

ECE 124 digital circuits and systems

Assignment #2

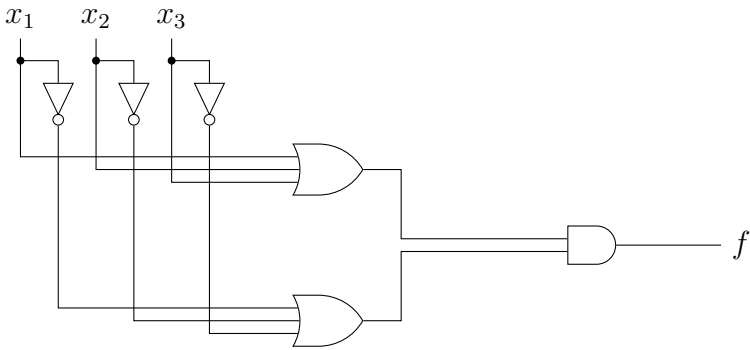
Q1: Design the simplest circuit that has 3 inputs, x_1 , x_2 and x_3 , which produces an output of 1 whenever exactly one or two of the inputs have the value of 1; otherwise, the output is 0.

Solution:

Write down a truth table that implements the verbal problem description.

x_3	x_2	x_1	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

From here, can write down (for example) a sum-of-products and simplify it which gives $f = x_2x_1' + x_3'x_1 + x_3x_2'$. This 2-level SOP has a cost of 13 (3, 2-input AND + 1, 3-input OR). We can also find a simplified POS which is $f = (x_1' + x_2' + x_3')(x_1 + x_2 + x_3)$. This 2-level POS has a cost of 11 (2, 3-input OR + 1, 2-input AND) and is the simplest circuit.



Q2: Consider a circuit with one output f and four inputs a_1 , a_0 , b_1 and b_0 . Let $A = a_1a_0$ be a binary representation of a number where the four possible values of A (00, 01, 10, and 11) represent the four unsigned integer values 0, 1, 2 and 3, respectively. Similarly, let $B = b_1b_0$ represent another number with the same four values. Assume that f should be 1 if the numbers represented by A and B are equal, otherwise f should be 0.

- (a) Provide the truth table for f ;
- (b) Determine the simplest product-of-sums expression for f .

Solution:

(a)

Write down a truth table that implements the verbal problem description. This function is a 4-input function. The numbers are equal when the individual bits are equal $a_1 = b_1$ and $a_0 = b_0$.

a_1	a_0	b_1	b_0	f
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

(b)

After some work, you can find that $f = (a_1 + b'_1)(a'_1 + b_1)(a'_0 + b_0)(a_0 + b'_0)$.

Q3: Consider a circuit with one output f and four inputs a_1 , a_0 , b_1 and b_0 . Let $A = a_1a_0$ be a binary representation of a number where the four possible values of A (00, 01, 10, and 11) represent the four unsigned integer values 0, 1, 2 and 3, respectively. Similarly, let $B = b_1b_0$ represent another number with the same four values. Assume that f should be 1 if $A \geq B$, otherwise f should be 0.

- (a) Provide the truth table for f ;
- (b) Determine the simplest sum-of-products expression for f .

Solution:

(a)

Write down a truth table that implements the verbal problem description. This function is a 4-input function. $A \geq B$ when $A = B$ or $A > B$. For $A = B$, see previous problem. For $A > B$, this is true when $a_1 > b_1$ or when $a_1 = b_1$ and $a_0 > b_0$.

a_1	a_0	b_1	b_0	$A = B$	$A > B$	$f = (A = B) + A > B$
0	0	0	0	1	0	1
0	0	0	1	0	0	0
0	0	1	0	0	0	0
0	0	1	1	0	0	0
0	1	0	0	0	1	1
0	1	0	1	1	0	1
0	1	1	0	0	0	0
0	1	1	1	0	0	0
1	0	0	0	0	1	1
1	0	0	1	0	1	1
1	0	1	0	1	0	1
1	0	1	1	0	0	0
1	1	0	0	0	1	1
1	1	0	1	0	1	1
1	1	1	0	0	1	1
1	1	1	1	1	0	1

(b)

