The model of Khan and Thomas (2008) with convex adjustment cost

Recursive formulation

Firm-side problem is as follows:

$$
\begin{aligned}
J(k,z;S) &= \pi(k,z;S) + (1-\delta)k + \int_0^{\bar{\xi}} \max\{R^*(k,z;S) - \xi w(S), R^c(k,z;S)\} \left(\frac{1}{\bar{\xi}}\right) d\xi \\
R^*(k,z;S) &= \max_{k'} -\gamma k' - \frac{\mu}{2}\left(\frac{k'}{k} - (1-\delta)\right)^2 k + \mathbb{E}_{z,S} M(S,S') J(k',z';S') \\
R^c(k,z;S) &= \max_{k^c \in [-\nu k, \nu k]} -\gamma k^c - \frac{\mu}{2}\left(\frac{k^c}{k} - (1-\delta)\right)^2 k + \mathbb{E}_{z,S} M(S,S') J(k^c,z';S') \\
\pi(k,z;S) &= \max_{L} Azk^\alpha L^\theta - w(S)L
\end{aligned}
$$

where γ is the aggregate growth rate adjuster as in Khan and Thomas (2008).

The household-side problem is as follows:

$$
\begin{aligned}
V(a;S) &= \max_{c,N,a'} \; log(c) - \eta N + \beta \mathbb{E} V(a';S') \\
\text{s.t.} \quad c &+ \int a'(S') d\Gamma_{S'} = a + w(S)N
\end{aligned}
$$

The stochastic discount factor $M(S,S')$ and the wage $w(S)$ are determined at the competitive market:

$$
\begin{aligned}
[M]: &\quad a(S) = \int J(k,z;S) d\Phi(k,z) \\
[w]: &\quad N(S) = \int \left(L(k,z;S) + \frac{\xi^*(k,z)^2}{2\bar{\xi}}\right) d\Phi(k,z)
\end{aligned}
$$

where $\xi^*(k, z; S)$ is the threshold rule of lumpy investment defined as follows:

$$\xi^*(k, z; S) = \frac{1}{w(S)} \left(R^*(k, z; S) - R^c(k, z; S) \right)$$