An RBC model with heterogeneous firms and irreversible investments

Each firm with an individual state (k,z) solves the following problem:

$$
\begin{aligned}
J(k,z;X) &= \max_{k'} \pi(k,z;X) + (1-\delta)k - k' + \mathbb{E}_{z,X} M(X,X') J(k',z';X') \\
&\text{s.t. } k' \geq \phi I_{ss} + (1-\delta)k \\
&\pi(k,z;X) = \max_{k,n} \ Ak^{\alpha} n^{\gamma} - w(X)n
\end{aligned}
$$

where k is individual capital stock; z is the idiosyncratic productivity; n is the labor demand; I_{ss} is the steady-state aggregate investment level.

The household-side problem is as follows:

$$
\begin{aligned}
V(a;X) &= \max_{c,a',N} \ log(c) - \eta N + \beta \mathbb{E} V(a';X') \\
\text{s.t.} \quad & c + \int a'(X') q(X,X') d\Gamma_{X'} = a + w(X)N
\end{aligned}
$$

where q is the stochastic discount factor; a' is the future equity portfolio.

The stochastic discount factor $M(X,X')$ and wage $w(X)$ is determined at the competitive market:

$$
\begin{aligned}
[M]: &\quad a(X) = J(k(X);X) \\
[w]: &\quad N(X) = \int n(k,z;X) d\Phi(k,z)
\end{aligned}
$$

The first-order condition of firm's problem is as follows:

$$
1 = \mathbb{E}_{z,X} M(X,X') J_1(k',z';X') + \lambda(k,z;X)
$$

where λ is the Lagrange multiplier. The envelope condition of firm's problem is as follows:

$$\begin{aligned} J_1(k, z; X) &= \pi_1(k, z; X) + (1-\delta) - \lambda(k, z; X)(1-\delta) \\ &= \pi_1(k, z; X) + (1-\delta)(1 - \lambda(k, z; X)) \end{aligned}$$

Combining the two conditions above, we have

$$1 = \mathbb{E}_{z,X} M(X, X')(\pi_1(k', z'; X') + (1-\delta)(1 - \lambda(k', z'; X'))) + \lambda(k, z; X),$$

which pins down the optimal future capital stock $k'(k, z, ; X)$.