An RBC model with endogenous labor supply and irreversible investment

Recursive formulation

The representative household solves the following problem:

$$
\begin{aligned}
V(a;S) = \max_{c,n,a'} \ & \frac{c^{1-\sigma}}{1-\sigma} - \frac{\eta}{1+\frac{1}{\chi}} n^{1+\frac{1}{\chi}} + \beta \mathbb{E} V(a';S') \\
\text{s.t. } & c + a' = a(1+r(S)) + w(S)n - T \\
& a' - (1-\delta)a \geq \phi I_{ss}
\end{aligned}
$$

where I_{ss} is the steady-state saving (investment) level. c is consumption, a is the wealth in the beginning of a period. N is the labor supply. T is the lump-sum subsidy. $\sigma > 0$ is the risk aversion parameter, and χ is the Frisch elasticity. ϕ is the parameter that governs the degree of the partial irreversibility.

We consider a production sector that operates using a CRS Cobb-Douglas production function:

$$
\max_{K,L} \ AK^{\alpha}L^{1-\alpha} - w(S)L - (r(S)+\delta)K,
$$

where A is the aggregate TFP, K and L are capital and labor input demands. The capital rent $r(S)$ and the wage $w(S)$ are determined at the competitive factor market by clearing conditions.

The aggregate state S is as follows

$$
S = \{K, A, G\}.
$$

K is the aggregate capital stock. A is TFP and G is government demand that follow log

AR(1) processes:

$$log(A') = \rho_A log(A) + \sigma_A \epsilon \quad \epsilon \sim_{iid} N(0,1)$$
$$log(G') = (1-\rho_G) log(\overline{G}) + \rho_G log(G) + \sigma_G \epsilon \quad \epsilon \sim_{iid} N(0,1)$$

where $\overline{G}$ is the average government demand level. Government budget is balanced by $T = G$ in each period.

Optimality conditions

The Largrangian is as follows:

$$\mathcal{L} = \frac{c^{1-\sigma}}{1-\sigma} - \frac{\eta}{1+\frac{1}{\chi}} L^{1+\frac{1}{\chi}} + \beta \mathbb{E} V(a'; S') + \mu(a(1+r(S)) + w(S)n - T - c - a')$$
$$+ \lambda(a' - (1-\delta)a - \phi I_{ss}),$$

where λ is the Lagrange multiplier. The first-order optimality conditions are as follows:

$$[c]: \quad c^{-\sigma} = \mu$$
$$[a']: \quad \mu = \beta \mathbb{E} V_1(a'; S') + \lambda$$
$$[N]: \quad N = \left(\frac{w}{\eta c^{\sigma}}\right)^{\chi}$$
$$[a]: \quad V_1(a; S) = \mu(1+r(S)) - (1-\delta)\lambda$$

Then, we obtain

$$c^{-\sigma} - \lambda = \beta \mathbb{E}\left[(c')^{-\sigma}(1+r(S)) - (1-\delta)\lambda'\right].$$