

Infrastructure Fiscal Multiplier and Corporate Taxations*

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Abstract

We study whether the infrastructure investment is beneficial by quantifying the public infrastructure investment multipliers. We build a heterogeneous firm general equilibrium model to provide a micro-foundation for the multipliers with firms' investment decisions. We estimate our general equilibrium model that informs us about the elasticity of substitution between public capital and private capital using the state-level variation in infrastructure spending. We find that private capital is a gross substitute for public capital at the firm level, while it is a gross complement of public capital at the state level when the non-rivalry nature of the public infrastructure is considered at the firm level. According to the estimated baseline model, the short-run fiscal multiplier with respect to output slightly exceeds unity. In addition, we illustrate that the multipliers are substantially different across financing methods such as lump-sum payments and corporate taxation. Heavier corporate taxation leads to dampened fiscal multipliers as firms' investment is crowded out. This crowding out of private investments is shown to be dominantly driven by the increase in the interest rate and the wage.

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Contents

1	Introduction	3
2	Empirical estimates of input elasticity	7
3	A simple theory on the firm-level and the local elasticity estimates	9
4	Model	12
4.1	Production Technology	12
4.2	Firm-level Investment	13
4.3	Government	13
4.4	Recursive Formulation	14
4.5	Equilibrium	16
5	Estimation	17
5.1	External Calibration	17
5.2	Estimation Method	18
5.2.1	The Limited-Information Bayesian Method	19
5.2.2	The Limited-Information Bayesian Method Augmented with General Equilibrium Conditions	20
5.3	Identification and Target Moments	20
5.3.1	Estimation Results	21
6	Analysis of fiscal multipliers	23
6.1	Fiscal multipliers and corporate taxation	24
6.2	The role of elasticity of substitution between private and public capital stocks	28
6.3	The role of the general equilibrium effect	29
6.4	Heterogeneous fiscal multipliers across the regions	30
6.5	The role of time to build	32
7	Conclusion	36

1 Introduction

Infrastructure spending and its effect on output and welfare have become central issue in recent policy discussions. Especially, the infrastructure Investment and Jobs Act includes more than \$2 trillion in transportation and other physical infrastructure spending over the decade 2022 to 2031. In this paper, we study whether the infrastructure investment is beneficial by quantifying the infrastructure investment multipliers. The multipliers rely on whether public capital and private capital are substitutes or complements in the aggregate production. We carefully estimate the elasticity of substitution and obtain the multipliers given a infrastructure investment plan. In addition, we also study how we should finance a infrastructure investment plan. We show that the multipliers can be different across financing methods such as lump-sum payments and corporate taxation as firms' investment decisions react to changes in policy.

To obtain the empirical estimates of input elasticity, we estimate a nested-CES production function with the two types of capital, private and public, along with labor input similar to [An, Kangur, and Papageorgiou \(2019\)](#). Using the micro data on state-level public infrastructure capital stock, the elasticity of substitution is estimated to be 0.44. The state-level variations indicate the complementarity between private and public capital. However, this result does not imply the complementarity between private and public capital at the firm-level. We provide a theoretical result showing that the private and public capitals can be gross substitutes at the firm level, whereas they are complements at the state level. Unfortunately, we do not have the firm-level data sufficient enough to estimate the input elasticity without relying on the structural model. Hence, we build a heterogeneous firm model and estimate the firm-level input elasticity as one of key model parameters.

Our model is a heterogeneous firm model with a nested-CES production function with private capital, public capital, and labor input. Public capital enters firms' production function in a non-rivalrous way as in [Glomm and Ravikumar \(1994\)](#). We attempt to provide a micro-foundation for infrastructure investment multipliers by modeling firms' decisions explicitly. Subject to idiosyncratic productivity shocks, firms make lumpy investment with a fixed adjustment cost and a convex adjustment cost ([Khan and Thomas, 2008](#); [Cooper and Haltiwanger, 2006](#); [Winberry, 2021](#)). Using a revenue financed from the household income tax and the corporate tax, government spends through infrastructure investment, lump-sum subsidy, and public employment. The infrastructure evolves with an exogenous law of motion with the same convex adjustment cost as the private investment. We model two regions that differ in infrastructure levels in order to link to variations across states from the micro data. The expenditure split between the poor infrastructure

region and the good infrastructure region is exogenously determined. We solve for the general equilibrium where the interest rate in the capital market and the wage in the labor market are endogenously determined.

To quantify the effect of an infrastructure plan via the lens of our general equilibrium model, it is important to get a reliable estimate for the model parameters. However, estimating a general equilibrium model with heterogeneous agents is known to be computationally challenging as market clearing prices have to be solved for each candidate value for the model parameters. We extend an existing estimation method in a novel way so that we can estimate a general equilibrium model with much reduced computational costs. Our method is closely related to a limited information Bayesian method to match the moments from the data. To handle the general equilibrium, we extend this method by including market clearing conditions as additional moments. In this paper, we estimate the parameters of our heterogeneous firm model as well as market clearing prices. The estimated parameters inform us about the lumpiness of firm investment and the firm-level elasticity of substitution between private and public capital. The elasticity is estimated to be around 1.1, which supports the Cobb-Douglas production function as a reasonable specification.

As validation of our model, we compute the state-level elasticity based on the model-simulated data and compare it to the empirical elasticity using the U.S. state-level data. In the model, the state-level elasticity is computed by aggregating firms' behaviors in two regions that differ in infrastructure levels (good vs. poor) and estimating the state-level production functions. The resulting elasticity is estimated to be 0.35 in comparison to the empirical counterpart 0.44. Hence, our results indicate that the public and the private capitals are gross complements at the state level, whereas they are substitutes at the firm level. This finding is consistent with the theoretical result we prove that the nature of substitution could flip as the micro-level (firm-level) input elasticity is aggregated up to the state-level counterpart.

Given this model, we conduct the quantitative analysis to compute fiscal multipliers with one-time unexpected infrastructure spending shock whose magnitude is 1% of steady-state GDP value. As our model has two regions of different infrastructure level, we take the weighted average of fiscal multipliers to obtain the aggregate multipliers. We consider three different corporate tax policies coupled with the lump-sum tax to sustain the infrastructure spending. The baseline policy is with a corporate tax rate of 27%. The short-run aggregate multiplier (2-year) is 1.042 and the long-run multiplier (5-year) is 1.844. The substantial increase in the public capital leads to a boost in the output. Next, we consider the scenario where the corporate tax is increased by 33% from the baseline

level, which resembles the plan included in the Biden administration’s agenda. The short-run multiplier turns out to be 0.92, not exceeding the unity, and the long-run multiplier is 1.615. Under heavier corporate tax burden, firms’ private investment is crowded out further. This reduces the multipliers in both short-run and long-run compared to the baseline. We show that the crowding out of the private investment is dominantly driven by the surge in the interest rate and wage. Lastly, we consider the scenario where the corporate tax is decreased by 33% from the baseline. The short-run multiplier is 1.164 and the long-run multiplier is 2.076. The private investment is not crowded out under this scenario.

Different micro-level elasticities of substitution lead to substantially different fiscal multipliers. Lower elasticity leads to a bigger fiscal multiplier as the private investment is crowded out less. As the elasticity matters for the responsiveness of private firms’ investment, we carefully estimate the elasticity parameter for our quantitative analysis. We also find that the fiscal multipliers vary with the inclusion of time-to-build assumption.¹ Time-to-build assumption influences the fiscal multipliers via two channels. First, there is a news effect that people behave differently as they expect a future increase in the infrastructure. Second, there is general equilibrium effect that the change in endogenous behaviors from the news effect leads to the change in the prices. Consistent with [Ramey \(2020\)](#), we find that the aggregate fiscal multiplier decreases compared to the one without the extended time-to-build assumption. Lastly, we compare fiscal multipliers between Good state and Poor state that differ in infrastructure levels. We find that Good state gets the most benefit from the increase in the infrastructure spending.

Our paper is related to several strands of literature. First, our paper is closely related to literature on government spending multiplier ([Ramey and Zubairy, 2018](#); [Chodorow-Reich, 2019](#); [Hagedorn, Manovskii, and Mitman, 2019](#); [Auerbach, Gorodnichenko, and Murphy, 2020](#); [Ramey, 2020](#); [Hasna, 2021](#)). Public investment is a specific class of government outlays, and our paper focuses on quantifying the infrastructure spending multipliers. There have been empirical attempts to estimate the output elasticity of the public investment ([An, Kangur, and Papageorgiou, 2019](#); [Espinoza, Gamboa-Arbelaes, and Sy, 2020](#); [Ramey, 2020](#), [An, Zhang, and Li, 2022](#)). In addition to empirical analysis using the data on the American Recovery and Reinvestment Act (ARRA), [Ramey \(2020\)](#) analyzes the impacts of government investment using a stylized neoclassical model and New Keynesian model. We quantify the infrastructure spending multipliers based on a heterogenous

¹[Suárez Serrato and Zidar \(2016\)](#) identify local incidence of corporate taxation using a spatial model where firms are heterogeneous in productivity and imperfectly mobile. If we had firms’ location choice in our model, time-to-build assumption would generate imperfectly mobile firms as firms cannot make decisions that perfectly insure themselves against idiosyncratic shocks.

firm model. We add a firm-level investment behavior that has not been included in the aforementioned papers. Our model enables us to study the change in fiscal multipliers as the corporate taxation changes and firms' investment reacts.

Second, our paper illustrates that fiscal multipliers are substantially different depending on how the public investment is financed. Given an infrastructure spending plan, we show that heavier corporate taxation leads to dampened fiscal multipliers as the private firms' investment is crowded out.² We also find that the crowding out is dominantly driven by the increase in the interest rate. We reach to this conclusion from using the parameter estimates of our general equilibrium model. We extend the limited-information estimation method by including market clearing conditions as additional moments to match. One of the key parameters we estimate is the firm-level elasticity of substitution between private and public capital stocks. We assume that the firm-level production function is identical across firms within a state. An exogenous variation in the public capital stock leads to variation in the private capital stock as firms endogenously change their investment. Hence, we use the difference in private capital stocks between the state with high infrastructure level and the state with low level in order to estimate the elasticity of substitution.

Lastly, we contribute to the literature that bridges the gap between the aggregate estimates and the micro estimates using a structural model. Similar to [Nakamura and Steinsson \(2018\)](#) and [Oberfield and Raval \(2021\)](#), we estimate the firm-level elasticity of substitution between private and public capital stocks using the cross-state variations. The firm-level elasticity of substitution is estimated to be 1.18. Then, we aggregate the capital stock within the states to measure the state-level elasticity of substitution from the model. From the model, the state-level elasticity is 0.35, which is close to the empirical estimate of 0.44 from the data. According to estimates, private capital is a gross substitute for public capital at the firm level, while it is a gross complement of public capital at the state level. At the state level, the elasticity of substitution intakes a public good nature of the infrastructure. Therefore, the non-rivalry of the infrastructure generates the complementarity between the state-level private capital and the public capital. We theoretically show the presence of the non-rivalry effect of infrastructure on the complementarity based on the state-level production function.

