The Spender of Last Resort: Global Equilibrium Dynamics under the Zero Lower Bound[†]

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Abstract

This paper develops a framework for a fiscally-backed zero lower bound (ZLB) that supports a globally well-defined recursive competitive equilibrium. We show that a state-contingent fiscal rule neutralizes liquidity traps, enabling unified short- and long-run equilibrium analysis. The model endogenously generates a concave Phillips curve that flattens during ZLB episodes, matching empirical evidence. While this fiscal anchor stabilizes inflation, it transfers volatility to the real economy, causing unemployment volatility to surge. Finally, we document a policy substitution effect: fiscal multipliers diminish at the ZLB because discretionary stimulus largely crowds out the endogenous fiscal support required to maintain the equilibrium.

Keywords: New Keynesian model, zero lower bound, monetary-fiscal interactions, Phillips curve, state dependence.

JEL codes: E32, E31, E52

1 Introduction

The zero lower bound (ZLB) on nominal interest rates has ceased to be a mere theoretical curiosity and has become a persistent feature of the global macroeconomic landscape. Standard New Keynesian models, however, face a fundamental difficulty in analyzing this regime: without specific stabilization mechanisms, the ZLB often generates global indeterminacy or explosive deflationary spirals, rendering the equilibrium mathematically ill-defined or non-ergodic. This paper develops a theoretical and quantitative framework to resolve this instability. We introduce a *fiscally-backed ZLB*—a regime where a state-contingent fiscal rule acts as a "spender of last resort"—to support a globally well-defined Recursive Competitive Equilibrium (RCE). By anchoring long-run expectations through an implicit fiscal guarantee, we neutralize the liquidity trap while preserving the nonlinear dynamics of the ZLB, enabling a unified analysis of short-run fluctuations and long-run stability within a single model.

This paper makes three distinct contributions to the literature on monetary-fiscal interactions and nonlinear business cycles. First, we provide a methodological contribution by constructing a recursive fiscal rule that restores global equilibrium determinacy. We show that the standard New Keynesian model with an occasionally binding ZLB is theoretically fragile; it lacks a stationary distribution for the endogenous state variables absent a fiscal anchor. We prove that a simple, state-dependent fiscal backing rule—which activates only when the ZLB binds—is sufficient to rule out deflationary traps and ensure the existence of a unique, ergodic distribution. This approach bridges the gap between local perturbation methods, which assume stability, and global solution methods, which often reveal instability, ensuring that short-run and long-run dynamics are consistent for all shock histories.

Second, we show that this framework endogenously generates a nonlinear Phillips

curve consistent with recent empirical evidence. In our model, the Phillips curve flattens markedly during ZLB episodes. This nonlinearity arises not from ad-hoc assumptions about price rigidity, but from the expectational channel of the fiscal backing. Because agents anticipate that the fiscal authority will intervene to prevent a deflationary collapse, inflation expectations remain anchored even as significant slack emerges. Consequently, inflation becomes desensitized to unemployment dynamics during liquidity traps, reproducing the "concave" Phillips curve observed in low-interest-rate environments.

Third, we document a novel policy implication regarding fiscal multipliers: the *policy substitution effect.* Contrary to the conventional wisdom that fiscal multipliers are strictly larger at the ZLB, we find that the marginal effectiveness of discretionary fiscal stimulus diminishes in a fiscally-backed regime. Because the economy is already supported by the endogenous fiscal rule (the "spender of last resort" mechanism), additional discretionary spending largely crowds out the emergency support that was implicitly in place. This result suggests that while fiscal policy is essential to sustain the ZLB equilibrium, the power of discretionary fine-tuning is more limited than previously understood.

The intuition behind our results relies on treating the fiscal backing not as an arbitrary model closure, but as a structural feature of modern policymaking. Much like a central bank acting as a lender of last resort, we model the fiscal authority as implicitly committed to preventing price-level indeterminacy. This commitment anchors the recursive equilibrium, preventing the economy from drifting into the non-stationary deflationary zones typical of standard models. By formalizing this mechanism, we bring the ZLB back within the scope of standard global equilibrium dynamics, allowing for a rigorous quantitative assessment of policy risks.

Related literature Our paper contributes to several strands of literature; first, to the large literature on New Keynesian (NK) models with the zero lower bound (ZLB) on the nominal interest rate. Early contributions such as Benhabib et al. (2001) show that a ZLB-constrained Taylor rule can generate multiple steady states and trap-like equilibrium selections, making purely forward-looking monetary policy fragile. We build on this insight by showing that, absent an explicit fiscal backstop, the induced global law of motion can be non-ergodic: the economy may enter a region in which the shadow rate remains negative and the state process is reducible. Introducing a simple, state-contingent fiscal backing rule eliminates these trap selections and restores a globally well-defined stationary equilibrium. Eggertsson and Woodford (2003) argue that monetary policy is largely ineffective at the ZLB unless it credibly shifts expectations, highlighting the limited role of conventional asset swaps. Our results echo this mechanism: when policy is constrained by the ZLB, standard monetary tools have limited traction on expected inflation and activity, whereas fiscal purchases directly affect demand and thereby shift expectations, raising inflation and shortening the expected duration of the binding regime. This underscores the distinctive role of fiscal policy in shaping global dynamics when monetary policy is constrained.

More recently, the literature has developed solution algorithms for occasionally binding ZLB constraints to study dynamics in low interest rate environments (Fernández-Villaverde et al., 2015; Guerrieri and Iacoviello, 2015; Eggertsson et al., 2021; Aruoba et al., 2021; Cao et al., 2023). While these methods advance nonlinear analysis, they typically ignore the possibility of multiple equilibria or global instability. An exception is Borağan Aruoba et al. (2018), who show that nonlinear NK models with an occasionally binding ZLB can generate prolonged stagnation or explosive dynamics without proper model closure, documenting multiple equilibria and global instability. Our contribution is to show that a simple fiscal spending rule elimi-

nates these explosive paths and deflationary traps, yielding a globally stable, ergodic process with a unique stationary distribution, in contrast to the non-stationarity or trapping behavior of models lacking fiscal intervention. The other closely related study is Ascari and Mavroeidis (2022), who generalize the conditions for existence and uniqueness of equilibria in NK models with an occasionally binding ZLB and highlight the fragility of existing algorithms. We contribute to this literature by (i) constructing a recursive, Markovian fiscal-backing rule that restores a globally defined recursive competitive equilibrium (RCE) under an occasionally binding ZLB, (ii) computing the minimal fiscal spending to eliminate negative shadow rates and satisfy the boundedness condition for all histories, and (iii) characterizing the resulting state-dependent dynamics—particularly how nonlinearities around the ZLB affect Phillips-curve slopes, volatilities, and fiscal multipliers. Unlike their focus on equilibrium fragility, our approach provides a microfounded instrument that resolves this fragility and enables quantitative policy evaluation. Moreover, we analyze state dependence that arises specifically from nonlinearities around the ZLB.

Second, our paper contributes to the literature on the monetary-fiscal policy mix (Davig and Leeper, 2011; Werning, 2011; Correia et al., 2013; Hofmann et al., 2021; Woodford and Xie, 2022; Bianchi et al., 2025). A seminal work by Leeper (1991) distinguishes between "active" and "passive" regimes and shows that equilibrium determinacy requires one policy to be active while the other remains passive. At the zero lower bound (ZLB), monetary policy is de facto passive, shifting the burden of determinacy to fiscal policy. We operationalize this logic in a dynamic setting with an occasionally binding ZLB by introducing a simple fiscal rule: fiscal policy becomes active during ZLB episodes and reverts to a passive role otherwise. This mechanism embeds his framework in a RCE and shows how modest, state-contingent fiscal interventions can ensure solvency and price-level determinacy across all states. Another

closely related study is Werning (2011), who characterizes optimal monetary and fiscal policy in a NK model with a ZLB constraint and shows that government spending is optimally above its natural level during liquidity-trap episodes. Our results align with this insight by quantifying the required state-contingent fiscal interventions that restore stability when monetary policy is constrained.

