Sorting between Real and Financial Constraints: Macroeconomic Implications*

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Abstract

A simple model predicts that financial intermediaries penalize firms with a greater fixed adjustment cost. We show the model prediction is consistent with the observed sorting patterns between the real and financial frictions using a unique data set covering the universe of Portuguese firms. Then, we incorporate the different cost structures and financial frictions into the heterogeneous-firm general equilibrium model to capture the observed sorting pattern. Using the model, we analyze how the recently strengthened sorting pattern affects capital misallocation and aggregate shock sensitivity.

Keywords: Financial frictions, Adjustment cost, Firm dynamics, Misallocation.

JEL Codes: D53, E44, D21

* All errors are our own.

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1 Introduction

A substantial amount of research has been focused on how firm's characteristics influence the borrowing limits the firm faces. Kiyotaki & Moore (1997) highlight how the amount of capital owned by the firm can be used as collateral to determined the total amount a firm can borrow. With the increase in intangible assets, some literature has also focused on how the different types of assets have distinct collateral values, which ends up affecting the financial conditions of the firm. More recently, Lian & Ma (2021) show that the borrowing limits are more dependent on the firm's cash flow than on the collateral the firm can provide. In this paper we show that the firm's cost structure also influences its borrowing limits.

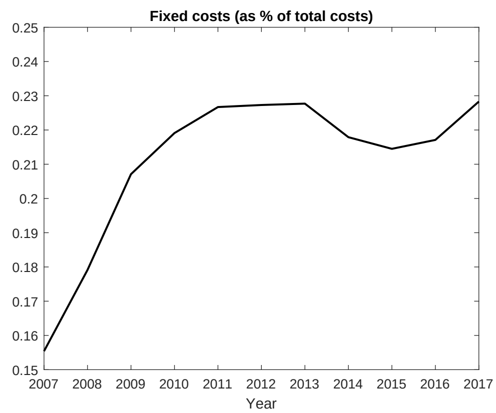

Figure 1: Fixed costs, as % of total costs, evolution over time.

In Figure 1 we can see that fixed costs, as a percentage of total costs, in the Portuguese economy has been increasing over the last decade.[1] This phenomenon is not unique to the Portuguese economy, as De Ridder (2019) finds similar patterns in the US and in France.

This paper provides evidence on the sorting pattern between the firms' cost structure and financial frictions, how both affect the firms' investment decisions and end up amplifying recessions during periods of turmoil.

[1] We follow De Ridder (2019) to calculate the share of fixed costs empirically. We use the micro data for the universe of Portuguese firms to find the average for the Portuguese economy.

We start by illustrating in a simple setting how the interaction between the real and financial frictions affects both the intensive and extensive margin of investment. In a setting where firms choose investment to maximize profits and face both convex and fixed adjustment costs of capital, as well as a borrowing limit, we illustrate two mechanisms: 1) the interaction between financial and real frictions affects the firms investment decision in the intensive and extensive margin; 2) this interaction is stronger, the larger the fixed cost. This happens has the presence of the fixed cost deters firms from investing. Only the highly productive firms will find it profitable to pay the fixed cost to invest. In the presence of the borrowing constraint, the amount a firm can invest if active is reduced, which will deter even more firms from investing, as it is not profitable to pay the fixed cost.

In addition to the mechanism that illustrates how financial frictions affect more firms facing fixed costs, we showcase that the borrowing constraints are also tighter for these firms. A simple bank problem micro-founding the firm's borrowing limit illustrates that banks are less willing to lend to firms with high fixed costs as they have a lower probability of actively investing. This creates a sorting pattern between real and financial frictions, with firms that face higher fixed costs being on average more financially constrained.

On top of this, with fixed costs increasing, the type of firms that will be more affected by financial constraints equally changes. While fixed costs are low, firms affected by the financial frictions are mainly at the bottom of the productivity distribution. When fixed costs start increasing, the more affected firms become the ones in the second half of the productivity distribution. For firms in the first half of the distribution the real constraint binds before the financial one, causing firms not to be affected by the presence of a borrowing limit. This is not the case for firms in the top half of the productivity distribution, where the financial constraint is the one that binds.

Next, using a unique data set covering both the balance sheet and credit situation of the universe of Portuguese firms, we take the predictions of the simple model to the data. We follow Ferreira et al. (2021) in identifying financially constraint firms. First, we validate an important model assumption, which is that fixed and variable costs are negatively correlated, and that firms are either high fixed cost type or high variable cost type.

Next, we find that a larger share of fixed costs as a percentage of total costs is positively and strongly correlated with the the percentage of constrained firms. This means that, firms that have a cost structure more dependent on fixed costs are on average more financially constrained. We equally find that firms with a lumpier investment profile are on average more constrained, in line with model predictions.

Lastly, we assess if the correlation between productivity and constrained firms depends on the share of fixed costs. Similar to model predictions, we find that for low fixed costs firms, the firms more affected by financial frictions are the ones at the bottom of the TFP distribution. For firms with high fixed costs, we find that firms in the top half of the TFP distribution are equally affected by financial frictions, in line with model predictions.

Then, making use of a heterogeneous-firm general equilibrium model, which includes the theoretical mechanisms from the simple model, we test how the sorting between real and financial frictions matter in the aggregate. The model generates a sorting pattern comparable to the empirical one, validating our structural approach.

2 Simple Theory

In this section we present a simple firm and bank model. The objectives are twofold: 1) at the firm level, show the interaction between real and financial frictions; 2) at the bank level, illustrate why banks are less willing to lend to firms which face higher fixed costs.