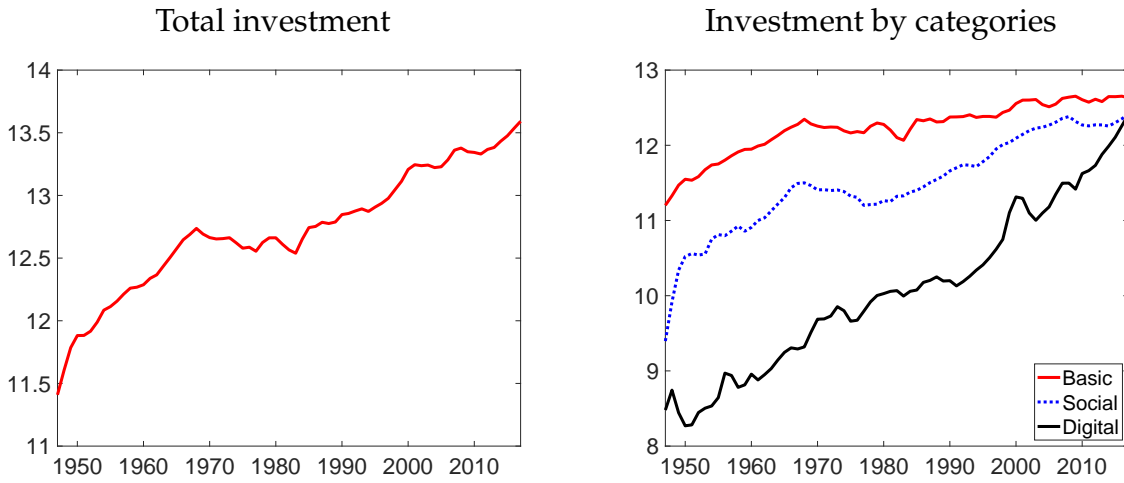
The rest of this paper proceeds as follows. Section 2 presents empirical estimates of input elasticity using the state-level data. Section 3 presents a theory showing that the

²[Hagedorn, Manovskii, and Mitman \(2019\)](#) quantify fiscal multipliers based on a dynamic heterogeneous agent New Keynesian (HANK) model. They model heterogeneous households instead of heterogeneous firms as we do. They also find that financing matters for fiscal multipliers, showing that the deficit-financed government spending multiplier is significantly larger than the tax-financed one (1.34 vs. 0.61).

nature of substitution between private and public capital flips with the aggregation from firm-level to state-level. Section 4 presents the model. Section 5 presents the estimation results. Section 6 presents the main quantitative analysis of fiscal multipliers under different levels of corporate tax. Section 7 concludes.

2 Empirical estimates of input elasticity

Figure 1: Logged real gross investment on infrastructure



Notes: Investment in millions of chained (2012) dollars.

In this section, we estimate the elasticity of substitution between private and public capital given a CES production technology. We closely follow [An, Kangur, and Papageorgiou \(2019\)](#) in which the elasticity is estimated using the non-linear least squares (NLLS) using the following:

$$\ln \left(\frac{Y_{it}}{Y_{i,t-1}} \right) = c + (1 - a) \ln \left(\frac{L_{it}}{L_{i,t-1}} \right) + \frac{a}{\psi} \ln \left[\frac{bK_{it}^\psi + (1 - b)N_{it}^\psi}{bK_{i,t-1}^\psi + (1 - b)N_{i,t-1}^\psi} \right] + (\epsilon_{it} - \epsilon_{i,t-1}).$$

i denotes the state, t denotes the time, and ϵ is the error term. Y is the output, K is the private capital stock, N is the public capital stock, and L is employment. ψ is the capital substitution parameter which implies a public-private capital elasticity of substitution (ES) of $1/(1 - \psi)$.

Using the state-level data, we compute local estimates of input elasticity. We first compute the net investment on public and private capital stocks. The state-level net public

investment is approximated by the portion of aggregate net infrastructure investment, where the weight is obtained by the state-level real public highway infrastructure investment from [Bennett, Kornfeld, Sichel, and Wasshausen \(2020\)](#).³ This is from the assumption that the fiscal spending at the state level for each of the different items (e.g., highway, water supply, etc.) is identically distributed across the states. The state-level net private investment is approximated by the portion of aggregate net non-residential fixed investment from NIPA (table 5.2.6), where the weight is obtained by the number of establishments at the state level from the Business Dynamics Statistics (BDS) at the US Census Bureau. This approximation is based on the assumption that the capital stock at each establishment does not vary significantly. After we obtain the net investment for public and private capital, we construct public and private capital stocks using the perpetual inventory method. For this approach, the initial capital stocks are needed for both public and private capital stocks. The state-level initial public capital stock is obtained by the portion of the aggregate public capital stock in 1977 from [Bennett, Kornfeld, Sichel, and Wasshausen \(2020\)](#), where the weight is from the highway infrastructure spending in 1977. The state-level initial private capital stock is from the portion of the aggregate private capital stock in 1977 from NIPA (table 4.1), where the weight is from the number of establishments in 1977. All the data is at the annual frequency. All real variables are chained in 2012 dollar value.

	Estimates	95% confidence interval
<i>a</i>	0.4022	[0.3415, 0.4630]
<i>b</i>	0.0698	[-0.1353, 0.2750]
<i>ES</i>	0.4406	[-0.1976, 1.0787]

Table 1: Parameter Estimates from NLLS

Notes: ES is calculated as $\frac{1}{(1-\psi)}$. Its confidence intervals are derived using the delta method.

Table 1 shows the estimation results from NLLS. The elasticity of substitution between public and private capital is estimated to be 0.44. In other words, the state-level variations indicate the complementarity between private and public capital. However, this result does not imply the complementarity between private and public capital at the firm level. In fact, the private and public capitals can be gross substitutes at the firm level, whereas

³The aggregate net infrastructure investment is also from [Bennett, Kornfeld, Sichel, and Wasshausen \(2020\)](#). In the state-level calculation, the weight is computed in the following way:

$$weight_{s,t} = \frac{\text{highway infrastructure investment}_{s,t}}{\sum \text{highway infrastructure investment}_{s,t}}$$

they are gross complements at the state level. The nature of substitution could flip as the micro-level (firm-level) input elasticities are aggregated upto the state-level as the following theoretical result illustrates.

3 A simple theory on the firm-level and the local elasticity estimates

In this section, we theoretically investigate the relationship between the micro and aggregate-level elasticities between private and public capital. In Section 2, the state-level elasticity is estimated to be 0.44, which implies the gross complementary relationship between the public and private capital. However, at the firm-level measurement, they become substitutive once the non-rivalrous nature of public infrastructure is considered at the firm level.

Consider a CES production function $F(K, N, L; \lambda, z)$ with CRS:

$$F(K, N, L; \lambda, z) = z(\theta K^{\frac{\lambda-1}{\lambda}} + (1-\theta)N^{\frac{\lambda-1}{\lambda}})^{\frac{\lambda}{\lambda-1}} L^{1-\alpha}$$

where λ is the elasticity of substitution between private and public capital; K is the capital input; L is the labor input; z is the productivity level. Then, we consider a static labor demand problem:

$$\max_L F(K, N, L; \lambda, z) - wL$$

Using the solution of this problem $L^* = z^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{w} \right)^{\frac{1}{\alpha}} (\theta K^{\frac{\lambda-1}{\lambda}} + (1-\theta)N^{\frac{\lambda-1}{\lambda}})^{\frac{\lambda}{\lambda-1}}$, we can rewrite the production function with the endogenous labor demand:

$$F(K, N, L(K, N; \lambda, z); \lambda, z) = f(K, N; \lambda, z) := z^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} (\theta K^{\frac{\lambda-1}{\lambda}} + (1-\theta)N^{\frac{\lambda-1}{\lambda}})^{\frac{\lambda}{\lambda-1}}$$

Then, we consider estimation of the elasticity λ at the firm level and at the state level using the production function f . Suppose we are given the firm-level data (k_1, k_2, y_1, y_2, N) , where the subscript $i \in \{1, 2\}$ indicates the firm-level observations at the same state. It is worth noting that state-level capital stock N is shared across the firms in the same state. In the firm-level estimation, we estimate the firm-level elasticity and the productivity (z, λ)

that satisfies

$$f(k_1, N; \lambda, z) = y_1$$

$$f(k_2, N; \lambda, 1) = y_2$$

where second firm's productivity is normalized to be unity.

In the state-level estimation, we estimate the state-level elasticity ξ that satisfies

$$f(k_1 + k_2, N; \xi, 1) = y_1 + y_2.$$

where state-level productivity is normalized to be unity.

In this estimation, due to the nonrivalrous nature of public good, firm-level estimate λ and state-level estimate ξ can be starkly different. Specifically, under the commonly observed conditions, which will be formally specified later, the private and public capitals are gross substitutes at the firm level, even if private and public capitals are gross complements at the state level.

The intuition behind the logic is that when the elasticity is estimated at the aggregated level, the non-rivalry of public capital stock is missing in the estimation. Therefore, in our paper's context, the state-level estimate supports a substantially stronger complementarity between private and public capital stocks than the firm-level estimate does. The following proposition formally states and proves this inconsistency in the firm-level and state-level estimates.

Proposition 1. *Suppose we are given with the micro-level data set (k_1, k_2, y_1, y_2, N) s.t.*

$$\exists i \in \{1, 2\}, \text{ s.t. } k_i < N, \quad N \leq k_1 + k_2, \quad \frac{y_1}{k_1} = \frac{y_2}{k_2}$$

Suppose the micro-level estimates (z, λ) and the aggregate-level estimate ξ are exactly identified by fitting the data into the production function as follows:

$$f(k_1, N; \lambda, z) = y_1$$

$$f(k_2, N; \lambda, 1) = y_2$$

$$f(k_1 + k_2, N; \xi, 1) = y_1 + y_2$$

Then, if the micro-level input elasticity satisfies $\lambda \geq 1$, then the aggregate-level input elasticity satisfies $\xi < 1$.

Proof. Without loss of generality suppose $k_1 > k_2$, $z > 1$, and let $k_2 < N$. From the

production functions, we have

$$\begin{aligned} y_1 &= z^{\frac{1}{\alpha}} B(\theta k_1^{\frac{\lambda-1}{\lambda}} + (1-\theta)N^{\frac{\lambda-1}{\lambda}})^{\frac{\lambda}{\lambda-1}} \\ y_2 &= B(\theta k_2^{\frac{\lambda-1}{\lambda}} + (1-\theta)N^{\frac{\lambda-1}{\lambda}})^{\frac{\lambda}{\lambda-1}} \\ y_1 + y_2 &= B(\theta(k_1 + k_2)^{\frac{\xi-1}{\xi}} + (1-\theta)N^{\frac{\xi-1}{\xi}})^{\frac{\xi}{\xi-1}} \end{aligned}$$

where $B := \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}}$. Therefore, the following relationships hold (from the second and the third equations above):

$$\begin{aligned} \left(\frac{y_2}{Bk_2}\right)^{\frac{\lambda-1}{\lambda}} &= \theta + (1-\theta) \left(\frac{N}{k_2}\right)^{\frac{\lambda-1}{\lambda}} \\ \left(\frac{y_1 + y_2}{B(k_1 + k_2)}\right)^{\frac{\xi-1}{\xi}} &= \theta + (1-\theta) \left(\frac{N}{k_1 + k_2}\right)^{\frac{\xi-1}{\xi}}. \end{aligned}$$

Now suppose that we are given with $\lambda \geq 1$. We will prove the proposition by contradiction, so we assume $\xi \geq 1$. As $N > k_2$, $\left(\frac{N}{k_2}\right)^{\frac{\lambda-1}{\lambda}} > 1$. Thus,

$$\left(\frac{y_2}{Bk_2}\right)^{\frac{\lambda-1}{\lambda}} = \theta + (1-\theta) \left(\frac{N}{k_2}\right)^{\frac{\lambda-1}{\lambda}} > 1.$$

Hence, $\frac{y_2}{Bk_2} > 1$. From the condition $\frac{y_1}{k_1} = \frac{y_2}{k_2}$,

$$1 < \frac{y_2}{Bk_2} = \frac{y_1 + y_2}{B(k_1 + k_2)}.$$

As $\xi \geq 1$, we have

$$1 < \left(\frac{y_1 + y_2}{B(k_1 + k_2)}\right)^{\frac{\xi-1}{\xi}} = \theta + (1-\theta) \left(\frac{N}{k_1 + k_2}\right)^{\frac{\xi-1}{\xi}}.$$

However, $N \leq k_1 + k_2$. Thus, $\left(\frac{N}{k_1 + k_2}\right)^{\frac{\xi-1}{\xi}} \leq 1$. This leads to

$$\theta + (1-\theta) \left(\frac{N}{k_1 + k_2}\right)^{\frac{\xi-1}{\xi}} \leq 1.$$

which is a contradiction. Therefore, if the micro-level input elasticity satisfies $\lambda \geq 1$, then the aggregate-level input elasticity satisfies $\xi < 1$. ■

As the firm-level data for the usage of public infrastructures is barely available, we build a heterogeneous-firm model which endogenously determines the usage of public infrastructures and estimates the micro-level input elasticity as one of the key model parameters. It is important to obtain a reliable estimate for the firm-level substitutability of public and private capital to compute the infrastructure investment multipliers. Whether the public capital crowd in or out the private capital will be crucial for the short-run and the long-run effect of infrastructure investment.