Finally, our findings connect to recent empirical studies of nonlinear Phillips curves (Nalewaik, 2016; Babb and Detmeister, 2017; Gagnon and Collins, 2019; Del Negro et al., 2020; Doser et al., 2023). Coibion and Gorodnichenko (2015) and Forbes et al. (2021) document that inflation responds asymmetrically to slack—flattening during recessions or when inflation is low, and steepening when the economy overheats. Our model naturally reproduces this behavior: during ZLB episodes with fiscal backing, inflation is pinned down by expectations and slack has little marginal effect. Away from the ZLB, price dynamics become more sensitive to output. This endogenous Phillips curve nonlinearity is a novel theoretical result and provides a mechanism linking occasionally binding constraints to the observed asymmetries in inflation dynamics.

2 Baseline model

We investigate the nonlinear nature of the canonical New Keynesian model with zero lower bound (ZLB) on the nominal interest rate. In this economy, a representative household supplies labor and purchases goods for consumption. The production side consists of a final good producer and a continuum of monopolistic competitive intermediate good producers. The intermediate good producers produce output using labor services supplied by the household. They face Rotemberg-type adjustment cost to change their price. The central bank sets the nominal interest rate following the

Taylor rule subject to the ZLB constraint. In addition, we assume a state dependent part of fiscal policy, which stimulate the economy during the ZLB binding periods. There are four exogenous shocks: TFP, preference, monetary policy and government spending shocks.

A representative household solves the following utility maximization problem.

$$\max \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \xi_t \left[\frac{c_t^{1-\sigma} - 1}{1 - \sigma} - \eta \frac{(n_t)^{1+1/\chi}}{1 + 1/\chi} \right] \tag{1}$$

$$s.t. \quad P_t c_t + \frac{1}{1 + i_t} B_{t+1} = P_t w_t n_t + B_t + P_t D_t - P_t T_t \tag{2}$$

where i_t is nominal interest rate, B_t is nominal bonds, D_t is real profit from firms owned by household and T_t is a lump-sum tax. ξ_t is a preference shock that follows

$$\ln \xi_t = \rho^\beta \ln \xi_{t-1} + \epsilon_t^\beta, \quad \epsilon_t^\beta \sim \mathcal{N}(0, \sigma^\beta) \tag{3}$$

with the initial condition $\xi_{-1} = 1$. Let $\beta_t := \beta \frac{\xi_t}{\xi_{t-1}}$. From the first-order condition of the household, we obtain

$$c_t^{-\sigma} = \mathbb{E}_t \left[\beta_{t+1} c_{t+1}^{-\sigma} \frac{1 + i_t}{1 + \pi_{t+1}} \right] \tag{4}$$

$$\frac{1}{1 + r_t} := \mathbb{E}_t \left[\beta_{t+1} \left(\frac{c_{t+1}}{c_t} \right)^{-\sigma} \right] \tag{5}$$

$$n_t = \left(\frac{w_t}{\eta c_t^\sigma} \right)^\chi \tag{6}$$

Final goods Y_t is assumed to be produced by a final good producer that uses the intermediary goods $y_t(j)$ as input and has CES production technology $Y_t = \left[\int_0^1 y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$. The profit maximization problem of the final good producer implies $P_t = \left[\int_0^1 p_t(j)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$.

There is a continuum of intermediate good producers with unit mass. Each intermediate producer produces differentiated output according to a constant return to scale technology in labor with a common productivity shock: $y_t(j) = A_t n_t(j)$. We assume that logarithm of A_t follows an AR(1) process $\ln A_t = \rho^A \ln A_{t-1} + \epsilon_t^A$, where $\epsilon_t^A \sim \mathcal{N}(0, \sigma^A)$.

In each period, each firm faces Rotemberg-type price adjustment cost $c(\{p_t(j), p_{t-1}(j)\}) := \frac{\psi}{2} (p_t(j)/p_{t-1}(j) - (1 + \overline{\pi}))^2 Y_t$, which leads to the following adjustment decision:

$$\epsilon - 1 = \epsilon mc_t - \psi(1 + \pi_t)(\pi_t - \overline{\pi}) + \psi \mathbb{E}_t \left[\beta_{t+1} \left(\frac{c_t}{c_{t+1}} \right)^\sigma (1 + \pi_{t+1})(\pi_{t+1} - \overline{\pi}) \frac{Y_{t+1}}{Y_t} \right]$$
(7)

where $mc_t = \frac{w_t}{A_t}$ is a marginal cost at period t.

The central bank sets the nominal interest rate following the standard Taylor rule with ZLB constraint:

$$i_t = \max \left\{ 0, \ i_t^N \right\},$$
(8)

$$i_t^N = (1 + i_{t-1})^{\rho_i} \left[(1 + \overline{r})(1 + \pi_t) \left(\frac{1 + \pi_t}{1 + \overline{\pi}} \right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^f} \right)^{\phi_Y} e^{\varepsilon_t^{MP}} \right]^{1 - \rho_i}$$
(9)

where Y_t^f is a potential output and $\overline{r}$ is the real interest rate at the steady state. We also assume an exogenous fiscal spending shocks $\widetilde{G}_t$ specified by

$$s_{G,t+1} = (1 - \rho^G) \overline{s}_G + \rho^G s_{G,t} + \sigma_G \varepsilon_t, \quad \varepsilon_t \sim_{iid} \mathcal{N}(0, 1)$$
(10)

$$\widetilde{G}_t = s_{G,t} \overline{Y}$$
(11)

where $\overline{Y}$ is a steady state value of total output. With zero net supply of government bond, the fiscal spending is financed by the contemporaneous period's lump-sum tax

to the households.

Finally, the goods market clearing condition is written as

$$Y_t = c_t + \widetilde{G}_t + \frac{\psi}{2}(\pi_t - \overline{\pi})^2 Y_t \qquad (12)$$

3 Recovering RCE through the fiscally-backed ZLB

Standard New Keynesian models face a fundamental theoretical challenge at the Zero Lower Bound (ZLB): the loss of stationarity. When the nominal interest rate is constrained, the economy is prone to deflationary spirals that act as absorbing states, violating the conditions necessary for a valid Recursive Competitive Equilibrium (RCE).

In this section, we first formally define the RCE and demonstrate why the standard model fails to support a global equilibrium. We then introduce our primary contribution: the *fiscally-backed ZLB*. By embedding a state-contingent fiscal rule that acts as a "spender of last resort," we restore ergodicity to the system, bridging the gap between local perturbation methods and global equilibrium theory.

3.1 Recursive competitive equilibrium (RCE)

We seek a recursive formulation of the sequential problem where equilibrium objects are functions of a sufficient aggregate state vector.

Assumption 1 (Principle of optimality and state space)**.**
The household and production-sector problems satisfy the principle of optimality (Stokey et al., 1989), so that a recursive representation is without loss in terms of an aggregate

state vector. We define the aggregate state as

$$X = [i_{-1}, S],\qquad(13)$$

where S collects the exogenous aggregate states (TFP, preference, monetary policy, and government-spending), and i_{-1} is the lagged nominal interest rate, the sole endogenous state variable inherited from the past. For the theoretical arguments and to remain consistent with the numerical implementation, we assume that S takes values in a compact set—for example, by approximating each exogenous process with a finite-state Markov chain (Tauchen or Rouwenhorst discretization).

The dynamics of the endogenous state are governed by the central bank's policy rule.

