① $E\{X\} = \mu. \Rightarrow E\{CX\} = \begin{bmatrix} c_1\mu_1 \\ \vdots \\ c_q\mu_q \end{bmatrix}$

$\text{Var Cov}\{X\} = \Sigma \Rightarrow \text{var}\{CX\} = C\cancel{X}C' = ((c_i c_j \sigma_{ij}))_{i,j=1,\dots,q}$

C be diagnal matrix $= \begin{bmatrix} c_1 & \cdots & 0 \\ 0 & \cdots & c_q \end{bmatrix}$.

$\text{Var}\{(\underline{X}-\underline{\mu})\} = E\{(\underline{x}-\underline{\mu})(X-\mu)'\} = LL' + \Psi$

$$\begin{aligned}\text{VarCov}\{C(X-\mu)\} &= E\{C(X-\mu)(x-\mu)'C'\}.\\ &= C\ \text{Var}\{LF + \underline{\varepsilon}\}C'\\ &= C[LL' + \Psi]C'\\ &= CLL'C' + C\Psi C'\\ &= L^{+}L^{+} + C\Psi C' \quad \text{where } L^{+} = CL\end{aligned}$$

② $E\{X_{q\times 1}\} = \mu$

$\text{VarCov}\{X_{q\times 1}\} = \Sigma_{q\times q} = \begin{bmatrix} \sigma_{11} & & 0 \\ 0 & \ddots & \\ & & \sigma_{qq} \end{bmatrix}$

OFM:

$X - \mu = LF + \varepsilon$

$\mu = X - LF - \varepsilon$

$$\Sigma = \underbrace{LL'}_{q\times m} + \Psi = E\{(X-\mu)(X-\mu)'\}$$

$$= E\{(LF+\varepsilon)(LF+\varepsilon)'\}$$

$$= E\{(LF+\varepsilon)((LF)'+\varepsilon')\}$$

$$= E\{LF(LF)' + \varepsilon(LF)' + LF\varepsilon' + \varepsilon\varepsilon'\}$$

$$= L\,E\{FF'\}L + E\{\varepsilon F'\}L' + L E\{F\varepsilon'\} + E\{\varepsilon\varepsilon'\}$$

$$= E\{\varepsilon\varepsilon'\} = \Psi$$

Scanned with CamScanner

(3)

$$S = 10^{-3}\begin{bmatrix} 11.072 & 8.019 & 8.160 \\ 8.019 & 6.417 & 6.005 \\ 8.160 & 6.005 & 6.773 \end{bmatrix}$$

$$L = \begin{bmatrix} .1022 \\ .0752 \\ .0765 \end{bmatrix}$$

(a)

$$\hat{L}\hat{L}' = \begin{bmatrix} .1022 \\ .0752 \\ .0765 \end{bmatrix}\begin{bmatrix} .1022 & .0752 & .0765 \end{bmatrix}$$

$$= \begin{bmatrix} 0.01044484 & 0.00768544 & 0.00781830 \\ 0.00768544 & 0.00565504 & 0.00575280 \\ 0.00781830 & 0.00575280 & 0.00585225 \end{bmatrix}$$

$$S = \hat{L}\hat{L}' + \Psi$$

$$\Rightarrow \Psi = S - \hat{L}\hat{L}' = 10^{-3}\begin{bmatrix} 11.072 & 8.019 & 8.160 \\ 8.019 & 6.417 & 6.005 \\ 8.160 & 6.005 & 6.773 \end{bmatrix} - \begin{bmatrix} 0.014 & 0.008 & 0.008 \\ 0.008 & 0.006 & 0.006 \\ 0.008 & 0.006 & 0.006 \end{bmatrix}$$

(b) Commonality : $l_{i1}^2 + l_{im}^2$

$$h_1^2 = l_{11}^2 = 0.1022^2$$

$$h_2^2 = l_{21}^2 = 0.0752^2$$

$$h_3^2 = l_{31}^2 = 0.765^2$$

③

ⓒ $\begin{pmatrix} \text{Proportion of total sample} \\ \text{variance due to } j^{th} \text{ factor} \end{pmatrix} = \dfrac{\hat{l}_{1j}^2 + \hat{l}_{2j}^2 + \dots + \hat{l}_{pj}^2}{s_{11} + s_{22} + \dots + s_{pp}}$.

$$\begin{pmatrix} \text{proportion of total sample} \\ \text{variance due to } 1^{st} \text{ factor} \end{pmatrix} = \frac{l_{11}^2 + l_{21}^2 + l_{31}^2}{s_{11} + s_{22} + s_{33}}$$

$$= \frac{.1022^2 + .0752^2 + .0765^2}{11.072 + 6.417 + 6.773}$$