An RBC model with heterogeneous firms and irreversible investments

Each firm with an individual state (k, z) solves the following problem:

$$
\begin{aligned}
J(k, z; X) &= \max_{k'} \pi(k, z; X) + (1-\delta)k - k' + \mathbb{E}_{z,X} M(X, X') J(k', z'; X') \\
&\text{s.t. } k' \geq \phi I_{ss} + (1-\delta)k \\
&\pi(k, z; X) = \max_{k,n} \; A k^{\alpha} n^{\gamma} - w(X) n
\end{aligned}
$$

where k is individual capital stock; z is the idiosyncratic productivity; n is the labor demand; I_{ss} is the steady-state aggregate investment level.

The household-side problem is as follows:

$$
\begin{aligned}
V(a; X) &= \max_{c, a', N} \; log(c) - \eta N + \beta \mathbb{E} V(a'; X') \\
\text{s.t.} \quad & c + \int a'(X') q(X') d\Gamma_{X'} = a + w(X) N
\end{aligned}
$$

where q is the Arrow-Debreu state price.

The stochastic discount factor $M(X, X')$ and wage $w(X)$ is determined at the competitive market:

$$
\begin{aligned}
[M]: \quad & a(X) = J(k(X); X) \\
[w]: \quad & N(X) = \int n(k, z; X) d\Phi(k, z)
\end{aligned}
$$

The first-order condition of firm's problem is as follows:

$$
1 = \mathbb{E}_{z,X} M(X, X') J_1(k', z'; X') + \lambda(k, z; X)
$$

where λ is the Lagrange multiplier. The envelope condition of firm's problem is as follows:

$$\begin{aligned} J_1(k,z;X) &= \pi_1(k,z;X) + (1-\delta) - \lambda(k,z;X)(1-\delta) \\ &= \pi_1(k,z;X) + (1-\delta)(1-\lambda(k,z;X)) \end{aligned}$$

Combining the two conditions above, we have

$$1 = \mathbb{E}_{z,X} M(X,X')(\pi_1(k',z';X') + (1-\delta)(1-\lambda(k',z';X'))) + \lambda(k,z;X),$$

which pins down the optimal future capital stock $k'(k,z,;X)$.