Lemma 1 (The aggregate law of motion)**.**
In any recursive competitive equilibrium, the induced law of motion for the endogenous state $\Gamma(i_{-1}) := i$ is given by the ZLB Taylor rule:

$$\log(1 + i^N(X)) = \rho_i \log(1 + i_{-1}) \qquad(14)$$
$$+ (1 - \rho_i) \log\left((1 + \bar{r})(1 + \pi(X))\left(\frac{1 + \pi(X)}{1 + \bar{\pi}}\right)^{\phi_\pi}\left(\frac{Y(X)}{Y^f(X)}\right)^{\phi_Y}\right) + MP(X).$$
$$i = \max\{i^N(X), 0\} \qquad(15)$$

Proof. See Appendix. ∎

We now define the standard Recursive Competitive Equilibrium. Note that we explicitly state the boundedness condition, which is often implicit but crucial in the context of liquidity traps.

Definition 1 (Recursive competitive equilibrium).

Let y be the vector of allocations and prices: $y := \{c, n, Y, w, \pi, mc, \Gamma\}$. A recursive competitive equilibrium is a set of functions $y(X)$ that satisfies:

(i) *Optimality conditions:*

$$c^{-\sigma} = \mathbb{E}\left[\beta' \left(c'\right)^{-\sigma} \frac{1+i}{1+\pi'}\right] \tag{16}$$

$$n = \left(\frac{w}{\eta c^\sigma}\right)^\chi \tag{17}$$

$$Y = c + \tilde{G} + \frac{\psi}{2}(\pi - \overline{\pi})^2 Y \tag{18}$$

$$Y = An \tag{19}$$

$$\epsilon - 1 = \epsilon mc - \psi(1+\pi)(\pi - \overline{\pi}) + \psi\mathbb{E}\left[\beta' \left(\frac{c}{c'}\right)^\sigma (1+\pi')(\pi' - \overline{\pi})\frac{Y'}{Y}\right] \tag{20}$$

$$mc = \frac{w}{A} \tag{21}$$

$$Y^f = \left(\frac{\epsilon - 1}{\eta\epsilon}\right)^{\frac{\chi}{1+\sigma\chi}} (1 - s_G)^{\frac{-\sigma\chi}{1+\sigma\chi}} A^{\frac{1+\chi}{1+\sigma\chi}} \tag{22}$$

(ii) *Consistency in the law of motion Γ: Households' and the government's expectations of the future aggregate state given X are consistent with the allocation implied by the optimality conditions.*

(iii) *Boundedness condition: We restrict attention to equilibria in which the household value is well-defined (Stokey et al., 1989):*

$$\sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 u(c_t, n_t) < \infty, \tag{23}$$

and allocations remain in a compact subset of the feasible set.

3.2 Non-ergodicity of the standard equilibrium with ZLB

In standard New Keynesian frameworks, the ZLB introduces a fundamental discontinuity. When the Taylor rule dictates a negative shadow rate ($i^N < 0$), the actual rate is constrained at zero. Absent additional stabilization, this constraint leaves open equilibrium selections under which global dynamics becomes non-ergodic and fails to admit a unique stationary distribution.

Proposition 1 (Trap equilibrium and non-ergodicity under a plain-vanilla ZLB).
*Under the ZLB-constrained Taylor rule $i(X) = \max\{0, i^N(X)\}$, there exists an equilibrium selection (*deflationary-trap expectations*) for which the set*

$$\mathcal{D} := \{X : i(X) = 0 \text{ and } i^N(X) < 0\} \tag{24}$$

is absorbing: if $X_t \in \mathcal{D}$, then $X_{t+k} \in \mathcal{D}$ for all $k \geq 1$ with probability one. Hence the induced Markov process is reducible and does not admit a unique invariant distribution over the full state space.

Proof. See Appendix. ∎

Proposition 1 isolates a precise sense in which a plain-vanilla ZLB Taylor rule does not deliver a globally well-behaved recursive dynamics. Under a deflationary-trap equilibrium selection, the economy can enter the region $\mathcal{D}$ in which the ZLB binds while the shadow rate remains negative. Once in $\mathcal{D}$, the state becomes trapped: the induced Markov process is reducible and does not admit a unique invariant distribution over the full state space. In other words, the standard policy specification leaves open equilibrium selections under which the global law of motion fails to be ergodic.

This observation has a substantive implication for the recursive formulation. A recursive competitive equilibrium is a primitive-level object: it is intended to provide

a stationary mapping $y(X)$ from the state to allocations and prices that generates a well-defined global stochastic dynamics. Trap selections of the kind characterized in Proposition 1 are incompatible with this objective: because the economy can remain indefinitely in a region with a negative shadow rate, key nominal objects drift persistently away from target and the induced process fails to possess a unique stationary distribution. Accordingly, throughout we restrict attention to *admissible* recursive equilibria that rule out such trap behavior and yield globally well-defined, stationary dynamics on the intended state space.

This perspective does not deny the usefulness of the deflationary-spiral benchmark as a diagnostic of destabilizing feedback at the ZLB. Rather, it emphasizes that ruling out persistent trap behavior requires an explicit policy regime that prevents the economy from remaining indefinitely in states with a negative shadow rate. In practice, such stabilization is provided by fiscal authorities through automatic or discretionary interventions. Our contribution is to model this fiscal backing explicitly and to quantify how it reshapes global equilibrium dynamics, state-dependent Phillips-curve behavior, and fiscal multipliers.

3.3 The fiscally-backed ZLB

To restore a globally defined RCE, we introduce an *Endogenous Fiscal Intervention*. This mechanism mirrors real-world policy responses—such as the American Recovery and Reinvestment Act (ARRA) of 2009 or the prolonged quantitative easing and fiscal support in Japan—where the government steps in precisely when monetary policy becomes powerless.

Unlike ad-hoc stimulus shocks often used in the literature, we model this as a structural rule. The government implicitly commits to a "fiscal backing" policy that

activates if and only if the monetary constraint binds.

Define the shadow nominal rate $i^N(X;G)$ as the rate implied by the Taylor rule absent the ZLB, given state X and government purchases G. We specify an automatic fiscal backstop that activates only when the ZLB binds:

$$G(X) = \begin{cases} 0, & \text{if } i(X) > 0, \\ G^*(X), & \text{if } i(X) = 0, \end{cases} \tag{25}$$

where $G^*(X)$ is the *minimal* additional government purchases that eliminate negative shadow rates:

$$G^*(X) := \inf\{g \geq 0 : i^N(X;g) \geq 0\}. \tag{26}$$

Equivalently, whenever the ZLB binds, the fiscal authority chooses $G^*(X)$ so that the shadow rate is exactly at the boundary, $i^N(X;G^*(X)) = 0$.

The fiscal-backing rule $G^*(X)$ is a within-period purchases policy, financed by contemporaneous lump-sum taxes as in our baseline fiscal environment (zero net bond supply). Hence the only feasibility requirements are those of the static resource constraint and non-negativity of private consumption. Throughout, we restrict attention to an admissible state space for $X = [i_{-1}, S]$ on which the model primitives and equilibrium mappings remain bounded, so that allocations satisfy $Y(X) < \infty$ and the Rotemberg resource loss is finite.

To see why the infimum in (26) is economically meaningful, note first that higher government purchases raise aggregate demand and, through the Euler equation and the New Keynesian Phillips curve, put upward pressure on inflation and activity. Because the notional Taylor rule is increasing in inflation and the output gap, this implies that $i^N(X;g)$ is (weakly) increasing in g for each fixed state X.[1] Second,

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[1]This monotonicity is the standard comparative-static implication of the NK block: higher pur-

on the admissible state space we assume there exists a finite "fiscal capacity" level $\bar{g}(X)$ such that the allocation implied by $g = \bar{g}(X)$ remains feasible (in particular $c(X; \bar{g}(X)) \geq 0$) and delivers $i^N(X; \bar{g}(X)) \geq 0$. Under these conditions the set $\{g \geq 0 : i^N(X; g) \geq 0\}$ is non-empty and $G^*(X)$ is finite.

Importantly, the minimality of $G^*(X)$ ensures that fiscal backing is not an arbitrary closure: it activates only in states where the shadow rate would otherwise be negative and selects the smallest purchases consistent with eliminating persistent negative-shadow-rate dynamics. This is precisely the sense in which fiscal backing rules out deflationary-trap behavior while remaining economically admissible.

