Homework 8

Hanao Li hl3202

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1.

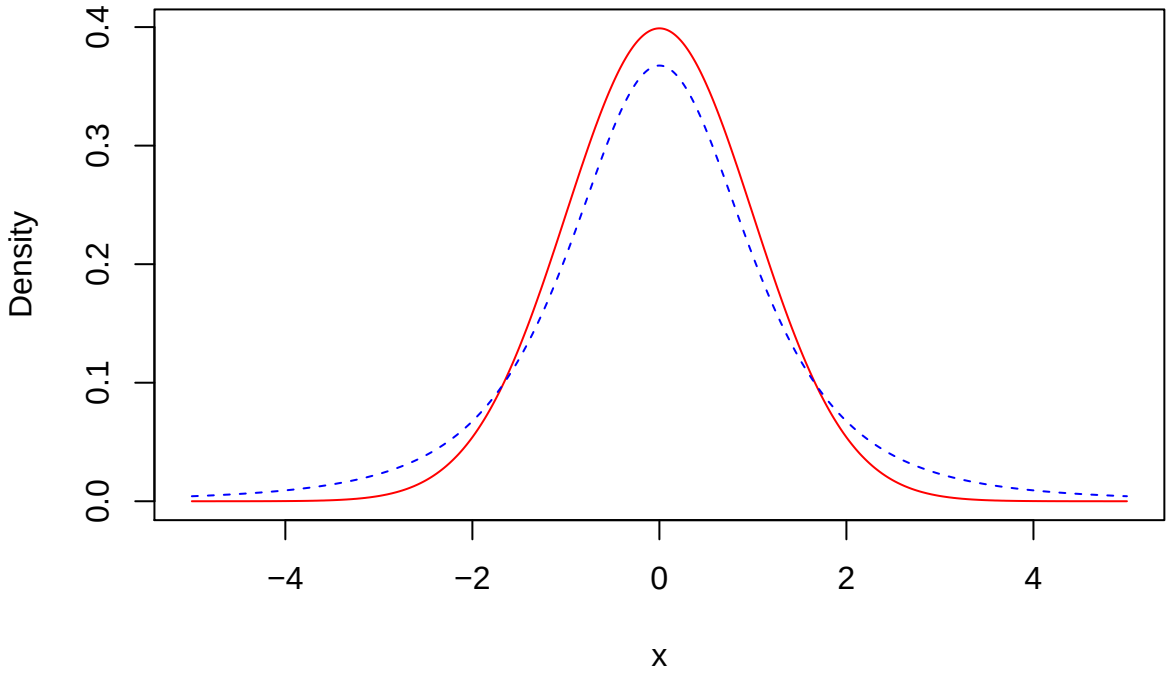
```
n <- 100
p <- 10
s <- 3
set.seed(0)
x <- matrix(rnorm(n*p), n, p)
b <- c(-0.7, 0.7, 1, rep(0, p-s))
y <- x %*% b + rt(n, df = 2)
cor(y, x)
```

```
##            [,1]      [,2]     [,3]       [,4]        [,5]      [,6]       [,7]
## [1,] -0.2526434 0.1239285 0.167384 -0.2522804 -0.03711618 0.1561141 -0.1175268
##             [,8]          [,9]      [,10]
## [1,] -0.08996818 -0.0002104895 0.05068511
```

```
# From the correlation between predictor variable x and response y, we are not able to pick out
# each of the 3 relevant variables.
```

2.

```
xp <- seq(-5, 5, 0.01)
plot(xp, dnorm(xp), type = 'l', col = "red", xlab = 'x', ylab = 'Density')
lines(xp, dt(xp, df = 3), lty = 2, col = "blue")
```




```
# From the plot, we could see that the t-distribution has thicker tails.
```

3.

```
psi <- function(r, c = 1){
  return(ifelse((r^2 > c^2), 2*c*abs(r)-c^2, r^2))
}

huber.loss <- function(b){
  sum(psi(y - x %*% b))
}
```

4.