2.1 Firm's investment problem

A firm with productivity z and capital k, chooses investment I in order to maximize the return on the investment. The firm-level investment problem is defined as follows:

$$I^*(z,k) = \arg\max_I -I - \frac{\mu}{2}\left(\frac{I}{k}\right)^2 k + \frac{1}{R} y(E(z'|z), k(1-\delta)+I)$$

where μ is the parameter governing the convex adjustment costs and y is the production function of the firm, which takes the form of a Cobb-Douglas where capital is the only input

$$y(z,k) = zk^\alpha \tag{1}$$

Define the expected net benefit with and without the investment as follows:

$$J^*(z,k) := -I^*(z,k) - \frac{\mu}{2}\left(\frac{I^*(z,k)}{k}\right)^2 k + \frac{1}{R}\mathbb{E}_z z'(k(1-\delta)+I^*(z,k))^\alpha$$
$$J^c(z,k) := \frac{1}{R}\mathbb{E}_z z'(k(1-\delta))^\alpha$$

We assume a (large-scale) investment incurs in a fixed cost $\xi \sim_{iid} Unif([0,\overline{\xi}])$.[2] Then, a firm needs to decide whether to invest or not as follows:

$$\max\{J^*(z,k) - \xi, J^c(z,k)\}$$

The condition under which a firm prefers investment is whenever the value of investing minus the fixed cost paid is larger than the value of not investing

$$J^*(z,k) - \xi > J^c(z,k),$$

which is equivalent to

$$J^*(z,k) - J^c(z,k) > \xi.$$

[2]We follow other papers in the literature and define large-scale investment when investment is greater than 20% of the beginning of period capital. See for example Cooper & Haltiwanger (2006), Gourio & Kashyap (2007) or Khan & Thomas (2008).

We characterize this cut-rule for ξ, defining ξ^* as follows:

$$\xi^*(z,k) := J^*(z,k) - J^c(z,k)$$

As such, the optimal investment policy of the firm will be defined by

$$\widehat{I}(z,k) = \begin{cases} I^* & \text{if } \xi \leq \xi^* \\ 0 & \text{if } \xi > \xi^* \end{cases} \tag{2}$$

The condition of making large-scale investment has a probabilistic interpretation as in Khan & Thomas (2008).

$$\xi \leq \xi^* \iff \xi \in [0, \xi^*],\ \xi \sim_{iid} Unif[0, \overline{\xi}]$$

Therefore, the probability of making large-scale investment p^* is defined as:

$$p^* := \mathbb{P}(\widehat{I} = I^*) = \frac{\min\{\xi^*, \overline{\xi}\}}{\overline{\xi}}$$

The minimum operator in the numerator is needed as ξ cannot be greater than $\overline{\xi}$ even if $\xi^* > \overline{\xi}$. Also, only with the minimum operator in the numerator, the right-hand side becomes weakly less than unity, consistent with the notion of the probability in the left-hand side.

Therefore,

$$\widehat{I} = \begin{cases} I^* & \text{with prob. } p^* = \frac{\min\{\xi^*, \overline{\xi}\}}{\overline{\xi}} \\ 0 & \text{with prob. } 1 - p^* \end{cases}$$

Standard borrowing limit: Before micro-founding the firm's borrowing limit with the financial intermediary's problem, let us assume a standard borrowing limit a la Kiyotaki & Moore (1997), where a firm can borrow up until a share θ of its capital

$$b = \theta k \tag{3}$$

From here, the firm's investment, if active, is defined as

$$I^{ex-post} = \begin{cases} I^* & \text{if } zk^\alpha + (1-\delta)k + b \geq I^* + \xi + \frac{\mu}{2}\left(\frac{I^*(z,k)}{k}\right)^2 k \\ I(b) & \text{if } zk^\alpha + (1-\delta)k + b < I(b) + \xi + \frac{\mu}{2}\left(\frac{I(b)}{k}\right)^2 k \end{cases}$$

if the firm's internal financing $zk^\alpha + (1-\delta)k$ plus the borrowing is enough to pay for the costs of implementing the optimal investment I^* the firm will do it. Otherwise, it will invest $I(b)$, that is the investment for which the firm exhaust all of its resources.

From this follows that the firm's investment policy is defined as

$$\widehat{I} = \begin{cases} I^{ex-post} & \text{with prob. } p^* \\ 0 & \text{with prob. } 1-p^* \end{cases}$$

Interaction between real and financial frictions: The previous firm investment problem joint with a standard borrowing limit already creates an interaction between the two frictions in the model. When fixed costs are higher, firms will have a harder time getting funding to pay the fixed cost and implement the optimal investment, which will decrease the probability of these firms being active. For high convex adjustment costs, firms will be equally limited by the borrowing limit in financing both the investment and the adjustment cost, which will decrease the size of the investment. These two forces result in Proposition 1.

Proposition 1 *If $I^* > I(b)$, financial frictions will have an impact both on the intensive and extensive margin of investment.*

Proof: The proof is provided in Appendix A.1.

The intuition for this proposition is as follows. If the firm is active, the amount it will invest is either when the financial or the real friction is binding. If the convex adjustment cost is too strong, then $I^* < I(b)$ and there is no interaction between the real and financial frictions.

The opposite case is the interesting one. When $I^* > I(b)$, then if the firm is active it will invest until the borrowing constraint is binding. In this case, given that the amount of investment the firm can do is limited, it will only be profitable for the firm to be active if it is highly productive. So, the financial friction, ends up not only affecting the amount invested when the firm is active, but also causes some firms not to invest at all.

Despite the impact that the borrowing limit has on both the intensive and extensive margins of investment, there is no sorting between the two forces when using a standard borrowing limit, as both high fixed costs or high variable costs firms may be financially constraint.

2.2 Financial intermediary's problem

Consider a firm's optimal lumpy invest decision as follows:

$$\widehat{I} = \begin{cases} I^* & \text{with prob. } p^* \\ 0 & \text{with prob. } 1 - p^* \end{cases}$$

A bank that is incentive-aligned with the firm provides funding b to the firm. Due to the competitive nature of the market, the bank receives zero net utility out of this lending as follows:

$$u(b) = p^* u(I^*) + (1 - p^*) u(0)$$

where u(b) is the utility a financial intermediary gets from lending b, with u being differentiable and concave. The utility the bank gets from lending to a firm will depend on the probability of the firm being active and implementing the investment p^*, and also on the size of the investment itself I^*.