Our notion of "global determinacy" is uniqueness within the class of bounded Markov recursive competitive equilibria under the specified policy regime. The fiscal-backing rule is part of the regime and eliminates the deflationary-trap region that otherwise supports non-ergodic equilibrium selections.

Proposition 2 (Recursivity of the law of motion under fiscal backing)**.**

The law of motion (15) *augmented with the fiscal backing rule G^* possesses a unique stationary (ergodic) time-series distribution for the nominal interest rate i.*

Proof. See Appendix. ∎

Definition 2 (Recursive competitive equilibrium with fiscal backing)**.** *Let y collect allocations and prices, $y := \{c, n, Y, w, \pi, mc, Y^f, G^*\}$. A recursive competitive equilibrium with fiscal backing is a set of functions $y(X)$ such that:*

(i) (Private optimality and market clearing). *Given the policy rules, the household and firms satisfy (16)–(22), and the goods market clears:*

$$Y(X) = c(X) + \tilde{G}(X) + G^*(X) + \frac{\psi}{2}\big(\pi(X) - \bar{\pi}\big)^2 Y(X).$$

chases increase demand, increase marginal costs, and raise inflation; the Taylor rule then prescribes a higher notional rate.

(ii) (Policy rules and consistency). *The nominal rate satisfies the ZLB Taylor rule (14)–(15). The fiscal authority follows (25)–(26) below, and expectations are consistent with the induced transition kernel.*

(iii) (Boundedness). *The boundedness condition in Definition 1 holds:*

$$\sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 u(c_t, n_t) < \infty,$$

and allocations remain in a compact subset of the feasible set.

Global–local consistency The introduction of G^* resolves a key tension in the New Keynesian literature. Standard perturbation methods ("local" solutions) typically ignore the explosive nature of the ZLB, implicitly assuming the economy will return to the steady state. Conversely, global solution methods often reveal that the ZLB leads to indeterminacy or collapse.

By embedding the G^* rule, we bridge this gap. For small shocks, $G^* \approx 0$, and the model behaves exactly like the standard New Keynesian framework. However, for large, persistent shocks that drive the economy deep into the ZLB, G^* activates to preserve global stability. This ensures that short-run analysis (where the ZLB binds occasionally) and long-run analysis (where the equilibrium must remain bounded) are consistent within a single Recursive Competitive Equilibrium.

4   Quantitative Analysis of the Fiscally-Backed ZLB

In this section, we quantify the theoretical framework developed above. We begin by detailing the calibration of the model to the U.S. economy. We then analyze the characteristics of the endogenous fiscal spending G^* required to sustain the equi-

librium, effectively measuring the "fiscal cost" of the ZLB. Finally, we explore the macroeconomic implications, focusing on the nonlinearities in the Phillips curve and the amplification of unemployment volatility during binding episodes.

4.1 Calibration

Table 1 summarizes the calibrated parameter values and their corresponding sources. We calibrate the parameter values based on the standard New Keynesian literature.

The discount factor β is set to 0.99 at a quarterly frequency, implying an annual risk-free rate of approximately 4 percent. The elasticity of substitution across firms, ϵ, is calibrated to 10, which corresponds to an average markup of 11 percent for intermediate goods producers. Target inflation rate $\bar{\pi}$ is set at an annual 2%.

A key parameter for our analysis is the price adjustment cost ψ. We choose the value of the Rotemberg-type price adjustment cost $\psi = 50.41$ such that the slope of the log-linearized Phillips curve matches that implied by a Calvo-type pricing friction with the parameterization in Smets and Wouters (2007).

Regarding the stochastic processes, we set $\sigma^\beta = 0.03$ and $\sigma^A = 0.009$. These values ensure that the simulated probability of hitting the ZLB constraint is approximately 5 percent, and the standard deviation of real output is around 1.5, consistent with U.S. business cycle data. The remaining parameters follow Fernández-Villaverde et al. (2015) and standard literature sources.

4.2 Solution method

To analyze global equilibrium dynamics, we solve the model using the repeated transition method (RTM) in Lee (2025). The key computational challenge is evaluating conditional expectations in the Euler equation and the New Keynesian Phillips curve

Table 1: Calibrated Parameters

Parameter	Value	Moment Matched/Source
Panel A. Households, firms		
Time discount factor (β)	0.9900	Quarterly frequency
Relative risk aversion (σ)	1.0000	Fernández-Villaverde et al. (2015)
Frisch elasticity (χ)	1.0000	Fernández-Villaverde et al. (2015)
Relative (dis)utility of labor (η)	2.5350	Employment = 0.66
Elasticity of substitution across firms (ϵ)	10.000	Firms' average markup
Rotemberg-type price adjustment cost (ψ)	50.4056	Smets and Wouters (2007)
Persistence of preference shocks (ρ^{β})	0.9000	Christiano et al. (2014)
S.D. of preference shocks (σ^{β})	0.0300	Probability of being at ZLB
Persistence of productivity shocks (ρ^{A})	0.9500	Cooley et al. (1995)
S.D. of productivity shocks (σ^{A})	0.0090	Std. of real output
Panel B. Fiscal, monetary policy		
Taylor rule: smoothing (ρ_i)	0.8500	Christiano et al. (2014)
Taylor rule: response to inflation (ϕ_{π})	1.5000	Fernández-Villaverde et al. (2015)
Taylor rule: response to output gap (ϕ_Y)	0.1250	Fernández-Villaverde et al. (2015)
S.D. of monetary shocks (σ^{MP})	0.0025	Guerron-Quintana (2010)
Steady State fiscal spending share ($\overline{s}_G$)	0.2000	Fernández-Villaverde et al. (2015)
Persistence of fiscal spending shocks (ρ^{G})	0.8000	Fernández-Villaverde et al. (2015)
S.D. of fiscal spending shocks (σ^{G})	0.0060	1% of stationary output level

Notes: The value of ψ is chosen so that the slope of the log-linearized Phillips curve matches that implied by a Calvo friction with the calibration in Smets and Wouters (2007).

over a large state space when the ZLB occasionally binds. RTM addresses this by combining (i) long simulations under candidate policy functions with (ii) repeated counterfactual one-step transitions that vary the exogenous aggregate shocks.

Concretely, let the aggregate state be $X_t = [i_{t-1}, S_t]$. Let $y(X)$ denote the collection of equilibrium objects we solve for (e.g., $c(X)$, $n(X)$, $\pi(X)$, and $Y(X)$). Given a candidate mapping $y(X)$, we simulate a long path $\{X_t\}_{t=1}^{T}$ and, at each date t, approximate conditional expectations by averaging over repeated one-step transitions that hold the predetermined endogenous component i_{t-1} fixed while varying the exogenous innovation to S_{t+1}. This delivers an accurate approximation to conditional expectations of the form $\mathbb{E}_t[h(y(X_{t+1})) \,|\, X_t]$, where $h(\cdot)$ collects the next-period terms

appearing in the Euler equation and the Phillips curve. We then update $y(\cdot)$ using these conditional expectations and iterate until convergence.
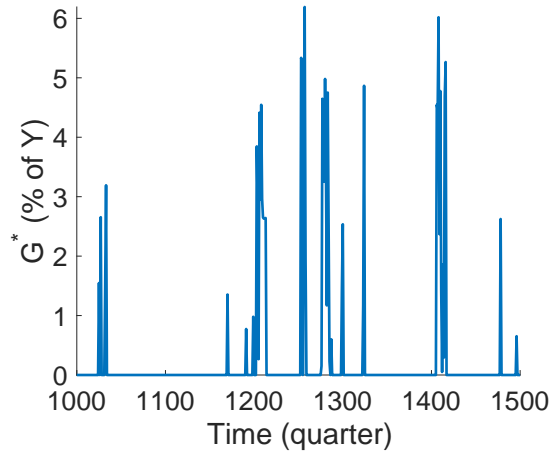
4.3   Fiscal cost of the ZLB

We first analyze the behavior of the endogenous fiscal spending G^*. This variable represents the fiscal cost required to sustain a globally well-defined equilibrium when the ZLB binds. In particular, $G^*(X)$ is the minimal state-contingent government purchases that eliminate negative shadow rates, $i^N(X; G^*(X)) = 0$, thereby ruling out deflationary-trap dynamics and restoring a stationary law of motion. Quantitatively, G^* measures how much fiscal capacity is required in crisis states to keep the economy from remaining in the ZLB-risk region.