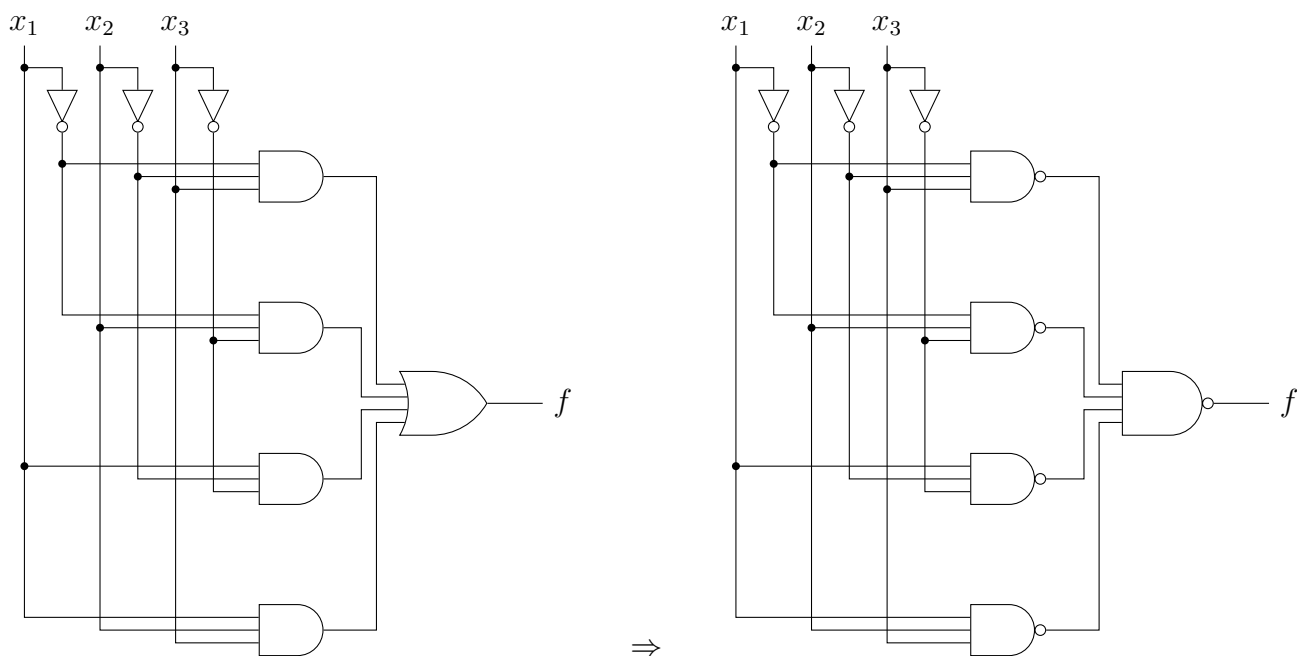
After some work, you can find that $f = a_1a_0 + b_1'b_0' + b_1'a_0 + a_1b_1' + a_1b_0'$.

Q4: Consider the following sum-of-products expression $f = x_1'x_2'x_3 + x_1'x_2x_3' + x_1x_2'x_3' + x_1x_2x_3$. Implement f as a 2-level circuit using only NAND gates.

Solution:

This can be done algebraically (double inversion) or graphically since we are starting from a SOP.

$$\begin{aligned} f &= x_1'x_2'x_3 + x_1'x_2x_3' + x_1x_2'x_3' + x_1x_2x_3 \\ f &= ((x_1'x_2'x_3 + x_1'x_2x_3' + x_1x_2'x_3' + x_1x_2x_3)')' \\ f &= ((x_1'x_2'x_3)'(x_1'x_2x_3')'(x_1x_2'x_3')'(x_1x_2x_3)')' \end{aligned}$$

