisher currently imposes no reserve price, the identification strategy is described assuming no reserve price exists.[35] First we characterize the identification strategy when the constraint does not bind ($\mu^* = 0$), then discuss the case of the binding constraint ($\mu^* > 0$).

5.1.1 Case1: $\mu^* = 0$

The equilibrium-bid function in Proposition 1 implies that advertisers bid truthfully when the minimum impressions goal does not bind. Thus, the probability density function of observed bids can directly be mapped to that of valuations. In other words, the distribution of bids

under the counterfactual. In this case, we are assuming that advertisers purchase as many impression as possible toward the minimum impressions goal while satisfying the participation constraint. Alternatively, we can specify the advertisers to drop-out all together from the exchange channel when participation constraint do not meet, but we think the former is more realistic in our context.

[35] When the reserve prices exits, the identification strategy will be similar, but the distribution of bids will represent a truncated distribution of valuations due to endogenous participation.

identifies the distribution of valuations such that

$$f_V^0(v) = f_B^0(b)$$
$$F_V^0(v) = F_B^0(b)$$

where f_V^0 and F_V^0 (f_B^0 and F_B^0) represent probability density function and cumulative density function of valuations (bids) respectively. The superscripts "0" denote the true population values.

Because only the payment data are used in estimation (see sub-section 3.2), the distribution of the observed payments needs to be linked to the distribution of valuations. Denoting D to be the winning payments, F_D^0 is the distribution of the second highest bids. The distribution of order statistics implies that

$$\begin{aligned} F_D^0(v) &= n(n-1)\int_0^{F_V^0(v)} u^{n-2}(1-u)du \\ &\equiv \varphi\left(F_V^0(v)|n\right) \\ f_D^0(v) &= nf_v^0(v)(n-1)F_V^0(v)^{n-2}\left(1-F_V^0(v)\right) \end{aligned}$$

In the last line, $nf_v^0(v)$ indicates that one of the n advertisers (i.e., $\begin{pmatrix} n \\ 1 \end{pmatrix} = n$) draws v exactly, and $(n-1)F_V^0(v)^{n-2}(1-F_V^0(v))$ indicates that $(n-2)$ out of the remaining $(n-1)$ advertisers (i.e., $\begin{pmatrix} n-1 \\ n-2 \end{pmatrix} = n-1$) draw valuations lower than v, $(F_V^0(v)^{n-2})$, and 1 advertiser draws valuation higher than v, $(1-F_V^0(v))$.

Since $\varphi(\cdot|n)$ is a strictly monotonic function given n, F_V^0 is identified from the distribution of observed winning payments F_D^0 when the number of potential bidders n is known (Paarsch, Hong, et al. 2006).

5.1.2 Case2: $\mu^* > 0$

When the minimum impressions constraint binds, the distribution of bids identifies the distribution of valuations up to a location constant μ^* such that

$$f_W^0(w) = f_B^0(b)$$
$$F_W^0(w) = F_B^0(b)$$

where $w = v + \mu^*$ and μ^* are the optimal Lagrangian multipliers. Thus $f_V(v) = f_W(v + \mu^*)$. Assuming we observe the advertiser type $\theta = (s, y)$, μ^* can be estimated from the conditions in Proposition 2, such that μ^* solves $y_\theta - \eta s_\theta E_{V,D}[\mathbf{1}\{V + \mu_\theta^* \geq D\}] = 0$ if $y_\theta \geq \eta s_\theta E_{V,D}[\mathbf{1}\{V \geq D\}]$.

Combining the argument in Case 1, F_V^0 is identified from the distribution of observed winning payments F_D^0 when the number of potential bidders n and the advertiser types (i.e., minimum impressions goal, campaign duration) are known.

5.2 Estimation

The estimation strategy is detailed in the next subsection.As in the preceding section, we discuss the case when the constraint does not bind ($\mu^* = 0$) first, then incorporate the case of the binding constraint ($\mu^* > 0$).

5.2.1 Case1: $\mu^* = 0$

Under the truth-telling scenario, F_D^0 can be estimated by substituting the sample analogue for the population quantity as

$$\begin{aligned}\hat{F}_D(v) &= \frac{1}{T}\sum_{t=1}^{T} \mathbf{1}\left[d_t \leq v\right] \\ &= n(n-1)\int_0^{\hat{F}_V(v)} u^{n-2}(1-u)du\end{aligned} \tag{3}$$

The asymptotic properties of the estimator $\hat{F}_D(v)$ (and the resulting $\hat{F}_V$) are discussed in Paarsch, Hong, et al. 2006.

Incorporating (discrete) covariates is possible as follows. For a given characteristic $z \in \boldsymbol{Z}$, the estimator of $F^0_{V|Z}(v|z)$ is specified as

$$\hat{F}_{D|Z}(v|z) = n(n-1)\int_0^{\hat{F}_{V|Z}(v|z)} u^{n-2}(1-u)du \tag{4}$$

Because the number of participants vary across auctions, n-specific non-parametric empirical cumulative distribution functions (ECDFs) are estimated first for a particular combination of the z. Then $\hat{F}_{V|Z}(v|z)$ is obtained by kernel smoothing n-specific ECDFs. The exceptionally large number of payments observed given a particular combination of ad characteristics facilitates inference in this context of display ad markets.[36]

5.2.2 Case2: $\mu^* > 0$

The key to incorporating the impressions goal constraint is estimating $F_{V|Z}$ together with μ^* for given z. To simplify notation, we drop the dependence on z. The estimation is done in two stages. In the first stage, we estimate location shifted distribution $F^0_V(w)$ where $w = v + \mu$. In the second stage, conditioned on the recovered distribution $F^0_V(w)$, μ is estimated using the condition in Proposition 2.

One challenge in estimating valuations when the impression constraint binds is that D, the distribution of maximum of competing bids, is a function others' bids. As such,

[36]The observed characteristics affecting the valuation are discrete in our context, such as site, ad type (ad location, ad size, device), and time (month). Although the dimension of $\boldsymbol{Z}$ considered is large, we also have many observations per given particular combination of Z enabling non-parametric estimation. If the covariates are continuous, semi-parametric approaches such as single-index models (e.g, the density-weighted derivative estimator in Powell et al. 1989, the maximum rank-correlation estimator in Han 1987) can instead be used to reduce the curse of dimensionality.

advertisers need to form beliefs about the bids of other advertisers. In estimation, because this maximum is observed and equivalent to the rational expectation of the advertisers' regarding the maximum of the competitors' bids. The estimation proceeds as follows:

Stage 1 Denoting $w = v + \mu$, $\hat{F}_W(w)$ and $\hat{F}_D(d)$ are non-parametrically estimated as described in Equations (3) and (4).

Stage 2 Using the optimality condition in Proposition 2, $\boldsymbol{\mu}^*$ is solved for the FMFE using the following algorithm.

1. Start with an arbitrary vector of multipliers $\boldsymbol{\mu}$. That is $\mu_\theta^0 = \mu_\theta, \forall \theta \in \Theta$;

2. Repeat

   (a) Using the estimates $\left(\hat{F}_D(d),\ \hat{F}_W(w)\right)$ obtained in the first stage, N_{sim} simulated values are drawn from the estimated distributions $v \sim \hat{F}_V = \hat{F}_W(v + \mu_\theta^i)$ and $d \sim \hat{F}_D(d)$ to construct

   $$\hat{h}(\mu_\theta; \mu_\theta^i) = \frac{y_\theta}{\eta s_\theta} - \frac{1}{N_{sim}} \sum \left[\mathbf{1}\left\{v_{sim} + \mu_\theta \geq d_{sim}\right\}\right] \tag{5}$$

   (b) μ_θ^{i+1} solves

   $$\arg\min_{\mu_\theta \geq 0} \hat{h}(\mu_\theta; \boldsymbol{\mu}^i), \quad \forall \theta \in \Theta$$

   where minimizing $\hat{h}(\mu_\theta; \boldsymbol{\mu}^i)$ over $\mu_\theta \geq 0$ ensures that the conditions in Proposition 2 are satisfied in equilibrium.