4 Model

4.1 Production Technology

Time is discrete and lasts forever. A measure one of ex-ante homogenous firms are considered. Each firm owns capital. It produces a unit of goods from the inputs of labor and capital. The production technology of a firm i located at a region j follows a CES form as specified below:

$$z_{i,t}x_{j,t}A_t f(k_{i,t}, l_{i,t}, \mathcal{N}_{j,t}) = z_{i,t}x_{j,t}A_t \left(\theta(k_{i,t})^{\frac{\lambda-1}{\lambda}} + (1-\theta)\mathcal{N}_{j,t}^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\lambda}{\lambda-1}\alpha} l_{i,t}^{\gamma}$$

where $k_{i,t}$ is capital input, $l_{i,t}$ is labor input, $z_{i,t}$ is idiosyncratic productivity, $\mathcal{N}_{j,t}$ is a region-specific infrastructure stock. $x_{j,t}$ is a region-specific productivity shock. λ is the elasticity of substitution between private capital and the infrastructure, A_t is aggregate TFP, α is capital share, and γ is labor share such that $\alpha + \gamma < 1$. Idiosyncratic productivity $z_{i,t}$ as specified below:

$$\ln(z_{i,t+1}) = \rho_z \ln(z_{i,t}) + \epsilon_{z,i,t+1}, \quad \epsilon_{z,i,t+1} \sim_{iid} N(0, \sigma_z)$$

where ρ_z and σ_z are persistence and standard deviation of *i.i.d* innovation in the process. The idiosyncratic shock process is discretized using the Tauchen method for computation. The aggregate TFP A_t is fixed at the unity in the baseline model, and an unexpected TFP shock is considered in the impulse response analysis.

In the economy, there are two regions $j \in \{P, G\}$ of which infrastructure levels and productivity levels are different from each other. We denote the poor infrastructure region as P and good infrastructure region as G : $N_G > N_P$. Firms switch from one region to

another following an exogenous Markov process:

$$\begin{bmatrix} p_{t+1}^P \\ p_{t+1}^G \end{bmatrix} = \begin{bmatrix} \pi_{PP} & \pi_{PG} \\ \pi_{GP} & \pi_{GG} \end{bmatrix}' \begin{bmatrix} p_t^P \\ p_t^G \end{bmatrix}$$

Using the production function, firms at a region j earn operating profit in each period by solving the following problem:

$$\pi(z_{i,t}, k_{i,t}, j; A_t, \mathcal{N}_t, w_t, r_t) = \max_{l_{i,t}} z_{i,t} x_{j,t} A_t f(k_{i,t}, l_{i,t}, \mathcal{N}_{j,t}) - w_t l_{i,t}$$

where w_t is the real wage.

4.2 Firm-level Investment

Firms make an investment decision as in [Khan and Thomas \(2008\)](#). A small-scale capital adjustment is specified as $\Omega(k_{i,t}) := [-\nu k_{i,t}, \nu k_{i,t}]$. When they make a large-scale capital adjustment, $I_{i,t} \notin \Omega(k_{i,t})$, they need to pay a fixed adjustment cost $\xi_{i,t}$, where $\xi_{i,t} \sim_{iid} Unif[0, \bar{\xi}]$. This cost is regarded as an labor overhead cost, so the actual cost is $w_t \xi_{i,t}$, where w_t is the real wage. If a firm makes a small-scale capital adjustment, $I_{i,t} \in \Omega(k_{i,t})$, it does not need to pay a fixed adjustment cost.⁴

Following [Cooper and Haltiwanger \(2006\)](#) and [Winberry \(2021\)](#), we assume all investments are subject to a convex adjustment cost, $C(I_{i,t}, k_{i,t}) = \frac{\mu}{2} \left(\frac{I_{i,t}}{k_{i,t}} \right)^2 k_{i,t}$. The convex adjustment cost plays an essential role in this paper, as it helps to capture the realistic elasticity of aggregate investment to the exogenous shocks such as fiscal policy shocks ([Zwick and Mahon, 2017](#); [Koby and Wolf, 2020](#); [Lee, 2022](#)).

4.3 Government

Government collects income tax from households at the rate of τ^h and corporate tax τ^c . Household income is the sum of labor income $w_t l_t$ and dividend income D_t . The tax rates are exogenously determined. Government issues a bond B_{t+1} which matures in one period and is discounted by the gross bond return, $1 + r_t^B$ and pays back the maturing bond, B_t . Using the revenue financed from the taxation and the net debt issuance, government spends through three channels: infrastructure investment, \mathcal{F}_t , public employment, $w_t \mathcal{E}_t$,

⁴As in [Khan and Thomas \(2008\)](#), there exists a threshold rule for the fixed cost shock ξ realization in the large-scale investment. For the brevity, we omit the detailed description.

and lump-sum subsidy, T_t .

$$\tau^h(w_t l_t + D_t) + \int \tau^c \pi(z_{i,t}, k_{i,t}, j; A_t, \mathcal{N}_t, w_t, r_t) d\Phi_t + \frac{B_{t+1}}{1 + r_t^B} - B_t = \mathcal{G}_t = \mathcal{F}_t + w_t \mathcal{E}_t + T_t$$

We assume the overhead fixed cost of infrastructure investment is covered by public sector workers, \mathcal{E}_t , without an extra cost. The public employment $\mathcal{E}_t = \mathcal{E}$ is exogenously determined. The split between the lump-sum subsidy and the infrastructure investment is determined exogenously at φ . Thus, for $\varphi > 0$, $\mathcal{F}_t = \varphi(\mathcal{G}_t - w_t \mathcal{E}_t)$, and $T_t = (1 - \varphi)(\mathcal{G}_t - w_t \mathcal{E}_t)$.

The country-level infrastructure $\mathcal{N}_{A,t}$ and state-level infrastructure $\mathcal{N}_{j,t}$ ($j \in \{P, G\}$) evolves in the following law of motion:

$$\begin{aligned} \mathcal{N}_{A,t+s} &= \mathcal{N}_{A,t+s-1}(1 - \delta_{\mathcal{N}}) + \mathcal{F}_t - \frac{\mu}{2} \left(\frac{\mathcal{F}_t}{\mathcal{N}_{A,t+s-1}} \right)^2 \mathcal{N}_{A,t+s-1} \\ \mathcal{N}_{j,t} &= \zeta_j \mathcal{N}_{A,t} \quad j \in \{P, G\} \end{aligned}$$

where the aggregate infrastructure $\mathcal{N}_{A,t}$ satisfies $\mathcal{N}_{A,t} = \mathcal{N}_{P,t} + \mathcal{N}_{G,t}$. The split between the poor infrastructure region and the good infrastructure region is exogenously determined by ζ_j which is calibrated to match the distribution of infrastructures described in Table 3. A positive integer s represents a time-to-build for the infrastructure investment. Infrastructure investment is subject to the same convex capital adjustment cost as the private investment.

To summarize the state variables, the individual state variables are idiosyncratic productivity shock, $z_{i,t}$ and individual capital stock, $k_{i,t}$. The aggregate state variables are productivity shock, A_t , the tuple of each region's infrastructure stocks, $\mathcal{N}_t = (\mathcal{N}_{P,t}, \mathcal{N}_{G,t})$, infrastructure spending history and plan, $\mathbb{F}_t = (\mathcal{F}_{t+\bar{s}})_{\bar{s}=-s}^{\infty}$, and the distribution of individual state variables, Φ_t .

4.4 Recursive Formulation

From this point on, for the brevity of notation in the recursive formulation, we drop the subscripts for each allocation. A representative household consumes, saves, and supplies labor. For brevity of the notation, we define $\mathcal{S} = (A, B, \Phi, \mathcal{N}, \mathbb{F})$. The recursive formula-

tion of the household problem is as follows:

$$\begin{aligned}
V(\Lambda; \mathcal{S}) &= \max_{c, \Lambda', L, B'} \log(c) - \frac{\eta}{1 + \frac{1}{\chi}} L^{1 + \frac{1}{\chi}} + \beta V(\Lambda'; \mathcal{S}') \\
\text{s.t.} \\
c + \frac{\int_{\mathcal{S}'} \Lambda'(s') J(s'; \mathcal{S}') ds'}{1 + r(\mathcal{S})} + \frac{B'}{1 + r^B(\mathcal{S})} &= w(\mathcal{S}) L (1 - \tau^h) \\
&\quad + \int_{\mathcal{S}} \Lambda(s) J(s; \mathcal{S}) ds + T(\mathcal{S}) + B
\end{aligned}$$

where c is consumption; Λ is the equity portfolio; L is labor supply; J is the individual firm value after the dividend tax; T is the lump-sum subsidy.

In the recursive formulation, a firm's problem is as follows:

$$\begin{aligned}
J(z, k, j; \mathcal{S}) &= \max_{I, I^c} \pi(z, k, j; \mathcal{S}) (1 - \tau^c) (1 - \tau^h) \\
&\quad + \int_0^{\bar{\xi}} \max\{(-I - w(\mathcal{S})\xi - C(I, k))(1 - \tau^h) \\
&\quad + \frac{1}{1 + r(\mathcal{S})} \mathbb{E}J(z', k', j'; \mathcal{S}'), \\
&\quad (-I^c - C(I^c, k))(1 - \tau^h) + \frac{1}{1 + r(\mathcal{S})} \mathbb{E}J(z', k^c, j'; \mathcal{S}')\} dG(\xi) \\
\text{s.t.} \\
k' &= (1 - \delta)k + I, \quad I \notin \Omega(k_t) = [-\nu k_t, \nu k_t] \\
k^c &= (1 - \delta)k + I^c, \quad I^c \in \Omega(k_t) \\
\mathcal{S}' &= G^{ALM}(\mathcal{S}) \\
dG(\xi) &= \frac{1}{\bar{\xi}} d\bar{\xi} \quad (\text{Uniform dist.}) \\
\pi(z, k, j; \mathcal{S}) &= \max_n z x_j A f(k, n, N_j) - w(\mathcal{S})n \\
C(I, k) &= \frac{\mu}{2} \left(\frac{I}{k}\right)^2 k
\end{aligned}$$

We assume the optimal dividend payout policy fully internalizes the income tax of households, τ^h . Without this assumption, there would be an inefficient allocation of dividends, which is beyond the scope of this paper. By allowing the fixed cost ξ to follow the *i.i.d* shock process, the value function becomes smooth without a kink. G^{ALM} is the aggregate law of motion that reflects the rational expectation for the future aggregate state allocations.