Sorting between real and financial frictions: Given that the bank's problem will depend on the probability of a firm being active, this will immediately create a sorting between the real and financial frictions. As firms with a higher fixed costs of investment are less likely to be active, this will cause this firms to have a tighter borrowing limit. This result is captured in the following proposition

Proposition 2 *A high fixed cost firm (low p^*) has a lower borrowing limit.*

Proof: The proof is provided in Appendix A.2.

As such, firms with higher fixed costs are more likely to be financially constrained. If this happens, even if the firms are active they will not be able to implement their optimal investment, which will decrease even more the probability of investing and consequently the borrowing limit.

The borrowing limit equally depends on how productive a firm is. On average, more productive firms have a higher probability of being active and will invest more, which loosens the borrowing limit. However, while this effect is linear for firms with a low fixed costs (high p^*), it may become nonlinear for high fixed cost firms. Imagine a firm with high fixed costs and low productivity. This firm has a zero probability of investing and so the real constraint is binding and it is not affected by the financial constraint. For firms that are more productive, with a strictly positive probability of investing, then the productivity positively affects the borrowing limit and highly productive firms are on average less constrained. This is capture in the following proposition

Proposition 3 *High productivity firms are on average less financially constrained. The exception happens for high fixed cost firms with low productivity, which have a zero probability of investing. For these firms the real constraint is binding before the financial constraint.*

Proof: The proof is provided in Appendix A.3.

2.3 Quadratic utility example

To provide more intuition on the previous results we know assume that the utility function of the financial intermediary takes the following form:

$$u(x) = -\gamma(x-\psi)^2 + \gamma\psi^2$$

where x is the amount the bank lends to the firm, γ governs the utility gain from the loan and ψ is the lending limit of the bank. Here we are assuming that the bank does not have infinite resources and can only lend up to the amount ψ. From here we get that the firm's borrowing limit will be

$$b = \psi - \sqrt{p^*(I^* - \psi)^2 + (1-p^*)\psi^2}$$

which depends positively both on the probability of investing and the amount of investment.

In Figure 2 we can see the effects described in Propositions 2 and 3. On both panels credit-rationing is plotted on the z-axis. Credit-rationing is measured as the share of internal financing the firm needs in order to implement the optimal investment. On the x-axis both panels have productivity. The left-hand side panel captures how credit rationing changes with the maximum threshold for the fixed cost shock $\overline{\xi}$ for a standard level of the convex adjustment cost parameter $\mu = 1$, while the right-hand side panel plots the credit rationing depending on the fixed cost parameter μ when $\overline{\xi} = 0.8$.

The main takeaway from the picture is very clear, in an economy that is moving towards a cost structure more dependent on fixed costs, firms are more likely to be constrained. Moreover, it is not the firms at the bottom of the productivity distribution that are going to be more affect by financial constraints as these firms will be inactive. The firms in the middle part of the distribution are the ones that will be more affect by the rise in the fixed costs and become more financially constrained. For low levels of fixed costs, even if the convex adjustment costs

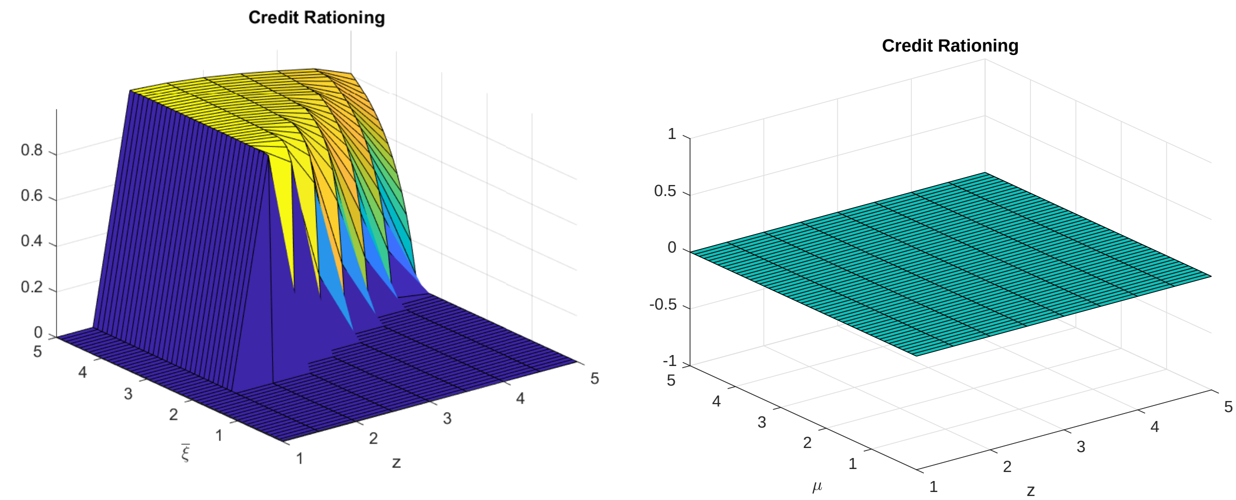

Figure 2

raise, the firms are less likely to be financially constrained as the real constrained binds before the financial one.

3 Data

We draw on a unique combination of data sets that cover the Portuguese economy between 2006 and 2017, all managed by the Bank of Portugal Microdata Research Laboratory.

