The model of Aiyagari (1994)

Recursive formulation

A continuum of measure one of households is considered, and they live forever in discrete time. Each household consumes, and saves. The labor endowment z is exogenously determined by an $AR(1)$ process. The recursive formulation of the household's problem is as follows:

$$\begin{aligned}
v(a,z;\Phi) = \max_{c,a'} \quad & log(c) + \beta \mathbb{E}_z v(a',z';\Phi) \\
\text{s.t.} \quad & c + a' = w(\Phi)z + a(1+r(\Phi)) \\
& a' \geq \underline{a} = 0 \\
& log(z') = \rho log(z) + \sigma\sqrt{1-\rho^2}\epsilon, \quad \epsilon \sim N(0,1)
\end{aligned}$$

where apostrophe indicates future allocations. Φ is the joint distribution of the individual states (a,z). The borrowing limit $\underline{a}$ is given as 0. The prices $w(\Phi)$ and $r(\Phi)$ are determined at the competitive labor and capital input markets. Now we consider a production sector that operates using the CRS Cobb-Douglas production function:

$$\max_{K,L} AK^{\alpha}L^{1-\alpha} - (r(\Phi)+\delta)K - w(\Phi)L$$

The aggregate TFP $A = 1$ fixed. The competitive input markets are cleared at the prices (w,r):

$$\begin{array}{rcc}
 & \underline{\text{Supply}} & \underline{\text{Demand}} \\
\text{[Capital market]} & \int a d\Phi = & K \\
\text{[Labor market]} & \int z d\Phi = & L
\end{array}$$

The parameters levels are set as in Aiyagari (1994), as follows:

$$\rho = 0.9, \quad \sigma = 0.2, \quad \alpha = 0.36, \quad \beta = 0.96, \quad \delta = 0.08.$$

The idiosyncratic labor endowment process is discretized by Tauchen method using 7 grid points covering ± 3 standard-deviation range.