4.5 Equilibrium

In the stationary recursive competitive equilibrium, the interest rate and the wage are determined in the competitive market. Specifically, the following condition determines each price.

$$\begin{aligned}
\text{[Capital Market]} \quad & \int \underbrace{\mathbb{E}J(z', k'(z, k); \mathcal{S}) d\Phi}_{\text{Capital Demand}} = \underbrace{a'(a; \mathcal{S})}_{\text{Capital Supply}} \\
\text{[Labor Market]} \quad & \int \underbrace{\left(n(z, k, j; \mathcal{S}) + \left(\frac{\xi^*(z, k, j; \mathcal{S})}{2} \right) \right) d\Phi}_{\text{Private Labor Demand}} + \underbrace{\mathcal{E}}_{\text{Public Labor Demand}} \\
& = \underbrace{L(a; \mathcal{S})}_{\text{Labor Supply}}
\end{aligned}$$

The aggregate dividend is a sum of individual after-corporate-tax operating profits net of investment, and the ex-dividend portfolio value $P(\mathcal{S})$ is a sum of all the firms' values after the dividend payout:

$$\begin{aligned}
\text{[Aggregate Dividends]} \quad D(\mathcal{S}) = & \int \left(\pi(z, k, j; \mathcal{S})(1 - \tau^c) \right. \\
& - I^*(z, k, j; \mathcal{S}) - C(I^*(z, k, j; \mathcal{S}), k) \\
& \left. - \mathbb{I}\{I^* \notin \Omega(k)\} w(\mathcal{S}) \xi \right) d\Phi
\end{aligned}$$

$$\text{[Ex-dividend Portfolio Value]} \quad P(\mathcal{S}) = \int J(z, k, j; \mathcal{S}) d\Phi - D(\mathcal{S})$$

And the government budget constraint and the spending constraint need to clear:

$$\begin{aligned}
\text{[Government Budget]} \quad & \mathcal{G}(\mathcal{S}) = \tau^h(w(\mathcal{S})L(a; \mathcal{S}) \\
& + D(\mathcal{S})) + \int \tau^c \pi(z, k, j; \mathcal{S}) d\Phi \\
& + \frac{B}{1 + r^B(\mathcal{S})} - B \\
\text{[Infrastructure Investment]} \quad & \mathcal{F}(\mathcal{S}) = \varphi(\mathcal{G}(\mathcal{S}) - w(\mathcal{S})\mathcal{E}) \\
\text{[Lump-sum Subsidy]} \quad & \mathcal{T}(\mathcal{S}) = (1 - \varphi)(\mathcal{G}(\mathcal{S}) - w(\mathcal{S})\mathcal{E})
\end{aligned}$$

From the law of motion of the infrastructure, the following stationary condition holds:⁵

$$\begin{aligned} \text{[Infrastructure]} \quad \mathcal{N}_A &= \frac{1 + \sqrt{1 - 2\mu\delta_N}}{2\delta_N} \mathcal{F}(\mathcal{S}) \\ \mathcal{N}_j &= \zeta_j \mathcal{N}_A \quad j \in \{P, G\} \end{aligned}$$

In the equilibrium, an arbitrage does not exist between the wealth return and the government bond return:

$$\text{[No Arbitrage]} \quad r(\mathcal{S}) = r^B(\mathcal{S})$$

5 Estimation

In this section, we postulate how we estimate the parameters of our general-equilibrium model with heterogeneous firms. This has been a computationally demanding task than estimating a partial-equilibrium model since the market clearing prices have to be solved for each candidate value for the model parameters. We provide a novel way to bypass this bottleneck by estimating market clearing prices simultaneously with the model parameters.

We first illustrate how we choose the values of externally calibrated parameters. We then provide a brief summary of the limited-information Bayesian method that we extend to estimate a general-equilibrium model. Our novelty lies in that we augment general equilibrium conditions as additional moments in the limited-information method and estimate market prices together with the model parameters. After we describe the estimation method, we provide identification arguments with targeted moments and report the estimation results.

5.1 External Calibration

We fix $\beta = 0.96, \alpha = 0.28, \gamma = 0.64$ for annual frequency. Some parameters are externally calibrated outside of the model, and their values are reported in Table 2.

For the average of household income tax rate, we use 0.15 as in [Krueger and Wu \(2021\)](#) where they compute the tax rate with the data from [Blundell, Pistaferri, and Saporta-Eksten \(2016\)](#). For corporate tax rate, we use 0.27 from [Gravelle \(2014\)](#) that is the effective tax paid after deductions and credits. We use 0.05 for the fraction of public employ-

⁵There are two fixed points in the stationary infrastructure stock. We focus only on the greater one, which is a stable fixed point.

Parameter	Description	Value
τ^h	household income tax rate (average)	0.15
τ^c	corporate tax rate	0.27
\mathcal{E}	public employment	0.05
φ	infrastructure spending	0.09
s	time to build	1
χ	Frisch elasticity	4
δ	depreciation rate	0.09
$\delta_{\mathcal{N}}$	depreciation rate	0.02
ρ_z	idiosyncratic shock persistence	0.75
σ_z	idiosyncratic shock volatility	0.13

Table 2: Externally Calibrated Parameters

Notes: Each period in the model corresponds to one year in the data.

ment, using the data on the government employees (*USGOVT*) and the private employees (*USPRIV*). We use 0.09 for the infrastructure spending out of tax revenue. This comes from the fact that the infrastructure spending as share of GDP is 2.4% and the tax revenue as share of GDP is 27.1%. We assume one year of time-to-build for the baseline analysis. We set Frisch elasticity to be 4 as in [Ramey \(2020\)](#). We use 0.09 for the private capital depreciation rate, and 0.02 for the public capital depreciation rate from the BEA depreciation data. Following [Lee \(2022\)](#), we use the Compustat estimates of the persistence and volatility of the idiosyncratic productivity shocks.⁶

Furthermore, our model captures state-variations by including two regions P, G that differ in infrastructure levels. To map this to the data pooled across years after detrending, we divide states into two groups by the median infrastructure level. Table 3 show some summary statistics between poor and good infrastructure groups. The transition probabilities from one relation to another are set to be persistent ($\pi_{PP} = 0.90, \pi_{GG} = 0.98$).⁷

5.2 Estimation Method

We estimate the remaining key parameters through a limited-information Bayesian approach augmented with general equilibrium conditions. We first explain the limited-information Bayesian method that uses a set of moments from the data for estimation. Then we illustrate our idea on augmenting general equilibrium conditions as additional

⁶[Lee \(2022\)](#) uses the methodology of [Akerberg, Caves, and Frazer \(2015\)](#) to estimate the firm-level TFP shock process.

⁷Transition probabilities are constructed based on the state-level data in Table A.2 in the appendix.

	Poor infrastructure	Good infrastructure
Infrastructure portion	0.19 (0.001)	0.81 (0.001)
Establishment (#) portion	0.17 (0.005)	0.83 (0.005)
Firm (#) portion	0.173 (0.006)	0.827 (0.006)
GDP (\$) portion	0.151 (0.005)	0.849 (0.005)

Table 3: Two state groups' infrastructure and the number of establishments and firms

moments.

5.2.1 The Limited-Information Bayesian Method

The limited-information Bayesian method, as described in [Kim \(2002\)](#) and later advocated by [Christiano, Trabandt, and Walentin \(2010\)](#) and [Fernández-Villaverde, Rubio-Ramírez, and Schorfheide \(2016\)](#) among others, can be viewed as the Bayesian version of the generalized method of moments (GMM). Similar to GMM, the limited-information Bayesian method only uses a set of moments from the data for parameter inference.

Let Θ denote the parameters of interest and $\hat{\mathbf{m}}$ denote the vector of M empirical moments from the data for estimation. The likelihood of $\hat{\mathbf{m}}$ conditional on Θ is approximately

$$f(\hat{\mathbf{m}}|\Theta) = (2\pi)^{-\frac{M}{2}} |S|^{-\frac{1}{2}} \exp \left[-\frac{1}{2} (\hat{\mathbf{m}} - \mathbf{m}(\Theta))' S^{-1} (\hat{\mathbf{m}} - \mathbf{m}(\Theta)) \right], \quad (1)$$

where $\mathbf{m}(\Theta)$ is the model's prediction for the moments under parameter Θ , and S is the covariance matrix of $\hat{\mathbf{m}}$. The covariance matrix S is often unknown but can be replaced by a consistent estimator of it, which can be obtained through bootstrap. Bayes' theorem tells us that the posterior density $f(\Theta|\hat{\mathbf{m}})$ is proportional to the product of the likelihood $f(\hat{\mathbf{m}}|\Theta)$ and the prior density $p(\Theta)$:

$$f(\Theta|\hat{\mathbf{m}}) \propto f(\hat{\mathbf{m}}|\Theta)p(\Theta), \quad (2)$$

and we can then apply the standard Markov Chain Monte Carlo (MCMC) techniques such as the Random-Walk Metropolis-Hastings (RWMH) algorithm to obtain a sequence of random samples from the posterior distribution.

Suppose we estimate parameters of the model in which market clearing conditions

need to be satisfied as general equilibrium conditions. Given each candidate parameter vector, the model is solved with an additional loop that makes sure the market clearing conditions become zero with numerical precision. This additional layer regarding general equilibrium conditions is likely to result in prohibitively high computational costs.

5.2.2 The Limited-Information Bayesian Method Augmented with General Equilibrium Conditions

In order to make the estimation procedure computationally feasible, we extend the limited-information Bayesian method by augmenting data moments with market clearing conditions. In other words, we treat market clearing prices as parameters to be estimated where the associated moments in estimation procedure are market clearing conditions.

With the standard estimation with RWMH, the computational bottleneck lies in that we need to satisfy market clearing conditions for each candidate parameter vector. Instead, our suggested method treats market clearing conditions as additional moments. We want these general equilibrium conditions to be zero as the RWMH chain runs, but we do not require them to hold for each candidate parameter. Given the lens of our model, we need to track the both of the market clearing prices: marginal utility, p and wage w . Thus, we treat (p, w) as additional parameters to estimate. From our model (given state \mathcal{S}), $p = 1/c(\mathcal{S})$, and $w = \eta L(\mathcal{S})^{\frac{1}{\lambda}} c(\mathcal{S}) / (1 - \tau^h)$.

We attach the following market clearing conditions as moments to be used in the limited-information Bayesian estimation:

$$\begin{bmatrix} p - 1/c(\mathcal{S}) \\ w - \eta L(\mathcal{S})^{\frac{1}{\lambda}} c(\mathcal{S}) / (1 - \tau^h) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Given candidates of the market clearing prices (p, w) , we compute the model-generated prices $(1/c(\mathcal{S}), \eta L(\mathcal{S})^{\frac{1}{\lambda}} c(\mathcal{S}) / (1 - \tau^h))$ after solving the model. Then we can check whether the difference between the model-generated prices and (p, w) is zero. As the RWMH chain runs, we will obtain posterior draws that render market clearing condition closer to zero.

5.3 Identification and Target Moments

The upper bound for fixed cost $\bar{\xi}$ is identified using the lumpy investment portion. The convex adjustment cost parameter μ is identified from the average investment to capital ratio. Parameter ν associated with the constrained investment region is identified from the standard deviation of investment to capital ratio. Private capital share parameter θ is iden-

tified from the private-to-public capital ratio. Productivity level parameter x is identified from the high region's y portion. Government spending level parameter G is identified from the government spending to output ratio. Labor distuility parameter η is identified from the employment rate. The elasticity of substitution parameter λ is identified from the high region's k portion. We assume that the firm-level production function is identical across firms within a state. An exogenous variation in the public capital stock leads to variation in the private capital stock as firms endogenously change their investment. Hence, we use the difference in private capital stocks between the state with high infrastructure level and the state with low level to identify λ .

Target moment	Data	Source
lumpy investment portion	0.140	Zwick and Mahon (2017)
mean(i/k)	0.100	Zwick and Mahon (2017)
standard deviation(i/k)	0.160	Zwick and Mahon (2017)
private-to-public capital ratio	0.750	Bureau of Economic Analysis
high region's k portion	0.830	Census Business Dynamics Statistics
high region's y portion	0.849	Bennett, Kornfeld, Sichel, and Wasshausen (2020)
government spending to output ratio	0.155	World Bank Database
Total hours	0.330	Current Employment Statistics

Table 4: Target Moments Used in Estimation

5.3.1 Estimation Results

We adopt uniform prior for each parameter. We apply the random-walk Metropolis-Hastings algorithm to simulate draws from the posterior density $f(\Theta|\hat{\mathbf{m}})$ given by (2), and the posterior distribution is characterized by a sequence of 2000 draws after a burn-in of 2000 draws.⁸

Table 5 reports the posterior means and the 95% credible intervals of preference and wage parameters from the Bayesian estimation, together with the uniform priors. The firm-level elasticity of substitution λ is estimated to be 1.18, which supports the Cobb-Douglas production function as a reasonable specification. High-region productivity is about two times higher than low-region productivity. It is worth noting that we do not consider endogenous evolution of productivity. Overall, the 95% credible intervals of the posterior distributions are much narrower than the uniform priors, suggesting that the variations from the data is useful to infer the parameters of interest. In addition, the market clearing price p turns out to be tightly pinned down. Table 6 shows the model fit for

⁸We initialize the chain at the point estimate from particle swam optimization routine from MATLAB.

