An RBC model with irreversible investment

Recursive formulation

The representative household solves the following problem:

$$V(a;S) = \max_{c,a'} \frac{c^{1-\sigma}}{1-\sigma} + \beta \mathbb{E} V(a';S')$$

$$\text{s.t. } c + a' - (1-\delta)a = Aa^{\alpha}$$

$$a' - (1-\delta)a \geq \phi I_{ss}$$

where I_{ss} is the steady-state investment level. The aggregate state S is as follows

$$S = [K, A].$$

K is the aggregate capital stock. A is TFP that follows the log AR(1) process:

$$log(A') = \rho log(A) + \sigma\epsilon, \quad \sigma \sim N(0,1).$$

c is consumption, a is the wealth in the beginning of a period. ϕ is the parameter that governs the degree of the partial irreversibility.

Optimality conditions

The Largrangian is as follows:

$$\mathcal{L} = \frac{c^{1-\sigma}}{1-\sigma} + \beta \mathbb{E} V(a';S') + \mu(Aa^{\alpha} + (1-\delta)a - c - a') + \lambda(a' - (1-\delta)a - \phi I_{ss}),$$

where λ is the Lagrange multiplier. The first-order optimality conditions are as follows:

$$\begin{aligned}
[c]: &\quad c^{-\sigma} = \mu \\
[a']: &\quad \mu = \beta \mathbb{E} V_1(a'; S') + \lambda \\
[a]: &\quad V_1(a; S) = \mu(\alpha A a^{\alpha-1} + (1-\delta)) - (1-\delta)\lambda
\end{aligned}$$

Then, we obtain

$$c^{-\sigma} - \lambda = \beta \mathbb{E}\left[(c')^{-\sigma}(\alpha A'(a')^{\alpha-1} + (1-\delta)) - (1-\delta)\lambda'\right].$$