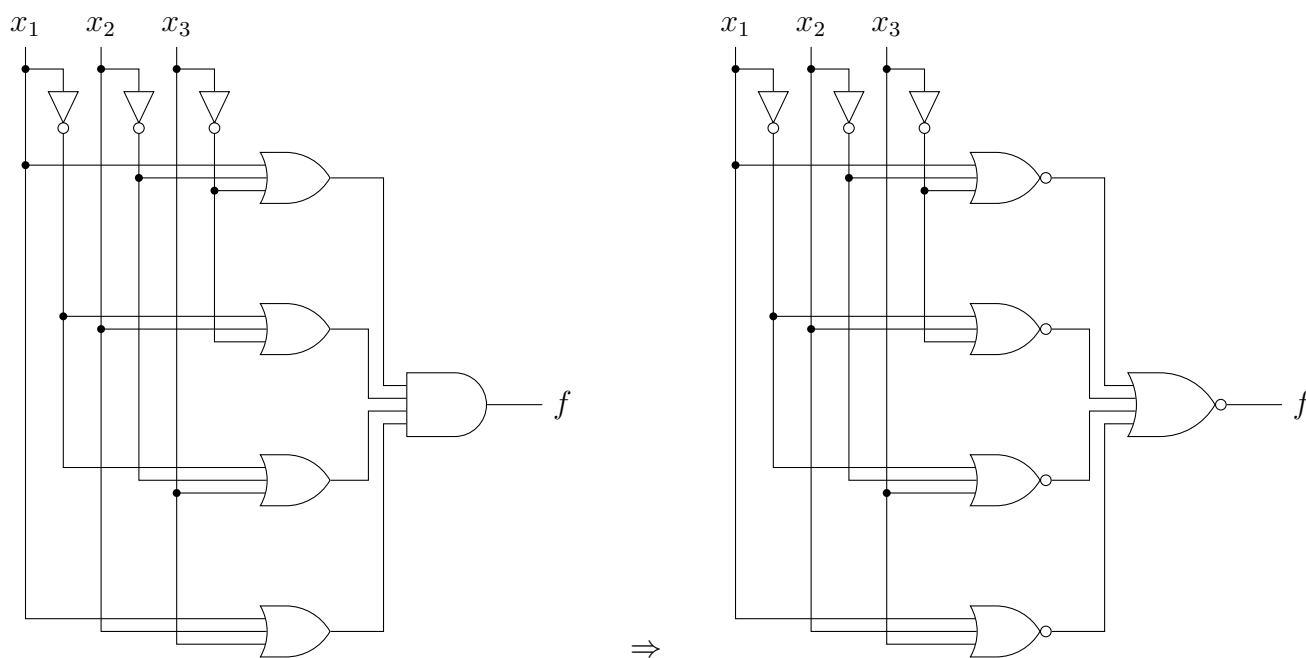


Q5: Consider the following product-of-sums expression $f = (x_1 + x'_2 + x'_3)(x'_1 + x_2 + x'_3)(x'_1 + x'_2 + x_3)(x_1 + x_2 + x_3)$. Implement f as a 2-level circuit using only NOR gates.

Solution:

This can be done algebraically (double inversion) or graphically since we are starting from a POS.

$$\begin{aligned}
 f &= (x_1 + x'_2 + x'_3)(x'_1 + x_2 + x'_3)(x'_1 + x'_2 + x_3)(x_1 + x_2 + x_3) \\
 f &= (((x_1 + x'_2 + x'_3)(x'_1 + x_2 + x'_3)(x'_1 + x'_2 + x_3)(x_1 + x_2 + x_3))')' \\
 f &= ((x_1 + x'_2 + x'_3)' + (x'_1 + x_2 + x'_3)' + (x'_1 + x'_2 + x_3)' + (x_1 + x_2 + x_3)')'
 \end{aligned}$$



Q6: Consider the logic function $f = (((x_1x_2') + (x_1'x_2))x_3) + (((x_1x_2') + (x_1'x_2))'x_4)$ which is shown below (note that inverters are shown explicitly) in Figure 1.

- Implement/convert this circuit to one that uses only NAND gates and NOT gates.
- Implement/convert this circuit to one that uses only NOR gates and NOT gates.