Parameter	Description	Posterior Distribution		Uniform Prior
		Mean	95% Interval	[Min, Max]
$\bar{\xi}$	fixed cost upper bound	0.5188	[0.5112,0.5230]	[0.001,1.900]
μ	convex adjustment cost	3.1244	[3.1175,3.1348]	[0.200,3.500]
ν	constrained investment region	0.0406	[0.0405,0.0407]	[0.001,0.080]
θ	private capital share	0.6674	[0.6658,0.6684]	[0.500,0.999]
λ	elasticity of substitution (private vs. public capital)	1.1848	[1.1800,1.1900]	[0.300,2.500]
x	productivity level of high infrastructure region	2.0637	[2.0421,2.0801]	[0.500,2.500]
G	government spending level	0.1030	[0.1014,0.1068]	[0.010,0.400]
η	labor disutility	2.8445	[2.8311,2.8597]	[2.100,3.500]

Table 5: Estimation Results

Target moment	Data	Model
lumpy investment portion	0.140	0.139
mean(i/k)	0.100	0.100
standard deviation(i/k)	0.160	0.160
private-to-public capital ratio	0.750	0.798
high region's k portion	0.830	0.870
high region's y portion	0.849	0.984
government spending to output ratio	0.155	0.154
Total hours	0.330	0.344

Table 6: Model Fit

the targeted moments. The model-generated moments fit the empirical moments from the data reasonably well.

As external validation, we compute the state-level elasticity from our model and compare it to the empirical estimates in Section 2. In our model, the infrastructure stock is shared among the firms in the same region. We conduct the state-level aggregation as follows: we fix the micro-level estimates except for the elasticity λ and spatial productivity heterogeneity x_1 .⁹ We estimate these two parameters under the state-level production models.¹⁰

$$\begin{bmatrix} x_1 \left(\theta k_1^{\frac{\lambda-1}{\lambda}} + (1-\theta) N_1^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\lambda}{\lambda-1}\alpha} l_1^\gamma \\ \left(\theta k_2^{\frac{\lambda-1}{\lambda}} + (1-\theta) N_2^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\lambda}{\lambda-1}\alpha} l_2^\gamma \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

⁹Since the production function in our model is decreasing returns to scale, there is no guarantee that the firm-level elasticity and productivity is aggregated to have the same value in the state-level.

¹⁰We cannot identify the public capital stock share, θ separately from the elasticity, λ in the state-level model. This is the main reason why we introduce the micro-level heterogeneity in our structural model. Therefore, in the state-level model, we fix the public capital stock share at the micro-level estimate.

where (x_1, λ) are unknown, while all the other allocations and parameters, $(y_1, y_2, k_1, k_2, N_1, N_2, l_1, l_2, \theta, \alpha, \gamma)$ are obtained from the estimated baseline model.¹¹ Using the same nonlinear optimization technique used in [An, Kangur, and Papageorgiou \(2019\)](#), we get the following new estimate of the state-level production function:¹²

$$(x_1, \lambda) = (1.7664, 0.3493)$$

The elasticity level is of similar order of magnitude to our state-level estimate from the US data, which is around 0.44.

It is worth noting that our model bridges the gap between the firm-level estimates and the state-level estimates. According to our estimates, private capital is a gross substitute for public capital at the firm level, while it is a gross complement of public capital at the state level. At the state level, the elasticity of substitution includes a good public nature of the infrastructure. Therefore, the non-rivalry of the infrastructure generates the complementarity between the state-level private capital and the public capital. This result is consistent with our theory in Section 3 showing that the presence of the non-rivalry effect of infrastructure on the complementarity based on the state-level production function.

6 Analysis of fiscal multipliers

In this section, we analyze the fiscal multipliers of fiscal spending based on our estimated structural model. We define the fiscal multiplier as follows:

$$\text{Fiscal Multiplier} = \frac{\sum_{t=1}^T \text{Present Value of } \Delta x_t}{\sum_{t=1}^T \text{Present value of } \Delta G_t}$$

where Δx_t is the deviation at period t of the equilibrium allocation of interest from the steady-state level; ΔG_t is the fiscal spending shock at period t .¹³ In the short run, we assume $T = 2$, and in the long run, we assume $T = 5$. In the baseline specification, we assume the fiscal spending shock is a one-time unexpected shock (MIT shock) without any persistence. The magnitude of the one-time shock is assumed at 1% of the steady-state output level as in Ramey (2020). As a baseline setup, we assume the sudden shock in the fiscal spending is through the infrastructure channel only.

¹¹The two parameters are obtained from the exact identification

¹²In the empirical estimation, we assume a CRS production function. If we also assume a CRS production function in the model side, the state-level estimates are $(x_1, \lambda) = (1.9233, 0.4822)$.

¹³The shock is deviation from the steady-state level.

The following laws of motion determine the time path of the public capital stocks after the fiscal spending shock ΔG at $t = 1$:

$$\begin{aligned}\mathcal{N}_{A,t+1} &= \mathcal{N}_{A,t}(1 - \delta_{\mathcal{N}}) + \mathcal{F}_t - \frac{\mu}{2} \left(\frac{\mathcal{F}_t}{\mathcal{N}_{A,t}} \right)^2 \mathcal{N}_{A,t} \\ \mathcal{N}_{j,t} &= \zeta_j \mathcal{N}_{A,t} \quad j \in \{P, G\} \\ F_t &= \begin{cases} F^{ss} + \Delta G & \text{if } t = 1 \\ F^{ss} & \text{otherwise} \end{cases}\end{aligned}$$

where F^{ss} is the stationary equilibrium level of infrastructure spending.

6.1 Fiscal multipliers and corporate taxation

We first compare the fiscal multipliers when the infrastructure spending is combined with different tax policies. Three different policies are considered. The first policy is decreasing the corporate tax rate by 33% from the baseline level (27%→18%). The second policy uses the baseline level (27%), and the last policy increases the corporate tax rate by 33% from the baseline level (27%→36%).¹⁴ The remaining balance in the fiscal budget after the change in taxation is financed by the lump-sum tax. Thus, the third policy collects the least amount of lump-sum tax among the three policies.¹⁵

Table 7 reports the fiscal multipliers across the three corporate tax policies. In the first policy with low corporate tax, the short-run multiplier is around 1.16, which is the greatest among the three. In the last policy with high corporate tax, the short-run multiplier is around 0.9210, which is the lowest among the three. The same ranking is observed for the long-run multipliers.

One of the main channels that cause the differences in the fiscal multipliers is the firm-level investment. When the fiscal spending is combined with the low corporate tax policy, due to the increased incentive of cumulating the future capital stock, the private investment crowds in, as can be seen from the positive investment multiplier of 0.0224. However, in other cases, the greater public capital stock crowds out the private capital invest-

¹⁴The third policy mimics the Biden administration's plan to increase the corporate tax rate by 33%. As our baseline tax level is 27% while the corporate tax rate of 2022 is 21%, there is a level difference in the tax rate.

¹⁵Our fiscal multiplier analysis is based on the impulse response to the MIT fiscal spending shock under perfect foresight. Therefore, the representative household becomes indifferent between lump-sum tax financing and debt financing as long as the lifetime income is unaffected. If the model considers household heterogeneity under the borrowing limit and frictional financial market, this indifference collapses, leading to divergent fiscal multipliers between tax financing and debt financing as in [Hagedorn, Manovskii, and Mitman \(2019\)](#).

Table 7: Fiscal multipliers

Fiscal multipliers	Low Corp. Tax	Baseline	High Corp. Tax
Output			
Short-run	1.1637	1.0416	0.9210
Long-run	2.0758	1.8439	1.6149
Short-run (2 years)			
Consumption	0.1308	0.1719	0.2123
Investment	0.0224	-0.0942	-0.2094
Labor income	0.8028	0.6590	0.5180
Long-run (5 years)			
Consumption	0.9822	0.9251	0.8684
Investment	0.0671	-0.0617	-0.1889
Labor income	1.5127	1.2904	1.0720

ment. A similar pattern is observed in the long-run fiscal multipliers of private investment.

The differences in the response of private capital investment to the fiscal policy lead to the differences in the labor income response. The greater the private investment, the greater the employment effect on the economy. In the low corporate tax policy, the labor income multiplier is 0.80; in the baseline corporate tax policy, the labor income multiplier is 0.66; in the high corporate tax policy, the labor income multiplier is 0.52. None of the labor income multipliers are greater than unity in the short run. As we show later, this low labor income multiplier is due to the general equilibrium effect. In the partial equilibrium, the three short-run labor income fiscal multipliers are all greater than the unity. In the long run, the multipliers are all greater than the unity, even in the general equilibrium.

However, the low corporate tax policy is not a free lunch. The low corporate tax policy leads to the lowest consumption multiplier of 0.13 in the short run. This is because this tax policy requires the greatest lump-sum tax to finance the spending shock. This clearly shows what is the trade-offs in corporate tax policies; the low tax policy sacrifices the short-run welfare to achieve long-run welfare. In the long run, due to the private investment and labor income channels, the fiscal multiplier is the greatest for the low corporate tax policy.

Figure 2 plots the impulse responses of the baseline fiscal policy. The dashed line in each panel shows the government expenditure changes from the steady-state level in per cent of the steady-state output. The solid line is the impulse response of the equilibrium allocations.

Private investment contemporaneously decreases. The response of private investment

is the outcome of two countervailing forces canceling out each other: 1) increase in the investment incentive and 2) increase in the general equilibrium effect. The increase in the investment incentive comes from the imperfect substitution between public and private capital stock. For a simple illustration, we consider a two-period model with the firm-level investment decision where the production functions are the same as in Proposition 1, and investment is subject to the convex adjustment cost. From the first-order condition of the investment, the following equation holds:

$$\underbrace{1 + \mu \left(\frac{k'}{k} - (1 - \delta) \right)}_{\text{LHS: marginal cost}} = \underbrace{\underbrace{\frac{1}{1+r}}_{\text{GE channel}} \underbrace{z^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} \left(\theta k'^{\frac{\lambda-1}{\lambda}} + (1-\theta) N^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\lambda}{\lambda-1}-1} k'^{-\frac{1}{\lambda}} \theta}_{\text{Future MPK}}}_{\text{RHS: marginal benefit = Discounted future MPK}}$$

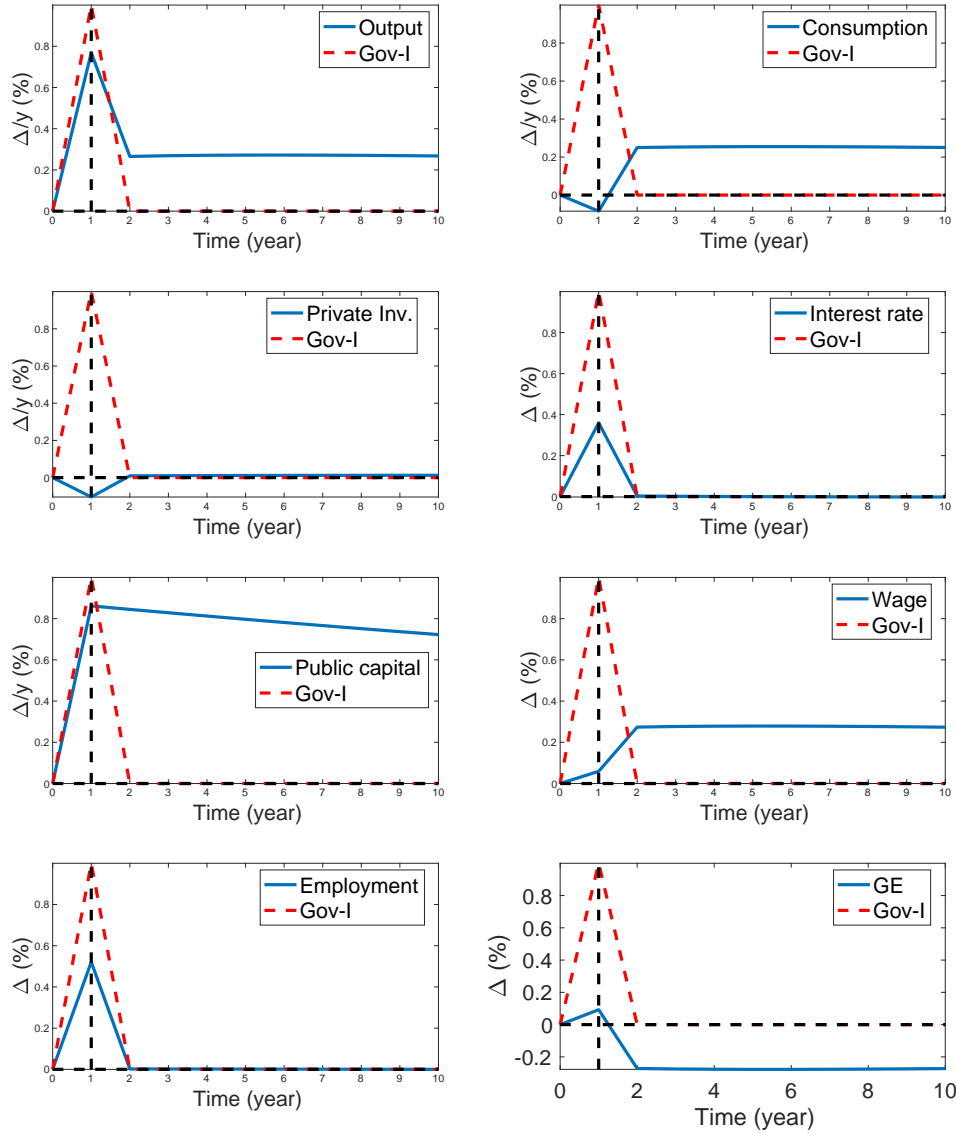
The left-hand side of the equation above is the marginal cost of the firm-level investment, and the right-hand side is the marginal benefit. To analyze how the increase in the public capital stock N affects the marginal benefit of firm-level investment, we take a partial derivative with respect to N .

