An RBC model with asset price

A represntative firm solves the following problem:

$$J(k;S) = \max_{k'} Ak^{\alpha} - k' + (1-\delta)k + \beta \mathbb{E} M(S,S')J(k';S')$$

The household-side problem is as follows:

$$\begin{aligned} V(a;S) &= \max_{c,a'} \; log(c) + \beta \mathbb{E} V(a';S') \\ \text{s.t.} \quad & c + \int q(S,S')a'(S')d\Gamma_{S'} = a \end{aligned}$$

The stochastic discount factor $M(S,S')$ is determined at the competitive market:

$$[M]: \quad a(S) = J(k(S);S)$$