```
library(numDeriv)
grad.descent <- function(f, x1, max.iter = 200, step.size = 0.001, stopping.deriv = 0.1){
  n <- length(x1)
  xmat <- matrix(0, n, max.iter)
  xmat[, 1] <- x1
  for (i in 2:max.iter){
    gradient <- grad(f, xmat[, i-1])

    if(all(abs(gradient) < stopping.deriv)){
      i <- i - 1
      break
    }
    xmat[, i] <- xmat[, i-1] - step.size * gradient
  }
```

```
  xmat <- xmat[, 1:i]
  return(list(x = xmat[, i], xmat = xmat, i = i))
}
gd <- grad.descent(huber.loss, x1 = rep(0, p), max.iter = 200, step.size = 0.001, stopping.deriv = 0.1)
gd$x
```

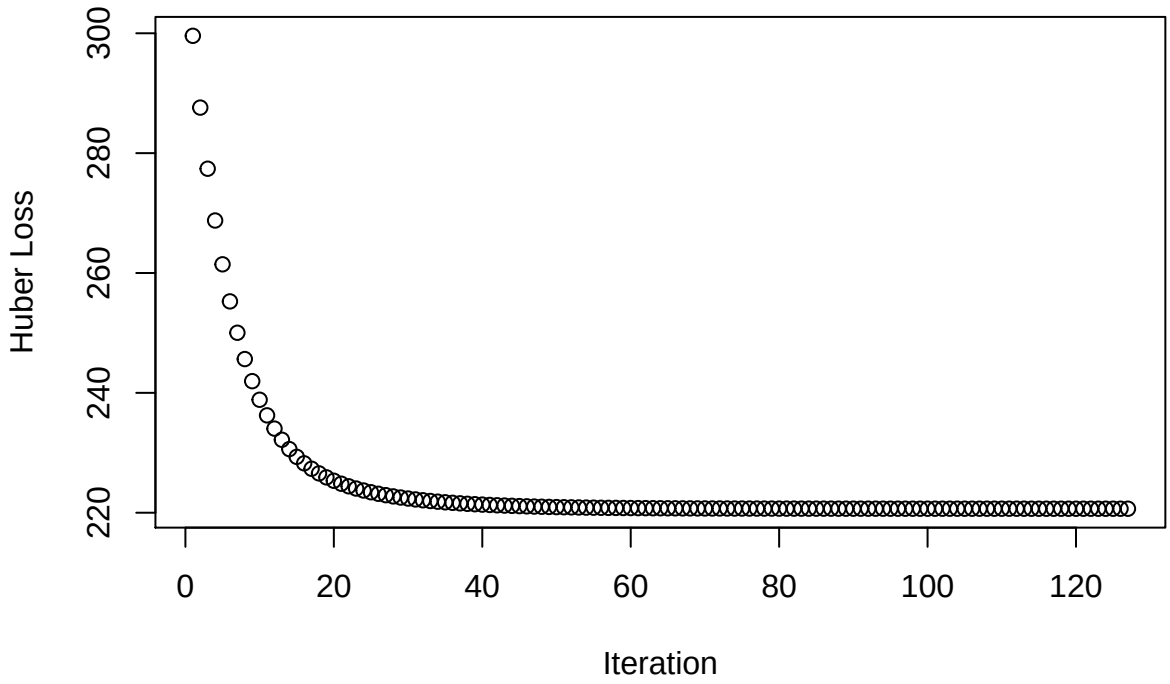
```
##  [1] -0.87346579  0.61828938  0.87989797 -0.04910821  0.07277491  0.10229815
##  [7] -0.12513246 -0.14559243 -0.11903666 -0.02250130
```

```
gd$i
```

```
## [1] 127
```

5.

```
obj <- apply(gd$xmat[, 1:gd$i], 2, huber.loss)
plot(1:gd$i, obj, xlab = 'Iteration', ylab = 'Huber Loss')
```



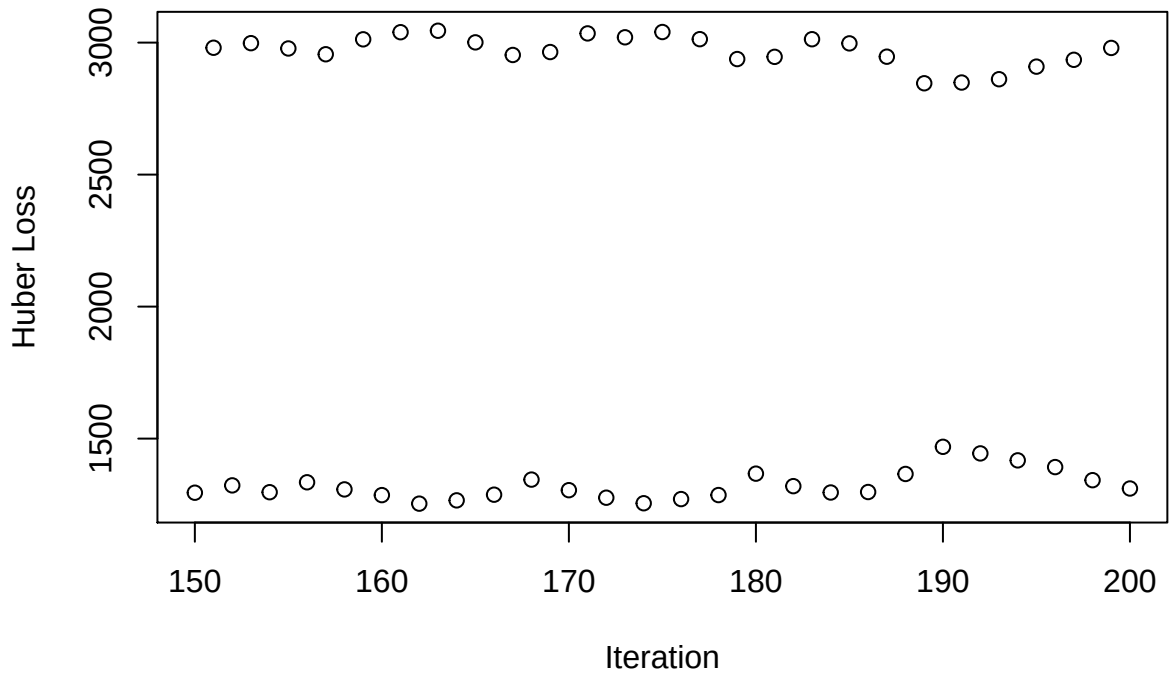

```
# From the plot, we could see that at the start the change from each iteration is significant,
# and the difference is becoming smaller and smllaer towards the end.
```

6.