   (c) Compute the difference $\triangle = \|\boldsymbol{\mu}^{i+1} - \boldsymbol{\mu}^i\|$ and update $i = i + 1$

3. Until $\triangle < \epsilon$

5.3 Institutional Details

This sub-section discusses four institutional aspects of the data that warrant additional attention; (i) metrics (e.g., payments, # impressions won) are only available as daily averages instead of at the auction (impression) level, (ii) the number of potential bidders, n, are not directly observed, (iii) how the minimum impressions goals and campaign lengths are operationalized, and (iv) which data points are included in Z.

Aggregate Data The metrics (e.g., payments, # impressions won) provided by the ad exchange are aggregated to day level, and are not available to this or other publisher at more granular levels. More specifically, an observational unit in the exchange data represents total payments, number of impressions won, number of clicks attained for each given advertiser-DSP-day-site-ad type. For each observational unit, the average (daily) CPM paid is calculated as (total payments / number of impressions won). This average (daily) CPM paid is used as d_t in Equation (3) in forming the estimator for $\hat{F}_D(v)$.

As each observation represents different number of impressions won, we weigh the average CPM paid by the number of impressions won when estimating the distribution of valuations. That is, a data point (y average CPM paid, x number of impressions won) is treated as if there are x number of observations with y CPM payment.[37]

Number of Potential Bidders The identification strategy discussed above requires that the number of potential bidders n is known. The number of potential bidders for a given observational unit (advertiser-DSP-day-site-ad type) will be

$$
\begin{aligned}
n = n_1 &(\text{\#advertisers with CPM payment, i.e., positive \#impressions won}) \\
&+ n_2(\text{\#advertisers with bids submitted, but zero impressions won})
\end{aligned}
$$

n_1 is observed in the data, while $n_2$is observed only for the opt-in advertisers. Thus, to operationalize n, we use the the following proxy:[38]

$$
\begin{aligned}
\hat{n} = n_1 &(\text{\#advertisers with CPM payment, i.e., positivie \#impressions won}) \\
&+ n_{2,opt-in}(\text{\# opt-in advertisers with bids submitted, but zero impressions won})
\end{aligned}
$$

Minimum Impressions Goal and Campaign Length When advertisers do not face binding minimum impressions constraints, advertisers bid truthfully and the distribution of valuations can be recovered without the knowledge of the minimum impressions goal or the campaign length. In the case the minimum impressions constraints bind for some advertisers, the identification strategy discussed in sub-section 5.1.2 requires more information from the researcher. More specifically, in forming the estimator in Equation (5), $\tau_\theta = \left(\frac{y_\theta}{\eta s_\theta}\right)$ term is needed as an input. This term reflects the minimum winning rate the advertiser aims to attain for a given campaign. Thus τ_θ is advertiser-campaign specific. In the estimation (and the counterfactual), we allow τ to vary across advertisers, but hold constant the target minimum winning rate within an advertiser across campaigns. Denoting k to be the advertiser, τ_k is

[37] The magnitude of the potential bias in using the aggregate data (as opposed to the impression level data) is to be examined. Our conjecture is that the variance of the valuation distribution will be biased downward when using the aggregate data, but its impact on the optimal reserve price level will be small (or minimal). We will provide a formal analysis at a later time.

[38] $n_{2,opt-out}$ will be small for two reasons. First, 82% of advertisers opted-in to share their bidding information (default setting). Second, the opt-out advertisers (18%) constitute about 84% of the total revenues, meaning opt-out advertisers are highly likely to be included in n_1, which is observed.

proxied by

$$\hat{\tau}_k = min\left(I_{k1}, ...I_{km}, ...I_{kM}\right),\ I_{km} > 0\,\forall m$$
$$I_{km} = \frac{1}{\sum_{day \in m} \mathbf{1}\left[i_{k,day} > 0\right] u_{day}} \sum_{day \in m} i_{k,day}$$

where $i_{k,day}$ represents the impressions won by advertiser k and u_{day} represents the impressions available for sale on a given day. I_{km} in the second line constructs the winning rate for a given month m, conditional on participating in the exchange. In other words, $\hat{\tau}_k$ is computed as the minimum of the monthly winning rates observed in the data, conditioned on the advertiser participating in the exchange. Accordingly, the optimal bidding strategy profile $\boldsymbol{\mu}|\boldsymbol{Z}$, will be recovered at the advertiser level, instead of advertiser-campaign level.

The solution to the publisher's optimal reserve price in Equation (2) involves the term $\delta_\theta = p_\theta s_\theta$ which weights the fraction of each type winning the auction (recall, δ_θ is the probability mass of for the bidding strategy profile $\boldsymbol{\mu}_\theta$, where this mass is determined as the product of type probability, p_θ, times the campaign length for this type, s_θ). We do not observe p_θ and s_θ, but can nonetheless compute δ_θ at the advertiser level (that is, set type $\theta = k$). We do this by approximating the weight for each advertiser by its observed share of participation. Thus, we use below proxy for $\delta_{\theta=k}$.

$$\hat{\delta}_k|Z = \frac{\sum_{day} \mathbf{1}\left[i_{k,day} > 0\right]}{\sum_k \sum_{day} \mathbf{1}\left[i_{k,day} > 0\right]}$$

where $i_{k,day}$ represents the impressions won by advertiser k on a given day. $\mathbf{1}\left[i_{k,day} > 0\right]$ takes value 1 if the advertiser k wins a positive amount of impressions on a given day. We include 'month' as the observable characteristics in Z, so the summation is done over days within a month. This $\hat{\delta}_k|Z$ is the weight advertiser k (with the bidding strategy profile μ_k) plays toward platform's revenue given Z.

Observed Covariates The distribution of valuations is estimated separately for each combination of Zs to control for heterogeneity. The observed covariates considered in Z are[39]

- Site: 20 U.S. based sites considered in our analysis
- Ad location: ATF, MID, BTF, no info
- Ad size: 300x250, (728x90, 970x66), 320x50, 300x600. These are the sizes conforming to Interactive Advertising Bureau (IAB) standard guideline, and are most commonly used by the advertisers.[40]

[39] Advertiser (or DSP)-specific distribution of valuations will be explored in the future to control for the heterogeneity across advertisers (or DSPs).

[40] https://www.iab.com/guidelines/

- Device: desktop, mobile, tablet
- Month: controls for seasonality

In sum, we estimate the distribution of valuations for each $11,520$ (20x4x4x3x12) combination of Zs.

6 Results

In this section we outline the results of the advertiser bidding model we use to infer the advertiser value distribution. The estimation is still in progress, thus this section discusses the model primitives to be estimated, how the results will be reported, and our predictions of the results.

Estimation Outcome Estimation will recover the distribution of the underlying advertiser valuation F_V, and the vector of optimal Lagrangian multiplier $\boldsymbol{\mu}^* = (\mu_1^*, .., \mu_\theta^*, ..\mu_\Theta^*)$ which reflects the tightness of the minimum impressions goal constraint advertisers face. The pair $(F_V, \boldsymbol{\mu}^*)$ will be obtained for each combination of observables $\boldsymbol{Z}$, which contain discrete space of (site-ad location-ad size-device-month).

Result Presentation

- Heterogeneity: The distribution of advertiser valuation will be plotted for a selection of $\boldsymbol{Z}$ to emphasize the heterogeneity present. Also the means and the standard deviations of the distribution will be regressed on $\boldsymbol{Z}$.
- Tightness of the Constraint: For a given combination of $\boldsymbol{Z}$, the optimal Lagrangian multiplier $\boldsymbol{\mu}^*$ will be plotted across advertisers to show the difference in tightness of the constraint across advertisers.

Predictions We expect that the distributions will show high variance in mean and standard deviation across $\boldsymbol{Z}$s, suggesting the benefit of setting heterogeneous reserve prices based on the observables. Likewise, we suspect some of the $\boldsymbol{\mu}^* = (\mu_1^*, .., \mu_\theta^*, ..\mu_\Theta^*)$ will deviate from zero, indicative that the truth-telling assumption is not valid in our context.

