Họ tên : Hàn Anh Tú

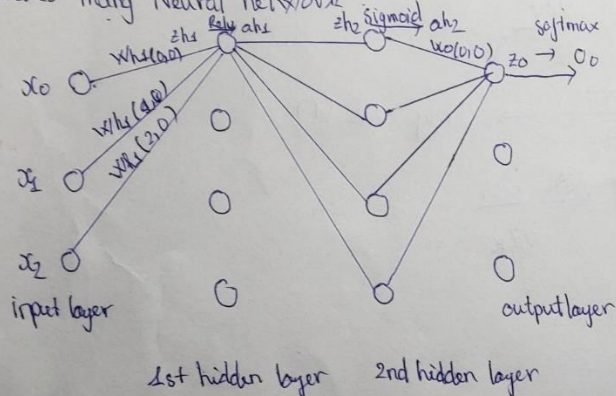
MSSV : 21161215

Lớp : AI - 11 - T4 - 8910

STT : 41

Bài làm :

Ta có mạng Neural network



bias tại 1st hidden layer 1 : $b_{h1,0}$, $b_{h1,1}$, $b_{h1,2}$, $b_{h1,3}$

bias tại 2nd hidden layer 2 : $b_{h2,0}$, $b_{h2,1}$, $b_{h2,2}$, $b_{h2,3}$

bias tại output layer : b_{z0}, b_{z1}.

Tại hidden layer 1 :

$$z_{h1} = \sum x\, w_{h1} + b_{h1}$$
$$\hookrightarrow a_{h1} = \text{relu}(z_{h1})$$

Tại hidden layer 2 :

$$z_{h2} = \sum A_{h1} W_{h2} + b_{h2}$$
$$\hookrightarrow a_{h2} = \text{sigmoid}(z_{h2})$$

Tại output layer :

$$z = \sum A_{h2} W_o + b_z$$
$$\hookrightarrow o = \text{softmax}(z).$$

⊛ Ứng dụng Gradient descent cập nhật trọng số trong mạng neural trên :

Hàm Cross - entropy loss : $\mathcal{L} = -\sum_{i=1}^{N} y_i \ln(O_i)$

N → số lượng classes ở ngõ ra.

y_i → ngõ ra sự thật

O_i → ngõ ra dự đoán

+) Tại output layer ứng dụng Gradient Descent cập nhật trọng số như sau :

$$W_o(t+1) = W_o(t) - \eta \frac{\partial \mathcal{L}}{\partial w_o}$$

$$\frac{\partial \mathcal{L}}{\partial w_0} = \frac{\partial \mathcal{L}}{\partial z} \cdot \frac{\partial z}{\partial w_0} \quad \text{Với:}$$

$$\frac{\partial z}{\partial w_0} = a_{h_2}.$$

$$\frac{\partial \mathcal{L}}{\partial z} = \frac{\partial \left(-\sum_{i=1}^{N} y_i \ln(o_i)\right)}{\partial z} = \frac{-\sum_{i=1}^{N} y_i \,\partial \ln(o_i)}{\partial z} \quad (1)$$

✱ Tính: $\dfrac{\partial \ln(o_i)}{\partial z} = \dfrac{\partial \ln(o_i)}{\partial o_i} \cdot \dfrac{\partial o_i}{\partial z}$

$$= \frac{1}{o_i} \cdot \frac{\partial o_i}{\partial z} \rightarrow \text{Thay vào (1)}$$

Ta được: $\rightarrow \dfrac{\partial \mathcal{L}}{\partial z} = -\sum_{i=1}^{N} \dfrac{y_i}{o_i} \cdot \dfrac{\partial o_i}{\partial z_k}$

Tính: $\dfrac{\partial o_i}{\partial z} = \dfrac{\partial \left(\dfrac{e^{z_i}}{\sum_{K=1}^{N} e^{z_K}}\right)}{\partial z}$

TH1: $i = k$

$$\frac{\partial o_i}{\partial z_k} = \frac{(e^{z_i})' \Sigma - e^{z_i}.(\Sigma)'}{\left(\sum_{K=1}^{N} e^{z_K}\right)^2} = \frac{e^{z_i}\Sigma - e^{z_i}.e^{z_i}}{\left(\sum_{K=1}^{N} e^{z_K}\right)^2}$$

$$= \frac{e^{z_i}}{\sum_{K=1}^{N} e^{z_K}} \cdot \left(1 - \frac{e^{z_i}}{\sum_{K=1}^{N} e^{z_K}}\right).$$

$$= o_i \cdot (1 - o_i) \quad \text{Với } i = k.$$

TH2: $i \neq k$

$$\frac{\partial o_i}{\partial z} = \frac{(e^{z_i})'.\Sigma - e^{z_i}.(\Sigma)'}{\left(\sum_{K=1}^{N} e^{z_K}\right)^2} = \frac{0.\Sigma - e^{z_i}.e^{z_K}}{\left(\sum_{K=1}^{N} e^{z_K}\right)^2}$$

$$= -\frac{e^{z_i}.e^{z_K}}{\left(\sum_{K=1}^{N} e^{z_K}\right)^2} = -\left(\frac{e^{z_i}}{\sum_{K=1}^{N} e^{z_K}}\right) \cdot \left(\frac{e^{z_K}}{\sum_{K=1}^{N} e^{z_K}}\right) = -o_i . o_K$$

$$\frac{\partial \mathcal{L}}{\partial z} = -\sum_{i=1, i\neq K}^{N} \frac{y_i}{o_i} \cdot (-o_i) o_K - \frac{y_i}{o_i} (o_i) \cdot (1-o_i)$$

$$\Rightarrow \frac{\partial \mathcal{L}}{\partial z} = \underbrace{\sum_{i=1, i\neq K}^{N} y_i o_K}_{i \neq K} - \underbrace{y_i \cdot (1-o_i)}_{i = K} \quad (2)$$

