Online appendix for "Endogenous Plucking Through Networks: The Plucking Paradox"

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A Derivations for the static equilibrium

This appendix derives the static equilibrium conditions reported in Section 2. We assume perfect competition in the simple sector and monopolistic competition for varieties sold to the retailer. All firms are symmetric within each sector.

Marginal costs. The network firm minimizes the cost of the Cobb–Douglas aggregator in (1). Let λ^N denote unit cost for a network firm and λ^S the unit cost for a simple firm. With competitive input prices and symmetry,

$$\ln \lambda^N = \alpha \ln \lambda^{N,\text{agg}} + (1-\alpha) \ln \lambda^{S,\text{agg}} - \ln A, \tag{1}$$

$$\ln \lambda^S = \ln w - \ln A. \tag{2}$$

Prices. Equating $\lambda^S, \lambda^N, \alpha$ in (2) with $\lambda^{S,\text{agg}}, \lambda^{N,\text{agg}}, \alpha^{\text{agg}}$ since the model features representative firms, we can solve for the sectoral prices as functions of A and w

$$\ln \lambda^{N,\text{agg}} = \ln w - \frac{2-\alpha^{\text{agg}}}{1-\alpha^{\text{agg}}} \ln A, \tag{3}$$

$$\ln \lambda^{S,\text{agg}} = \ln w - \ln A. \tag{4}$$

Retailer's cost minimization. The retailer faces constant-elasticity demand within each block, so the sectoral price indices satisfy

$$P = (p^{N,\text{agg}})^{\zeta} (p^{S,\text{agg}})^{1-\zeta} = \left(\frac{\sigma}{\sigma-1}\right)^{\zeta} (\lambda^{N,\text{agg}})^{\zeta} (\lambda^{S,\text{agg}})^{1-\zeta}. \tag{5}$$

Normalizing $P \equiv 1$ and substituting (1)–(2) yields

$$\ln w = \left(\frac{\zeta}{1-\alpha^{\text{agg}}} + 1\right) \ln A - \zeta \ln \frac{\sigma}{\sigma-1}, \tag{6}$$

which is Equation (9) in the text.

Accounting and profits. Let TC denote total cost of network intermediates used by network firms. A fraction αTC is network goods and $(1-\alpha)TC$ is simple goods. Total cost of simple goods is wL, so the retailer spends $wL-(1-\alpha)TC$ on simple goods and, by CES expenditure shares, $\frac{\zeta}{1-\zeta}(wL-(1-\alpha)TC)$ on network goods. Converting retailer spending on network goods into costs using the sectoral markup implies

$$\alpha TC + \frac{\zeta}{1-\zeta}\frac{\sigma-1}{\sigma}\big(wL-(1-\alpha)TC\big) = TC, \tag{7}$$

so

$$TC = \frac{\zeta(\sigma-1)}{(1-\alpha)(\sigma-\zeta)}wL. \tag{8}$$

Retailer spending on network goods is then $\frac{\sigma\zeta}{\sigma-\zeta}wL$, and network profits equal revenue minus cost,

$$\pi^N = \frac{\zeta}{\sigma-\zeta}wL, \tag{9}$$

while perfect competition in the simple sector implies

$$\pi^S = 0. \tag{10}$$

Finally, total consumption equals total revenue,

$$C = \frac{\sigma}{\sigma-\zeta}wL = wL + \pi^N, \tag{11}$$

which matches the aggregate consumption defined in (8).

Firm-specific profits. Let $X = [\alpha^{\text{agg}}, A]$ and consider a firm that chooses its own network intensity α while taking aggregate prices as given. Using (1) with aggregate prices (3)–(4) and (6), the firm's unit cost satisfies

$$\ln \lambda^N(\alpha; X) = \left(\frac{\zeta - \alpha}{1 - \alpha^{\text{agg}}} - 1\right) \ln A - \zeta \ln \frac{\sigma}{\sigma - 1}, \tag{12}$$

The aggregate $\lambda^{N,\text{agg}}(X)$ satisfies

$$\ln \lambda^{N,\text{agg}}(X) = \left(\frac{\zeta - 1}{1 - \alpha^{\text{agg}}}\right) \ln A - \zeta \ln \frac{\sigma}{\sigma - 1}. \tag{13}$$

Under CES demand, revenue is proportional to $(p(\alpha; X)/p^{N,\text{agg}}(X))^{1-\sigma}$, implying

$$\begin{aligned}
\pi^N(\alpha; X) &= \pi^{N,\text{agg}}(X) \left(\frac{p(\alpha; X)}{p^{N,\text{agg}}(X)}\right)^{1-\sigma} && (14)\\
&= \pi^{N,\text{agg}}(X) \left(\frac{\lambda^N(\alpha; X)}{\lambda^{N,\text{agg}}(X)}\right)^{1-\sigma} && (15)\\
&= \frac{\zeta}{\sigma - \zeta} \left(\frac{\sigma - 1}{\sigma}\right)^{\zeta} A^{\frac{\zeta + (1-(1-\delta)\alpha)(1-\sigma)}{1-(1-\delta)\alpha^{\text{agg}}} + \sigma} L, && (16)
\end{aligned}$$

which is consistent with the main text.