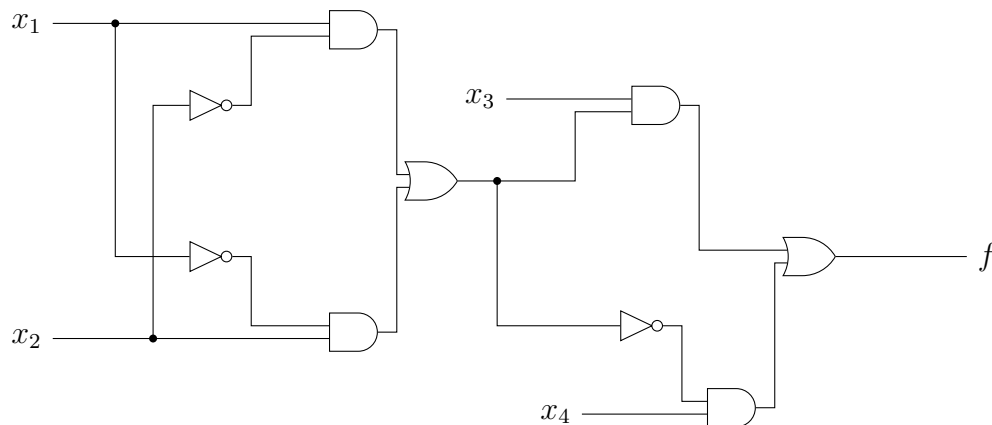
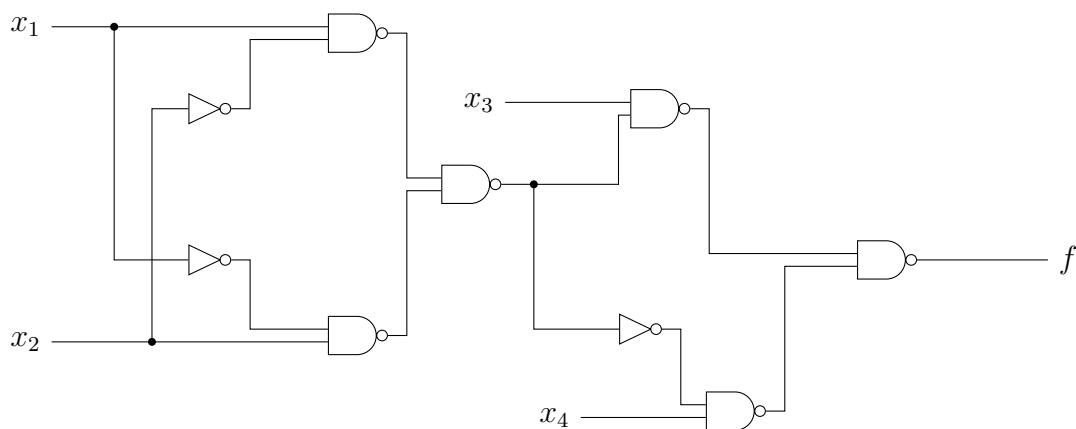


Figure 1: Circuit for Q5.

Solution:

(a)



The diagram shows a logic circuit with four inputs: x_1 , x_2 , x_3 , and x_4 . The circuit is composed of the following gates and connections:

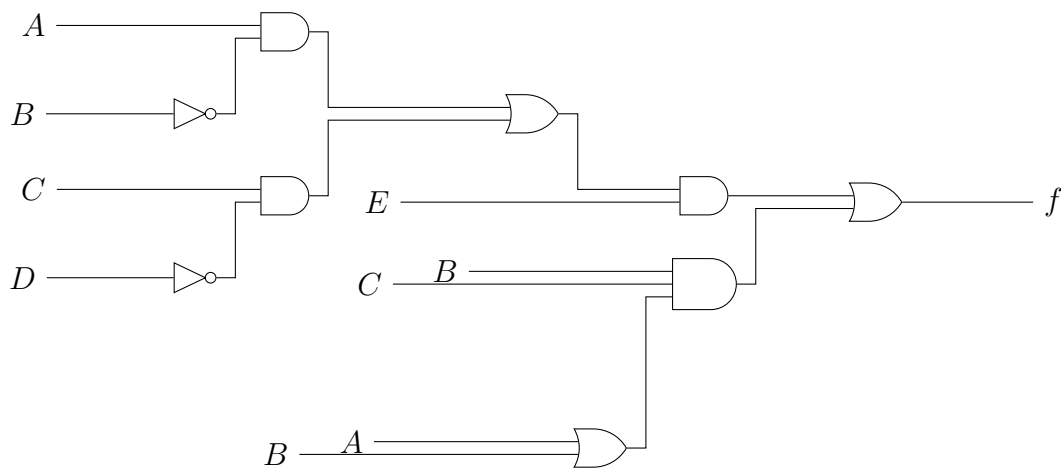
- Input x_1 is connected to a NOT gate and an OR gate.
- Input x_2 is connected to an OR gate and a NOT gate.
- The output of the NOT gate for x_1 is connected to the OR gate that also takes x_2 as input.
- The output of the NOT gate for x_2 is connected to the OR gate that also takes x_1 as input.
- The outputs of these two OR gates are connected to a single AND gate.
- Input x_3 is connected to a NOT gate.
- The output of the NOT gate for x_3 is connected to an OR gate.
- The output of the AND gate from the first stage is connected to the OR gate that also takes the output of the NOT gate for x_3 as input.
- The output of this OR gate is connected to a final OR gate.
- Input x_4 is connected to a NOT gate.
- The output of the NOT gate for x_4 is connected to an AND gate.
- The output of the final OR gate is connected to the AND gate that also takes the output of the NOT gate for x_4 as input.
- The output of this final AND gate is connected to a NOT gate, which produces the final output f .