Figure 1 displays a simulated sample path of G^* (expressed as a percentage of steady-state output). The intervention is highly state-dependent and episodic. Away from the ZLB, G^* is identically zero, as monetary policy is sufficient to determine equilibrium. However, during deep recessionary shocks that trigger the ZLB, the fiscal backing activates automatically. These spikes represent the government acting as a "spender of last resort" to prevent deflationary spirals.

Table 2 provides summary statistics for these interventions. While the unconditional average of G^* is small (0.14% of output), this masks the intensity of the policy during crises. Conditional on the ZLB binding ($G^* > 0$), the average fiscal spending rises to 2.65% of output. This magnitude suggests that while the fiscal backing is a "modest" intervention in a long-run sense, it requires significant fiscal capacity during liquidity traps.

Furthermore, the volatility of G^* is substantial (23.34% relative to output volatility), and it is strongly pro-cyclical with respect to the crisis itself (correlation with

Figure 1: The fiscal cost of ZLB

output is 0.79 during binding periods). This confirms that G^* functions as a targeted automatic stabilizer that scales with the severity of the aggregate demand deficiency.

Table 2: Summary statistics of G^* dynamics

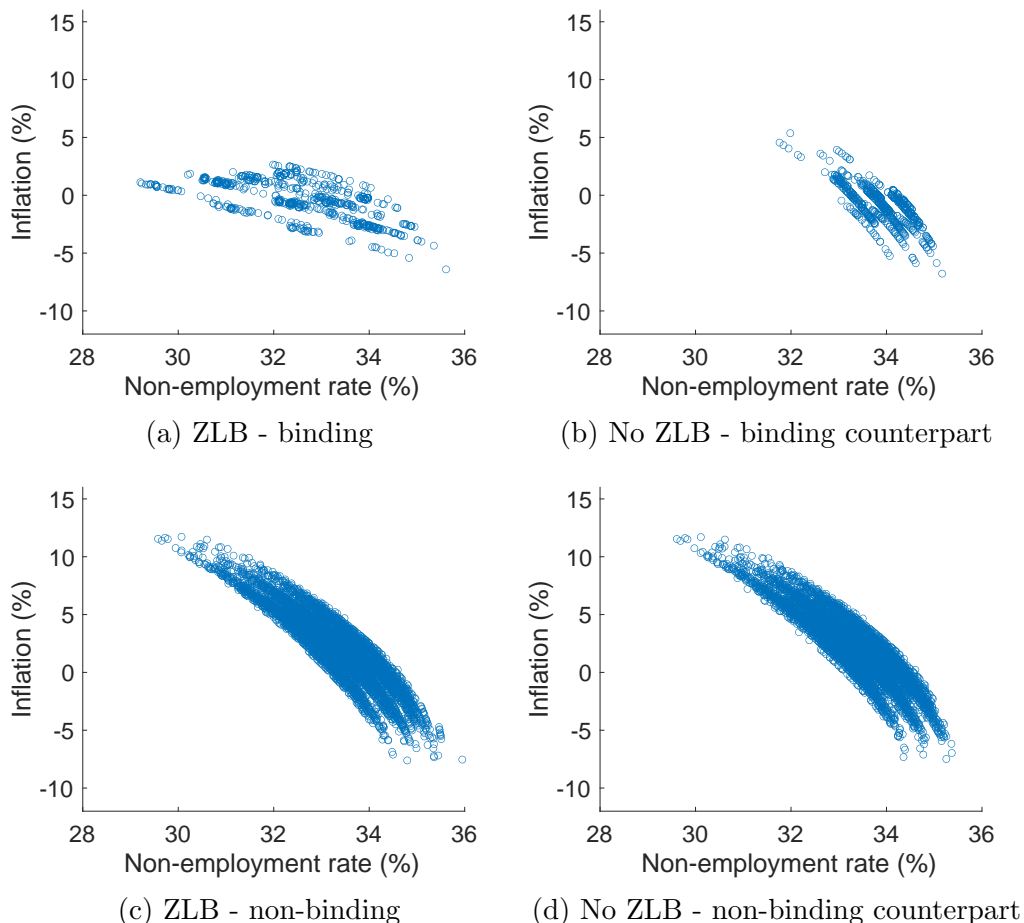|	G^*	$G^*|_{G^*>0}$
Average relative to average output (%)	0.14	2.65
Volatility relative to output volatility (%)	23.34	53.49
Correlation with output	0.29	0.79
Correlation with inflation	-0.13	0.25

4.4 Macroeconomic dynamics: flattened Phillips curve

The fiscal backing mechanism does not merely stabilize the model mathematically; it fundamentally alters the economy's structural dynamics, generating a state-dependent trade-off between inflation and real activity.

Figure 2 illustrates the Phillips curve relationship—inflation plotted against the

non-employment rate—across different policy regimes. Panel (a) captures the economy under the fiscally-backed ZLB. The most striking feature is the marked flattening of the slope during binding episodes (blue circles). In this regime, inflation becomes largely insensitive to labor market slack.

(a) ZLB - binding

(b) No ZLB - binding counterpart

(c) ZLB - non-binding

(d) No ZLB - non-binding counterpart

Notes: Each panel plots the realized pair of inflation and non-employment rates along a simulated sample path, grouped by the contemporaneous interest rate level. Panels (a) and (c) are simulated with ZLB constraint, while panels (b) and (d) are simulated without the constraint.

Figure 2: The Phillips curves

This flattening arises from the *expectational channel* of the fiscal-backing rule. When the economy hits the ZLB, agents anticipate that the fiscal authority will activate state-contingent purchases G^* to prevent persistent negative shadow rates and

the associated deflationary-trap dynamics. These expectations stabilize expected inflation and thereby attenuate the response of realized inflation to further deteriorations in activity. Consequently, even as slack rises sharply (moving right along the horizontal axis), inflation declines only modestly (moving down along the vertical axis). The Phillips curve is therefore highly state dependent: inflation responds strongly to activity in tight conditions but becomes markedly less sensitive when slack is elevated, as in ZLB episodes. This mechanism generates a "missing disinflation" pattern reminiscent of the Great Recession, in which inflation remained relatively stable despite substantial slack.

Contrast this with Panel (b), which plots the binding counterpart for a hypothetical economy without the ZLB constraint (where nominal rates can go negative). Here, the curve retains its standard steep slope. Without the ZLB constraint, the central bank aggressively cuts rates to stabilize the output gap, maintaining the standard link between prices and slack. The comparison between Panels (a) and (b) confirms that the flattening is not driven by the shock itself, but by the interaction between the ZLB constraint and the fiscal backing.

Table 3 quantifies this regime shift. The slope of the Phillips curve flattens dramatically from -3.01 in the non-binding regime to -0.80 in the ZLB-binding regime.

Table 3: Equilibrium statistics around ZLB

| | ZLB | | Without ZLB | |
	Binding	Non-binding	Binding	Non-binding
Slope of the Phillips curve	-0.80	-3.01	-2.31	-3.01
Average inflation (%)	-0.44	2.03	-0.92	2.08
Inflation volatility (%)	1.68	2.76	1.68	2.72
Average non-employed $(1-n)$ (%)	32.58	33.34	33.88	33.33
Non-employed volatility (%)	1.30	0.80	0.54	0.79

Notes: All statistics are computed by simulating the model for 10,000 periods with and without the ZLB constraints and automatic fiscal backing, using the same realized path of the exogenous shocks.