```
gd1 <- grad.descent(huber.loss, x1 = rep(0, p), max.iter = 200, step.size = 0.1, stopping.deriv = 0.1)
gd1$i
```

```
## [1] 200
```

```r
obj1 <- apply(gd1$xmat[, 1:gd1$i], 2, huber.loss)
plot(150:200, obj1[150:200], xlab = 'Iteration', ylab = 'Huber Loss')
```




```r
# From the plot, we could see that the huber loss is oscillating between 1300 and 3000.
# It is not converging at the end.
gd$xmat
```

```
##       [,1]         [,2]         [,3]         [,4]         [,5]         [,6]
##  [1,]    0 -0.057333921 -0.110579829 -0.159594790 -0.204266836 -0.245042778
##  [2,]    0  0.041772705  0.081319260  0.118460144  0.153250180  0.185778855
##  [3,]    0  0.069072391  0.134130588  0.195155692  0.252503191  0.306170906
##  [4,]    0 -0.027494865 -0.051034153 -0.070570818 -0.086468331 -0.099024379
##  [5,]    0 -0.017602248 -0.029911400 -0.037434068 -0.041221526 -0.041936337
##  [6,]    0  0.034044352  0.064721825  0.092226665  0.116679834  0.138509798
##  [7,]    0 -0.012249313 -0.024290334 -0.036154803 -0.047245096 -0.057384457
##  [8,]    0 -0.010539868 -0.019859263 -0.028033369 -0.035210482 -0.041566707
##  [9,]    0  0.000947101  0.001949435  0.002587417  0.002600211  0.002039122
## [10,]    0  0.011872905  0.022540225  0.032307821  0.041373715  0.049372874
##               [,7]         [,8]         [,9]        [,10]       [,11]
##  [1,] -0.2823619104 -0.316674020 -0.348134386 -0.377028824 -0.40360242
##  [2,]  0.2160656230  0.243819222  0.269225266  0.292577522  0.31395747
##  [3,]  0.3562707827  0.402896991  0.445949210  0.485915410  0.52253519
##  [4,] -0.1086035524 -0.115754196 -0.120860072 -0.124242027 -0.12621620
##  [5,] -0.0405649495 -0.038270083 -0.035335494 -0.032047423 -0.02868154
##  [6,]  0.1578665491  0.174614138  0.189044599  0.201568743  0.21242552
##  [7,] -0.0665288764 -0.074518358 -0.081825407 -0.088531631 -0.09462105
```

```
##  [8,] -0.0471992566 -0.052343858 -0.057026155 -0.061085879 -0.06491873
##  [9,]  0.0007297179 -0.001263384 -0.003938169 -0.007286654 -0.01143206
## [10,]  0.0564393797  0.062694988  0.067909509  0.071795346  0.07418110
##             [,12]       [,13]       [,14]       [,15]       [,16]       [,17]
##  [1,] -0.42834985 -0.45137369 -0.47279262 -0.49295728 -0.51197172 -0.52991299
##  [2,]  0.33363967  0.35175446  0.36839395  0.38357055  0.39744785  0.41017620
##  [3,]  0.55623832  0.58713821  0.61525469  0.64039011  0.66287432  0.68302669
##  [4,] -0.12721477 -0.12749890 -0.12715490 -0.12634647 -0.12517856 -0.12372983
##  [5,] -0.02545450 -0.02243009 -0.01952703 -0.01666959 -0.01384328 -0.01104847
##  [6,]  0.22177375  0.22941793  0.23547838  0.23995866  0.24308425  0.24505476
##  [7,] -0.10004372 -0.10454326 -0.10830986 -0.11138975 -0.11389784 -0.11593914
##  [8,] -0.06872042 -0.07244589 -0.07590442 -0.07900556 -0.08180370 -0.08434487
##  [9,] -0.01639069 -0.02180915 -0.02750306 -0.03306763 -0.03844621 -0.04360256
## [10,]  0.07525630  0.07512216  0.07422749  0.07260390  0.07040966  0.06778211
##              [,18]        [,19]        [,20]         [,21]        [,22]
##  [1,] -0.546844370 -0.562836897 -0.577938460 -0.5921945341 -0.605672919
##  [2,]  0.421885707  0.432689683  0.442678301  0.4519309182  0.460506067
##  [3,]  0.701131496  0.717424072  0.732110155  0.7453701777  0.757366557
##  [4,] -0.122058317 -0.120223751 -0.118274207 -0.1162490701 -0.114192368
##  [5,] -0.008293601 -0.005577412 -0.002910344 -0.0003013271  0.002245468
##  [6,]  0.246037185  0.246187825  0.245652254  0.2445555961  0.242982873
##  [7,] -0.117607049 -0.118969882 -0.120065654 -0.1209274262 -0.121616172
##  [8,] -0.086663280 -0.088794706 -0.090780513 -0.0926541107 -0.094413982
##  [9,] -0.048504334 -0.053144502 -0.057536796 -0.0616946691 -0.065594497
## [10,]  0.064831843  0.061650934  0.058302248  0.0548378185  0.051364008
##             [,23]       [,24]        [,25]       [,26]       [,27]       [,28]
##  [1,] -0.618436266 -0.63052234 -0.641972313 -0.65284187 -0.66318490 -0.67303228
##  [2,]  0.468452426  0.47585970  0.482791784  0.48930628  0.49545608  0.50127220
##  [3,]  0.768240210  0.77812673  0.787134192  0.79534482  0.80283117  0.80966658
##  [4,] -0.112140343 -0.11006865 -0.107978712 -0.10589219 -0.10382811 -0.10179499
##  [5,]  0.004723413  0.00713391  0.009475495  0.01176333  0.01401529  0.01622506
##  [6,]  0.241007424  0.23866964  0.236027667  0.23314779  0.23008616  0.22688835
##  [7,] -0.122184026 -0.12263823 -0.122994684 -0.12326681 -0.12346304 -0.12359705
##  [8,] -0.096055588 -0.09761661 -0.099116904 -0.10055121 -0.10191121 -0.10320620
##  [9,] -0.069218276 -0.07259098 -0.075730291 -0.07864140 -0.08132891 -0.08380895
## [10,]  0.047972155  0.04470098  0.041564696  0.03856009  0.03568201  0.03292826
##             [,29]       [,30]       [,31]       [,32]       [,33]       [,34]
##  [1,] -0.68241236 -0.69135130 -0.69987235 -0.70798988 -0.71572554 -0.72309949
##  [2,]  0.50678189  0.51200918  0.51697461  0.52169095  0.52617638  0.53044706
##  [3,]  0.81591635  0.82163869  0.82688617  0.83171141  0.83615394  0.84024928
##  [4,] -0.09979951 -0.09784682 -0.09594175 -0.09409561 -0.09230875 -0.09058105
##  [5,]  0.01838746  0.02049835  0.02255487  0.02455773  0.02650345  0.02838960
##  [6,]  0.22359310  0.22023327  0.21683613  0.21342047  0.21000675  0.20661201
##  [7,] -0.12368025 -0.12372217 -0.12373277 -0.12373734 -0.12373846 -0.12373815
##  [8,] -0.10444404 -0.10563137 -0.10677568 -0.10789928 -0.10900301 -0.11008747
##  [9,] -0.08609693 -0.08820744 -0.09015419 -0.09194984 -0.09360684 -0.09513669
## [10,]  0.03029588  0.02778134  0.02538175  0.02310159  0.02093367  0.01887142
##             [,35]       [,36]       [,37]       [,38]        [,39]        [,40]
##  [1,] -0.73013056 -0.73683705 -0.74323478 -0.74933863 -0.755162724 -0.760720493
##  [2,]  0.53451742  0.53840041  0.54210650  0.54564529  0.549025738  0.552256203
##  [3,]  0.84402933  0.84749018  0.85066247  0.85357661  0.856259166  0.858733410
##  [4,] -0.08891203 -0.08730420 -0.08575724 -0.08427014 -0.082841572 -0.081469947
##  [5,]  0.03021457  0.03196896  0.03365477  0.03527480  0.036831467  0.038326939
##  [6,]  0.20325041  0.19993718  0.19668095  0.19348840  0.190364992  0.187315073
```

```
##  [7,] -0.12373795 -0.12373926 -0.12374386 -0.12375298 -0.123767377 -0.123787446
##  [8,] -0.11115302 -0.11217038 -0.11314268 -0.11407544 -0.114973052 -0.115839078
##  [9,] -0.09654996 -0.09786062 -0.09907667 -0.10020512 -0.101252647 -0.102225496
## [10,]  0.01690879  0.01501052  0.01317698  0.01141071  0.009712749  0.008083068
##              [,41]        [,42]        [,43]        [,44]         [,45]
##  [1,] -0.766044334 -0.771153906 -0.776059388 -0.780771718 -0.7852989545
##  [2,]  0.555336984  0.558273821  0.561075677  0.563749417  0.5663032635
##  [3,]  0.860993717  0.863050937  0.864926215  0.866636617  0.8681990793
##  [4,] -0.080151863 -0.078885145 -0.077668319 -0.076500493 -0.0753798500
##  [5,]  0.039783175  0.041208034  0.042599908  0.043956647  0.0452778483
##  [6,]  0.184325103  0.181391727  0.178518650  0.175708111  0.1729625230
##  [7,] -0.123797046 -0.123790880 -0.123771083 -0.123740752 -0.1237015628
##  [8,] -0.116684244 -0.117513501 -0.118327708 -0.119125507 -0.1199078624
##  [9,] -0.103151114 -0.104040434 -0.104893663 -0.105711486 -0.1064947471
## [10,]  0.006517696  0.005014687  0.003573192  0.002191324  0.0008681929
##              [,46]        [,47]        [,48]        [,49]        [,50]
##  [1,] -0.789613632 -0.793729237 -0.797657922 -0.801392747 -0.804927321
##  [2,]  0.568769287  0.571149313  0.573445357  0.575646964  0.577721612
##  [3,]  0.869621513  0.870917432  0.872099167  0.873167673  0.874106084
##  [4,] -0.074310396 -0.073288544 -0.072311155 -0.071378388 -0.070469545
##  [5,]  0.046583457  0.047866723  0.049122688  0.050314752  0.051453642
##  [6,]  0.170276072  0.167652099  0.165093054  0.162600859  0.160173040
##  [7,] -0.123680385 -0.123672906 -0.123675755 -0.123679646 -0.123691361
##  [8,] -0.120679901 -0.121440474 -0.122188763 -0.122926182 -0.123649788
##  [9,] -0.107251123 -0.107980211 -0.108681901 -0.109348526 -0.109966211
## [10,] -0.000359774 -0.001505688 -0.002580062 -0.003600874 -0.004568266
##              [,51]        [,52]        [,53]        [,54]        [,55]
##  [1,] -0.808273172 -0.811442329 -0.814445861 -0.817293971 -0.819996078
##  [2,]  0.579676330  0.581521515  0.583266216  0.584918347  0.586484866
##  [3,]  0.874927610  0.875647031  0.876277164  0.876829137  0.877312624
##  [4,] -0.069584328 -0.068724763 -0.067892195 -0.067087434 -0.066310870
##  [5,]  0.052542782  0.053582667  0.054574202  0.055518598  0.056417284
##  [6,]  0.157812334  0.155521277  0.153301534  0.151154045  0.149079146
##  [7,] -0.123710361 -0.123734897 -0.123763563 -0.123795231 -0.123829000
##  [8,] -0.124359760 -0.125056537 -0.125740338 -0.126411217 -0.127069118
##  [9,] -0.110537782 -0.111067445 -0.111558954 -0.112015665 -0.112440585
## [10,] -0.005484138 -0.006351378 -0.007172733 -0.007950809 -0.008688069
##              [,56]       [,57]       [,58]       [,59]       [,60]       [,61]
##  [1,] -0.822560893 -0.82499648 -0.82731096 -0.82951165 -0.83160492 -0.83359667
##  [2,]  0.587971916  0.58938495  0.59072823  0.59200577  0.59322159  0.59437937
##  [3,]  0.877736048  0.87810675  0.87843042  0.87871227  0.87895740  0.87917024
##  [4,] -0.065562561 -0.06484231 -0.06415006 -0.06348549 -0.06284790 -0.06223654
##  [5,]  0.057271842  0.05808395  0.05885497  0.05958646  0.06028028  0.06093829
##  [6,]  0.147076670  0.14514604  0.14328600  0.14149516  0.13977217  0.13811551
##  [7,] -0.123864149 -0.12390011 -0.12393700 -0.12397482 -0.12401323 -0.12405196
##  [8,] -0.127713904 -0.12834540 -0.12896253 -0.12956460 -0.13015159 -0.13072349
##  [9,] -0.112836411 -0.11320557 -0.11355031 -0.11387268 -0.11417446 -0.11445729
## [10,] -0.009386835 -0.01004929 -0.01067775 -0.01127423 -0.01184042 -0.01237795
##              [,62]       [,63]       [,64]       [,65]       [,66]       [,67]
##  [1,] -0.83549245 -0.83729739 -0.83901631 -0.84065373 -0.84221388 -0.84370071
##  [2,]  0.59548245  0.59653391  0.59753656  0.59849302  0.59940569  0.60027682
##  [3,]  0.87935472  0.87951427  0.87965191  0.87977029  0.87987175  0.87995834
##  [4,] -0.06165058 -0.06108916 -0.06055141 -0.06003644 -0.05954337 -0.05907134
##  [5,]  0.06156224  0.06215389  0.06271489  0.06324684  0.06375128  0.06422966
```

```
##  [6,]  0.13652353  0.13499448  0.13352653  0.13211783  0.13076649  0.12947058
##  [7,] -0.12409076 -0.12412943 -0.12416781 -0.12420575 -0.12424315 -0.12427990
##  [8,] -0.13128029 -0.13182202 -0.13234872 -0.13286046 -0.13335734 -0.13383947
##  [9,] -0.11472262 -0.11497179 -0.11520601 -0.11542636 -0.11563385 -0.11582939
## [10,] -0.01288835 -0.01337307 -0.01383349 -0.01427091 -0.01468656 -0.01508160
##             [,68]       [,69]       [,70]       [,71]       [,72]       [,73]
##  [1,] -0.84511797 -0.84646915 -0.84775757 -0.84898634 -0.85015840 -0.85127651
##  [2,]  0.60110850  0.60190269  0.60266122  0.60338580  0.60407805  0.60473950
##  [3,]  0.88003188  0.88009396  0.88014600  0.88018922  0.88022473  0.88025349
##  [4,] -0.05861949 -0.05818699 -0.05777303 -0.05737684 -0.05699766 -0.05663477
##  [5,]  0.06468337  0.06511374  0.06552202  0.06590940  0.06627701  0.06662591
##  [6,]  0.12822822  0.12703750  0.12589656  0.12480354  0.12375665  0.12275410
##  [7,] -0.12431593 -0.12435118 -0.12438559 -0.12441913 -0.12445176 -0.12448348
##  [8,] -0.13430699 -0.13476006 -0.13519888 -0.13562365 -0.13603459 -0.13643193
##  [9,] -0.11601380 -0.11618785 -0.11635223 -0.11650757 -0.11665448 -0.11679348
## [10,] -0.01545711 -0.01581414 -0.01615366 -0.01647658 -0.01678377 -0.01707604
##             [,74]       [,75]       [,76]       [,77]       [,78]       [,79]
##  [1,] -0.85234331 -0.85336128 -0.85433276 -0.85525996 -0.85614501 -0.85698989
##  [2,]  0.60537159  0.60597567  0.60655304  0.60710492  0.60763246  0.60813677
##  [3,]  0.88027636  0.88029408  0.88030733  0.88031669  0.88032267  0.88032574
##  [4,] -0.05628746 -0.05595508 -0.05563697 -0.05533253 -0.05504115 -0.05476229
##  [5,]  0.06695711  0.06727157  0.06757018  0.06785379  0.06812321  0.06837919
##  [6,]  0.12179418  0.12087521  0.11999554  0.11915360  0.11834785  0.11757681
##  [7,] -0.12451427 -0.12454411 -0.12457302 -0.12460098 -0.12462801 -0.12465413
##  [8,] -0.13681594 -0.13718688 -0.13754502 -0.13789066 -0.13822408 -0.13854559
##  [9,] -0.11692507 -0.11704972 -0.11716785 -0.11727984 -0.11738608 -0.11748688
## [10,] -0.01735418 -0.01761890 -0.01787090 -0.01811082 -0.01833927 -0.01855685
##             [,80]       [,81]       [,82]       [,83]       [,84]       [,85]
##  [1,] -0.85779705 -0.85856906 -0.85930748 -0.86001382 -0.86068950 -0.86133588
##  [2,]  0.60861965  0.60908317  0.60952799  0.60995480  0.61036425  0.61075699
##  [3,]  0.88032636  0.88032498  0.88032182  0.88031710  0.88031101  0.88030373
##  [4,] -0.05449534 -0.05423963 -0.05399460 -0.05375972 -0.05353450 -0.05331851
##  [5,]  0.06862174  0.06885058  0.06906672  0.06927103  0.06946431  0.06964730
##  [6,]  0.11683861  0.11613125  0.11545350  0.11480419  0.11418216  0.11358633
##  [7,] -0.12467919 -0.12470302 -0.12472571 -0.12474734 -0.12476796 -0.12478762
##  [8,] -0.13885583 -0.13915560 -0.13944519 -0.13972486 -0.13999486 -0.14025546
##  [9,] -0.11758185 -0.11767027 -0.11775274 -0.11782977 -0.11790183 -0.11796934
## [10,] -0.01876389 -0.01896067 -0.01914772 -0.01932557 -0.01949470 -0.01965559
##             [,86]       [,87]       [,88]       [,89]       [,90]       [,91]
##  [1,] -0.86195426 -0.86254588 -0.86311193 -0.86365354 -0.86417178 -0.86466769
##  [2,]  0.61113366  0.61149486  0.61184121  0.61217327  0.61249161  0.61279678
##  [3,]  0.88029544  0.88028628  0.88027637  0.88026586  0.88025485  0.88024343
##  [4,] -0.05311131 -0.05291254 -0.05272181 -0.05253880 -0.05236318 -0.05219465
##  [5,]  0.06982065  0.06998497  0.07014082  0.07028871  0.07042911  0.07056245
##  [6,]  0.11301560  0.11246896  0.11194540  0.11144397  0.11096375  0.11050385
##  [7,] -0.12480638 -0.12482428 -0.12484135 -0.12485764 -0.12487318 -0.12488799
##  [8,] -0.14050690 -0.14074944 -0.14098333 -0.14120881 -0.14142612 -0.14163551
##  [9,] -0.11803266 -0.11809213 -0.11814805 -0.11820068 -0.11825027 -0.11829705
## [10,] -0.01980866 -0.01995434 -0.02009301 -0.02022504 -0.02035077 -0.02047053
##             [,92]       [,93]       [,94]       [,95]       [,96]       [,97]
##  [1,] -0.86514225 -0.86559639 -0.86603100 -0.86644695 -0.86684506 -0.86722609
##  [2,]  0.61308929  0.61336966  0.61363838  0.61389592  0.61414273  0.61437925
##  [3,]  0.88023170  0.88021973  0.88020760  0.88019538  0.88018311  0.88017084
##  [4,] -0.05203290 -0.05187768 -0.05172872 -0.05158577 -0.05144858 -0.05131693
```

```
##  [5,]  0.07068916  0.07080959  0.07092410  0.07103303  0.07113666  0.07123530
##  [6,]  0.11006342  0.10964165  0.10923774  0.10885095  0.10848055  0.10812585
##  [7,] -0.12490212 -0.12491559 -0.12492842 -0.12494065 -0.12495231 -0.12496341
##  [8,] -0.14183722 -0.14203147 -0.14221850 -0.14239854 -0.14257180 -0.14273852
##  [9,] -0.11834121 -0.11838294 -0.11842239 -0.11845973 -0.11849508 -0.11852858
## [10,] -0.02058462 -0.02069335 -0.02079697 -0.02089574 -0.02098991 -0.02107972
##             [,98]       [,99]      [,100]      [,101]      [,102]      [,103]
##  [1,] -0.86759079 -0.86793988 -0.86827403 -0.86859389 -0.86890009 -0.86919320
##  [2,]  0.61460590  0.61482308  0.61503118  0.61523059  0.61542164  0.61560470
##  [3,]  0.88015862  0.88014649  0.88013448  0.88012262  0.88011093  0.88009943
##  [4,] -0.05119061 -0.05106939 -0.05095308 -0.05084148 -0.05073441 -0.05063170
##  [5,]  0.07132920  0.07141862  0.07150379  0.07158494  0.07166226  0.07173597
##  [6,]  0.10778619  0.10746094  