# An RBC model with irreversible investment

The social planner solves the following problem:

$$V(a;X) = \max_{c,a'} \frac{c^{1-\sigma}}{1-\sigma} + \beta \mathbb{E} V(a';X')$$

$$\text{s.t. } c + a' - (1-\delta)a = Aa^{\alpha}$$

$$a' - (1-\delta)a \geq \phi I_{ss}$$

where $V$ is the value function of a household. The value function's arguments are wealth $a$ and the aggregate state $X$. $c$ is consumption and $\sigma$ is the risk-aversion parameter. $I_{ss}$ is the steady-state investment level. $\phi$ is the parameter that governs the degree of the irreversibility. $\delta$ is the depreciation rate, and $\alpha$ is the capital share in the production function $F = Aa^{\alpha}$. An apostrophe indicates a future allocation. The aggregate state $X$ is as follows

$$X = [K, A].$$

$K$ is the aggregate capital stock, satisfying $a = K$ immediately in the solution from the social planner's perspective. $A$ is TFP that follows the log AR(1) process:

$$log(A') = \rho log(A) + \sigma\epsilon, \quad \sigma \sim N(0,1).$$

## Optimality conditions

The Largrangian is as follows:

$$\mathcal{L} = \frac{c^{1-\sigma}}{1-\sigma} + \beta \mathbb{E} V(a';S') + \mu(Aa^{\alpha} + (1-\delta)a - c - a') + \lambda(a' - (1-\delta)a - \phi I_{ss}),$$

where $\lambda$ is the Lagrange multiplier. The first-order optimality conditions are as follows:

$$\begin{aligned}
[c]: &\quad c^{-\sigma} = \mu \\
[a']: &\quad \mu = \beta \mathbb{E} V_1(a'; S') + \lambda \\
[a]: &\quad V_1(a; S) = \mu(\alpha A a^{\alpha-1} + (1-\delta)) - (1-\delta)\lambda
\end{aligned}$$

Then, we obtain

$$c^{-\sigma} - \lambda = \beta \mathbb{E}\left[(c')^{-\sigma}(\alpha A'(a')^{\alpha-1} + (1-\delta)) - (1-\delta)\lambda'\right].$$