The *Informação Empresarial Simplificada* (IES) Central Balance Sheet Database (CBSD) is based on annual accounting data of individual firms. Portuguese firms have to fill out mandatory financial statements in order to comply with their statutory obligation. Consequently, this data set covers the population of virtually all non-financial corporations in Portugal from 2006 onward. We combine this data set with the Central Credit Register (CCR), which contains monthly information on the actual and potential credit above 50 euros extended to individuals and non-financial corporations, reported by all financial institutions in Portugal.[3] Actual credit includes loans that are truly taken up, such as mortgages,

[3]Given that the firm balance sheet data is of yearly frequency, we consider the month in which

Table 1: Descriptive statistics of Portuguese firms between 2006 and 2017

Variable	Mean	Median	Std. Dev.	Size group median			
				0 - 90th	90th-99th	99-99.5th	>99.5th
Total Assets (€ mio.)	3.48	0.31	92.92	0.26	5.34	45.44	145.00
Turnover (€ mio.)	2.06	0.25	35.74	0.22	3.45	20.73	29.07
Potential credit (€ mio.)	0.21	0.01	4.76	0.00	0.16	1.01	3.12
Effective credit (€ mio.)	0.57	0.05	6.18	0.04	1.22	7.34	13.00
Leverage	0.28	0.20	0.41	0.20	0.24	0.16	0.08
Liquidity ratio	0.13	0.06	0.19	0.06	0.02	0.01	0.01
Age	16	13	12	12	21	23	22
Employees	15	5	140	4	26	98	99
Bank relationships	3	2	2	2	4	4	5

consumer loans, overdrafts, and others. Potential credit encompasses all irrevocable commitments to the subject that have not materialized into actual credit, such as available credit on credit cards, credit lines, pledges granted by participants, and other credit facilities. We then merge these two databases on the firm level. Moreover, we also add the Monetary Financial Institutions Balance Sheet Database in order to gain information on the balance sheets of banks that extend credit to non-financial institutions. We merge this data set on a firm level using the bank identifier and the share of loans extended by one firm to arrive at our detailed data set.

We restrict the set of firms in this panel data set to those with at least five consecutive observations and to firms that are in business at the time of reporting. Furthermore, we only consider privately or publicly held firms and drop micro firms, i.e., those with overall credit amounts of less than 10,000 €. Descriptive statistics for the relevant variables can be found in Table 1.

3.1 Measures of financial constraints

We follow Ferreira et al. (2021), who use the credit information in the data to construct several binary measures indicating whether a firm is financially con-

the balance sheet data was reported. Results were robust to shifting and averaging the monthly credit data.

strained. Financial constraints are most commonly conceived as a supply side phenomenon. Firms that could potentially obtain credit in perfect credit markets are unable to do so due to asymmetric information considerations on the supply side. For example, a firm that has a profitable investment project that requires external financing cannot realise it as the bank is not satisfied with the creditworthiness of that firm. This may happen either via the price dimension, ie. a risk premium on the interest rate, or on the quantity dimension ie. the credit is denied altogether.

In this paper, we classify constrained firms along the quantity dimension, using the credit information for each firm. Given that credit allowances are changing over time, this provides us with a time-varying and firm-specific measure for being financially constrained. It should be noted, however, that while credit information offers a far more detailed notion of a firm being constrained compared to standard financial ratios such as leverage or liquidity, it is still a proxy.

Measure As outlined above, potential credit summarizes all the irrevocable commitments by credit institutions. Even though this measure enables an understanding of whether firms have drawn down their credit lines and are thus potentially constrained it also encompasses a lot of noise. One problem might be that although firms have exhausted their committed credit line they could still successfully apply for a short- or long-term loan. To account for this, in our baseline definition, we consider a firm to be credit constrained at time t, if it has no potential credit available at time t and neither its short- nor long-term credit (ie. effective credit) is growing:

$$\text{Constrained I} := \mathbf{1}_{\text{Potential credit}_t = 0 \ \& \ \Delta\text{Effective credit}_t \leq 0}.$$

Fixed cost One important dimension that is not directly reported by the firms are the fixed and variable costs. To estimate the fixed costs we follow De Ridder (2019) and identify fixed costs as

$$\frac{f_{it}}{py_{it}} = \left(1 - \frac{1}{\mu_{it}}\right) - \frac{\pi_{it}}{py_{it}} \tag{4}$$

where f_{it} is the fixed cost of firm i in year t, py_{it} is revenues and π_{it} the firm's profits. μ_{it} is the firm's markup, which is calculated following De Loecker et al. (2020). For more details please see Appendix B. The idea is that the profit ratio captures the average profits of the firms, while the markup captures the marginal profit. The two only differ in the presence of fixed costs.

3.2 Empirical Results

Using the aforementioned data, we test the implications of the simple model. First, we validate an important assumption we make when testing the implications of the model: that fixed and convex adjustments costs are negatively correlated. This is, that firms do not have both fixed and convex adjustments costs both high or low at the same time. To test this assumption empirically we measure both fixed and variable costs as a share of the firm's turnover. Figure 3 validates our assumption that fixed and variable costs are negatively correlated.

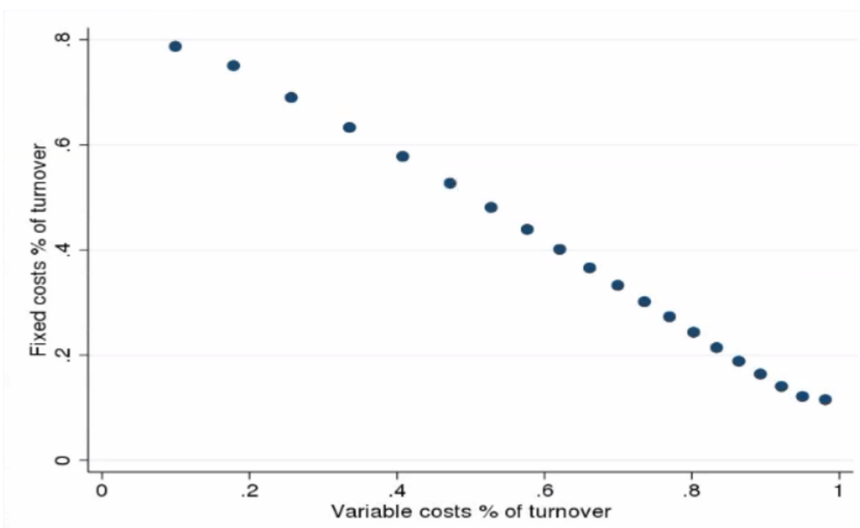

Figure 3: On the y-axis the fixed costs as a % of turnover. On the x-axis variable costs as a % of turnover.

