ECE 124 digital circuits and systems
Assignment #3

Q1: Find minimum SOP and POS expressions for the following functions using 3-variable Kmaps.

(a) $f(a,b,c) = \sum(0,2,6,7)$

(b) $f(a,b,c) = \Pi(1,2,4)$

Q2: Find minimum SOP and POS expressions for the following functions using 3-variable Kmaps.

(a) $f = xy + x'y'z' + x'yz'$

(b) $f = A'B + BC' + B'C'$

(c) $f = (x' + y + z)(x' + y + z')(x' + y' + z)$

Q3: Find minimum SOP and POS expressions for the following functions using 4-variable Kmaps.

(a) $f = w'z + xz + x'y + wx'z.$

(b) $f = B'D + A'BC' + AB'C + ABC'$

(c) $f = (A' + B' + D')(A + B' + C')(A' + B + D')(B + C' + D').$

Q4: Find minimum SOP and POS expressions for the following functions using 4-variable Kmaps.

(a) $f(w,x,y,z) = \sum(0,1,2,3,4,12,13,14,15)$

(b) $f(w,x,y,z) = \Pi(0,1,2,4,5,7,8,9,10,12,14,15)$

Q5: Find minimum SOP and POS expressions for the following functions using 5-variable Kmaps.

(a) $f(x_1,x_2,x_3,x_4,x_5) = \sum(0,1,4,5,16,17,21,25,29)$

(b) $f = x_1'x_2'x_3x_4' + x_1'x_2'x_3'x_4' + x_2'x_4'x_5' + x_2'x_3x_4' + x_3x_4x_5' + x_2x_4x_5'.$

(c) $f(x_1,x_2,x_3,x_4,x_5) = \Pi(1,4,6,7,9,12,15,17,20,21,22,23,28,31)$

Q6: Find minimum SOP and POS expressions for the following functions together with the don't care conditions D.

(a) $f(w,x,y,z) = \sum(0,2,8,9,10,15) + D(1,3,6,7)$
(b) $f(w,x,y,z) = \Pi(1,3,5,7,13) + D(0,2,15)$

Q7: Find the prime implicants for the following functions. Determine which prime implicants are essential.

(a) $f(a,b,c,d) = \sum(0,2,4,5,6,7,8,10,13,15)$
(b) $f(a,b,c,d) = \sum(1,3,4,5,10,11,12,13,14,15)$

Q8: A four-variable logic function that equals 1 if any three or all four input variables are equal to 1 is called a *majority* function. Design a minimum cost SOP circuit that implements the majority function.

Q9: Derive a minimum cost 2-level circuit for a four variable function that is equal to 1 if exactly two or exactly three of its input variables are equal to 1; otherwise it is equal to 0.

Q10: A circuit with two outputs is required to implement the two logic functions f and g given by

$$f(a,b,c,d) = \sum(0,2,4,6,7,9) + D(10,11)$$

and

$$g(a,b,c,d) = \sum(2,4,9,10,15) + D(0,13,14)$$

where D denotes the don't cares for each function.

(a) Design a minimum SOP for both f and g separately and compute cost of each function assuming each gate costs 1 and each gate input costs 1. You may assume circuit inputs are available in both complemented and uncomplemented forms.
(b) Design a minimum cost circuit that implements both f and g as SOPs. Determine the cost of the circuit.

Q11: Repeat problem 10 for the following functions.

$$f(x_1,x_2,x_3,x_4,x_5) = \sum(1,4,5,11,27,28) + D(10,12,14,15,20,31)$$

and

$$g(x_1,x_2,x_3,x_4,x_5) = \sum(0,1,2,4,5,8,14,15,16,18,20,24,26,28,31) + D(10,11,12,27)$$

Q12: Derive a minimum cost circuit that implements the function $f(a,b,c,d) = \sum(4,7,8,11) + D(12,15)$. You are not restricted to 2-level circuits and may use any sort of logic gate.

Q13: Find a minimum cost circuit that implements the function $f(a,b,c,d) = \sum(0,4,8,13,14,15)$. The input variables are available in uncomplemented form only.

Q14: Find the simplest realization of the function $f(a,b,c,d) = \sum(0,3,4,7,9,10,13,14)$ assuming you can only use logic gates with a maximum of 2-inputs.