ECE 124 digital circuits and systems
Assignment #1

Note: In many cases, there are different ways to get to the same final answer.

Q1: Use algebraic manipulation to show that:

(a) $(x+y)(x+y') = x$

(b) $xy + yz + x'z = xy + x'z$

Solution:

(a)

$$\begin{aligned} f &= (x+y)(x+y') \\ &= xx + xy' + xy + yy' \\ &= x + xy' + xy + 0 \\ &= x(1 + y' + y) \\ &= x \end{aligned}$$

(b)

$$\begin{aligned} f &= xy + yz + x'z \\ &= xy + (x + x')yz + x'z \\ &= xy + xyz + x'yz + x'z \\ &= xy(1+z) + x'z(y+1) \\ &= xy + x'z \end{aligned}$$

Q2: Use algebraic manipulation to simply the following Boolean expressions as much as possible:

(a) $(x'y' + z)' + z + xy + wz$ (**Hint:** This expression simplifies to 3 literals).

(b) $A'B(D' + C'D) + B(A + A'CD)$ (**Hint:** This expression simplifies to 1 literal).

Solution:

(a)

$$\begin{aligned}
f &= (x'y' + z)' + z + xy + wz \\
&= (x'y')'z' + z + xy + wz \\
&= (x + y)z' + z + xy + wz \\
&= xz' + yz' + xy + z + wz \\
&= xz' + yz' + xy + z(1 + w) \\
&= xz' + yz' + xy + z \\
&= xz' + yz' + xy + z(x + x') + z(y + y') \\
&= xz' + yz' + xy + xz + x'z + yz + y'z + xz + yz \\
&= xz' + xz + yz' + yz + xy + xz + x'z + yz + y'z \\
&= x(z' + z) + y(z' + z) + xy + z(x + x' + y + y') \\
&= x + y + xy + z \\
&= x + y(1 + x) + z \\
&= x + y + z
\end{aligned}$$

(b)

$$\begin{aligned}
f &= A'B(D' + C'D) + B(A + A'CD) \\
&= B(A'(D' + C'D) + (A + A'CD)) \\
&= B(A'D' + A'C'D + A + A'CD) \\
&= B(A'D' + A'D(C' + C) + A) \\
&= B(A'D' + A'D + A) \\
&= B(A'(D' + D) + A) \\
&= B(A' + A) \\
&= B
\end{aligned}$$

Q3: Determine the truth tables for each of the following functions:

(a) $(xy + z)(y + xz)$

(b) $(A' + B)(B' + C)$

(c) $y'z + wxy' + wxz' + w'x'z$

There are different ways to get to a truth table. One way is to evaluate the function for all possible input values, but that will take a lot of time. Another way is to find sum of minterms expressions and then fill in the truth table. There are other ways too.

Solution:

(a)

$$
\begin{aligned}
f &= (xy + z)(y + xz) \\
&= xy + xyz + yz + xz \\
&= xy(z + z') + xyz + yz(x + x') + xz(y + y') \\
&= xyz + xyz' + xyz + xyz + x'yz + xyz + xy'z \\
&= xyz + xyz' + x'yz + xy'z \\
&= m_7 + m_6 + m_3 + m_5
\end{aligned}
$$

x	**y**	**z**	**f**
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

(b)

$$
\begin{aligned}
f &= (A' + B)(B' + C) \\
&= A'B' + A'C + BB' + BC \\
&= A'B' + A'C + BC \\
&= A'B'C + A'B'C' + A'BC + A'B'C + A'BC + ABC \\
&= m_1 + m_0 + m_3 + m_1 + m_3 + m_7 \\
&= m_0 + m_1 + m_3 + m_7
\end{aligned}
$$

x	**y**	**z**	**f**
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

(c) $f = y'z + wxy' + wxz' + w'x'z$. (Only the truth table is shown).

w	**x**	**y**	**z**	**f**
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

Q4: Draw logic diagrams for each of the following Boolean expressions:

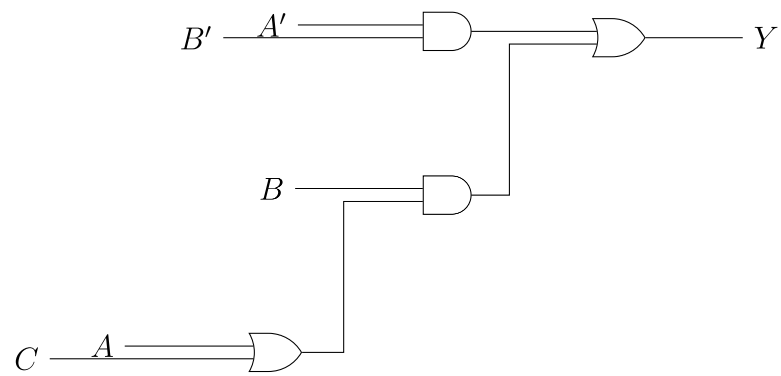
(a) $Y = A'B' + B(A + C)$

(b) $Y = A' + CD$

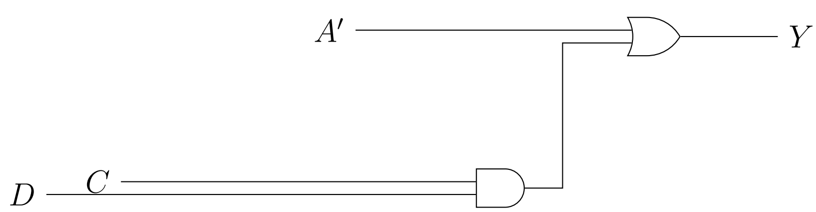
(c) $Y = (A + B')(C' + D)(A' + B + D)$

Solution:

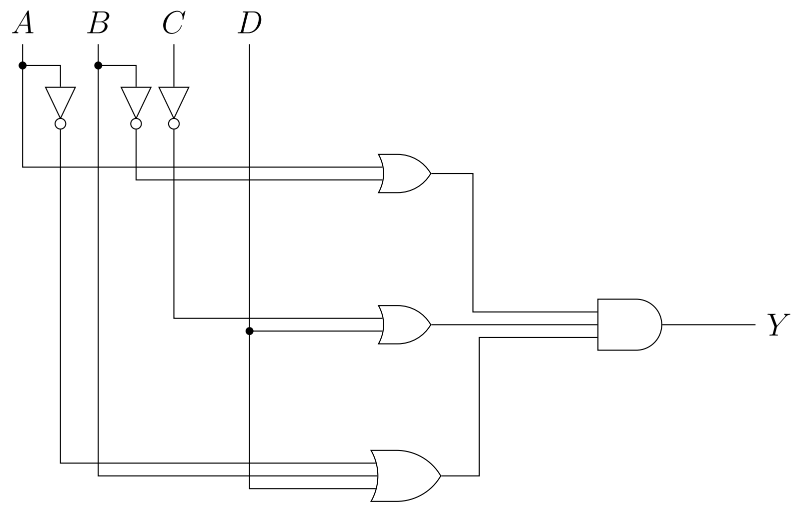
(a) $Y = A'B' + B(A + C)$



(b) $Y = A' + CD$



(c) $Y = (A + B')(C' + D)(A' + B + D)$



Q5: Use algebraic manipulation to find the minimum product-of-sums (POS) expressions for the following functions:

(a) $f = (x_1 + x_3 + x_4)(x_1 + x_2^{'} + x_3)(x_1 + x_2^{'} + x_3^{'} + x_4)$

(b) $f = x_2 + x_1 x_3 + x_1^{'} x_3^{'}$

Solution:

(a)

$$
\begin{aligned}
f &= (x_1 + x_3 + x_4)(x_1 + x_2^{'} + x_3)(x_1 + x_2^{'} + x_3^{'} + x_4) \\
f &= (x_1 + x_3 + x_4)[(x_1 + x_2^{'}) + ((x_3)(x_3^{'} + x_4))] \\
f &= (x_1 + x_3 + x_4)[(x_1 + x_2^{'}) + (x_3 x_4)] \\
f &= (x_1 + x_3 + x_4)(x_1 + x_2^{'} + x_3)(x_1 + x_2^{'} + x_4)
\end{aligned}
$$