We are now ready to test the predictions of the simple model. Following the predictions from the model, we test if firms with a higher share of fixed costs (as a % of total costs) are more financially constrained. Figure 4 shows that, in line with the simple model predictions, firms with higher fixed costs, as a % of total costs, are on average more financially constraint.

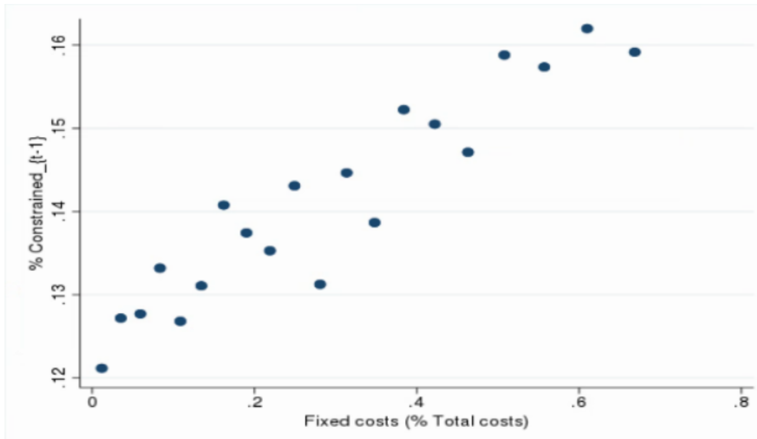

Figure 4: On the y-axis the % of the constrained firm at the end of period $t-1$. On the x-axis fixed costs, as a % of total costs.

Next we check if the firm's investment profile is equally in accordance with the model predictions and if it is correlated with the financial situation of the firm. From the simple model we know that firms with a more lumpy investment will invest less often, as the threshold for a firm to become active is higher. In the data, we follow the literature and classify a firm as active if it has an investment rate above 20% in any given period. On the right panel of Figure 5 it is possible to observe that, indeed, firms that have a lumpier investment profile on average are active for fewer periods. At the same time, these firms, are, on average, more financially constrained, as can be seen on the left panel.

Lastly, we test if the relation between TFP and constrained firms depends on the share of fixed costs, as predicted by the model. We estimate TFP following Ackerberg et al. (2015). In Figure 6 on the right-hand side it is possible to observe the strong and negative correlation between TFP and share of constrained firms, when fixed costs are low, whit the more productive firms being less financially constrained. On the left-hand side, the non-linear relation between TFP and financial constraints for high fixed costs firms is plotted. When fixed costs are high, the firms in the second half of the productivity distribution are more affected by financial constraints, as predicted by the model.

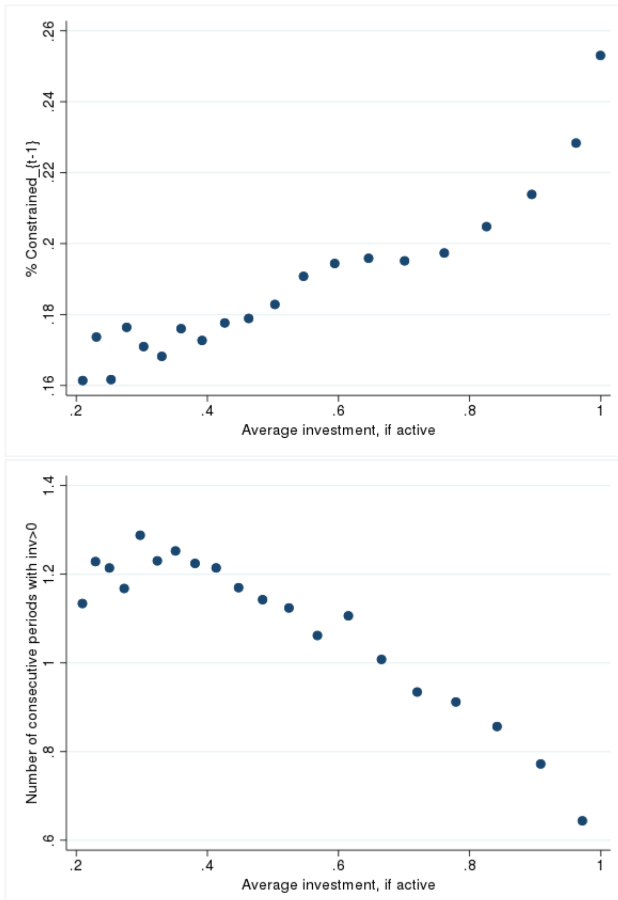

Figure 5: On the x-axis in both figures, the investment rate is conditional on a firm being active. On the left panel, on the y-axis, the % of constrained firms, and on the right panel, the number of consecutive periods a firm is active.