$$\begin{aligned}
 \frac{\partial}{\partial N} \text{Marginal benefit} &= \left(\frac{1}{1+r} \right) \times \frac{\partial}{\partial N} \text{Future MPK} + \overbrace{\text{Future MPK} \times \frac{\partial}{\partial N} \left(\frac{1}{1+r} \right)}^{\text{GE effect}} \quad (3) \\
 \frac{\partial}{\partial N} \text{Future MPK} &= \frac{\partial}{\partial N} \left(\theta k'^{\frac{\lambda-1}{\lambda}} + (1-\theta) N^{\frac{\lambda-1}{\lambda}} \right)^{\frac{1}{\lambda-1}} F(\Theta) \\
 &= \left(\frac{1}{\lambda-1} \right) \left(\frac{\lambda-1}{\lambda} \right) (1-\theta) N^{-\frac{1}{\lambda}} \left(\theta k'^{\frac{\lambda-1}{\lambda}} + (1-\theta) N^{\frac{\lambda-1}{\lambda}} \right)^{\frac{2-\lambda}{\lambda-1}} F(\Theta) \\
 &= \frac{1}{\lambda} (1-\theta) N^{-\frac{1}{\lambda}} \left(\theta k'^{\frac{\lambda-1}{\lambda}} + (1-\theta) N^{\frac{\lambda-1}{\lambda}} \right)^{\frac{2-\lambda}{\lambda-1}} F(\Theta) > 0
 \end{aligned}$$

where F is a function of the parameters, Θ . If the elasticity of substitution λ is a finite positive number, the marginal benefit of firm-level investment increases in N through the increased future marginal productivity of capital, given the general equilibrium effect is fixed. However, if λ goes to infinity, the marginal benefit of investment does not depend on N . It is worth noting that the marginal benefit increases in N regardless of whether the public and private capital stocks are gross complements ($\lambda < 1$) or substitutes ($\lambda > 1$).

However, a fiscal policy affects the prices, which we denote as the general equilibrium effect. In this, one of the most important channels is lump-sum taxation: the household reduces consumption to pay this lump-sum tax. If the interest rate substantially increases due to the increase in the marginal utility from the reduced consumption, despite the

Figure 2: The impulse responses to the fiscal policy shock



increase in the marginal benefit coming out of the large public capital stock, the firm-level investment is crowded out by the infrastructure spending.¹⁶

In the baseline impulse response, the interest rate increases by 0.39 percent after a fiscal spending shock. This general effect dominates the increase in the marginal benefit from the greater public capital stock, leading to a net crowding out of the private investment as

¹⁶There is no precautionary saving motivation from the households, as the economy is abstract from the aggregate uncertainty.

observed from the impulse response.

In contrast, the employment response is not dampened by the general equilibrium (wage) effect. As the positive infra structure shock hits, employment increases by 0.52%. Consumption and the GE effect are the mirror image of each other as the GE effect refers to the inverse of consumption under the log utility. In the shock period, consumption decreases strongly due to the lump-sum taxation, but it increases in the following period from the strong consumption smoothing motivation: all the expected future gains out of the infrastructure spending smoothly shift up the consumption level.

6.2 The role of elasticity of substitution between private and public capital stocks

The elasticity of substitution between private and public capital stock plays a key role in determining the marginal benefit of firm-level investment given a fiscal expenditure shock. Analytically, the change in the marginal benefit of firm-level investment over the elasticity given the fiscal spending shock can be captured by the cross derivative $\frac{\partial^2}{\partial \lambda \partial N}$ Marginal benefit in the two-period simple model. Using Equation (3), we have the following equation:

$$\frac{\partial^2}{\partial \lambda \partial N} \text{Marginal benefit} = \underbrace{\frac{\partial}{\partial \lambda} \left[\frac{1}{\lambda} \right]}_{\text{Direct}} \underbrace{\frac{\left(\frac{1}{1+r} \right) MPK}{\left(\theta k' \left(\frac{N}{k'} \right)^{\frac{1}{\lambda}} + (1-\theta)N \right)}}_{\text{Indirect}} + \frac{\partial}{\partial \lambda} GEeffect$$

As displayed in the equation above, the elasticity of substitution affects the response of marginal benefit through two channels: 1) direct and 2) indirect channels. The direct channel refers to newly added capital being relatively less valuable when the public capital stocks are more substitutable with the private capital. The indirect channel refers to a change in marginal benefit of investment due to the change in the relative values of the existing public and private capital stocks. The direct channel predicts the marginal benefit of firm-level investment decreases in the elasticity, while the sign of the indirect channel cannot be analytically determined.¹⁷

Therefore, the effect of elasticity on the aggregate marginal benefit needs a quantitative analysis. For this analysis, we separately compute the stationary equilibria under the

¹⁷The sign of the effect also depends on the firm-level capital stock.

Table 8: Fiscal multipliers with the different elasticities of substitutions

Fiscal multipliers	High ($\lambda = 3$)	Estimated ($\lambda = 1.18$)	Low ($\lambda = 0.5$)
Output			
Short-run	0.6129	1.0416	1.3296
Long-run	0.7734	1.8439	2.6087
Short-run (2 years)			
Consumption	-0.0799	0.1719	0.2553
Investment	-0.2257	-0.0942	0.0532
Labor income	0.3178	0.6590	0.9362
Long-run (5 years)			
Consumption	0.1835	0.9251	1.3078
Investment	-0.3087	-0.0617	0.2242
Labor income	0.4372	1.2904	1.9937

different elasticities of substitution.¹⁸ Then, we measure the fiscal multipliers by tracking impulse responses of equilibrium allocations separately for these economies.

Table 8 reports the fiscal multipliers for different elasticities of substitutions between public and private capital stocks. As can be seen from the table, the low firm-level elasticity ($\lambda = 0.5$) leads to 28% greater output fiscal multipliers compared to our estimated baseline. Especially when the elasticity is low, the private investment is not crowded out even in the presence of the general equilibrium effect. Due to the greater response from the private sector, in the low λ economy, consumption and labor income responses are also greater than in the high λ or baseline economy both in the short run and the long run.

Depending on the firm-level elasticity level, the fiscal multiplier varies significantly. Therefore, this analysis shows that the sharp measurement of the elasticity has the first-order importance for the fiscal multiplier analysis.

6.3 The role of the general equilibrium effect

In this section, we analyze the role of the general equilibrium effect on fiscal multipliers. For this analysis, we measure the fiscal multipliers by tracking impulse responses of equilibrium allocations to the fiscal spending shock while the prices are exogenously fixed at the stationary equilibrium level.

Table 9 reports the fiscal multipliers for different corporate tax policies without the general equilibrium effect. In the absence of the general equilibrium effect, the fiscal spending

¹⁸Other parameters are assumed to be at the same level as the baseline estimates except for the prices. Therefore, it is similar to the comparative statics.

shock combined with any of the corporate tax policies studied in the previous section leads to fiscal multipliers greater than unity. For the baseline policy, the multiplier jumped by around 79% ($1.04 \rightarrow 1.86$) without the general equilibrium effect. Therefore, it is obvious that the general equilibrium effect significantly dampens the fiscal multipliers.

Table 9: Fiscal multipliers in partial equilibrium

Fiscal multipliers	Low Corp. Tax	Baseline	High Corp. Tax
Output			
Short-run	2.0504	1.8582	1.6728
Long-run	5.6476	5.0314	4.4360
Short-run (2 years)			
Consumption	0.5569	0.6052	0.6518
Investment	0.3614	0.1892	0.0227
Labor income	1.5112	1.2935	1.0851
Long-run (5 years)			
Consumption	3.7878	3.4216	3.0675
Investment	0.6649	0.4769	0.2952
Labor income	3.9647	3.4781	3.0098

Especially, none of the tax policies leads to a net crowding out of the private investment. Thus, it is followed by the large-scale labor income and consumption multiplier effect.

One important caveat in the analysis is that without the general equilibrium analysis, the fiscal multiplier can be significantly biased upward. Therefore, the general equilibrium framework is essential for analyzing the fiscal multiplier. However, models with micro-level heterogeneity are often abstract from the general equilibrium effect due to their high computational cost during the estimation. We overcome this problem through a new estimation technique that obtains the prices in the general equilibrium simultaneously with the parameters to be estimated. The detailed explanation is in section 5.2.

6.4 Heterogeneous fiscal multipliers across the regions

In this section, we analyze the heterogeneous fiscal multipliers of the baseline infrastructure spending policy across Poor and Good states. Table 10 reports the fiscal multipliers for Poor state (column 1), for Good state (column 2), and for the aggregate level (column 3). As there is only a representative household in this economy, the state-level consumption is defined after the following assumption:

- All the incomes are state-specific, and there is no cross-state transfer.

- Each equity is exclusively owned by the state's household.
- Bond holding and lump-sum subsidies are attributed to each state proportionately to the exogenous fiscal spending ratio.

After these assumptions, the state-level consumption can be properly defined due to the separate budget clearing across the states. One can introduce two households in the model to capture Poor and Good households separately, but this can be done only at a high computational cost and the model complication.

As can be seen from the table, there are large asymmetries in the fiscal multipliers between the two states. In Good state, the fiscal multiplier exceeds the unity, while Poor state's multiplier is less than one-tenth of the Good state. This is due in part to the productivity difference and to the difference in the public capital stock between the two states.

Table 10: Fiscal multipliers across the states

Fiscal multipliers	Poor state	Good state	Aggregate
Output			
Short-run	0.0819	1.2667	1.0416
Long-run	0.1370	2.2443	1.8439
Short-run (2 years)			
Consumption	-0.5322	0.3749	0.1719
Investment	-0.0104	-0.1139	-0.0942
Labor income	0.2390	0.8019	0.6590
Long-run (5 years)			
Consumption	0.0096	1.2719	0.9251
Investment	-0.0040	-0.0753	-0.0617
Labor income	0.7498	1.5725	1.2904

Importantly, the private investment in Good state is more severely crowded out by the public infrastructure spending than in Poor state. As we will clarify in Table 11, this is due to the large general equilibrium effect in the Good state. However, consumption and labor income multipliers are substantially greater in Good state than in Poor state.

Table 11 reports the fiscal multipliers across the states without the general equilibrium effect. Without the general equilibrium effect, the private investment fiscal multiplier is greater in Good state than in Poor state. This reflects that Good state has higher productivity and greater public capital stocks. Thus, the marginal benefit of investment is greater in Good state if the general equilibrium effect is turned off.¹⁹ However, once the general

¹⁹In section 6.1, we show that the marginal benefit of firm-level investment increases in the public capital

equilibrium is considered, the Good state's private investment is more severely crowded out than the Poor state's private investment. This shows that the general equilibrium effect asymmetrically affects the Poor state's and the Good state's equilibrium allocations.