Proposal Defense Presentation For the purposes of the proposal defense, the estimation results will be depicted for the experimental data. That is, the advertiser valuation distributions and the multipliers will be recovered for the observable $\boldsymbol{Z}$s considered in the experiments (see Table 9 in online Appendix).

7 Policy Simulation

The counterfactual we consider in this proposal is changing the reserve price in order to improve publisher's revenues in the exchange channel. Based on the recovered advertiser distribution F_V, the optimal reserve can be obtained by solving the publisher's optimization problem prescribed in Equation (2). The results of the policy simulation will provide the optimal pricing scheme for the publisher, taking into account that advertisers may be constraint by the minimum impressions goal and the participation constraints.

7.1 Solving Optimal Reserve Price

This sub-section discusses the numerical approach used to solve for the optimal reserve price. For each combination of $\boldsymbol{Z}$, we compute two reserve prices. The first reserve price, r^*_{nc}, is calculated under the assumption that advertisers bid truthfully (i.e., $\boldsymbol{\mu}^* = 0$). This can be simply done by solving the implicit function in Equation (6) in online Appendix.

The second reserve price, r^*, is calculated with the minimum impressions goal in place by solving the publisher's optimization problem in Equation (2). The second reserve price r^* takes into account advertisers' best responses with the minimum impressions constraint in FMFE. Therefore we have to update advertisers' beliefs on D to ensure that advertisers' beliefs are consistent with the bidding profile $\boldsymbol{\mu}$ and the reserve price r considered. Thus solving the optimal reserve price r^* with the minimum impressions goal involves embedding the iterative best-response algorithm to find FMFE under the new reserve price. That is, given the recovered F_v,

1. Start with an arbitrary r^j for $j = 0$ (we start with $r^0 = r^*_{nc}$, the optimal reserve when advertisers bid truthfully)

2. Repeat: r-step

   (a) Start with an arbitrary vector of multipliers $\boldsymbol{\mu}$. That is $\mu^0_\theta = \mu_\theta, \forall \theta \in \Theta$;

   (b) Repeat: $\boldsymbol{\mu}$-step

      i. Obtain $F_D\left(\cdot|\boldsymbol{\mu^i}\right)$

      ii. μ^{i+1}_θ solves

$$\arg\min_{\mu_\theta \geq 0} h(\mu_\theta; \boldsymbol{\mu}^i), \quad \forall \theta \in \Theta$$
$$h(\mu_\theta; \boldsymbol{\mu}^i) = \frac{y_\theta}{\eta s_\theta} - E_{V,D}\left[\mathbf{1}\left\{V + \mu_\theta \geq D\right\}\right]$$

      iii. Check the participation constraint

      For θ with $0 > E_{V,D}\left[\mathbf{1}\left\{V + \mu_\theta \geq D\right\}(V - D)\right]$, update $\mu^{i+1}_\theta = \bar{\mu}_\theta$ where

$$\bar{\mu}_\theta = \text{argmax}_{\mu_\theta \geq 0}\left[0 \leq \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V + \mu_\theta \geq D\right\}(V - D)\right]\right]$$

iv. Compute the difference $\triangle = \|\boldsymbol{\mu}^{i+1} - \boldsymbol{\mu}^{i}\|$ and update $i = i + 1$

(c) Until $\triangle < \epsilon$

(d) Optimize the Equation (2) to compute the new reserve price r^{j+1}

3. Until the global maximum is found

7.2 Policy Simulation Results

Policy Simulation Outcome The first set of reserve prices r^*_{nc} corresponding to the observables $\boldsymbol{Z}$, will be obtained under the assumption that advertisers bid truthfully (i.e., $\boldsymbol{\mu}^* = 0$). The second set of reserve prices r^* as well as the Lagrangian multipliers $\boldsymbol{\mu}^*$ in FMFE will be obtained.

Result Presentation

- Difference in r^*_{nc} and r^*: The difference in the reserve price under the truth-telling bidding strategy (r^*_{nc}) and the reserve price with the minimum impressions goal constraint (r^*) will be compared. This will also provide the first empirical support, to our knowledge, for this type of constraint in bidding behavior.

- Difference in publisher's revenues: The difference in the publisher's revenues when setting the truth-telling reserve price $\pi(r^*_{nc})$ will be compared to the the case of setting the optimal reserve price $\pi(r^*)$ to gauge into the profit loss of ignoring the minimum impressions constraint.

- Difference in prediction errors: Experimental data provide an opportunity to further verify the modeling assumption of the minimum impressions constraint. We compare the prediction error in platform's revenues when using the model with the minimum impressions constraint $\pi_{actual} - \hat{\pi}(r^*_{nc} = \text{post period reserve})$, to the prediction error when using the truth-telling model $\pi_{actual} - \hat{\pi}_{nc}(r^*_{nc} = \text{post period reserve})$.

Predictions We expect that r^* will be higher in general than r^*_{nc}. This is because, advertisers bid higher with respect to increase in r, thus mitigating the negative effect of the reducing the selling probability when increasing r. We further expect the profit loss to be substantial when the minimum impressions constraint is ignored. Finally the prediction error in using the model with the minimum impressions constraint will be smaller in the 'post' experimental period, providing another support for the proposed model.

Proposal Defense Presentation For the purposes of the proposal defense, the policy simulation results will be shown for the experimental data.

8 Conclusion

With the rapid growth in display advertising markets, there is increasing value in characterizing the value created and apportioned across players, and how these are affected by participants' strategies in this market. We take the view of the publisher and consider how the i) publisher's pricing decisions affect its advertising revenues, ii) how ad inventory should be allocated across the direct and exchange channels, and iii) the role of information asymmetry across publishers and advertisers and across direct and exchange channels in publisher and advertiser welfare.

In this proposal we focus on the first question, though it is our intent to address the remaining questions subsequently. We further focus on the question of reserve prices in the exchange market, taking the decisions made in the direct sales channel as given (e.g., price, number of impressions bought and sold in direct). Taking the direct channel as given is an assumption that mirrors the structure of this market where inventory is allocated to the exchange channel after it does not sell in the direct channel. The pricing decision in the exchange channel therefore distills into finding the optimal reserve price, which requires information on the underlying advertiser valuations. Though our initial focus is limited at reserve pricing in display markets, we find this focus remains economically consequential (in a series of experiments, we increased publisher advertising revenues by nearly a third).

The optimal reserve is a function of advertiser valuations, and auction theory has established that bidding one's true valuation is the weakly dominant strategy in second-price, sealed-bid auctions with standard assumptions. Though the truth-telling strategy is tractable and often assumed in models of display markets (e.g., Sayedi 2017, Celis et al. 2014), our discussions with industry participants indicate reasons that bidders face impression constraints when bidding in the exchange auctions that would lead their bids to deviate from truth-telling.

To discriminate between truth-telling and other practical constraints, we conducted a series of experiments where we manipulated reserve prices to test the extent to which this constraint affects advertisers' bidding behaviors. A key finding of this paper is that the truth-telling assumption is not supported by our data, as an increase in reserve prices leads to an increase in advertiser bids.

Specifically, we find that setting the reserve price at an optimal level based on a truth-telling assumption increased publisher's revenues by 32%. We expect that setting reserves based on the more correct assumption of impression constraints will yield even greater gains. In addition, increasing the reserve price increased bid CPM and the total payment, while it decreased the number of impressions won. These observations afford further evidence of a minimum impressions constraint.

Subsequently, we construct an advertiser bidding model that incorporate the minimum impressions goal and departs from the commonly adopted truth-telling strategy. The model builds on the notion of a fluid mean-field equilibrium developed in Balseiro et al. 2015, which approximates the rational behavior of thousands of advertisers competing in the display advertising markets well.

