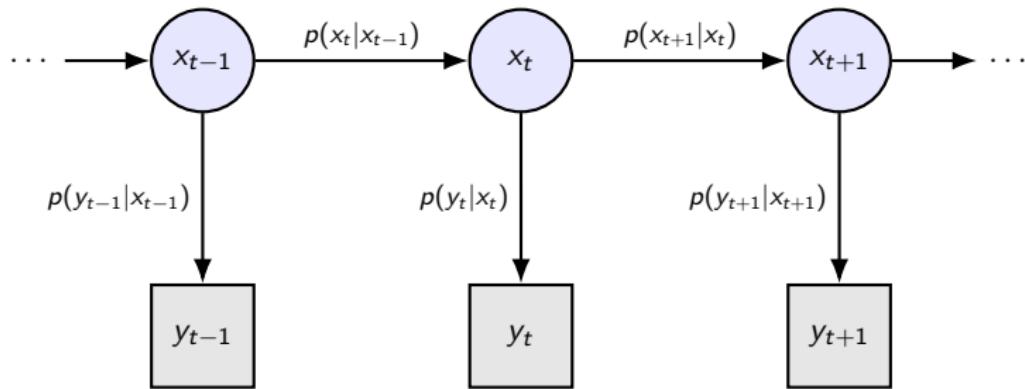


# Comparing POMP to Birth-Death-Immigration ODE

Kunyang He and Hanbin Lee

December 4, 2025

# State Space Model Structure



**Top Layer:** Latent process evolving via transition density.

$$p(x_t | x_{t-1}) \sim \frac{1}{2N_t} \text{Binomial}(2N_t, x_{t-1}), \quad x_t \in [0, 1] \quad (1)$$

**Bottom Layer:** Observation process conditioned on latent state.

$$p(y_t | x_t) \sim \text{Binomial}(n, x_t), \quad y_t \in \{0, \dots, n\} \quad (2)$$

# Approximate Derivation of the BDI

The ordinary differential equation for the birth-death-immigration process is

$$\begin{aligned}\frac{d}{dt} p_k(t) = & (f + (k - 1)b(t))p_{k-1}(t) \\ & - (f + k(b(t) + d(t)))p_k(t) \\ & + (k + 1)d(t)p_{k+1}(t), \quad k \geq 0\end{aligned}\tag{3}$$

This is derived from

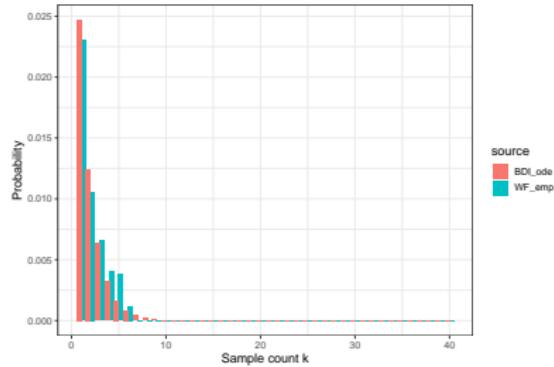
$$p_k(t) = \underbrace{\int_0^1 \binom{n}{k} x^k (1-x)^{n-k} p_t(x) dx}_{\text{Poisson approximation}} \underbrace{\qquad\qquad\qquad}_{\text{Diffusion}}\tag{4}$$

which makes the integration analytic, hence the recursion in equation (3).

# Results

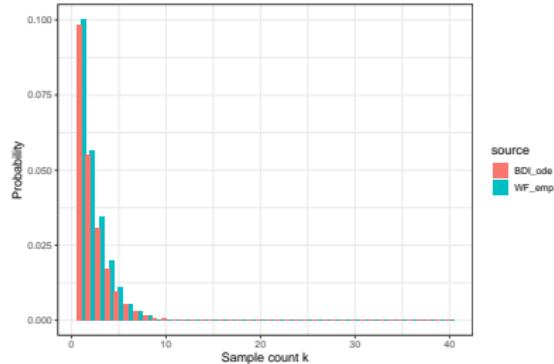
WF vs BDI  $p_0 = 0.001$

Population Size N = 5000, Sample Size n = 100, Particles  $n_{\text{rep}} = 5000$



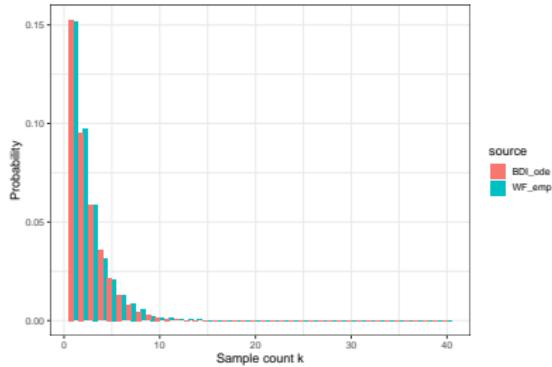
WF vs BDI  $p_0 = 0.005$

Population Size N = 5000, Sample Size n = 100, Particles  $n_{\text{rep}} = 5000$



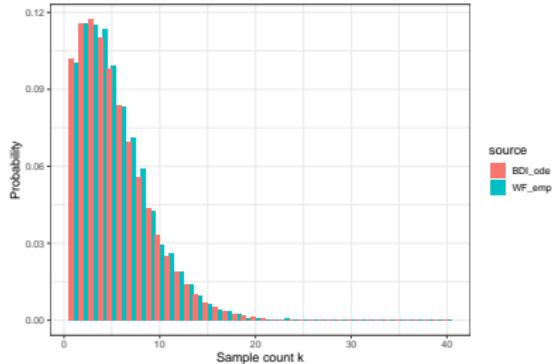
WF vs BDI  $p_0 = 0.01$

Population Size N = 5000, Sample Size n = 100, Particles  $n_{\text{rep}} = 5000$



WF vs BDI  $p_0 = 0.05$

Population Size N = 5000, Sample Size n = 100, Particles  $n_{\text{rep}} = 5000$



# Thank you for listening

Questions?