

An RBC model with irreversible investment

Recursive formulation

The representative household solves the following problem:

$$\begin{aligned} V(a; S) &= \max_{c, a'} \frac{c^{1-\sigma}}{1-\sigma} + \beta \mathbb{E} V(a'; S') \\ \text{s.t. } c + a' - (1 - \delta)a &= Aa^\alpha \\ a' - (1 - \delta)a &\geq \phi I_{ss} \end{aligned}$$

where I_{ss} is the steady-state investment level. The aggregate state S is as follows

$$S = [K, A].$$

K is the aggregate capital stock. A is TFP that follows the log AR(1) process:

$$\log(A') = \rho \log(A) + \sigma \epsilon, \quad \sigma \sim N(0, 1).$$

c is consumption, a is the wealth in the beginning of a period. ϕ is the parameter that governs the degree of the partial irreversibility.

Optimality conditions

The Lagrangian is as follows:

$$\mathcal{L} = \frac{c^{1-\sigma}}{1-\sigma} + \beta \mathbb{E} V(a'; S') + \mu (Aa^\alpha + (1 - \delta)a - c - a') + \lambda (a' - (1 - \delta)a - \phi I_{ss}),$$

where λ is the Lagrange multiplier. The first-order optimality conditions are as follows:

$$[c] : \quad c^{-\sigma} = \mu$$

$$[a'] : \quad \mu = \beta \mathbb{E} V_1(a'; S') + \lambda$$

$$[a] : \quad V_1(a; S) = \mu(\alpha A a^{\alpha-1} + (1 - \delta)) - (1 - \delta)\lambda$$

Then, we obtain

$$c^{-\sigma} - \lambda = \beta \mathbb{E} \left[(c')^{-\sigma} (\alpha A' (a')^{\alpha-1} + (1 - \delta)) - (1 - \delta)\lambda' \right].$$