In our counterfactual, the optimal reserve price with the minimum impressions constraint is computed to maximize the publisher's revenues. By comparing the difference in the publisher's revenues when setting the truth-telling reserve price (i.e., optimal reserve price assuming advertisers bid truthfully) and the optimal reserve price in our model, we can assess the magnitude of the profit loss in ignoring the minimum impressions constraint.

Going forward, this dissertation will extend the exchange model to a dual channel model to incorporate the direct channel. Using a two stage game framework, the first stage will represent the direct channel where impressions are bundled and sold at a fixed price, and the second stage will represent the exchange channel where the unsold impressions from the direct channel are sold via auctions. By building a dual channel model, we can find the optimal pricing (fixed price in the direct, reserve price in the exchange) by understanding the interrelationship between the selling channels. Also as a counterfactual, the dual channel model can contrast the waterfall and dynamic allocation strategies by manipulating the allocation mechanism.

Eventually, this research will consider the role of information about impressions in display advertising markets. The information available to advertisers about the publisher's inventory differs markedly across selling channels, which has ramifications for publishers in provisioning information to advertisers. Research questions in interests include how coarsely should impressions be bundled in the direct channel and what information should be shared about a given impression in the exchange channel. The dual channel pricing, inventory allocation, information provision will be completed in future drafts.

To the best of our knowledge, this paper is amongst the first to empirically consider the issues of pricing, distribution and information in display advertising markets. Given our initial results and the growth in these markets, we hope this research will continue to yield economically meaningful implications for publishers in these rapidly growing markets and lead to more research in this area.

References

Arnosti, N., M. Beck, and P. Milgrom. 2016. "Adverse selection and auction design for internet display advertising." *The American Economic Review* 106 (10): 2852–2866.

Athey, S., E. Calvano, and J. S. Gans. 2017. "The impact of consumer multi-homing on advertising markets and media competition." *Management Science, forthcoming.*

Athey, S., and P. A. Haile. 2002. "Identification of standard auction models." *Econometrica* 70 (6): 2107–2140.

Bajari, P., and A. Hortacsu. 2003. "The winner's curse, reserve prices, and endogenous entry: Empirical insights from eBay auctions." *RAND Journal of Economics:* 329–355.

Balseiro, S. R., O. Besbes, and G. Y. Weintraub. 2015. "Repeated auctions with budgets in ad exchanges: Approximations and design." *Management Science* 61 (4): 864–884.

Balseiro, S. R., J. Feldman, V. Mirrokni, and S. Muthukrishnan. 2014. "Yield optimization of display advertising with ad exchange." *Management Science* 60 (12): 2886–2907.

Balseiro, S., and Y. Gur. 2017. "Learning in repeated auctions with budgets: Regret minimization and equilibrium." Working paper.

Barajas, J., R. Akella, M. Holtan, and A. Flores. 2016. "Experimental designs and estimation for online display advertising attribution in marketplaces." *Marketing Science* 35 (3): 465–483.

Cai, H., K. Ren, W. Zhang, K. Malialis, J. Wang, Y. Yu, and D. Guo. 2017. "Real-time bidding by reinforcement learning in display advertising." In *Proceedings of the 10th international conference on web search and data mining,* 661–670. ACM.

Celis, L. E., G. Lewis, M. Mobius, and H. Nazerzadeh. 2014. "Buy-it-now or take-a-chance: Price discrimination through randomized auctions." *Management Science* 60 (12): 2927–2948.

Chen, J., and J. Stallaert. 2014. "An economic analysis of online advertising using behavioral targeting." *Mis Quarterly* 38 (2): 429–449.

Chen, Y.-J. 2017. "Optimal dynamic auctions for display advertising." *Operations Research* 65 (4): 897–913.

Choi, H., C. F. Mela, S. Balseiro, and A. Leary. 2017. "Online Display Advertising Markets: A Literature Review and Future Directions." *Available at SSRN 3070706.*

De Corniere, A., and R. De Nijs. 2016. "Online advertising and privacy." *The RAND Journal of Economics* 47 (1): 48–72.

Edelman, B., M. Ostrovsky, and M. Schwarz. 2007. "Internet advertising and the generalized second-price auction: Selling billions of dollars worth of keywords." *The American Economic Review* 97 (1): 242–259.

Fu, H., P. Jordan, M. Mahdian, U. Nadav, I. Talgam-Cohen, and S. Vassilvitskii. 2012. “Ad auctions with data.” In *Conference on computer communications workshops,* 184–189. IEEE.

Gallego, G., and G. Van Ryzin. 1994. “Optimal dynamic pricing of inventories with stochastic demand over finite horizons.” *Management science* 40 (8): 999–1020.

Ghosh, A., P. McAfee, K. Papineni, and S. Vassilvitskii. 2009. “Bidding for representative allocations for display advertising.” In *International workshop on internet and network economics,* 208–219. Springer.

Ghosh, A., B. Rubinstein, S. Vassilvitskii, and M. Zinkevich. 2009. “Adaptive bidding for display advertising.” In *Proceedings of the 18th international conference on world wide web,* 251–260. ACM.

Goldfarb, A., and C. Tucker. 2011. “Privacy regulation and online advertising.” *Management science* 57 (1): 57–71.

Han, A. K. 1987. “Non-parametric analysis of a generalized regression model: the maximum rank correlation estimator.” *Journal of Econometrics* 35 (2-3): 303–316.

Hoban, P. R., and R. E. Bucklin. 2015. “Effects of internet display advertising in the purchase funnel: Model-based insights from a randomized field experiment.” *Journal of Marketing Research* 52 (3): 375–393.

Hong, H., and M. Shum. 2006. “Using price distributions to estimate search costs.” *The RAND Journal of Economics* 37 (2): 257–275.

Hummel, P., and R. P. McAfee. 2016. “When does improved targeting increase revenue?” *Transactions on Economics and Computation* 5 (1): 4.

Iyer, K., R. Johari, and M. Sundararajan. 2014. “Mean field equilibria of dynamic auctions with learning.” *Management Science* 60 (12): 2949–2970.

Johnson, G. A., R. A. Lewis, and E. I. Nubbemeyer. 2016. “The online display ad effectiveness funnel & carryover: A meta-study of predicted ghost ad experiments.” Working paper.

Johnson, G. A., R. A. Lewis, and D. Reiley. 2016. “Location, location, location: Repetition and proximity increase advertising effectiveness.” Working paper.

Krishna, V. 2009. *Auction theory.* Academic press.

Laffont, J.-J., and E. Maskin. 1980. “Optimal reservation price in the vickrey auction.” *Economics Letters* 6 (4): 309–313.

Lambrecht, A., and C. E. Tucker. 2017. “Algorithmic bias? An empirical study into apparent gender-based discrimination in the display of STEM career ads.” Working paper.

Lee, K.-C., A. Jalali, and A. Dasdan. 2013. “Real time bid optimization with smooth budget delivery in online advertising.” In *Proceedings of the 7th international workshop on data mining for online advertising,* 1. ACM.

Levin, J., and P. Milgrom. 2010. "Online advertising: Heterogeneity and conflation in market design." *The American Economic Review* 100 (2): 603–607.

Lewis, R. A., and J. M. Rao. 2015. "The unfavorable economics of measuring the returns to advertising." *The Quarterly Journal of Economics* 130 (4): 1941–1973.

Ostrovsky, M., and M. Schwarz. 2011. "Reserve prices in internet advertising auctions: A field experiment." In *Proceedings of the 12th conference on electronic commerce,* 59–60. ACM.

Paarsch, H. J., H. Hong, et al. 2006. "An introduction to the structural econometrics of auction data." *MIT Press Books* 1.

Powell, J. L., J. H. Stock, and T. M. Stoker. 1989. "Semiparametric estimation of index coefficients." *Econometrica: Journal of the Econometric Society:* 1403–1430.

Riley, J. G., and W. F. Samuelson. 1981. "Optimal auctions." *The American Economic Review* 71 (3): 381–392.