(b)

$$
\begin{aligned}
f &= x_2 + x_1 x_3 + x_1^{'} x_3^{'} \\
f &= (x_2 + x_1)(x_2 + x_3) + x_1^{'} x_3^{'} \\
f &= ((x_2 + x_1)(x_2 + x_3) + x_1^{'})((x_2 + x_1)(x_2 + x_3) + x_3^{'}) \\
f &= (x_2 + x_1 + x_1^{'})(x_2 + x_3 + x_1^{'})(x_2 + x_1 + x_3^{'})(x_2 + x_3 + x_3^{'}) \\
f &= (x_2 + x_3 + x_1^{'})(x_2 + x_1 + x_3^{'})
\end{aligned}
$$

Q6: Use algebraic manipulation to find the minimum sum-of-products (SOP) expressions for the following functions:

(a) $f = x_1x_2'x_3' + x_1x_2x_4 + x_1x_2'x_3x_4'$

(b) $f = x_1'x_2'x_3 + x_1x_3 + x_2x_3 + x_1x_2x_3'$

Solution:

(a)

$$\begin{aligned}
f &= x_1x_2'x_3' + x_1x_2x_4 + x_1x_2'x_3x_4' \\
f &= x_1x_2'(x_3' + x_3x_4') + x_1x_2x_4 \\
f &= x_1x_2'(x_3' + x_4') + x_1x_2x_4 \\
f &= x_1x_2'x_3' + x_1x_2'x_4' + x_1x_2x_4
\end{aligned}$$

(b)

$$\begin{aligned}
f &= x_1'x_2'x_3 + x_1x_3 + x_2x_3 + x_1x_2x_3' \\
f &= (x_1'x_2' + x_1)x_3 + x_2x_3 + x_1x_2x_3' \\
f &= (x_2' + x_1)x_3 + x_2x_3 + x_1x_2x_3' \\
f &= x_2'x_3 + x_1x_3 + x_2x_3 + x_1x_2x_3' \\
f &= x_2'x_3 + x_2x_3 + x_1(x_3 + x_2x_3') \\
f &= x_2'x_3 + x_2x_3 + x_1(x_3 + x_2) \\
f &= x_2'x_3 + x_2x_3 + x_1x_3 + x_1x_2 \\
f &= (x_2' + x_2 + x_1)x_3 + x_1x_3 + x_1x_2 \\
f &= (1 + x_1)x_3 + x_1x_3 + x_1x_2 \\
f &= x_3 + x_1x_2
\end{aligned}$$

Q7: Determine the simplest sum-of-products circuit that implements the function $f(x_1, x_2, x_3) = \sum m(1, 3, 4, 6, 7)$.

Solution:

$$\begin{aligned}
f &= x_1'x_2'x_3 + x_1'x_2x_3 + x_1x_2'x_3' + x_1x_2x_3' + x_1x_2x_3 \\
&= x_1'x_2'x_3 + x_1'x_2x_3 + x_1x_2'x_3' + x_1x_2x_3' + x_1x_2x_3' + x_1x_2x_3 \\
&= x_1'x_3(x_2' + x_2) + x_1(x_2' + x_2)x_3' + x_1x_2(x_3' + x_3) \\
&= x_1'x_3 + x_1x_3' + x_1x_2
\end{aligned}$$

Q8: Determine the simplest product-of-sums circuit that implements the function $f(x_1, x_2, x_3) = \Pi M(0, 2, 5)$.

