The model of Krusell and Smith (1998) with endogenous labor supply and convex adjustment cost

Recursive formulation

The heterogeneous household's problem is as follows:

$$
\begin{aligned}
V(a,z;S) = \max_{c,l,a'} \; & log(c) - \frac{\eta}{1+\frac{1}{\chi}} l^{1+\frac{1}{\chi}} + \beta \mathbb{E} V(a',z';S') \\
\text{s.t.} \quad & c + a' + \Psi(a',a) = (1+r(S))a + w(S)zl \\
& a' \geq 0 \\
& S' = \Gamma_S(S) \quad \text{(Aggregate law of motion)} \\
& z' \sim \pi(z'|z) \quad \text{(Idiosyncratic productivity)}
\end{aligned}
$$

where $S = \{\Phi, A\}$ is the aggregate state. c is consumption, a is the wealth in the beginning of a period. The adjustment cost occurs for any wealth adjustment: $\Psi(a',a) = \frac{\mu}{2}\left(\frac{a'-a}{a}\right)^2 a$.

The production side is as follows:

$$
\max_{K,L} AK^{\alpha}L^{1-\alpha} - w(S)L - (r(S)+\delta)K
$$

The capital rent $r(S)$ and the wage $w(S)$ are determined at the competitive market:

$$
\begin{aligned}
[r]: \quad & \int a'(a,z;S) d\Phi(S) = K'(S) \\
[w]: \quad & \int zl(a,z;S) d\Phi(S) = L(S)
\end{aligned}
$$