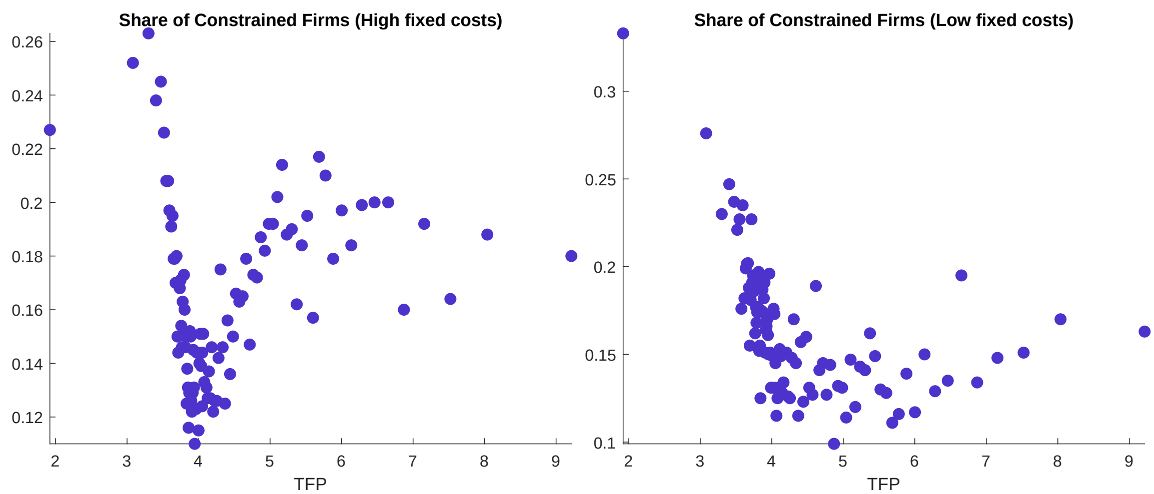

Figure 6: On the x-axis in both figures, the estimated Total Factor Productivity (TFP) at the firm level, following Ackerberg et al. (2015). On the left-hand side the share of constrained firms with high fixed costs (firms in the top quartile of the fixed cost distribution). On the right-hand side the share of constrained firms with low fixed costs (firms in the bottom quartile of the fixed cost distribution).

4 Baseline Model

In this section, we describe the full quantitative baseline model that includes the theoretical mechanism described in the previous section. We consider a heterogeneous-firm general equilibrium model, where a measure of one of the heterogeneous firms belongs to two different types: F (fixed cost) or C (convex adjustment cost) types. Each type is distinguished from the other by the nature of its real friction in capital adjustment: F type firms with greater fixed adjustment costs and C type firms with greater convex adjustment costs. This difference leads to a difference in financial frictions, which we call as the sorting between the real and financial frictions. A detailed explanation of the types follows after the production function. In the model, time is discrete and lasts forever.

4.1 Production Technology

At the beginning of each period, a firm is given the capital stock k_t that is determined from the previous period's investment decision and an idiosyncratic productivity z_t. The firm (manager of the firm) is aware of the distribution Φ_t of the individual states (k_t, z_t) and the aggregate productivity A_t, rationally expecting the dynamics of these aggregate allocations. Each firm operates using the Cobb-Douglas production function, where the inputs are labor and capital stock. The labor demand n_t is determined in the static operating profit maximization problem as follows:

$$\max_{n_t} A_t z_t k_t^{\alpha} n_t^{\gamma} - w_t n_t$$

where w_t is wage at period t. The logged idiosyncratic productivity follows an AR(1) process that is discretized by the Tauchen method:

$$log(z_{t+1}) = \rho_z log(z_t) + \sigma_z \epsilon_{t+1}, \quad \epsilon_{t+1} \sim_{iid} N(0,1).$$

Similarly, the aggregate productivity process also follows an AR(1) process, which we discretize in the quantitative analysis:

$$log(A_{t+1}) = \rho_A log(At) + \sigma_A \epsilon_{t+1}, \quad \epsilon_{t+1} \sim_{iid} N(0,1).$$

4.2 Investment decision and real and financial constraints

Each firm owns the capital stock and makes an investment decision. Each firm is subject to two real frictions: a fixed adjustment cost and a convex adjustment cost. The fixed adjustment cost occurs when the investment is beyond a range Ω that is proportional to the existing capital stock. The Ω is defined as follows:

$$\Omega = [-\nu k, \nu k].$$

A capital adjustment is subject to convex adjustment cost regardless of the size.

There are two types of firms in the economy: F (fixed cost) type and C (convex adjustment cost) type. Each firm's type follows an exogenous Markov process, represented by the following transition kernel Γ_{type}:

$$\Gamma_{type} = \begin{bmatrix} p_{type} & 1 - p_{type} \\ 1 - p_{type} & p_{type} \end{bmatrix}$$

F-type firms are subject to a greater fixed adjustment cost parameter than C-type firms: $\overline{\xi}_F > \overline{\xi}_C$. For each firm, the fixed cost is assumed to follow an $i.i.d.$ random uniform shock:

$$\xi \sim_{iid} Unif([0, \overline{\xi}_j]), \quad j \in \{F, C\}$$

Therefore, the expected fixed adjustment cost is greater for F-type firms than C-type firms. We assume the fixed cost is a labor overhead cost, so the total cost is computed by combining the wage and the fixed cost, $w\xi$ (Khan & Thomas, 2008). In contrast, F-type firms' convex adjustment cost parameter is smaller than C-type firms' convex adjustment cost parameter: $\mu_F < \mu_C$. The convex adjustment cost follows the conventional form in the literature:

$$\frac{\mu_j}{2}\left(\frac{I}{k}\right)^2 k, \quad j \in \{F, C\}.$$

On top of the two real constraints, firm-level investments are subject to financial constraints. Following Lian & Ma (2021), we assume financial intermediaries impose the cash-flow-based borrowing limits:

$$I \leq \theta_j \pi(k, z; S), \quad j \in \{F, C\},$$

where $\pi(k, z; S)$ is the operating profit of a firm with capital stock k and productivity z in the aggregate state S. Importantly, we assume financial intermediaries discriminate F-type firms due to the illiquid nature of the investment: $\theta_F < \theta_C$. Although we do not explicitly model the intermediary sector in the full baseline model, the exogenously designed sorting between financial and real frictions that captures the intermediaries' incentives helps properly capture the observed patterns in the firm-level investments.

4.3 Household

We close the model by introducing the representative household, who consumes, saves, and supplies labor. The household specification closely follows Khan & Thomas (2008) and Bachmann et al. (2013). Specifically, we assume a log utility and disutility for indivisible labor supply in the following form:

$$log(c) - \frac{\eta}{1+\frac{1}{\chi}} L^{1+\frac{1}{\chi}},$$

where c is consumption; L is the labor supply; η is the scaling parameter; χ is the Frisch elasticity of the labor supply. In the current calibration, we assume $\chi \to \infty$, but we plan to adopt the level in the empirically supported range in future work.