Solution:

$$\begin{aligned}
f &= (x_1 + x_2 + x_3)(x_1 + x_2' + x_3)(x_1' + x_2 + x_3') \\
&= ((x_1 + x_3) + (x_2 x_2'))(x_1' + x_2 + x_3') \\
&= ((x_1 + x_3) + 0)(x_1' + x_2 + x_3') \\
&= (x_1 + x_3)(x_1' + x_2 + x_3')
\end{aligned}$$

Q9: Convert each of the following Boolean expressions into both sum-of-products and product-of-sums:

(a) $(AB + C)(B + C'D)$

(b) $x' + x(x + y')(y + z')$

Can write down a (any) sum-of-products. Then, we can use inversion to find a product-of-sums (by finding a sum-of-products for Y' and then complementing to get back to Y). We can simplify if we think it helps.

Solution:

(a) $Y = (AB + C)(B + C'D) = AB + ABC'D + BC + CC'D = AB + ABC'D + CB = AB + BC$

So $Y = AB + BC$ is a sum-of-products.

$Y = (Y')' = ((AB)'(BC)')' = ((A' + B')(B' + C'))' = (A'B' + A'C' + B' + B'C')' = (B' + A'C')'$. Finally, doing the last inversion gives $Y = (B)(A + C)$.

So $Y = (B)(A + C)$ is a product-of-sums.

(b) $f = x' + x(x + y')(y + z') = x' + (x + xy')(y + z') = x' + xy + xz' + xy'y + xy'z' = x' + xy + xz' + xy'z' = x' + xy + xz' = x' + y + z'$.

So $f = (x') + (y) + (z')$ is a sum-of-products.

$f = (f')' = (xy'z')'$. Finally, doing the last inversion gives $f = x' + y + z'$.

So $f = (x' + y + z')$ is also a product-of-sums.

Q10: Express $f(x_1, x_2, x_3, x_4) = x_2^{'}x_4 + x_1^{'}x_4 + x_2x_4$ as both a sum-of-minterms and as a product-of-maxtems.

We have a sum-of-products so it is easy to find the sum-of-minterms. Then, we can easily find the product-of-maxterms.

Solution:

$$
\begin{aligned}
f &= x_2^{'}x_4 + x_1^{'}x_4 + x_2x_4 \\
&= x_2^{'}x_4(x_1 + x_1^{'})(x_3 + x_3^{'}) + x_1^{'}x_4(x_2 + x_2^{'})(x_3 + x_3^{'}) + x_2x_4(x_1 + x_1^{'})(x_3 + x_3^{'}) \\
&= x_1x_2^{'}x_4(x_3 + x_3^{'}) + x_1^{'}x_2^{'}x_4(x_3 + x_3^{'}) \\
&\quad + x_1^{'}x_2x_4(x_3 + x_3^{'}) + x_1^{'}x_2^{'}x_4(x_3 + x_3^{'}) \\
&\quad + x_1x_2x_4(x_3 + x_3^{'}) + x_1^{'}x_2x_4(x_3 + x_3^{'}) \\
&= x_1x_2^{'}x_3x_4 + x_1x_2^{'}x_3^{'}x_4 + x_1^{'}x_2^{'}x_3x_4 + x_1^{'}x_2^{'}x_3^{'}x_4 \\
&\quad + x_1^{'}x_2x_3x_4 + x_1^{'}x_2x_3^{'}x_4 + x_1^{'}x_2^{'}x_3x_4 + x_1^{'}x_2^{'}x_3^{'}x_4 \\
&\quad + x_1x_2x_3x_4 + x_1x_2x_3^{'}x_4 + x_1^{'}x_2x_3x_4 + x_1^{'}x_2x_3^{'}x_4 \\
&= m_{11} + m_9 + m_3 + m_1 \\
&\quad + m_7 + m_5 + m_3 + m_1 \\
&\quad + m_{15} + m_{13} + m_7 + m_5 \\
&= m_1 + m_3 + m_5 + m_7 + m_9 + m_{11} + m_{13} + m_{15} \\
&= \sum(1, 3, 5, 7, 9, 11, 13, 15) \\
&= \Pi(0, 2, 4, 6, 8, 10, 12, 14) \\
&= M_0 + M_2 + M_4 + M_6 + M_8 + M_{10} + M_{12} + M_{14}
\end{aligned}
$$