B Derivations for the optimal fiscal policy

This appendix derives the tax rule that decentralizes the planner allocation by equating the competitive-equilibrium (CE) and social-planner (SPP) Euler equations.

Competitive Equilibrium Euler Equation. Let $\tilde{c}(S)$ denote household consumption in state S. The firm Euler in CE with a proportional tax/subsidy τ on network profits is

$$\mathbb{E}\left[\beta \frac{(\tilde{c}'(S'))^{-\rho}}{(\tilde{c}(S))^{-\rho}} \left(\pi_1(\alpha'; S')(1 + \tau') - \Phi_1(\alpha', \alpha'')\right) \middle| S\right] = \Phi_2(\alpha, \alpha') - \lambda_{CE} + \mu_{CE}, \tag{17}$$

where, using the static equilibrium,

$$w(S) = \left(\frac{\sigma-1}{\sigma}\right)^{\zeta} A^{\frac{\zeta}{1-(1-\delta)\alpha}+1}, \tag{18}$$

$$n(S) = \left(\frac{w(S)}{\eta}\right)^{\chi} = \eta^{-\chi}\left(\frac{\sigma-1}{\sigma}\right)^{\zeta\chi} A^{\left(\frac{\zeta}{1-(1-\delta)\alpha}+1\right)\chi}, \tag{19}$$

$$\pi(\alpha; S) = \frac{\zeta}{\sigma-\zeta}\left(\frac{\sigma-1}{\sigma}\right)^{\zeta} A^{\frac{\zeta}{1-(1-\delta)\alpha}+1} n(S). \tag{20}$$

Thus

$$\pi_1(\alpha; S) = \pi(\alpha; S)\,\log(A)\,\frac{(\sigma-1)(1-\delta)}{1-(1-\delta)\alpha}. \tag{21}$$

$$= \frac{\zeta}{\sigma-\zeta}\left(\frac{\sigma-1}{\sigma}\right)^{\zeta} A^{\frac{\zeta}{1-(1-\delta)\alpha}+1}\eta^{-\chi}\left(\frac{\sigma-1}{\sigma}\right)^{\zeta\chi} A^{\left(\frac{\zeta}{1-(1-\delta)\alpha}+1\right)\chi}\log(A)\,\frac{(\sigma-1)(1-\delta)}{1-(1-\delta)\alpha}, \tag{22}$$

$$= \frac{\zeta}{\sigma-\zeta}\left(\frac{\sigma-1}{\sigma}\right)^{\zeta(1+\chi)} \eta^{-\chi} A^{\left(\frac{\zeta}{1-(1-\delta)\alpha}+1\right)(1+\chi)}\log(A)\,\frac{(\sigma-1)(1-\delta)}{1-(1-\delta)\alpha}, \tag{23}$$

Planner's Euler Equation. The SPP Euler equation is

$$\beta\mathbb{E}\Bigg[(\tilde{c}')^{-\rho}\Bigg(\left(\frac{\sigma}{\sigma-\zeta}\right)^{1+\chi}\left(\frac{\sigma-1}{\sigma}\right)^{\zeta(1+\chi)}\eta^{-\chi}(A')^{\left(\frac{\zeta}{1-(1-\delta)\alpha'}+1\right)(1+\chi)}\log(A')\,\frac{\zeta(1-\delta)}{(1-(1-\delta)\alpha')^{2}}$$
$$-\,\Phi_1(\alpha', \alpha'')\Bigg)\Bigg]$$
$$= (\tilde{c})^{-\rho}\Phi_2(\alpha, \alpha') - \lambda_{SP} + \mu_{SP}. \tag{24}$$

Tax rule. Equating (17) and (24) (and matching multipliers via $(\tilde{c})^{\rho}\lambda_{SP} = \lambda_{CE}$ and $(\tilde{c})^{\rho}\mu_{CE} = \mu_{SP}$) yields

$$1+\tau' = \frac{\sigma-\zeta}{\sigma-1}\left(\frac{\sigma}{\sigma-\zeta}\right)^{1+\chi}\frac{1}{1-(1-\delta)\alpha'} \tag{25}$$

Budget balance. The subsidy is rebated lump-sum to households. Using $c = wn + \pi(1+\tau) - \Phi(\alpha, \alpha') - \tau\pi$, the resource constraint reduces to

$$c = wn + \pi - \Phi(\alpha, \alpha'), \tag{26}$$

so the tax leaves aggregate resources unchanged.