Ta có: $\sum_{i=1}^{N} y_i = 1$

$$\sum_{i=1, i\neq K}^{N} y_i + y_K = 1 \rightarrow \sum_{i=1, i\neq K}^{N} y_i = (1-y_K) \quad \text{Thay vào (2)}$$

$$(2) \Leftrightarrow (1-y_K) \cdot o_K - y_K (1-o_K)$$

$$= o_K - o_K y_K - y_K + y_K o_K$$

$$= o_K - y_K$$

Vậy $\frac{\partial \mathcal{L}}{\partial w_o} = (o-y) \cdot ah_2$.

$\hookrightarrow w_o(t+1) = w_o(t) - \eta (o-y)\, ah_2$.

+) Tại ~~hidd~~ 2nd hidden layer:

$$w_{h_2}(t+1) = w_{h_2}(t) - \eta \frac{\partial \mathcal{L}}{\partial w_{h_2}}$$

Tính: $\frac{\partial \mathcal{L}}{\partial w_{h_2}} = \frac{\partial \mathcal{L}}{\partial z} \cdot \frac{\partial z}{\partial ah_2} \cdot \frac{\partial ah_2}{\partial zh_2} \cdot \frac{\partial zh_2}{\partial w_{h_2}}$

- $\frac{\partial \mathcal{L}}{\partial z} = o - y$

(chứng minh ở lớp ngõ ra phía trên)

- $\frac{\partial z}{\partial ah_2} = \frac{\partial(\sum Ah_2 w_o + b_z)}{\partial ah_2} = w_o$

- $\frac{\partial ah_2}{\partial zh_2} = \frac{\partial(\text{Sigmoid}(zh_2))}{\partial zh_2} = \frac{\partial\left(\frac{1}{1+e^{-zh_2}}\right)}{\partial zh_2} = \left(\frac{1}{1+e^{-zh_2}}\right)' = \frac{e^{-zh_2}}{(1+e^{-zh_2})^2} \quad (3)$

~~Đặt~~ Với $a_{h_2} = \frac{1}{1+e^{-z_{h_2}}} \Rightarrow (a_{h_2})^2 = \left(\frac{1}{1+e^{-z_{h_2}}}\right)^2$ Thay vào (3)

$e^{-z_{h_2}} = \frac{1}{a_{h_2}} - 1 = \frac{1-a_{h_2}}{a_{h_2}}$ Thay vào (3)

$\Rightarrow \frac{\partial a_{h_2}}{\partial z_{h_2}} = a_{h_2} \cdot (1 - a_{h_2})$

- $\frac{\partial z_{h_2}}{\partial w_{h_2}} = \frac{\partial(\sum A_{h_1} W_{h_2} + b_{h_2})}{\partial W_{h_2}} = A_{h_1}$

Vậy $W_{h_2}(t+1) = W_{h_2}(t) - \eta\,(o-y) W_o\, a_{h_2}(1-a_{h_2}) A_{h_1}$

+) Tại 1st hidden layer. $W_{h_1}(t+1) = W_{h_1}(t) - \eta \frac{\partial L}{\partial W_{h_1}}$.

$$\frac{\partial L}{\partial w_{h_1}} = \underbrace{\frac{\partial L}{\partial z} \cdot \frac{\partial z}{\partial a_{h_2}} \cdot \frac{\partial a_{h_2}}{\partial z_{h_2}}}_{\text{Đã chainrule ở layer trước đó.}} \cdot \frac{\partial z_{h_2}}{\partial a_{h_1}} \cdot \frac{\partial a_{h_1}}{\partial z_{h_1}} \cdot \frac{\partial z_{h_1}}{\partial w_{h_1}}$$

- $\frac{\partial z_{h_2}}{\partial a_{h_1}} = \frac{\partial(\sum A_{h_1} W_{h_2} + b_{h_2})}{\partial a_{h_1}} = W_{h_2}$

- $\frac{\partial a_{h_1}}{\partial z_{h_1}} = \frac{\partial(\text{relu}(z_{h_1}))}{\partial z_{h_1}} = \begin{cases} 0 & z_{h_1} \le 0 \\ 1 & z_{h_1} > 0 \end{cases}$

- $\frac{\partial z_{h_1}}{\partial w_{h_1}} = X$.

Vậy $W_{h_1}(t+1) = W_{h_1}(t) - \eta \frac{\partial L}{\partial z_{h_2}} \cdot W_{h_2} \cdot X \cdot \begin{cases} 0 & z_{h_1} \le 0 \\ 1 & z_{h_1} > 0 \end{cases}$