Table 11: Fiscal multipliers across the states in partial equilibrium

Fiscal multipliers	Poor state	Good state	Aggregate
Output			
Short-run	0.1441	2.2603	1.8582
Long-run	0.3670	6.1256	5.0314
Short-run (2 years)			
Consumption	-0.5184	0.8700	0.6052
Investment	0.0255	0.2276	0.1892
Labor income	0.1063	1.5735	1.2935
Long-run (5 years)			
Consumption	0.3040	4.1586	3.4216
Investment	0.0667	0.5732	0.4769
Labor income	0.2831	4.2343	3.4781

The consumption multiplier of Good state is also strongly dampened. In Good state, the consumption multiplier reduces down by around 57% ($0.87 \rightarrow 0.37$), while it reduces down by around 3% ($-0.52 \rightarrow -0.53$) in Poor state. In contrast, the output multipliers and the labor income multipliers are almost proportionately dampened by the general equilibrium effect across the states.

6.5 The role of time to build

In this section, we analyze the role of time to build on the fiscal multiplier. On top of the one-year time to build in the baseline, we assume there is an extra year of time to build for capital stock to be utilizable after the investment as in [Ramey \(2020\)](#) (two years, in total).

stock if $\lambda \in (0, \infty)$.

Therefore, the law of motion of the public capital stock is as follows:²⁰

$$\begin{aligned}\mathcal{N}_{A,t+2} &= \mathcal{N}_{A,t+1}(1 - \delta_{\mathcal{N}}) + \mathcal{F}_t - \frac{\mu}{2} \left(\frac{\mathcal{F}_t}{\mathcal{N}_{A,t+1}} \right)^2 \mathcal{N}_{A,t+1} \\ \mathcal{N}_{j,t+1} &= \zeta_j \mathcal{N}_{A,t+1} \quad j \in \{P, G\} \\ F_t &= \begin{cases} F^{ss} + \Delta G & \text{if } t = 1 \\ F^{ss} & \text{otherwise} \end{cases}\end{aligned}$$

where F^{ss} is the stationary equilibrium level of fiscal spending on infrastructure. Due to the time lag between the fiscal policy shock and the arrival of the public capital stock, there exists a news component in the policy, which will be analyzed further in this section.

Table 12: Fiscal multipliers across the states under the time to build of two year

Fiscal multipliers	Poor state	Good state	Aggregate
Output			
Short-run	0.0831	1.1179	0.9213
Long-run	0.1362	2.0521	1.6881
Short-run (2 years)			
Consumption	-0.5586	0.3387	0.1438
Investment	-0.0199	-0.1962	-0.1627
Labor income	0.1995	0.6643	0.5472
Long-run (5 years)			
Consumption	-0.0370	1.1951	0.8636
Investment	-0.0160	-0.1609	-0.1333
Labor income	0.6885	1.4050	1.1542

For this analysis, the fiscal multiplier is measured by the sum of the present values over the first three years for the short run and over the six years for the long run after the initial fiscal spending shock.²¹

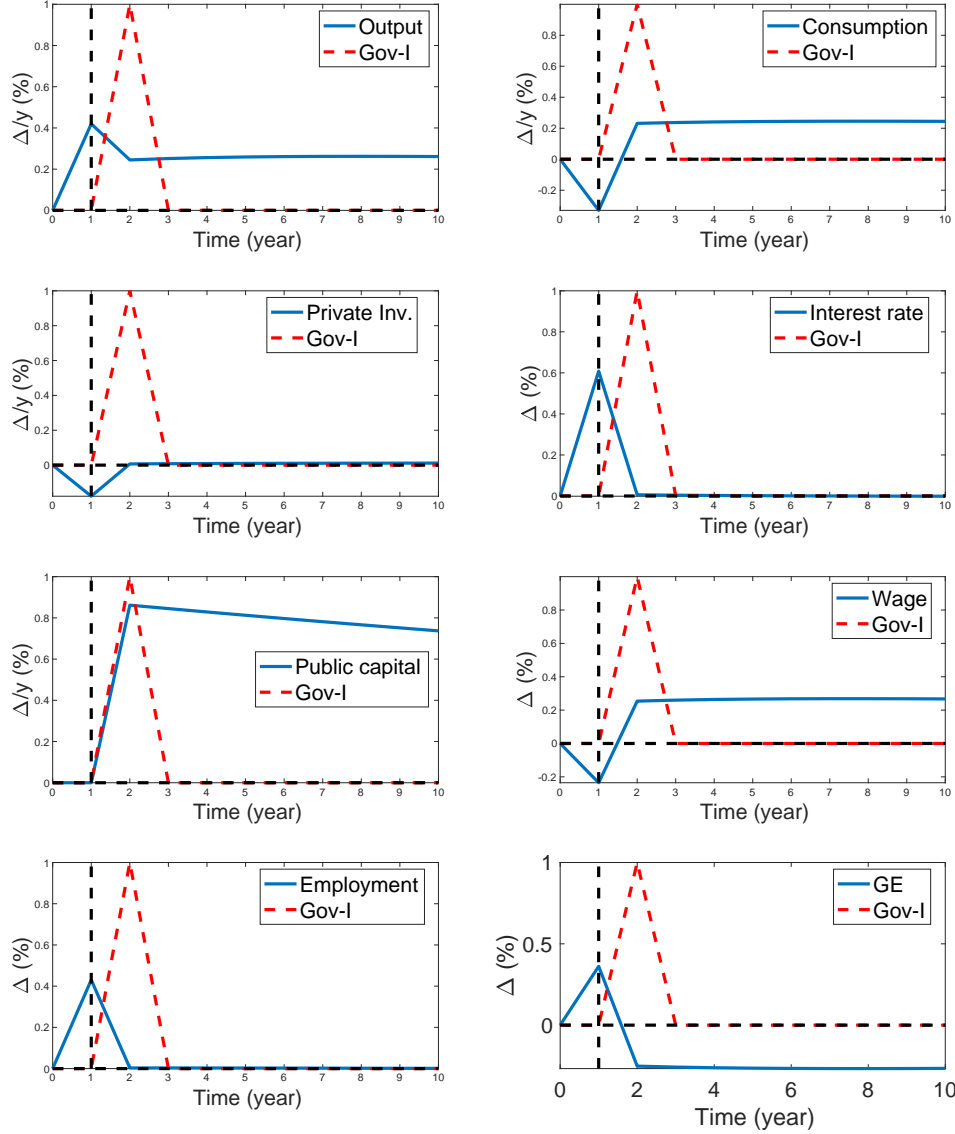
Table 12 reports the heterogeneous fiscal multipliers across Poor and Good states when there is a time to build of one year. The aggregate fiscal multiplier decreases compared to the one without the extended time-to-build assumption (Table 10), consistent with Ramey (2020). In the short (long) run, the fiscal multiplier decreases by 12% (8%) at the aggregate

²⁰For the consistency in the notation with the previous formulations, we leave the time index of the future public capital stock to be $t + 1 + s$ where $s = 1$.

²¹Previously, it was 2 years for the short run and 5 years for the long run without the extended time to build.

level. Especially, the time to build strongly dampens the fiscal multiplier of Good state: 12% in the short run and 9% in the long run.

Figure 3: The impulse responses to the fiscal policy shock under the extended time to build



To illustrate the mechanism under the time to build, Figure 3 plots the impulse responses of equilibrium allocations in the consistent format with Figure 2. Due to the presence of the time to build, the capitalized government expenditure in the dashed line spikes one year after the beginning of the endogenous responses in the equilibrium allocations.

As the fiscal spending shock hits, consumption immediately drops as the lump-sum tax immediately puts downward pressure on the household's consumption. This makes the household more willing to supply the labor. On the other hand, the production side does not face any change in the infrastructure until one year after the shock. Therefore, the increased labor supply at the period of shock ($t = 1$) leads to a lower wage and greater employment. Then, this feeds back into increased output at $t = 1$. The interest rate increases as the marginal utility (=GE) at $t = 1$ increases, resulting in a decrease in private investment. After the infrastructure spending becomes capitalized, the demand for labor increases while the willingness for the labor supply decreases (income effect). This leads to an increase in the wage while the employment stays almost unchanged from the stationary equilibrium level.

Table 13: Fiscal multipliers across the states under the time to build of two years in the partial equilibrium

Fiscal multipliers	Poor state	Good state	Aggregate
Output			
Short-run	0.1503	2.4710	2.0301
Long-run	0.3836	6.5785	5.4015
Short-run (2 years)			
Consumption	-0.5111	0.9130	0.6414
Investment	0.0394	0.3484	0.2897
Labor income	0.1143	1.7653	1.4504
Long-run (5 years)			
Consumption	0.3606	4.4284	3.6506
Investment	0.0835	0.7036	0.5858
Labor income	0.2992	4.5795	3.7604

To quantify the general equilibrium effect, we compute the fiscal multipliers under the extended time to build assumption in the partial equilibrium, where the price is fixed at the stationary equilibrium level. Without the general equilibrium, the Good State's fiscal multiplier is 2.47, which is greater than when there is one year of time-to-build friction (2.2603 in Table 11). The Poor state's fiscal multipliers are also amplified more with the extended time-to-build friction in the partial equilibrium, but the difference is not dramatic as in Good state.

This is due to the news effect that allows the agents with the rational expectation to adjust their allocations optimally even before the spending shock is capitalized. However, this effect is dominated by changes in the price once we consider the general equilibrium

effect, as can be seen from the large difference between Table 12 and Table 13. The agents' adjustment before the shock capitalization results in wage and interest rate adjustment, dampening the fiscal multiplier even in a greater magnitude than the one-year time-to-build. This is because the interest adjustment occurs at one time due to the absence of aggregate uncertainty, and the resulting increase in the cost of investment at the period before the spending shock leads to a lowered capital stock. Under real friction, such as the convex adjustment cost, the lowered capital stock leads to a greater adjustment cost in the following period when the fiscal spending shock is materialized, leading to a substantially dampened fiscal multiplier. Therefore, this is an outcome of the interaction between the news and the real friction.

7 Conclusion

This paper attempts to quantify the infrastructure investment multipliers. With a heterogeneous firm model, we provide a micro-foundation for the multipliers with firms' investment decisions. We estimate our general equilibrium model that informs us about the elasticity of substitution between public capital and private capital. We find that private capital is a gross substitute for public capital at the firm level, while it is a gross complement of public capital at the state level. The non-rivalry of the infrastructure generates the complementarity between the state-level private capital and the public capital.

In addition, we illustrate that the multipliers are substantially different across financing methods such as lump-sum payments and corporate taxation. Heavier corporate taxation leads to dampened fiscal multipliers as firms' investment is crowded out. This crowding out of private investments is shown to be dominantly driven by the increase in the interest rate and the wage.

A potential topic for future research is to study the optimal corporate taxation given an infrastructure spending plan. In the short run, a lower corporate tax is likely to increase private investment but reduce consumption as households need to pay more in lump-sum. We will be able to quantify these trade-offs and solve optimal corporate taxation problems given a planner's welfare criterion.

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A Appendix

A.1 Residualized infrastructure investment

	Dependent variable: Infrastructure _{s,t}		
	log(real_public_inv)		
	Lag 1 (1)	Lag 2 (2)	Lag 3 (3)
Infrastructure _{s,t-1}	0.955*** (0.029)	1.624*** (0.116)	1.624*** (0.116)
Infrastructure _{s,t-2}		-0.718*** (0.094)	-0.711*** (0.106)
Infrastructure _{s,t-3}			-0.002 (0.053)
State fixed effect	Yes	Yes	Yes
Year fixed effect	Yes	Yes	Yes
Two-way cluster	Yes	Yes	Yes
Observations	1,275	1,224	1,173
R ²	0.996	0.998	0.998
Adjusted R ²	0.996	0.998	0.998
Residual Std. Error	0.058 (df = 1199)	0.041 (df = 1148)	0.040 (df = 1097)

Note: *p<0.1; **p<0.05; ***p<0.01

Table A.1: Residualization results

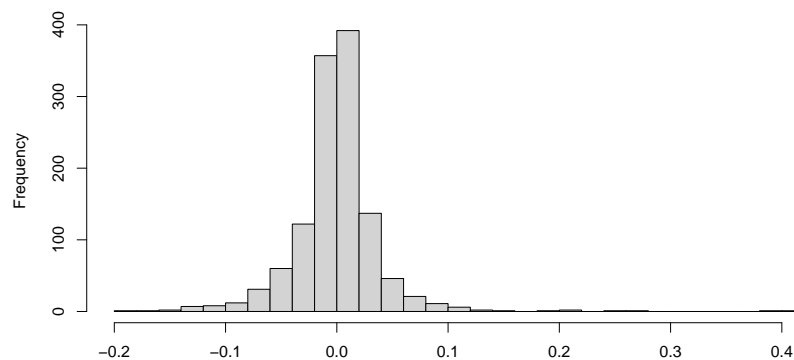


Figure A.1: Histogram of residualized infrastructure investments

A.2 Notes on the market clearing conditions

In the model, there are two centralized markets: the capital market and the labor market. Thus, there are two prices to be determined endogenously.

