(1) $e_1, \dots, e_q \in \mathbb{R}^q$ eigenvectors of $[S_p^2]^{-1} B$

eigenvalues $\lambda_1 \geq \dots \geq \lambda_q \geq 0$.

Show that: $\max f(a) = \max \dfrac{a_1' B a_1}{a_1' S_p^2 a_1}$ is attained at $a_1 = e_1$

use the fact that

$$\operatorname{argmax} g(a) = e \quad \text{where} \quad g(a) = f(|S_p^2|^{-1/2} a)$$

Proof:

$$g(\underline{a}) = f\left((S_p^2)^{-1/2}\,\underline{a}\right)$$

$$= \frac{\left((S_p^2)^{-1/2}\,\underline{a}\right)' B \left((S_p^2)^{-1/2}\,\underline{a}\right)}{\left((S_p^2)^{-1/2}\,\underline{a}\right)' S_p^2 \left((S_p^2)^{-1/2}\,\underline{a}\right)}$$

$$= \frac{\underline{a}' (S_p^2)^{-1/2} B (S_p^2)^{-1/2} \underline{a}}{\underline{a}'\underline{a}}$$

$$= \frac{\underline{a}' \left(\sum_{j=1}^{q} \lambda_j \underline{e}_j \underline{e}_j'\right) \underline{a}}{a'a}$$

$$\operatorname{argmax} g(a) = \underline{e}$$

Scanned with CamScanner