The recursive formulation of the household's problem is as follows:

$$
\begin{aligned}
V(a;S) = \max_{c,a',L} \quad & log(c) - \frac{\eta}{1+\frac{1}{\chi}} L^{1+\frac{1}{\chi}}, + \beta \mathbb{E} V(a';S') \\
\text{s.t. } & c + \int \Gamma_{A,A'} q(S,S') a'(S') dS' = w(S) l_H + a(S) \\
& G_\Phi(S) = \Phi', \quad \mathbb{P}(A'|A) = \Gamma_{A,A'}, \quad S = \{\Phi, A\}
\end{aligned}
$$

where a is the state-contingent equity portfolio value; A is the aggregate productivity; Φ is the joint cumulative distribution of the individual state variable; q is the state-contingent price; Γ is the transition kernel of the aggregate productivity; G_Φ is the expected dynamics of the individual state distribution Φ.

From the first order condition with respect to state contingent saving a', we characterize the state price as follows:

$$q(S,S') = \beta \frac{C(S)}{C(S')}.$$

As in Khan & Thomas (2008), we define $P(S) := 1/C(S)$, which we use for normalizing the firm's value function for easier computation.

4.4 Recursive Formulation of the firm's problem

In the analysis of the baseline model economy, we assume a stationary environment, $S = \{A, \Phi\}$, where $A(=1)$ is the fixed aggregate productivity, and Φ is the stationary distribution of the individual firms. However, in the business cycle analysis, we analyze the recursive competitive equilibrium where the aggregate productivity shocks generate the dynamics in the aggregate state variables.

A firm is given with a type $j \in \{F, C\}$, and there are type-specific fixed cost $\overline{\xi}_j$, convex adjustment cost μ_j and financial constraint θ_j.

$$\begin{aligned}
J(k,z,j;S) = \quad & \pi(k,z;S) + (1-\delta)k \\
& + \int_0^{\overline{\xi}_j} \max\left\{R^*(k,z;S) - \xi w(S), R^c(k,z,j;S)\right\} dG_\xi(\xi) \\
R^*(k,z,j;S) = & \max_{k' \in \Theta} \; -k' - c(k,k',j) + \mathbb{E}q(S)J(k',z',j';S) \\
R^c(k,z,j;S) = & \max_{k^c \in \Omega \cap \Theta} \; -k^c - c(k,k^c,j) + \mathbb{E}q(S)J(k^c,z',j';S)
\end{aligned}$$

The following lines explain the details of each component in the value function.

$$\begin{aligned}
\text{(Operating profit)} \quad & \pi(z,k;S) := \max_{n_d} zAk^\alpha n_d^\gamma - w(S)n_d \ (n_d\text{: labor demand}) \\
\text{(Convex adjustment cost)} \quad & c(k,k',j) := \left(\mu_j^I/2\right)\left((k' - (1-\delta)k)/k\right)^2 k, \quad j \in \{F,C\} \text{ and } \mu_F < \mu_C \\
\text{(Fixed adjustment cost)} \quad & \xi \sim_{iid} Unif[0,\overline{\xi}_j], \quad j \in \{F,C\} \text{ and } \overline{\xi}_F > \overline{\xi}_C \\
\text{(Real constraint of investment)} \quad & \Omega(k;\nu) := [-k\nu, k\nu] \quad (\nu < \delta) \\
\text{(Financial constraint of investment)} \quad & \Theta(k,z,j;\theta) := (-\infty, \theta_j\pi(k,z) + (1-\delta)k], \quad j \in \{F,C\} \text{ and } \theta_F < \theta_C \\
\text{(Idiosyncratic productivity)} \quad & z' = G_z(z) \text{ (AR(1) process)} \\
\text{(Firm type transition)} \quad & j' = G_j(j) \text{ (Markov chain)} \\
\text{(Discount factor)} \quad & q(S) = \beta \\
\text{(Aggregate states)} \quad & S = \{A, \Phi\}
\end{aligned}$$

As a benchmark model to the baseline, we also consider an economy without the type-specific heterogeneity in the frictions. The benchmark is separately calibrated from the baseline model to be properly compared with the baseline with respect to the data patterns in the firm-level investment.

5 Quantitative Analysis

5.1 Sorting between the real and financial frictions in the baseline model

In this section, we analyze the firm dynamics of constrained firms in the simulated data in comparison with the observed pattern from the data. The 10,000 firms are simulated using the equilibrium allocations in the baseline model over 100 periods after the initial 100 periods of burn-in periods that are discarded from the sample. Consistent with the data counterpart, we define financially constrained firms as those with a binding financial constraint. We define active firms as firms making a large-scale investment greater than 20% of existing capital stock, interchangeably with lumpy investment, following the literature on firm-level investment.

Figure 7 is the baseline-model counterpart of Figure 5. The right-hand side panel of Figure 7 is the firm-level (*not* observation-level) scatter plot along the two dimensions: one is the intensive margin in the investment when a firm makes a large-scale investment, and the other is a portion (%) of constrained firms at the last period. The figure displays a significant positive sorting pattern between the portion of constrained firms and the average size of the lumpy investments. If a firm is financially constrained in the last period, the firm's lumpy investment plan must have been delayed to the next period due to the lack of funding to implement the investment. Therefore, the more firms are financially constrained in the last period, the more likely the firms are to make lumpy investments in the following period, implementing the delayed plan in the last period. This effect can generate synchronized investment patterns over the business cycle, leading to endogenous fluctuations in aggregate investment (Lee, 2022).

