(5) $q_j = \frac{1}{\sqrt{\lambda_j}} Z e_j$ where e_j is an element of the set of orthonormal eigenvectors of $Z'Z$ and λ_j its corresponding eigenvalue. Show that $q_j' q_j = 1$, thus showing that its inverse is also 1

Solution: $q_j' q_j = \left(\frac{1}{\sqrt{\lambda_j}} Z e_j \right)' \left(\frac{1}{\sqrt{\lambda_j}} Z e_j \right)$

$$= \frac{1}{\lambda_j} e_j' Z' Z e_j$$

$$= \frac{1}{\lambda_j} e_j' E D E' e_j$$

Since $Z'Z$ is symmetric matrix, there exists decomposition $ZZ' = EDE'$ with D is diagonal matrix

$$q_j' q_j = \frac{1}{\lambda_j} e_j' [e_1 \ \dots \ e_{r+1}] D E' e_j$$

$$= \frac{1}{\lambda_j} [\underbrace{e_j' e_1 \ \dots}_{0} \underbrace{e_j' e_j}_{1} \underbrace{\dots \ e_j' e_{r+1}}_{0}] D E' e_j$$

$$= \frac{1}{\lambda_j} [0 \ \dots \ 1 \ \dots \ 0] \begin{bmatrix} \lambda_1 & & & 0 \\ & \ddots & \lambda_j & \\ & & \ddots & \\ 0 & & & \lambda_{r+1} \end{bmatrix} \begin{bmatrix} e_1' \\ \vdots \\ e_j' \\ \vdots \\ e_{r+1} \end{bmatrix} e_j$$

$$= \frac{1}{\lambda_j} [0 \ \dots \ \lambda_j \ \dots 0] \begin{bmatrix} e_1' \\ \vdots \\ e_j' \\ \vdots \\ e_{r+1} \end{bmatrix} e_j$$

$$= \frac{1}{\lambda_j} \lambda_j e_j' e_j = 1$$ since $e_j' e_j = 1$

Therefore its inverse is also 1

Scanned with CamScanner