This newfound nominal stability, however, comes at a cost. Because prices (inflation) do not adjust downward to clear the market during the ZLB, the adjustment burden falls entirely on quantities. This creates a "hydraulic" trade-off: by anchoring inflation via fiscal backing, the volatility is transferred to the real economy. As shown in Table 3, while inflation volatility remains comparable across regimes (1.68%), the volatility of the non-employment rate surges to 1.30% during ZLB episodes—more than double the 0.54% volatility observed in the without-ZLB counterfactual. Thus, the fiscally-backed ZLB prevents nominal collapse but exacerbates real-side instability.

5 State-dependent short-run dynamics

We now examine how the presence of the fiscally-backed ZLB alters the transmission of aggregate shocks. A key prediction of our framework is that the economy's response to structural shocks is highly state-dependent, governed by whether the fiscal backing rule is active.

To illustrate this, we compute generalized impulse response functions (GIRFs) conditional on two distinct initial states:

- *Normal Regime:* The economy is at the deterministic steady state with positive interest rates.

- *ZLB Regime ($i \cong 0$):* The economy is in a liquidity trap where the zero lower bound binds, and the endogenous fiscal backing G^* is active.

5.1 Monetary Policy Shocks

Figure 3 displays the impulse responses to a one-standard-deviation contractionary monetary policy shock (a positive innovation to the Taylor rule residual by 25 basis points).
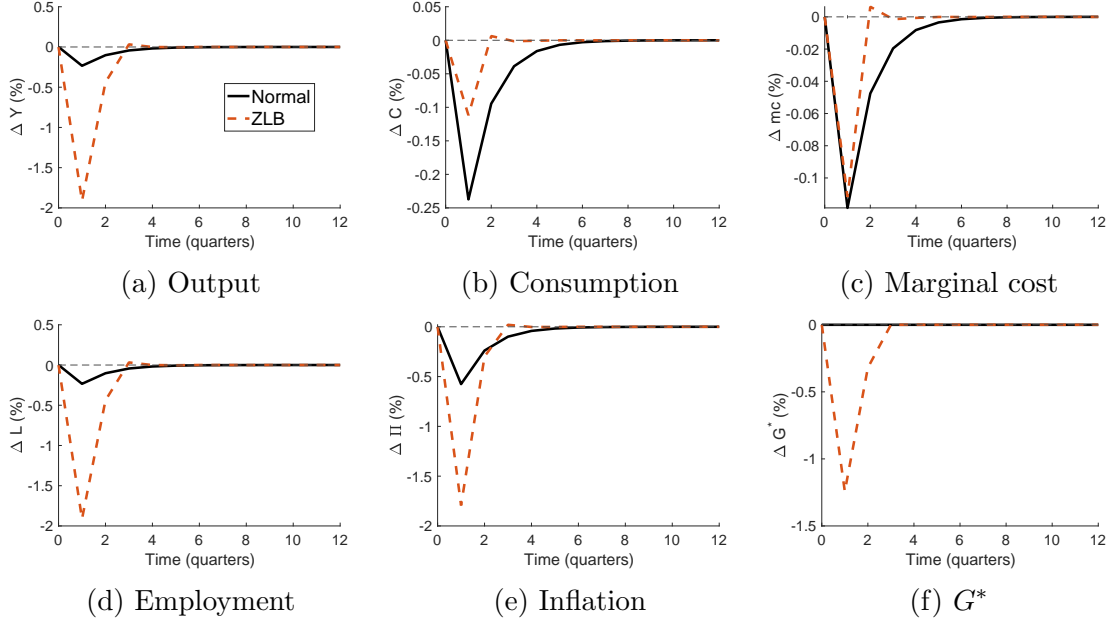
In the *Normal Regime* (solid black lines), the shock induces standard New Keynesian dynamics: the nominal interest rate rises (not shown), leading to an immediate contraction in output (Panel a) and consumption (Panel b). Inflation falls (Panel e) via the Phillips curve, and marginal costs decline. In the *ZLB Regime* (dashed red lines), the transmission is markedly different. Although the actual nominal rate is constrained at zero, the shock raises the *shadow* nominal rate, effectively signaling a "tougher" policy stance that delays the exit from the ZLB.

Crucially, observe Panel (f): the endogenous fiscal backing G^* decreases in response to the monetary-policy shock. Intuitively, G^* is pinned to a shadow-rate boundary condition, not to output stabilization; the monetary-policy shock shifts that boundary mechanically. Because the shock enters the shadow Taylor rule, it raises i^N holding X fixed. Since G^* is defined as the minimal purchases required to satisfy $i^N(X; G^*) = 0$, the increase in i^N reduces the required fiscal backstop—even though the shock remains contractionary for activity.

5.2 Productivity Shocks

Figure 4 plots the response to a negative productivity shock (a decrease in the marginal productivity of labor by one standard deviation), representing a negative supply disturbance.

In the *ZLB Regime* (dashed red lines), an adverse supply shock depresses consumption; as a result, the Euler gap that discretionary fiscal backing G^* is designed

(a) Output   (b) Consumption   (c) Marginal cost

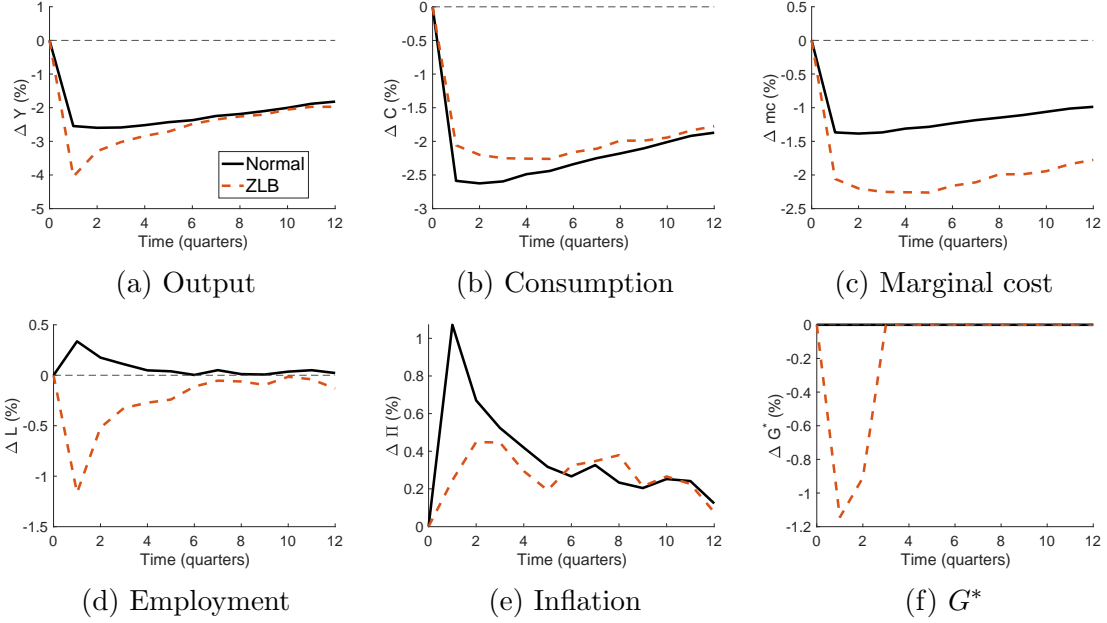(d) Employment   (e) Inflation   (f) G^*

Notes: Solid black lines denote the response starting from the steady state (*Normal* regime). Dashed red lines denote the response starting from a ZLB episode.

Figure 3: State-dependent impulse responses to a positive monetary policy shock

to offset becomes smaller, so the required level of G^* falls on impact. This endogenous reduction in G^* then feeds back into aggregate demand, mechanically lowering output Y and amplifying the output contraction relative to the *Normal Regime*. Importantly, this demand feedback can be strong enough to alter the labor-market transmission: with weaker goods demand, the induced movements in labor demand and equilibrium labor outcomes can flip.