Sahni, N. S. 2015. "Effect of temporal spacing between advertising exposures: Evidence from online field experiments." *Quantitative Marketing and Economics* 13 (3): 203–247.

Sahni, N. S., and H. Nair. 2016. "Native advertising, sponsorship disclosure and consumer deception: Evidence from mobile search-ad experiments." Working paper.

Sayedi, A. 2017. "Real-time bidding in online display advertising." Working paper.

Tucker, C. 2014. "Social networks, personalized advertising, and privacy controls." *Journal of Marketing Research* 51 (5): 546–562.

Weintraub, G. Y., C. L. Benkard, and B. Van Roy. 2008. "Markov perfect industry dynamics with many firms." *Econometrica* 76 (6): 1375–1411.

Xu, J., K.-c. Lee, W. Li, H. Qi, and Q. Lu. 2015. "Smart pacing for effective online ad campaign optimization." In *Proceedings of the 21th international conference on knowledge discovery and data mining,* 2217–2226. ACM.

Yuan, S., J. Wang, and X. Zhao. 2013. "Real-time bidding for online advertising: Measurement and analysis." In *Proceedings of the 7th international workshop on data mining for online advertising,* 3. ACM.

Online Appendix

A Field Experiment

A.1 Experiment Design

A series of experiments were designed to manipulate the reserve prices in the exchange channel. Data were collected for the period 07/01/2017 - 02/11/2018, where 07/01/2017 - 10/15/2018 constitute 'Prior' period and 10/19/2017 - 02/11/2018 constitute 'Post' period where the change in reserve took place for the treatment group. The experiments were conducted on two chosen websites that were closest in terms of contents, user demographics, revenues, and number of impressions (user visits).

A.1.1 Treatment and Control Groups

The experiments were designed to run across different ad types including (i) site (site1 and site2), (i) device (desktop, mobile, tablet, no info), (iii) ad location (ATF, MID, BTF, no info, front door), and (iv) size (300x250, 728x90, 970x66, 320x50, 300x600, 970x250). Twelve experiments were constructed; each constituted a pair, closest in ad characteristics, that was randomized either into the treatment or the control group. For example, (site1, desktop, BTF, 300x250) and (site1, desktop, ATF, 300x250) were paired for the first experiment, and the randomly chosen (site1, desktop, BTF, 300x250) was assigned to the treatment group ,whereas (site1, desktop, ATF, 300x250) was assigned to the control group. Table 9 shows the full list of the experiments and the corresponding treatment and control group characteristics.

Table 9: Treatment and Control Groups

Id	Device	Site and Position		Inventory Size	
		Treatment	Control	Treatment	Control
1	Desktop	Site1 BTF	Site1 ATF	300x250	300x250
2	Desktop	Site2 MID	Site2 ATF, BTF	300x250	300x250
3	Mobile	Site1 No Info	Site2 MID, BTF	300x250	300x250
4	Mobile	Site2 ATF	Site2 MID, BTF	300x250	300x250
5	Desktop	Site1 ATF	Site1 BTF	728x90, 970x66	728x90, 970x66
6	Tablet	Site1 ATF, BTF	Site1 No Info	728x90, 970x66	728x90, 970x66
7	Mobile	Site2 MID, BTF	Site1 No Info, Site2 ATF	320x50	320x50
8	Desktop	Site1 ATF	Site2 ATF	300x600	300x600
9	All	Site1 Front Door BTF	Site1 Front Door BTF	300x250	728x90
10	Desktop	Site1 ATF	Site1 BTF	970x250	970x250
11	Tablet	Site1 No Info	Site2 ATF, MID, BTF	300x250	300x250
12	Desktop	Site2 BTF	Site2 ATF	728x90, 970x66	728x90, 970x66

A.1.2 Optimal Reserve Price

For purposes of the experiment, advertiser valuations are estimated assuming advertisers use truth-telling strategies, and the optimal reserve prices are calculated conditioned on this assumption.

Under the standard second-price, sealed-bid auction, where advertisers bid truthfully, their valuation distribution can be identified (sub-section 5.1.1) and estimated (sub-section 5.2.1) from the observed payment data for each given ad characteristics.

The publisher can maximize the revenue from the exchange channel by choosing the reserve price optimally. The optimal reserve price r^*_{nc} can be expressed as

$$r^*_{nc} = c + \frac{[1 - F_V(r^*_{nc})]}{f_V(r^*_{nc})} \tag{6}$$

where c is publisher's valuation (Riley and Samuelson 1981). F_V and f_V are cdf and pdf of advertiser valuation distribution. For each of the twelve experiments, we use the estimated distribution $\hat{F}_{V|Z}$ and $\hat{f}_{V|Z}$ to calculate $r^*_{nc}(Z)$.

A.2 Experimental Results on eCPM

Table 10: Treatment Effect on eCPM

	Model 1		Model 2		Model 3	
DV= eCPM	Estimate	SE	Estimate	SE	Estimate	SE
Treated× Post	0.14*	0.03	0.12*	0.02	0.11*	0.02
Treated	0.07*	0.01	0.09*	0.01	0.11*	0.01
Post	–		–		–	
Day fixed effect	yes		yes		yes	
Group fixed effect	–		yes		yes	
Site-Adtype controls	–		–		yes	
Adjusted R^2	0.04		0.26		0.64	
$N = 8,764$, $^*p < 0.01$						

Next, the results from a formal difference-in-difference (DID) analysis on eCPM are reported, in which observables are controlled for. The outcome measure considered is *eCPM* (multiplied by a common, multiplicative constant for confidentiality) which is estimated to be positive and significant.[41] Based on the DID estimate, the increase in revenue is

$$\text{Increase in revenue} = \frac{0.11}{0.38} \simeq 30\%$$

[41]From the experiment, we have 5,937,287 observations at the advertiser-DSP-day-site-ad type level. Aggregation is necessary for this DID analysis, because the denominator is in eCPM, i.e., 'impressions supplied to exchange', can only be defined at day-site-ad type level. Thus, the data are aggregated across advertisers and DSPs, leaving us 8,764 observations. Each observation is weighted by the number of 'impressions supplied to exchange' in running the weighted least square regression.

where 0.38 is the baseline eCPM (again scaled for confidentiality) for the treatment group in the prior period.

B Model

B.1 Proofs

B.1.1 Proposition 1

The proof for Proposition 1 follows the steps established in Balseiro et al. 2015 A.1.. First, the dual of the primal problem in equation 1 is introduced using a Lagrange multiplier for the minimum impressions goal. Second, the first-order conditions are derived to determine the solution for the dual problem.

Step 1: The Lagrangian for type θ is denoted as

$$\begin{aligned}\mathcal{L}_\theta(b,\mu) &= \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{b(V)\geq D\right\}(V-D)\right] + \mu\left[\eta s_\theta E_{V,D}\left[\mathbf{1}\left\{b(V)\geq D\right\}\right] - y_\theta\right] \\ &= \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{b(V)\geq D\right\}(V-D+\mu)\right] - \mu y_\theta\end{aligned} \tag{7}$$

where a Lagrange multiplier for the minimum impressions goal is $\mu \geq 0$. The dual problem (converting from maximizing the advertiser's objective function given its minimal goal constraint (see Equation 1) to minimizing the Lagrangian multipliers given its maximizing the objective function) is given by

$$\begin{aligned}\Psi_\theta(\mu) =& \inf_{\mu\geq 0}\sup_{b(\cdot)}\mathcal{L}_\theta(b,\mu) \\ =& \inf_{\mu\geq 0}\left\{\eta s_\theta \sup_{b(\cdot)}\left\{E_{V,D}\left[\mathbf{1}\left\{b(V)\geq D\right\}(V+\mu-D)\right]\right\} - \mu y_\theta\right\} \\ =& \inf_{\mu\geq 0}\left\{\eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V\geq D-\mu\right\}(V-(D-\mu))\right] - \mu y_\theta\right\}\end{aligned} \tag{8}$$

the inf is the Lagrangian minimization step and the sup is the goal maximization step. The equality in the last line comes from the fact that the inner optimization problem is similar to an advertiser's problem who faces value $v+\mu$ and seeks to maximize its expected utility in the second-price auction so that bidding truthfully becomes optimal (consider Equation 1 without the constraints). That is for any given multiplier $\mu \geq 0$, the inner expectation term is maximized with the policy $b(V) = V + \mu$. Further, the term within the expectation in the last line is convex in μ, and the expectation preserves convexity, leading to a convex dual problem.