A.2.1 Interest rate and capital market

Define $p := U'(c(\mathcal{S}))$. Then, from the Euler equation of the representative household,

$$\begin{aligned} \beta \mathbb{E} \frac{U'(c(\mathcal{S}'))}{U(c(\mathcal{S}))} (D(\mathcal{S})) &= \frac{1}{1 + r^B(\mathcal{S})} \\ \iff \beta \mathbb{E} \frac{p(\mathcal{S}')}{p(\mathcal{S})} &= \frac{1}{1 + r^B(\mathcal{S})} = \frac{1}{(1 + r(\mathcal{S}))(1 - \tau^h \mathcal{M})} \end{aligned}$$

where $\mathcal{M} = \mathbb{E} \frac{D(\mathcal{S}')}{D(\mathcal{S}') + P(\mathcal{S}')}.$

Then, define a modified value function $\tilde{J}(z, k, j; \mathcal{S}) = p(\mathcal{S})J(z, k, j; \mathcal{S}).$

In the following original recursive formulation,

$$\begin{aligned} J(z, k, j; \mathcal{S}) &= \max_{I, I^c} \pi(z, k, j; \mathcal{S})(1 - \tau^c) \\ &\quad + \int_0^{\bar{\xi}} \max\{-I - \xi w(\mathcal{S}) - C(I, k) \\ &\quad + \frac{1}{1 + r(\mathcal{S})} \mathbb{E} J(z', k', j'; \mathcal{S}'), \\ &\quad - I^c - C(I^c, k) + \frac{1}{1 + r(\mathcal{S})} \mathbb{E} J(z', k^c; \mathcal{S}')\} dG(\xi) \end{aligned}$$

replace $\frac{1}{1 + r(\mathcal{S})}$ with $\beta(1 - \tau^h \mathcal{M}) \mathbb{E} \frac{p(\mathcal{S}')}{p(\mathcal{S})}$. So we have,

$$\begin{aligned} J(z, k, j; \mathcal{S}) &= \max_{I, I^c} \pi(z, k, j; \mathcal{S})(1 - \tau^c) \\ &\quad + \int_0^{\bar{\xi}} \max\{-I - \xi w(\mathcal{S}) - C(I, k) \\ &\quad + \beta(1 - \tau^h \mathcal{M}) \mathbb{E} \frac{p(A', B', \Phi', \mathcal{N}')}{p(\mathcal{S})} \mathbb{E} J(z', k', j'; \mathcal{S}'), \\ &\quad - I^c - C(I^c, k) + \beta(1 - \tau^h \mathcal{M}) \mathbb{E} \frac{p(A', B', \Phi', \mathcal{N}')}{p(\mathcal{S})} \mathbb{E} J(z', k^c; \mathcal{S}')\} dG(\xi) \end{aligned}$$

Then, multiply $p(\mathcal{S})$ to both sides. It leads to

$$\begin{aligned}
p(\mathcal{S})J(z, k, j; \mathcal{S}) &= \max_{I, I^c} p(\mathcal{S})\pi(z, k, j; \mathcal{S}) \\
&+ \int_0^{\bar{\xi}} \max\{-p(\mathcal{S})I - p(\mathcal{S})w(\mathcal{S})\xi \\
&\quad - p(\mathcal{S})C(I, k) \\
&\quad + \beta(1 - \tau^h \mathcal{M})\mathbb{E}p(\mathcal{S}')J(z', k', j'; \mathcal{S}'), \\
&\quad - p(\mathcal{S})I^c - p(\mathcal{S})C(I^c, k) \\
&\quad + \beta(1 - \tau^h \mathcal{M})\mathbb{E}p(\mathcal{S}')J(z', k^c; \mathcal{S}')\}dG(\xi)
\end{aligned}$$

Thus, we have

$$\begin{aligned}
\tilde{J}(z, k, j; \mathcal{S}) &= \max_{I, I^c} p(\mathcal{S})\pi(z, k, j; \mathcal{S}) \\
&+ \int_0^{\bar{\xi}} \max\{-p(\mathcal{S})I - p(\mathcal{S})w(\mathcal{S})\xi \\
&\quad + \beta(1 - \tau^h \mathcal{M})\mathbb{E}\tilde{J}(z', k', j'; \mathcal{S}'), \\
&\quad - p(\mathcal{S})I^c + \beta(1 - \tau^h \mathcal{M})\mathbb{E}\tilde{J}(z', k^c; \mathcal{S}')\}dG(\xi)
\end{aligned}$$

Therefore, a firm's problem is perfectly characterized by the price

$$p(\mathcal{S}) = U'(c(\mathcal{S}), \mathbb{F}) = 1/c(\mathcal{S}).$$

Also, the interest rate can be obtained from $\beta(1 - \tau^h \mathcal{M})\mathbb{E}\frac{p(\mathcal{S}')}{p(\mathcal{S})} = \frac{1}{1+r(\mathcal{S})}$.

A.2.2 Wage and labor market

From the representative household's intra-temporal optimality condition (with respect to the labor supply),

$$\eta L^{\frac{1}{\chi}} = U'(c(\mathcal{S}))w(\mathcal{S})(1 - \tau^h)$$

Therefore,

$$\eta L^{\frac{1}{\chi}} = p(\mathcal{S})w(\mathcal{S})(1 - \tau^h) \implies w(\mathcal{S}) = \frac{\eta}{p(\mathcal{S})(1 - \tau^h)}L^{\frac{1}{\chi}}$$

The optimal labor supply L depends upon w , and w can be determined only when the labor supply L is known. Therefore, w needs to be tracked together with p .

A.3 State-level data on infrastructure

State	Avg. Rank (Infra.)	# Good Group	Portion (Infra.)	Portion (GDP)	Avg. Rank (Estab.)	Portion (Estab.)
New York	1.708	24.000	0.072	0.081	2.439	0.072
California	1.833	24.000	0.071	0.133	1.000	0.114
Texas	2.458	24.000	0.071	0.079	2.659	0.069
Florida	4.000	24.000	0.064	0.050	4.000	0.060
Illinois	5.000	24.000	0.049	0.046	5.341	0.044
Ohio	6.542	24.000	0.035	0.036	7.000	0.039
New Jersey	7.125	24.000	0.034	0.034	8.415	0.033
Georgia	8.458	24.000	0.032	0.029	11.171	0.027
Pennsylvania	8.708	24.000	0.032	0.040	5.561	0.044
Massachusetts	9.708	24.000	0.030	0.027	12.341	0.024
Minnesota	10.458	24.000	0.029	0.018	18.561	0.019
North Carolina	12.208	24.000	0.025	0.027	9.976	0.028
Wisconsin	13.083	24.000	0.025	0.017	17.439	0.020
Washington	14.250	24.000	0.024	0.024	14.561	0.022
Virginia	14.458	24.000	0.024	0.027	12.585	0.025
Michigan	16.083	24.000	0.022	0.030	9.024	0.032
Tennessee	16.917	24.000	0.021	0.018	19.195	0.019
Missouri	18.167	24.000	0.019	0.018	15.171	0.021
Indiana	18.833	24.000	0.018	0.019	15.171	0.021
Kentucky	20.292	24.000	0.018	0.011	27.415	0.013
Louisiana	21.333	24.000	0.017	0.014	22.805	0.015
Iowa	21.625	24.000	0.017	0.010	28.951	0.012
Arizona	22.875	24.000	0.016	0.017	23.756	0.016
Colorado	25.625	15.000	0.015	0.017	19.439	0.018
Kansas	25.833	13.000	0.014	0.009	30.829	0.011
Alabama	26.000	23.000	0.015	0.012	24.951	0.014
Maryland	26.042	11.000	0.015	0.020	20.415	0.018
Connecticut	26.542	10.000	0.014	0.016	25.951	0.014
Oklahoma	29.458	0.000	0.012	0.010	27.634	0.013
Mississippi	30.208	0.000	0.011	0.006	33.317	0.009
Oregon	30.500	0.000	0.011	0.011	25.659	0.014
South Carolina	31.917	0.000	0.011	0.011	26.634	0.013
Nevada	33.083	0.000	0.010	0.008	38.000	0.006
Nebraska	34.417	0.000	0.010	0.006	34.927	0.007
Arkansas	34.708	0.000	0.010	0.007	32.439	0.009
New Mexico	35.542	0.000	0.010	0.006	37.000	0.006
West Virginia	37.000	0.000	0.009	0.004	37.244	0.006
Utah	38.375	0.000	0.008	0.007	34.122	0.008
Alaska	39.167	0.000	0.007	0.003	51.000	0.002
Hawaii	39.458	0.000	0.007	0.005	41.854	0.005
Idaho	41.667	0.000	0.006	0.004	40.512	0.005
Montana	41.958	0.000	0.006	0.002	42.512	0.004
Delaware	42.375	0.000	0.006	0.004	47.317	0.003
Wyoming	44.167	0.000	0.005	0.002	49.707	0.003
South Dakota	45.042	0.000	0.005	0.002	45.073	0.003
Rhode Island	46.083	0.000	0.004	0.003	42.963	0.004
Maine	47.208	0.000	0.004	0.004	39.098	0.005
North Dakota	47.500	0.000	0.004	0.002	47.146	0.003
New Hampshire	49.000	0.000	0.003	0.004	39.963	0.005
District of Columbia	50.000	0.000	0.002	0.007	47.927	0.003
Vermont	51.000	0.000	0.002	0.002	47.829	0.003

Table A.2: State-level summary

Notes: Avg. Rank (Infra.) is the average time-series ranking of infrastructure (this variable is the sorting variable). # Good Group is how many times the state belonged to the good infrastructure group (Max:24). Portion (Infra.) is the portion of infrastructure on average. Avg. Rank (Estab.) is the average time-series ranking of the number of establishments. Portion (Estab.) is the portion of establishments on average.

A.4 Nonlinearity in the fiscal multipliers along the spending amount

Table A.3: Fiscal multipliers across the different spending levels (per GDP (%))

	0.2%	0.4%	0.6%	0.8%	1.0%	1.2%	1.4%	1.6%	1.8%	2.0%
Output										
Short-run	1.0942	1.0805	1.0669	1.0540	1.0416	1.0293	1.0173	1.0054	0.9936	0.9820
Long-run	1.9696	1.9370	1.9050	1.8741	1.8439	1.8141	1.7847	1.7556	1.7268	1.6984
Short-run (2 years)										
Consumption	0.2195	0.2073	0.1954	0.1836	0.1719	0.1603	0.1489	0.1375	0.1263	0.1152
Investment	-0.0905	-0.0916	-0.0929	-0.0936	-0.0942	-0.0947	-0.0952	-0.0956	-0.0959	-0.0963
Labor income	0.7003	0.6895	0.6787	0.6687	0.6590	0.6495	0.6401	0.6310	0.6219	0.6129
Long-run (5 years)										
Consumption	1.0411	1.0112	0.9820	0.9534	0.9251	0.8972	0.8695	0.8421	0.8150	0.7881
Investment	-0.0545	-0.0565	-0.0586	-0.0603	-0.0617	-0.0631	-0.0643	-0.0656	-0.0668	-0.0680
Labor income	1.3886	1.3631	1.338	1.3139	1.2904	1.2672	1.2444	1.2218	1.1995	1.1774