The left-hand side panel of Figure 7 is another firm-level scatter plot along the two dimensions: one is the intensive margin in the investment when a firm makes a large-scale investment, and the other is the number of consecutive periods with a positive investment. When a firm is strongly constrained by the convex adjustment cost, the firm needs to split the size of the investment out of the

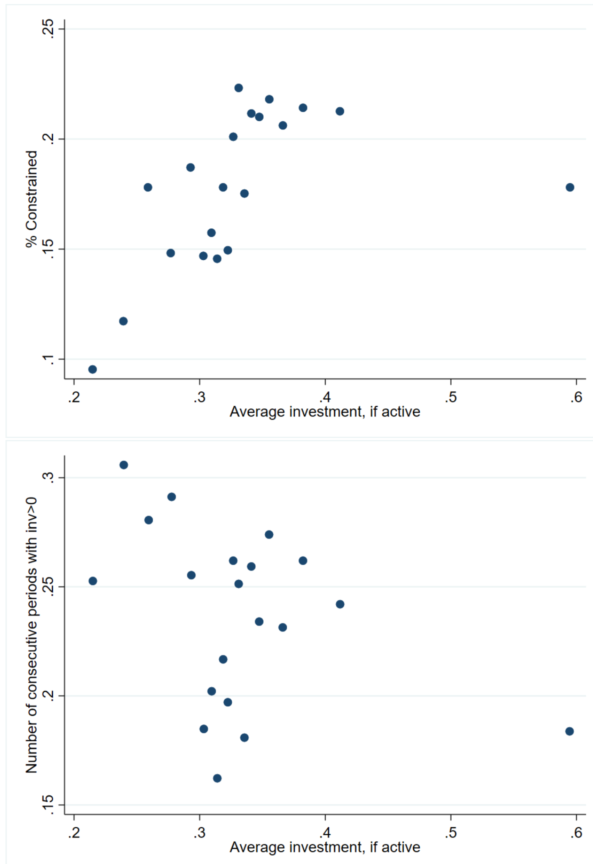

Figure 7: On the x-axis in both figures, the investment rate is conditional on a firm being active. On the left panel, on the y-axis, the % of constrained firms at the last period, and on the right panel, the number of consecutive periods a firm is active.

concern of reducing the marginal cost of investment. This makes firms invest more frequently with a smaller size. Therefore, as can be seen from Figure 7, the number of consecutive periods of positive investment is significantly negatively correlated with the average size of the lumpy investment. Importantly all these patterns in the simulated data are consistent with the observed patterns in the data, validating our structural approach at the firm level.

6 Concluding remarks and future plan

The starting point of this paper is the change in firm-level operating cost structure: the importance of fixed cost is rising. By investigating how the real frictions associated with the fixed cost interplay with the other real frictions (convex adjustment cost) and the financial frictions, we (plan to) analyze how the macroeconomic change affects the economy through resource misallocation and aggregate shock propagation.

From a simple theory, we establish that when the fixed adjustment cost meets financial friction, the lumpiness of firm-level investment becomes more severe. On top of that, due to the option value of the liquid investment project in comparison with the illiquid investment project, financial intermediaries tend to prefer lending money to convex-adjustment-cost type firms than fixed-adjustment-cost type firms, which intensifies the lumpiness of the latter type's investment even further. The theory is consistent with the empirical patterns we analyze in the unique Portuguese firm-level data set.

To investigate the macroeconomic impact of the rising fixed cost, we introduce a heterogeneous-firm general equilibrium model, where firms are subject to real and financial frictions with the sorting pattern between the two frictions, as our simple theory predicts. From the simulated data based on the stationary equilibrium, we compare model-implied firm-level investment patterns and the data counterpart, validating our structural approach.

In future work, we plan to sharpen the theoretical points on the financial intermediaries' incentive on lending money to two different types of firms: fixed-cost types and convex-adjustment-cost types. This will also help us build the micro-foundation of the full model better, allowing us a richer analysis of the counterfactual. In particular, we plan to analyze the capital misallocation in the stationary equilibrium that arises due to the sorting pattern between the real and financial frictions. Going a step further, we will measure how much of additional misallocation occurs due to the rising fixed adjustment cost, as we showed as a motivating fact. Then, we will analyze the business cycle implication of the sorting between the two frictions. Due to the strengthened lumpiness in the investment pattern, we expect the sorting leads to a stronger tendency for synchronization of investments across the firms, resulting in substantial amplification of the aggregate investment fluctuations at the cost of households' welfare.

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A Proofs

A.1 Proposition 1

Taking the derivative of the threshold for z with respect to the investment we have

$$\frac{dE_{\bar{z}}(z')}{dI} = \frac{1+\mu\frac{I}{k}}{\beta^F\left[((1-\delta)k+I)^{\alpha}-((1-\delta)k)^{\alpha}\right]} - \frac{I(b)+\xi+\frac{\mu}{2}\left(\frac{I(b)}{k}\right)^2 k}{\beta^F\alpha\left((1-\delta)k+I\right)^{\alpha-1}}$$

Notice that this can also be rewritten as

$$\frac{dE_{\bar{z}}(z')}{dI} = \frac{MC(I)}{TB(I)} - \frac{TC(I)}{MB(I)}$$

Reorganizing we get

$$\frac{dE_{\bar{z}}(z')}{dI} = \frac{MB(I)}{TB(I)} - \frac{TC(I)}{MC(I)}$$

Which is smaller than zero as the first term is smaller than 1 (marginal benefit of investment is lower or equal to the total benefit) and the second term is higher than 1 (total cost of investment is higher or equal to the marginal cost). This means that if the financially constrained level of investment $I(b)$ is lower than I^*, the financial constraints will have both an impact on the intensive and extensive margin, as the productivity threshold for active investment will be higher, causing some firms not to invest. The presence of the convex adjustment cost will have a similar effect given that $I^c \leq I^*$.

A.2 Proposition 2

Taking the derivative with respect to p^*, we obtain

$$u(I^*) - u(0) = u'(b)\frac{\partial b}{\partial p^*}$$

Thus,

$$\frac{\partial b}{\partial p^*} = \frac{u(I^*) - u(0)}{u'(b)} > 0$$

Consider a concave u, so $u'' < 0$. Then,

$$\frac{\partial^2 b}{\partial (p^*)^2} = -\frac{u(I^*) - u(0)}{u'(b)^2} u''(b) \frac{\partial b}{\partial p^*} > 0.$$

This implies that a risk-averse bank would convexly reduce the lending amount for firms with a high fixed cost (firms with low p^*).

A.3 Proposition 3

B Markup Estimation