5.3 The Substitution Effect of Fiscal Policy

Finally, we analyze the effectiveness of discretionary fiscal policy. Figure 5 plots the response to a positive exogenous government spending shock ($\tilde{G}$). This exercise highlights the *diminishing marginal effectiveness* of discretionary stimulus within a fiscally-backed regime.

(a) Output   (b) Consumption   (c) Marginal cost

(d) Employment   (e) Inflation   (f) G^*

Notes: Solid black lines denote the response starting from the steady state (*Normal* regime). Dashed red lines denote the response starting from a ZLB episode.
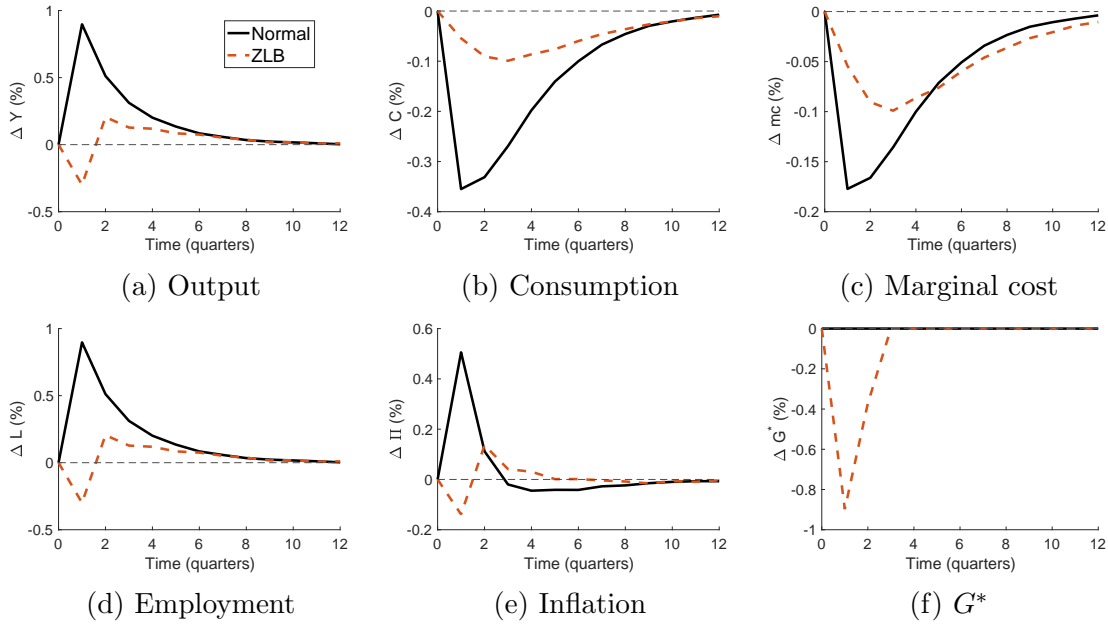
Figure 4: State-dependent impulse responses to a negative productivity shock

Standard New Keynesian models typically predict that fiscal multipliers are largest at the ZLB. The conventional argument is that because the central bank does not raise rates to offset the stimulus, the resulting increase in inflation expectations lowers real interest rates, thereby crowding in private consumption.

However, our framework introduces a crucial countervailing force: the *policy substitution effect*. As shown in the *ZLB Regime* (dashed red lines), the output response (Panel a) is not just dampened—it initially turns negative. This occurs because the discretionary shock ($\tilde{G}$) effectively displaces the endogenous fiscal backing (G^*).

The mechanism driving this result relies on the state-contingent nature of the fiscal rule. Recall that G^* acts as an automatic stabilizer designed to close the Euler gap and prevent deflationary spirals. When a positive discretionary shock $\tilde{G}$ hits the economy, it directly stimulates aggregate demand, thereby reducing the severity of the deflationary pressure. Consequently, the endogenous requirement for the emer-

gency backstop G^* diminishes. As shown in Panel (f), the automated spending drops precipitously—almost one-for-one—upon the arrival of the exogenous shock. The government effectively substitutes "discretionary" spending for "emergency" spending. If the retrenchment in G^* is sufficiently sharp, the *net* change in aggregate demand can become negligible or even contractionary. Thus, while fiscal policy remains essential to sustain the equilibrium at the ZLB, the marginal benefit of additional discretionary stimulus is severely undercut by the endogenous retreat of the automatic stabilizers.

(a) Output   (b) Consumption   (c) Marginal cost

(d) Employment   (e) Inflation   (f) G^*

Notes: Solid black lines denote the response starting from the steady state (*Normal* regime). Dashed red lines denote the response starting from a ZLB episode. The drop in G^* in Panel (f) illustrates the substitution between discretionary and endogenous fiscal support.

Figure 5: State-dependent impulse responses to a positive fiscal spending shock

To quantify this substitution effect, we compute fiscal multipliers directly from the state-dependent impulse response functions shown in Figure 5. We define the fiscal multiplier at horizon H as the average output deviation relative to the no-shock

baseline over the horizon:

$$\mathcal{M}_H = \frac{\sum_{h=0}^{H} \left(Y_{t+h}^{shock} - Y_{t+h}^{no\ shock} \right)}{\sum_{h=0}^{H} \left(G_{t+h}^{shock} - G_{t+h}^{no\ shock} \right)} \tag{27}$$

where the exogenous government spending shock is exactly 1% of GDP on impact.

Table 4 reports the results. In the *Normal* regime, the multiplier is positive and sizable (0.90 on impact). This is consistent with standard theoretical predictions where an active Taylor rule creates partial crowding-out effects through higher real interest rates, keeping the output response positive but below the one-for-one benchmark.

Table 4: State-Dependent Fiscal Multipliers (Computed from IRFs)

Horizon	Normal Regime	ZLB Regime
Impact ($H = 0$)	0.893	-0.299
4 Quarters ($H = 4$)	0.196	0.013
8 Quarters ($H = 8$)	0.093	0.015

Notes: Multipliers are computed as cumulative multipliers from the state-dependent impulse responses: $M_H = \frac{\sum_{h=0}^{H}(Y_{t+h}^{shock} - Y_{t+h}^{no\ shock})}{\sum_{h=0}^{H}(G_{t+h}^{shock} - G_{t+h}^{no\ shock})}$, conditional on a 1% (of contemporaneous output) discretionary government spending shock $\tilde{G}$ on impact.

In the *ZLB (Fiscally Backed)* regime, however, the dynamic is fundamentally different. As shown in the third column, the impact multiplier is not merely dampened but turns negative (-0.30). This result highlights the severity of the *saturation* of liquidity support. Because the private sector already expects the government to act as a "spender of last resort" via the endogenous G^* rule, the introduction of a discretionary shock $\tilde{G}$ triggers a sharp retrenchment in the automatic fiscal stabilizer. This substitution effect is strong enough to initially generate a net contractionary impulse, before stabilizing near zero at longer horizons. This finding nuances the conventional wisdom that fiscal efficacy is strictly higher at the Zero Lower Bound;

in a fiscally-backed regime, discretionary stimulus faces diminishing—and potentially negative—marginal returns.

Why can the discretionary fiscal multiplier be negative? A distinctive implication of our fiscally-backed framework is that the measured multiplier of an *exogenous* government-spending shock $\tilde{G}_t$ can be close to zero and even negative during ZLB episodes. The reason is policy substitution. Total government purchases are the sum of an exogenous (discretionary) component and an endogenous backstop component,

$$G_t^{\text{tot}} = \tilde{G}_t + G_t^*(X_t), \tag{28}$$

where $G_t^*(X_t)$ is determined *minimally* so as to prevent the economy from remaining in states with a negative shadow rate. This feature makes G^* "option-like": it is active only in bad states and switches off endogenously as soon as conditions improve.