0.10714947  0.10685121  0.10656560  0.10629210
##  [7,] -0.12497398 -0.12498404 -0.12499363 -0.12500275 -0.12501143 -0.12501969
##  [8,] -0.14289890 -0.14305315 -0.14320149 -0.14334411 -0.14348122 -0.14361299
##  [9,] -0.11856035 -0.11859048 -0.11861909 -0.11864626 -0.11867207 -0.11869660
## [10,] -0.02116536 -0.02124706 -0.02132499 -0.02139936 -0.02147032 -0.02153805
##            [,104]      [,105]      [,106]      [,107]      [,108]      [,109]
##  [1,] -0.86947381 -0.86974244 -0.86999962 -0.87024585 -0.87048158 -0.87070728
##  [2,]  0.61578009  0.61594813  0.61610912  0.61626336  0.61641113  0.61655269
##  [3,]  0.88008814  0.88007708  0.88006625  0.88005567  0.88004534  0.88003527
##  [4,] -0.05053315 -0.05043862 -0.05034794 -0.05026097 -0.05017754 -0.05009753
##  [5,]  0.07180623  0.07187323  0.07193712  0.07199807  0.07205621  0.07211170
##  [6,]  0.10603020  0.10577940  0.10553924  0.10530925  0.10508902  0.10487813
##  [7,] -0.12502756 -0.12503504 -0.12504216 -0.12504893 -0.12505537 -0.12506150
##  [8,] -0.14373962 -0.14386130 -0.14397819 -0.14409048 -0.14419833 -0.14430190
##  [9,] -0.11871993 -0.11874212 -0.11876325 -0.11878335 -0.11880250 -0.11882074
## [10,] -0.02160270 -0.02166442 -0.02172334 -0.02177960 -0.02183333 -0.02188465
##            [,110]      [,111]      [,112]      [,113]      [,114]      [,115]
##  [1,] -0.87092337 -0.87113027 -0.87132837 -0.87151806 -0.87169968 -0.87187359
##  [2,]  0.61668831  0.61681823  0.61694269  0.61706192  0.61717614  0.61728556
##  [3,]  0.88002546  0.88001592  0.88000665  0.87999764  0.87998890  0.87998042
##  [4,] -0.05002079 -0.04994720 -0.04987663 -0.04980895 -0.04974406 -0.04968184
##  [5,]  0.07216464  0.07221518  0.07226342  0.07230948  0.07235346  0.07239546
##  [6,]  0.10467617  0.10448277  0.10429757  0.10412022  0.10395039  0.10378775
##  [7,] -0.12506733 -0.12507288 -0.12507815 -0.12508317 -0.12508794 -0.12509248
##  [8,] -0.14440135 -0.14449683 -0.14458850 -0.14467648 -0.14476094 -0.14484199
##  [9,] -0.11883811 -0.11885467 -0.11887046 -0.11888551 -0.11889987 -0.11891357
## [10,] -0.02193366 -0.02198048 -0.02202521 -0.02206794 -0.02210878 -0.02214780
##            [,116]      [,117]      [,118]      [,119]      [,120]      [,121]
##  [1,] -0.87204011 -0.87219957 -0.87235226 -0.87249847 -0.87263849 -0.87277256
##  [2,]  0.61739037  0.61749077  0.61758695  0.61767909  0.61776734  0.61785188
##  [3,]  0.87997221  0.87996425  0.87995655  0.87994911  0.87994190  0.87993494
##  [4,] -0.04962218 -0.04956498 -0.04951014 -0.04945756 -0.04940716 -0.04935884
##  [5,]  0.07243558  0.07247390  0.07251050  0.07254548  0.07257890  0.07261083
##  [6,]  0.10363200  0.10348285  0.10334002  0.10320323  0.10307225  0.10294681
##  [7,] -0.12509680 -0.12510091 -0.12510482 -0.12510854 -0.12511207 -0.12511544
##  [8,] -0.14491977 -0.14499440 -0.14506601 -0.14513472 -0.14520062 -0.14526385
##  [9,] -0.11892663 -0.11893910 -0.11895100 -0.11896236 -0.11897321 -0.11898356
## [10,] -0.02218509 -0.02222073 -0.02225479 -0.02228735 -0.02231848 -0.02234824
##            [,122]      [,123]      [,124]      [,125]      [,126]      [,127]
##  [1,] -0.87290095 -0.87302391 -0.87314165 -0.87325440 -0.87336238 -0.87346579
##  [2,]  0.61793286  0.61801043  0.61808473  0.61815590  0.61822408  0.61828938
##  [3,]  0.87992822  0.87992173  0.87991547  0.87990942  0.87990359  0.87989797
```

```
##  [4,] -0.04931253 -0.04926813 -0.04922557 -0.04918478 -0.04914568 -0.04910821
##  [5,]  0.07264135  0.07267052  0.07269841  0.07272506  0.07275055  0.07277491
##  [6,]  0.10282668  0.10271163  0.10260146  0.10249596  0.10239492  0.10229815
##  [7,] -0.12511864 -0.12512168 -0.12512458 -0.12512734 -0.12512997 -0.12513246
##  [8,] -0.14532449 -0.14538265 -0.14543844 -0.14549194 -0.14554324 -0.14559243
##  [9,] -0.11899345 -0.11900290 -0.11901193 -0.11902055 -0.11902878 -0.11903666
## [10,] -0.02237669 -0.02240389 -0.02242990 -0.02245477 -0.02247855 -0.02250130
```