Step 2: The first order condition of $\Psi_\theta(\mu)$ with respect to μ (that is, the FOC for the $\inf_{\mu\geq 0}\{\cdot\}$) is given by the third line in Equation (8):

$$(d/d\mu)\Psi_\theta(\mu) = \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V\geq D-\mu\right\}\right] - y_\theta = 0$$

To explain this FOC, we begin by noting that $\Psi_\theta(\mu)$ is convex in μ. If the constraint does not bind (i.e., $\eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V\geq D\right\}\right] \geq y_\theta$), then $(d/d\mu)\Psi_\theta \geq 0$ at $\mu = 0$. This condition implies the function $\Psi_\theta(\mu)$ is increasing in μ for all $\mu \geq 0$, such that the function is minimized at $\mu^* = 0$ (has a corner solution). Intuitively, the Lagrangian multiplier can be interpreted as the cost of the impressions constraint; if the constraint does not bind, the constraint is costless.

On the other hand when the constraint binds (i.e., the optimal unconstrained number of impressions is less than the impression constraint, that is $\eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V \geq D\right\}\right] < y_\theta$), then $(d/d\mu)\Psi_\theta(\mu)$ takes a negative value at $\mu = 0$. As $\mu \to \infty$, $(d/d\mu)\Psi_\theta(\mu)$ converges to a positive value $\eta s_\theta - y_\theta > 0$.[42] Thus, there exists a unique interior solution $\mu^* > 0$ for $(d/d\mu)\Psi_\theta(\mu^*) = 0$ as Ψ_θ is a convex function in μ.

Moreover, the complementary slackness conditions hold with the bidding function $b^*(V) = \beta_\theta^F = v + \mu^*$ and the optimal multiplier μ^* such that:

$$\mu^* \left[\eta s_\theta E_{V,D}\left[\mathbf{1}\left\{\beta_\theta^F(V) \geq D\right\}\right] - y_\theta\right] = 0,$$

That is, either i) the optimal number of impressions is constrained or ii) the Lagrangian multiplier, μ^* is 0.

Lastly, there is no duality gap (that is, the solution to the dual problem (Equations 8) is optimal for the original problem (Equation 1)). That is

$$\begin{aligned}
&\eta s_\theta E_{V,D}\left[\mathbf{1}\left\{b(V) \geq D\right\}(V - D)\right] \\
=&\mathcal{L}_\theta(\beta_\theta^F, \mu^*) - \mu^* \left[\eta s_\theta E_{V,D}\left[\mathbf{1}\left\{\beta_\theta^F(V) \geq D\right\}\right] - y_\theta\right] \\
=&\mathcal{L}_\theta(\beta_\theta^F, \mu^*) \\
=&\Psi_\theta(\mu^*)
\end{aligned}$$

where the second equality follows from the complementary slackness conditions, and the last equality follows from the fact that $\Psi_\theta(\mu^*) = \sup_{b(\cdot)} \mathcal{L}_\theta(b, \mu^*)$ and the optimal bid function β_θ^F solves the latter problem.

B.1.2 Proposition 2

Once we establish the optimal bidding function and the optimal multiplier as above, Proposition 2 follows from Proposition 4.1 in Balseiro et al. 2015, in which they characterize the equilibrium. Since we analyze the minimum impressions constraint without the participation constraint, the existence and the characterization of the equilibrium are valid only when the participation constraints do not bind in equilibrium. Readers are referred to their proof in the supplemental material (http://dx .doi.org/10.1287/mnsc.2014.2022).

[42] $(\eta s_\theta - y_\theta)$ is a finite positive number as we constrain $\eta s_\theta > y_\theta$ when defining the optimization problem (i.e., the minimum impressions goal is lower than the total available impressions) in sub-section 4.3.

B.2 Theory Predictions

In this appendix, we outline the intuition for the predictions in Table 3 in section 3. Recall, these predictions indicate how advertiser behavior changes as the reserves increase from 0 to a reserve level r^*_{nc}, that is the optimal reserve price under the assumption that advertisers do not face binding impression constraints.[43]

In the subsequent analysis, we consider several cases: i) when the constraint binds neither at 0 nor at r^*_{nc} (not bind, not bind), ii) when the constraint does not bind at 0 but does bind at r^*_{nc} (not bind, bind), and iii) when the constraint binds at both reserve levels (bind, bind). The impressions goal constraint and the budget constraint are each considered in isolation. That is when we consider the budget constraint, we assume that the minimum impression constraint does not bind in both ($r = 0$) and ($r^*_{nc} > 0$).

B.2.1 No Binding Impression or Budget Constraint (Not Bind, Not Bind)

When the underlying state is (not bind, not bind), the advertiser bidding model collapses to the standard second-price auction without the constraint. In this case, advertisers bid their true valuations, and the distribution of bids will be invariant regardless of the reserve price level. The probability of winning an impression decreases at higher r^*_{nc}, but the total payment increases as r^*_{nc} maximizes publisher's profits.

B.2.2 When Impression Constraints Bind

Effect of the Reserve Price on the Optimal Bid The equilibrium multiplier μ^* increases monotonically with the increase in r (until the participation constraint binds). To see this, when the minimum impressions goal binds, the optimal $\mu^* > 0$ satisfies $y_\theta - \eta s_\theta E_{V,D}\left[\mathbf{1}\left\{V + \mu^* \geq D\right\}\right] = 0$. An increase in the reserve price will increase D (the steady-state maximum of the competitors' bids), and to offset this effect, μ^* needs to increase as well to satisfy the equality constraint. As the optimal bidding strategy is derived as $\beta^F_\theta = v + \mu^*$, an increase in r increases μ^*, which in turn increases bids.

Effect of the Reserve Price on the #Impressions Won and the Total Payment When at least some advertisers face the case of (not binding, binding) , the number of impressions won by advertisers will decrease as the reserve prices increase. This is because some advertisers will face a binding impression constraint at r^*_{nc} so they cannot buy as many impressions as they used to when there was no reserve, $r = 0$.

The direction of the total payments across the advertisers is ambiguous when reserves are increased in the (not binding, binding) condition because there are opposing effects.

[43]In the fields experiments, we chose this reserve price level as a first approximation to a solution that might reflect advertiser constraints.

Although the total number of impressions won decreases , advertisers increase their bids with the increase in the reserve price leading to a higher payment CPM per impression sold.

Finally, when the advertiser faces the (binding, binding) condition, the number of impressions won do not change, as the advertiser cannot decrease the number of impressions to buy with the increase in the reserve price (as the constraint does not allow this).[44] Accordingly, the total payment will increase with the increase in the reserve price, which also raises advertisers' bids.

B.2.3 Budget Constraints

Optimal Bidding Strategy with Maximum Budget Constraint Balseiro et al. 2015 establish that the optimal bidding strategy when advertisers face the maximum budget constraint is

$$\beta_{\theta}^{F}\left(v|F_{D}\right)=\frac{v}{1+\mu^{*}}$$

where μ^* is

$$\begin{cases}\mu^{*}=0 & \text{if } b_{\theta}>\eta s_{\theta}E_{V,D}\left[\mathbf{1}\left\{V\geq D\right\}D\right]\\ \eta s_{\theta}E_{V,D}\left[\mathbf{1}\left\{V/(1+\mu^{*})\geq D\right\}D\right]-b_{\theta}=0 & \text{if } b_{\theta}\leq\eta s_{\theta}E_{V,D}\left[\mathbf{1}\left\{V\geq D\right\}D\right]\end{cases}$$

where b_θ is the maximum (constrained) budget.