A positive innovation to $\tilde{G}_t$ raises current demand and inflation, thereby increasing the shadow rate and reducing the expected duration of the ZLB/backstop regime. Consequently, it crowds out the endogenous backstop not only on impact but also, and more importantly, in expectation over future horizons:

$$\Delta\mathbb{E}_t\big[G_{t+h}^*(X_{t+h})\big] < 0 \qquad (h \geq 1). \tag{29}$$

When $\tilde{G}_t$ is persistent (in our benchmark, the quarterly persistence is $\rho = 0.8$), this expectation channel is amplified: the initial discretionary impulse shifts beliefs about demand and inflation for several quarters, which can trigger an earlier exit from the backstop region and hence eliminate multiple future periods of G^* activation. In such cases the cumulative reduction in endogenous backstop spending can exceed the

cumulative increase in discretionary spending,

$$\sum_{h=0}^{H} \Delta G_{t+h}^{*} \ < \ -\sum_{h=0}^{H} \Delta \tilde{G}_{t+h}, \tag{30}$$

so that total fiscal support falls, $\sum_{h=0}^{H} \Delta G_{t+h}^{\text{tot}} < 0$, even though $\tilde{G}$ increases. Output then declines relative to the baseline path, implying a negative multiplier when output is normalized by the discretionary spending change.

Importantly, this does not mean that government purchases are contractionary in the liquidity trap *per se*. Rather, it reflects a regime-specific accounting: with an active fiscal backstop, discretionary stimulus largely replaces (and can over-replace) the endogenous stabilization that would otherwise occur. Put differently, the relevant object is the multiplier with respect to *total* government purchases G^{tot}, which remains positive; the negative sign arises for the conditional multiplier of the discretionary component $\tilde{G}$ given that the backstop rule responds endogenously.

6   Concluding remarks

This paper resolves the global instability of the New Keynesian model at the Zero Lower Bound by introducing a state-contingent fiscal backing rule. We show that this "spender of last resort" mechanism restores global determinacy, ensuring that short-run ZLB dynamics are consistent with long-run equilibrium. This framework yields two key economic insights: it endogenously flattens the Phillips curve, explaining the "missing disinflation" puzzle, and it reveals a policy substitution effect where discretionary multipliers turn negative because they crowd out the endogenous fiscal support already sustaining the equilibrium.

These results open several avenues for future research. While we focus on a rep-

resentative agent framework, extending this global consistency analysis to Heterogeneous Agent New Keynesian (HANK) models could reveal how the distributional consequences of the ZLB interact with the fiscal backing rule. Furthermore, our framework provides a rigorous platform for re-evaluating optimal policy mixes in an era where the ZLB is a recurring feature rather than a rare anomaly. By treating the liquidity trap not as a model failure but as a distinct, fiscally-supported regime, we successfully bring the ZLB back within the scope of unified, global equilibrium dynamics.

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A   Appendix: Proofs

A.1   Proof of Lemma 1 (The aggregate law of motion)

Proof. The state vector is given by $X = [i_{-1}, S]$. Since the exogenous state vector S follows a finite-state Markov chain with transition matrix for preference, technology, and fiscal shocks, its transition probability $\Gamma_S(S'|S)$ is well-defined and independent of endogenous variables.

The evolution of the endogenous state variable, i_{-1}, is governed entirely by the central bank's policy rule. The Taylor rule in the model is specified as:

$$i_t = \max\{0, i_t^N\} \tag{31}$$

where i_t^N is a function of the current endogenous variables π_t and Y_t, and the lagged interest rate i_{t-1}. In a recursive equilibrium, the allocations $\pi(X)$ and $Y(X)$ are functions of the state X. Substituting these policy functions into the Taylor rule yields the law of motion for the nominal interest rate:

$$\log(1 + i') = \rho_i \log(1 + i) + (1 - \rho_i) \log\left(\ldots \pi(X) \ldots Y(X) \ldots\right) + MP(X) \tag{32}$$

Thus, the transition of the endogenous state is fully characterized by the policy rule and the equilibrium policy functions, denoted as Γ_i. ∎

A.2   Proof of Proposition 1 (Reducibility of the law of motion)

Proof. Fix an equilibrium selection ("deflationary-trap expectations") with the following property: whenever $X_t \in \mathcal{D}$ (i.e. $i_t = 0$ and $i_t^N < 0$), private-sector expecta-

tions satisfy (i) $\mathbb{E}_t[\pi_{t+1}] < \pi$ and (ii) $\mathbb{E}_t[Y_{t+1}] \leq Y_t$.

Take any $X_t \in \mathcal{D}$. Since $i_t^N < 0$, the ZLB binds and $i_t = \max\{0, i_t^N\} = 0$. Under the maintained deflationary-trap selection, expected inflation remains below target and expected activity does not improve, $\mathbb{E}_t\pi_{t+1} < \bar{\pi}$ and $\mathbb{E}_t Y_{t+1} \leq Y_t$. With $i_t = 0$, lower expected inflation raises the expected real rate relative to its target-consistent benchmark, which provides the standard NK intuition for depressed demand. The key step, however, is the maintained selection together with the Taylor rule: as long as inflation and activity remain weak, the notional rule continues to prescribe a negative shadow rate, so $i^N(X_{t+1}) < 0$ and hence $X_{t+1} \in D$.

$$\mathbb{E}_t[r_t] \equiv \mathbb{E}_t[i_t - \pi_{t+1}] = -\mathbb{E}_t[\pi_{t+1}] > -\pi. \tag{33}$$

Therefore the Taylor rule *absent* the ZLB continues to prescribe an interest rate below the target level. In particular, because the shadow rule is strictly increasing in (expected) inflation and activity,[2] and these objects remain weak under the trap selection, we obtain $i_{t+1}^N < 0$. Since the ZLB binds whenever $i_{t+1}^N < 0$, it follows that $i_{t+1} = 0$ and hence $X_{t+1} \in \mathcal{D}$. Iterating yields $X_{t+k} \in \mathcal{D}$ for all $k \geq 1$, so $\mathcal{D}$ is absorbing.

Because $\mathcal{D}$ is absorbing under this equilibrium selection while its complement is reachable under other histories/selections, the induced Markov process is reducible. Reducibility implies the process does not admit a unique invariant distribution over the full state space. ∎

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[2]This is immediate under the standard Taylor specification with $\phi_\pi > 1$ and $\phi_y \geq 0$.

A.3 Proof of Proposition 2 (Recursivity/Ergodicity under fiscal backing)

Proof. We establish ergodicity for the finite-state Markov approximation used in the quantitative analysis. Under Assumption 1, the exogenous aggregate state S is approximated by a finite-state Markov chain with transition matrix Γ_S that is irreducible and aperiodic.[3] Hence the aggregate state space for $X = [i_{-1}, S]$ is finite.

Under fiscal backing, whenever the ZLB binds $(i(X) = 0)$, the fiscal authority sets $G^*(X)$ so that the notional rate satisfies $i^N(X; G^*(X)) \geq 0$ by construction. Therefore the deflationary-trap region $D = \{X : i(X) = 0, i^N(X) < 0\}$ is not visited along equilibrium paths under the backed regime, and the induced law of motion for X evolves on a recurrent class that excludes D.

Because the state space is finite, it suffices to show that the induced Markov chain on its recurrent class is irreducible and aperiodic. Irreducibility follows from the irreducibility of Γ_S together with the fact that from any admissible X the model's equilibrium mapping and shock transitions allow the process to reach any other state in the recurrent class with positive probability in finitely many steps. Aperiodicity follows from the aperiodicity of Γ_S (e.g., strictly positive probability of remaining in the same S state) and the induced transition structure for i_{-1} on the grid. Hence the chain admits a unique invariant distribution on the recurrent class, and it is ergodic: time averages converge almost surely to expectations under that invariant distribution.

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[3]In addition, in the numerical implementation the predetermined endogenous state i_{-1} is represented on a finite grid. RTM computes equilibrium objects on a grid for i_{-1} and a finite Markov chain for S, which is the state space over which conditional expectations are evaluated.