```
# From the xmat, we could see that the coefficients are not converging either.
# They are oscillating between positive and negative values.
```

7.

```
sparse.grad.descent <- function(f, x1, max.iter = 200, step.size = 0.001, stopping.deriv = 0.1){
  n <- length(x1)
  xmat <- matrix(0, n, max.iter)
  xmat[, 1] <- x1
  for (i in 2:max.iter){
    gradient <- grad(f, xmat[, i-1])

    if(all(abs(gradient) < stopping.deriv)){
      i <- i - 1
      break
    }
    new <- xmat[, i-1] - step.size * gradient
    new[abs(new) < 0.05] <- 0
    xmat[, i] <- new
  }
  xmat <- xmat[, 1:i]
  return(list(x = xmat[, i], xmat = xmat, i = i))
}
gd.sparse <- sparse.grad.descent(huber.loss, x1 = rep(0, p))
gd.sparse$x
```

```
##  [1] -0.8944804  0.6332991  0.8823860  0.0000000  0.0000000  0.0000000
##  [7]  0.0000000  0.0000000  0.0000000  0.0000000
```

8.

```
for (i in 1:10){
  assign(paste("X", i, sep = ""), x[, i])
}
fit <- lm(y ~ X1 + X2 + X3 + X4 + X5+ X6 + X7 + X8 + X9 + X10 + 0)
fit$coefficients
```

```
##            X1            X2            X3            X4            X5
## -0.9477210986  0.4864220270  0.5875664655 -0.7416200316  0.0008874065
##            X6            X7            X8            X9           X10
##  0.3149846567 -0.3994729398 -0.2712937636 -0.1445449407  0.0788007924
```

```
mean((b - fit$coefficients)^2)
```

```
## [1] 0.1186581
```

```
mean((b - gd$x)^2)
```

```
## [1] 0.01208955
```

```r
mean((b - gd.sparse$x)^2)
```

```
## [1] 0.005610471
```

```r
# From the results, we could see that gd.sparse is the best.
```

9.

```r
set.seed(10)
y <- x %*% b + rt(n, df = 2)
gd <- grad.descent(huber.loss, x1 = rep(0, p))
gd.sparse <- sparse.grad.descent(huber.loss, x1 = rep(0, p))
gd$x
```

```
##  [1] -0.46329748  0.92390614  0.92287242 -0.06526259  0.24633002 -0.04406371
##  [7]  0.01858892 -0.18921630  0.19479185 -0.18395820
```

```r
gd.sparse$x
```

```
##  [1] 0.0000000 0.7850744 0.9398727 0.0000000 0.0000000 0.0000000 0.0000000
##  [8] 0.0000000 0.0000000 0.0000000
```

```r
mean((b - gd$x)^2)
```

```
## [1] 0.02869228
```

```r
mean((b - gd.sparse$x)^2)
```

```
## [1] 0.0500853
```

```r
# Gradient descent has a smaller mean squared error.
# We can deduce that the variability of the sparse method is higher than the regular method.
```

10.

```r
gdmean <- c()
gdsmean <- c()
for (i in 1:10){
  y <- x %*% b + rt(n, df = 2)
  gd <- grad.descent(huber.loss, x1 = rep(0, p))
  gd.sparse <- sparse.grad.descent(huber.loss, x1 = rep(0, p))
  gdmean[i] <- mean((b - gd$x)^2)
  gdsmean[i] <- mean((b - gd.sparse$x)^2)
}
summary(gdmean)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.01431 0.01824 0.02283 0.02495 0.02692 0.04358
```

```r
summary(gdsmean)
```

```
##      Min.   1st Qu.    Median      Mean   3rd Qu.      Max.
## 0.0006265 0.0044639 0.0061948 0.0265082 0.0532420 0.0795591
```

```r
# Gradient descent has a lower average mean squared error.
# Sparsified gradient descent has a lower minimum mean squared error.
# This is in line with our interpretation of the variability in the previous question.
```