Q7: Consider the logic function $f = (AB' + CD')E + BC(A + B)$.

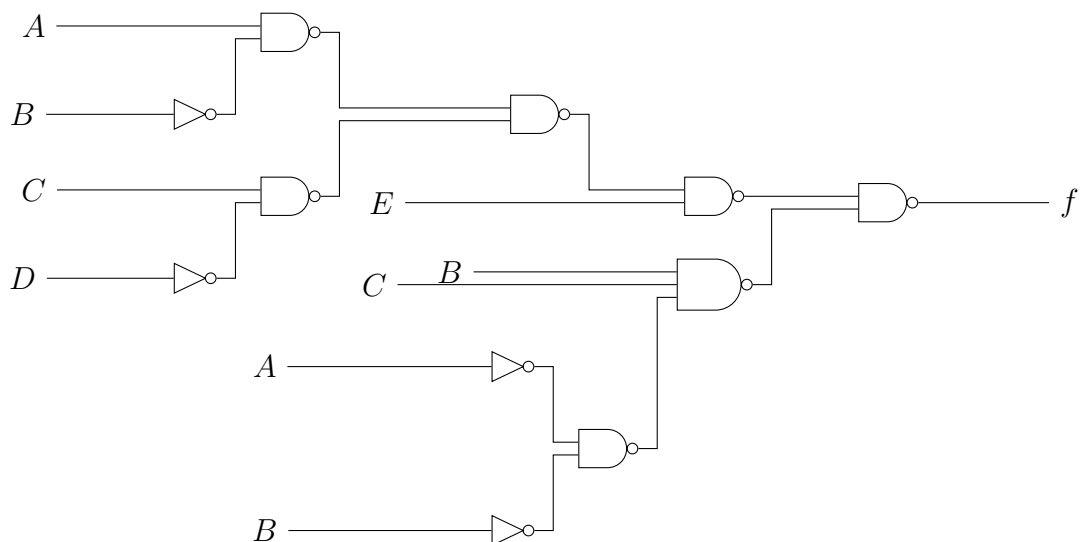
- (a) Draw this multi-level circuit using AND, OR and NOT gates.
 (b) Convert and draw a multi-level circuit implementation that uses only NAND gates and NOT gates.

Solution:

(a)

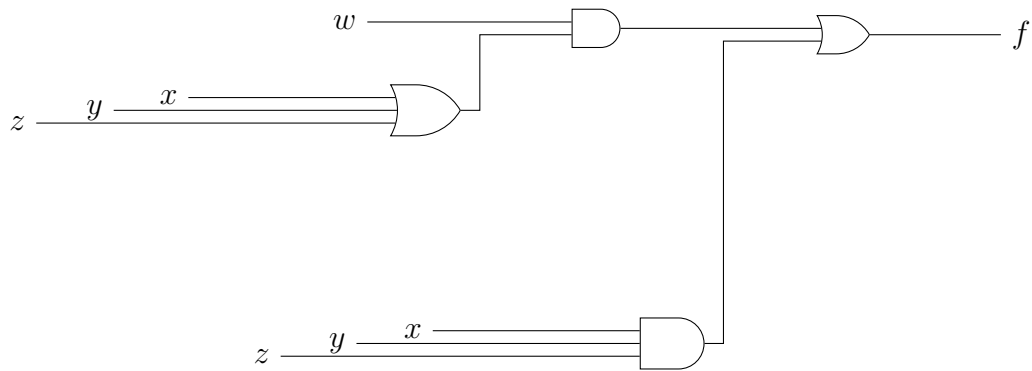


(b)



- Draw this multi-level circuit using AND, OR and NOT gates.
- Convert and draw a multi-level circuit implementation that uses only NOR gates and NOT gates.

(a)



The top diagram shows a logic circuit with four inputs: w , x , y , and z . The output is f . The circuit consists of the following components and connections:

- Input w is connected to a NOT gate.
- Inputs x and y are connected to an OR gate.
- The output of the NOT gate on w and the output of the OR gate on x and y are connected to a second OR gate.
- The output of this second OR gate is connected to a third OR gate.
- Input z is connected to the third OR gate.
- The output of the third OR gate is connected to a NOT gate, which produces the final output f .

The bottom diagram shows a logic circuit with three inputs: x , y , and z . The output is f . The circuit consists of the following components and connections:

- Inputs x , y , and z are each connected to a NOT gate.
- The outputs of these three NOT gates are connected to a single OR gate.
- The output of this OR gate is connected to a NOT gate, which produces the final output f .