Effect of Reserve Price on Optimal Bid When the advertiser faces the case of a (not bind, bind) budget constraint as the reserve price increases from 0 to r_{nc}^*, the bidding strategy will change from v to $\frac{v}{1+\mu^*}$ where $\mu^* > 0$. Thus the bid will decrease in this case.

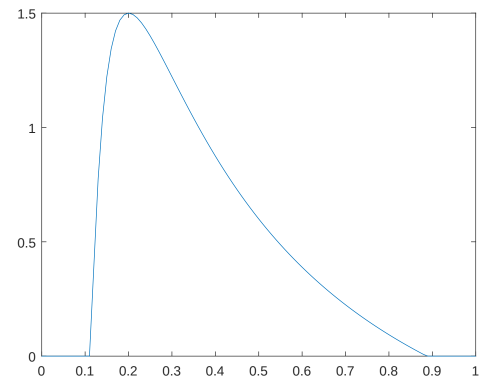
When the advertiser faces (bind, bind) constraint as the reserve price increases from 0 to r_{nc}^*, the effect on the bid is ambiguous. This is because $\frac{\partial \mu^*}{\partial r}$ is not monotonic. For example, Figure 7 shows the change in μ^* (y-axis) with respect to the change in r (x-axis), when $v \sim U[0,1]$ and $b = 0.1$, when there is one advertiser. The plot shows that μ^* first increases than decreases in the range $[0, r_{nc}^*] = [0, 0.5]$.

Effect of Reserve Price on #Impressions Won and Total Payment When the advertiser faces the case of a (not binding, binding) budget constraint as the reserve price increases from 0 to r_{nc}^*, the number of impressions won will decrease as advertisers now shade bids and the reserve price increases. The total payment across advertisers increases, because the budget constraint becomes binding and advertisers spend more.

When the advertiser faces the case of a (binding, binding) budget constraint, the total payment will stay the same (at the binding budget), but the effect on the number of impressions won is ambiguous because bid CPM may or may not increase with respect to the

[44] The number of impressions can decrease if the participation constraint binds for some advertisers.

Figure 7: Budget Constraint: Change in μ^* with respect to Change in r



change in the reserve price.

B.2.4 Other Constraints

When advertisers set pacing options, we expect the results to be similar to the budget constraint case, as pacing plays a similar role to setting an even budget over a specified time period. Finally, advertisers' learning of their own (or others') valuations can either decrease or increase bid CPM depending on the signals received.[45]

B.3 Dual Channel Model

This section develops a simple theoretical model, to serve as a foundation for the empirical, structural dual channel model. The model provides a rationale for why advertiser's willingness to pay for the same impression might differ across channels; specifically that advertiser's valuation uncertainty in the direct channel (where there is less impression-level information) can affect advertisers' channel choice and publisher pricing in equilibrium.

B.3.1 Model Set-Up

To capture the sequential nature of the market in which our data are collected, we employ the two-stage game framework in Sayedi 2017.

The basic model consists of one publisher, one advertiser, and one impression for sale. We assume that both the publisher and the advertiser are risk neutral and that they are profit/utility maximizers. We further assume that the publisher's outside option value (of not selling the impression) is zero without loss of generality. To focus on the nature of the dual channel structure, we abstract away from the practical constraints such as budget and

[45] If advertisers have a common value component (Bajari and Hortacsu 2003, Hong and Shum 2006), bids are also expected to decrease (shade more) when setting the (secret) reserve price, as the seller with the reserve price is treated as one additional competitor.

impressions goal.

Information States The advertiser's valuation for the impression is drawn from $v \sim N(\mu,\ \sigma)$. We assume that this valuation is private, in that the publisher does not know the realized true value, and only knows the distribution. For the advertiser, there exists information asymmetry across selling channels, in that the advertiser has more information about his/her own valuation in the exchange channel than in the direct channel. To facilitate the exposition, we consider an extreme case, where the advertiser knows only the value distribution in the direct channel (same as the publisher), but the true valuation is fully revealed only in the exchange channel. This assumption is adopted as advertisers in the direct channel buy a bundle of impressions at a fixed price based on their expectations about the impressions to be delivered in the future, whereas in the exchange channel advertisers target/buy each impression separately based on the information revealed (e.g., with cookies).

Timing of the Game The timing of the game is as follows.

- Stage 1: The publisher announces a per-impression fixed price p for the direct channel.
- Stage 2: The advertiser decides whether or not to buy the impression at the fixed price p. If the advertiser buys, the impression is delivered to the advertiser with probability 1 (i.e. guaranteed) and the game ends. If the advertiser does not buy, then the game continues.
- Stage 3: In Stage 3, the publisher and the advertiser enter the exchange channel. The auction mechanism is exogenously given as a second-price, sealed-bid auction. The publisher sets the optimal reservation price r, so that the impression is sold if the advertiser bids above or equal to r.
- Stage 4: The true valuation v is revealed to the advertiser. The advertiser decides how much to bid for the impression.

B.3.2 Analysis

We use backward induction to find the sub-game perfect equilibria of the game.

Fourth Stage Bidding own true valuation is the dominant strategy, as the impression is sold in a standard second-price, sealed-bid auction with private, independent value drawn from a continuous distribution satisfying the regularity condition (Krishna 2009). The expected utility of the advertiser a, in buying in the exchange channel n, is

$$\pi^{a,n}(\mu,\ \sigma,\ r) = \int_r^{\infty} (v - r)\,\frac{1}{\sigma}\phi\left(\frac{v-\mu}{\sigma}\right) dv$$

where $\phi(x) = \frac{1}{\sqrt{2\pi}} exp\left(-\frac{x^2}{2}\right)$. Note that the payment is r in this basic setting as there is only one advertiser.

B.3.3 Third Stage

Given the advertiser's optimal strategy of bidding own valuation, the publisher p sets an optimal reserve price in the exchange channel n, that maximizes publisher's revenues.

$$\pi^{p,n}(\mu,\ \sigma,\ r) = r \times \left(1 - \frac{1}{\sigma}\Phi\left(\frac{r-\mu}{\sigma}\right)\right)$$

where $\pi^{p,n}(\mu,\ \sigma,\ r)$ indicates publisher's expected revenue if the impression is sold in the exchange channel via auction. The publisher receives r if advertiser's realized valuation is above r. The optimal reserve price r^* satisfies the first order condition

$$1 - \frac{1}{\sigma}\Phi\left(\frac{r^*-\mu}{\sigma}\right) - \frac{r^*}{\sigma}\phi\left(\frac{r^*-\mu}{\sigma}\right) = 0 \tag{9}$$

B.3.4 Second Stage

Denoting the fixed price offered by the publisher in the first stage to be p, the advertiser in the second stage will buy in the direct channel at the fixed price over the exchange channel if and only if

$$E(v) - p \geq \pi^{a,n}(\mu,\ \sigma,\ r^*)$$
$$p \leq \mu - \pi^{a,n}(\mu,\ \sigma,\ r^*)$$

where the left hand side in the first line denotes the expected profit in buying in the direct channel at the fixed price, whereas the right hand side in the first line denotes the expected profit in buying in the exchange channel through auction.

