(2) Show that $PX'X = QX'X \Rightarrow PX' = QX'$ for any conforming matrices P, Q and X. Use this to show that $(X'X)^- X'$ is a generalized inverse of X

(*) Solution:

$$PX'X = QX'X$$

$$\Rightarrow (PX' - QX')X = 0$$

$$\Rightarrow PX' - QX' = 0$$

$$PX' = QX'$$

For $(X'X)^- X'$ to be generalized inverse of X, we need to prove:

$$X(X'X)^- X'X = X$$

$$\Leftrightarrow X'X(X'X)^- X'X = X'X$$

$$\Leftrightarrow \underbrace{X'X(X'X)^-}_{P} X'X = \underbrace{I}_{Q} X'X$$

Let $X'X(X'X)^- = P$ and $I = Q$.

Hence $PX'X = QX'X$

As shown above $\Rightarrow PX' = QX'$

$$\Leftrightarrow X'X(X'X)^- X' = X'$$

Transposing this, we have

$$X(X'X)^- X'X = X$$

$\therefore$ $(X'X)^- X'$ is a generalized inverse of X