A canonical DMP model with exogenous separation

Household A representative household composed of a unit measure of a homogeneous labor force is considered. The employed labor force earns wage w that is bilaterally determined, and an unemployed labor force engages in home production $b > 0$. From the national identity (resource constraint), the produced output after the wasted total vacancy posting cost.

$$c(S) = Y(S) - \kappa v(S) = n(S)z - \kappa v(S)$$

where Y is aggregate output, n is the employment, z is aggregate TFP, $\kappa > 0$ is the vacancy posting cost, and v is the vacancy. We assume a temporal CRRA utility with risk aversion $\sigma > 0$ and discount factor $\beta \in (0, 1)$.

Aggregate states The aggregate state S is as follows:

$$S = \{z, n_{-1}\}$$

where z is aggregate TFP, and n_{-1} is the past employment level. The aggregate productivity follows a log AR(1) process.

Matching function A Cobb-Douglas matching function M is considered:

$$M(u, v) = mu^{\alpha}v^{1-\alpha}$$

where $u = 1 - n$ is unemployment level, v is the total vacancy, and m is the matching efficiency.

Unemployment/employment dynamics The employment level evolves in the following dynamics:

$$n = (1-\lambda)n_{-1} + vq$$

where $q = \frac{M}{v} = m\theta^{-\xi}$ is the vacancy-filling probability. $\theta = u/v$ is the market tightness. $\lambda \in (0,1)$ is the exogenous job separation rate. Due to the Cobb-Douglass specification, q can sometimes take a value greater value than unity over the business cycle. We truncate q at unity. The repeated transition method is not confined by a specific form of the matching function, but the Cobb-Douglass specification is considered for illustrative purposes.

Equilibrium condition The standard vacancy posting condition leads to the following equilibrium condition:

$$\frac{\kappa}{q(S)} = (1-\lambda)\beta\mathbb{E}\left[\left(\frac{c(S)}{c(S')}\right)^{\sigma}\left(z' - w(S') + \frac{\kappa}{q(S')}\right)\right]$$

where the wage $w = w(S)$ is determined by the following Nash bargaining outcome:

$$w(S) = (1-\eta)b + \eta(A + \kappa\theta(S)).$$

$b > 0$ is the home production, and $\eta \in (0,1)$ is the bargaining weight for household.