Krusell and Smith (1998) with endogenous labor supply, investment irreversibility, and fiscal spending shock

Recursive formulation

The recursive formulation of a household's problem is as follows:

$$V(a,z;X) = \max_{c,n,a'} \; log(c) - \frac{\eta}{1+\frac{1}{\chi}} n^{1+\frac{1}{\chi}} + \beta \mathbb{E} V(a',z';X') \tag{1}$$

$$\text{s.t.} \quad c + a' = (1+r(X))a + w(X)zn - T(X) \tag{2}$$

$$a' - (1-\delta)a \geq \phi I^{ss} \tag{3}$$

$$X' = \Gamma_X(X) \quad \text{(Aggregate law of motion)} \tag{4}$$

$$z' \sim \pi(z'|z) \tag{5}$$

where V is the value function of a household; r and w are capital rent and wage that are determined at the competitive input factor markets. I_{ss} is the steady-state aggregate saving (investment) level. T is the lump-sum tax. χ is the Frisch elasticity parameter, and η is the labor disutility parameter. ϕ is the parameter that governs the degree of the saving irreversibility. Γ_X is the aggregate law of motion. The idiosyncratic productivity z follows a Markov process, where $\pi(z'|z)$ governs the transition probability.

We consider a production sector that operates using a CRS Cobb-Douglas production function:

$$\max_{K,L} \; AK^{\alpha}L^{1-\alpha} - w(X)L - (r(X)+\delta)K, \tag{6}$$

where A is the aggregate TFP, K and L are capital and labor input demands.

The aggregate state X includes following three components:

$$X = \{\Phi, A, G\}. \tag{7}$$

where Φ is the distribution of the individual states, A is TFP, and G is government demand. The first is endogenous aggregate state, and the others follow exogenous log AR(1) processes specified as follows:

$$log(A') = \rho_A log(A) + \sigma_A \epsilon \quad \epsilon \sim_{iid} N(0,1) \tag{8}$$

$$log(G') = (1-\rho_G) log(\overline{G}) + \rho_G log(G) + \sigma_G \epsilon \quad \epsilon \sim_{iid} N(0,1) \tag{9}$$

where $\overline{G}$ is the steady-state government demand. For $j \in \{A, G\}$, ρ_j is the persistence parameter for the exogenous processes, and σ_j is the volatility parameter. These processes are discretized by the Tauchen method in the computation. I assume the simplest government setup where the budget is balanced by lump-sum tax collection: $T(X) = G$. By assuming this, the symmetric lump-sum tax is collected from heterogeneous households. For computation, I use the standard parameter levels in the literature, which are available in Appendix C.

The recursive competitive equilibrium is defined based on the following market-clearing conditions:

$$\text{(Labor market)} \quad L(X) = \int zn(a,z;X) d\Phi \tag{10}$$

$$\text{(Capital market)} \quad K(X) = \int a d\Phi. \tag{11}$$