B.3.5 First Stage

If the publisher wants to sell in the exchange channel, the publisher can set a high enough fixed price p in the direct channel, under which the advertiser would wait and enter the auction instead. The expected revenue will be $\pi^{p,n}(\mu,\ \sigma,\ r^*)$ where r^* is the optimal reserve price satisfying the first order condition in Equation (9). On the other hand, if the publisher wants to sell in the direct channel, the maximum fixed price the publisher can charge is

$$p^* = max\left[\mu - \pi^{a,n}(\mu,\ \sigma,\ r^*),\ 0\right] \tag{10}$$

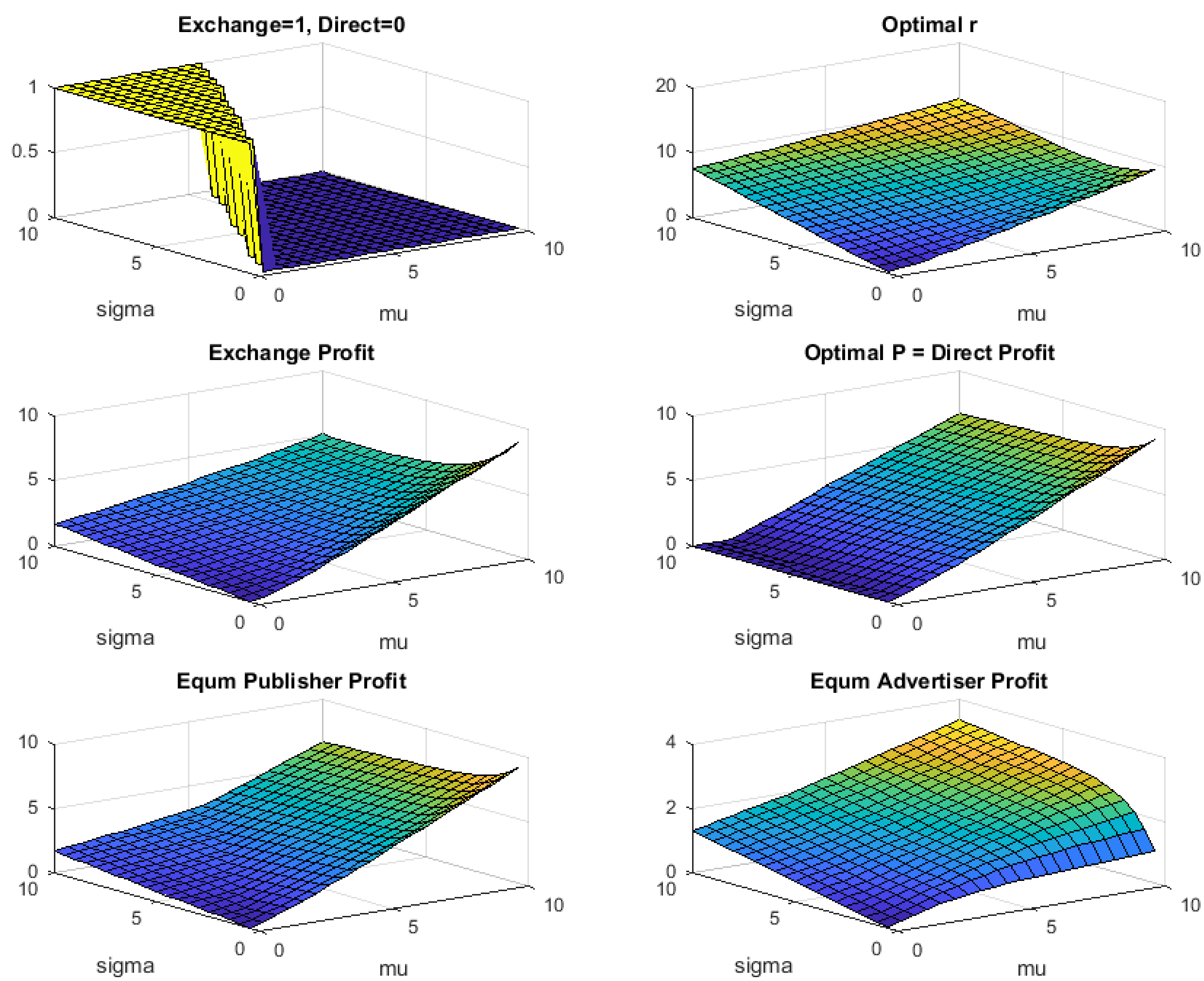
which will be equal to the profit from selling in the direct channel.[46] The publisher will compare $\pi^{p,n}(\mu, \sigma, r^*)$ and p^* in choosing the selling channel. Note that the advertiser receives $\pi^{a,n}(\mu,\ \sigma,\ r^*)$ in equilibrium as an expected utility whether he/she buys at the fixed price or in the auction.

[46] We assume that the advertiser will buy in the direct channel at the fixed price if the expected utilities are the same in both selling channels.

B.3.6 Results

The solution (advertiser choice and publisher prices) to the theoretical model is solved numerically for varying values of (μ, σ), and the results are shown in Figure 8. The x-axis and y-axis in these plots indicate different values of (μ, σ). The first plot shows the equilibrium channel structure where $z = 1$ if the impression is sold in the exchange, and 0 if it is sold in the direct channel. The plot in the first row, second column shows the optimal reserve price in the exchange channel. The two plots in the second row indicate (potential) exchange profit, and (potential) direct channel profit which equals the optimal fixed price. The first plot in the third row shows the publisher's equilibrium profit based on the equilibrium selling channel choice. The plot in the third row, second column shows the advertiser's profit (utility) in equilibrium.

Figure 8: Dual Channel Equilibrium with $N(\mu,\, \sigma)$



Before the existence of the exchange channel (in the absence of Stage 3 and Stage 4), the publisher's optimal pricing strategy is to offer take-it-or-leave-it fixed price, $p = \mu$. In this setting, the publisher's expected profit is $\pi^p = \mu$ and the advertiser's expected profit is $\pi^a = 0$. When both the publisher and the advertiser are uninformed of the true valuation, the publisher who makes the take-it-or-leave-it offer has a greater bargaining power, extracting advertiser's surplus.

The introduction of RTB technology (existence of Stage 3 and Stage 4) has two implications. One is that the auction mechanism with properly set reserve price maximizes publisher's revenue (than the fixed price). Second is that more information is revealed to the advertiser in the exchange, hence publisher's bargaining power in the direct channel is reduced because the exchange channel "competes" with the direct channel. As the value of the information revealed in the exchange channel increases, the advertiser will prefer to buy in the exchange channel, all else equal. More specifically, the equilibrium channel choice (first plot) shows that advertiser's selling channel choice depends on the coefficient of variation $\left(\text{CV}, \frac{\sigma}{\mu}\right)$ of the valuation distribution; advertisers with higher CV prefer the exchange over the direct channel, because they have the higher option value of waiting.

From the publisher's perspective, leveraging both selling channels (Stage 1 through Stage 4) yields higher revenue than selling only through the exchange channel (Stage 3, Stage 4), especially when the value of information (CV) is low, as the publisher can exploit the fact that the advertiser is uninformed when buying in the direct channel.[47] From the first plot, the direct channel equilibrium region (blue region) is where the publisher benefits from the dual channel structure (over exchange only). For this region the exchange profit (first plot in the second row) is lower than the equilibrium publisher profit (first plot in the third row).

In setting the optimal fixed price in the direct channel, $p^* = max\left[\mu - \pi^{a,n}(\mu,\, \sigma,\, r^*),\, 0\right] < \mu$, the publisher shades a value from the mean (relative to the direct only channel solution) to make the advertiser indifferent between buying in the direct channel and the exchange channel. We characterize this shading value, $\pi^{a,n}(\mu,\, \sigma,\, r^*)$ = equilibrium advertiser profits, as an "*information rent*". As the figure in third row, second column suggests, this information rent increases with level of uncertainty, σ. All else equal, the equilibrium selling channel moves from direct to exchange as the uncertainty, σ, increases. This is because as the shading value (information rent) increases with σ (as there is more information to resolve uncertainty in the exchange channel). At certain point, the publisher becomes better off letting the

[47]Selling only through the direct selling channel (Stage 1, Stage 2), with the existence of the exchange technology, is not a credible commitment. When the impression is not sold in the direct selling channel, the publisher will end up selling it in the non-guaranteed selling channel in the sub-game.

advertiser enter the exchange channel than trying to sell in the direct channel.

Of note, even when the publisher uses “waterfalling” allocation method (as opposed to dynamic allocation), leveraging both selling channels yields higher revenue for the publisher than selling only via exchange auctions. This result complements the finding in Sayedi 2017, where he shows that selling in the direct channel as well as in the exchange maximizes publisher’s profits when using dynamic allocation.