ECE 124 digital circuits and systems
Assignment #2

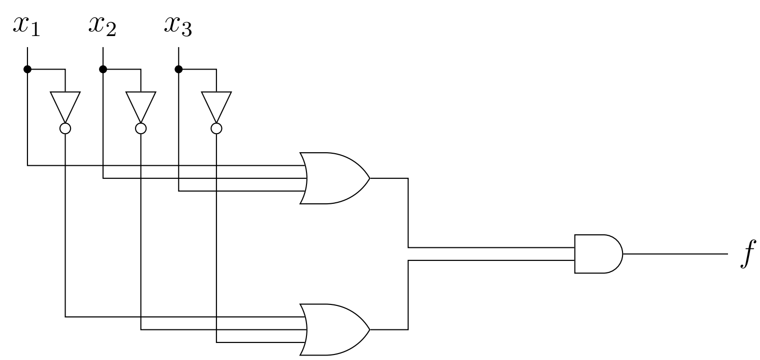
Q1: Design the simplest circuit that has 3 inputs, x_1, x_2 and x_3, which produces an output of 1 whenever exactly one or two of the inputs have the value of 1; otherwise, the output is 0.

Solution:

Write down a truth table that implements the verbal problem description.

$\mathbf{x_3}$	$\mathbf{x_2}$	$\mathbf{x_1}$	**f**
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

From here, can write down (for example) a sum-of-products and simplify it which gives $f = x_2x_1' + x_3'x_1 + x_3x_2'$. This 2-level SOP has a cost of 13 (3, 2-input AND + 1, 3-input OR). We can also find a simplified POS which is $f = (x_1' + x_2' + x_3')(x_1 + x_2 + x_3)$. This 2-level POS has a cost of 11 (2, 3-input OR + 1, 2-input AND) and is the simplest circuit.


x_1
x_2
x_3
f

Q2: Consider a circuit with one output f and four inputs a_1, a_0, b_1 and b_0. Let $A = a_1a_0$ be a binary representation of a number where the four possible values of A (00, 01, 10, and 11) represent the four unsigned integer values 0, 1, 2 and 3, respectively. Similarly, let $B = b_1b_0$ represent another number with the same four values. Assume that f should be 1 if the numbers represented by A and B are equal, otherwise f should be 0.

(a) Provide the truth table for f;

(b) Determine the simplest product-of-sums expression for f.

Solution:

(a)

Write down a truth table that implements the verbal problem description. This function is a 4-input function. The numbers are equal when the individual bits are equal $a_1 = b_1$ and $a_0 = b_0$.

$\mathbf{a_1}$	$\mathbf{a_0}$	$\mathbf{b_1}$	$\mathbf{b_0}$	**f**
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

(b)

After some work, you can find that $f = (a_1 + b_1^{'})(a_1^{'} + b_1)(a_0^{'} + b_0)(a_0 + b_0^{'})$.

Q3: Consider a circuit with one output f and four inputs a_1, a_0, b_1 and b_0. Let $A = a_1 a_0$ be a binary representation of a number where the four possible values of A (00, 01, 10, and 11) represent the four unsigned integer values 0, 1, 2 and 3, respectively. Similarly, let $B = b_1 b_0$ represent another number with the same four values. Assume that f should be 1 if $A \geq B$, otherwise f should be 0.

(a) Provide the truth table for f;

(b) Determine the simplest sum-of-products expression for f.

Solution:

(a)

Write down a truth table that implements the verbal problem description. This function is a 4-input function. $A \geq B$ when $A = B$ or $A > B$. For $A = B$, see previous problem. For $A > B$, this is true when $a_1 > b_1$ or when $a_1 = b_1$ and $a_0 > b_0$.

$\mathbf{a_1}$	$\mathbf{a_0}$	$\mathbf{b_1}$	$\mathbf{b_0}$	$\mathbf{A = B}$	$\mathbf{A > B}$	$\mathbf{f = (A = B) + A > B}$
0	0	0	0	1	0	1
0	0	0	1	0	0	0
0	0	1	0	0	0	0
0	0	1	1	0	0	0
0	1	0	0	0	1	1
0	1	0	1	1	0	1
0	1	1	0	0	0	0
0	1	1	1	0	0	0
1	0	0	0	0	1	1
1	0	0	1	0	1	1
1	0	1	0	1	0	1
1	0	1	1	0	0	0
1	1	0	0	0	1	1
1	1	0	1	0	1	1
1	1	1	0	0	1	1
1	1	1	1	1	0	1

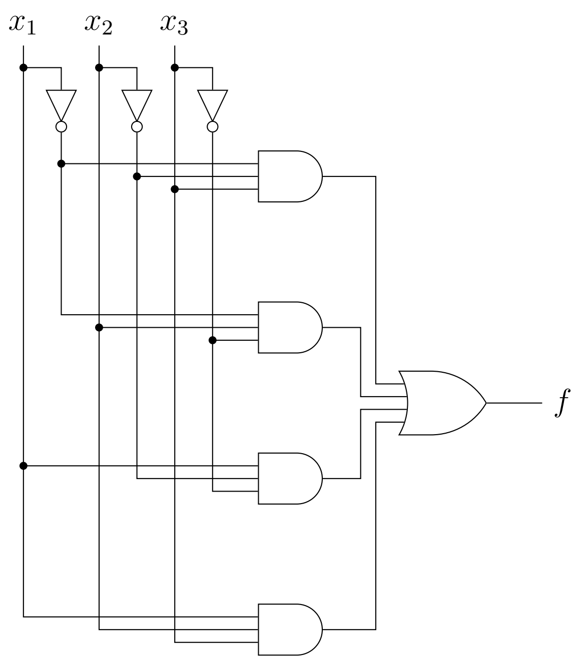
(b)

After some work, you can find that $f = a_1 a_0 + b_1' b_0' + b_1' a_0 + a_1 b_1' + a_1 b_0'$.

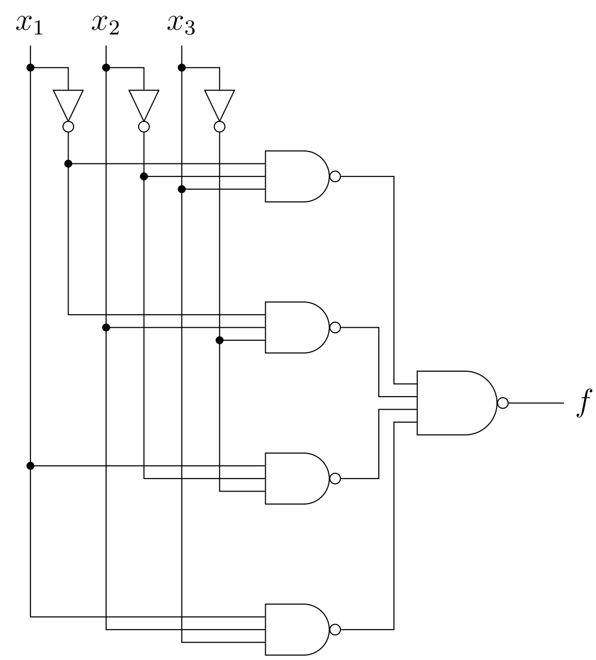
Q4: Consider the following sum-of-products expression $f = x_1'x_2'x_3 + x_1'x_2x_3' + x_1x_2'x_3' + x_1x_2x_3$. Implement f as a 2-level circuit using only NAND gates.

Solution:
This can be done algebraically (double inversion) or graphically since we are starting from a SOP.

$$\begin{aligned} f &= x_1'x_2'x_3 + x_1'x_2x_3' + x_1x_2'x_3' + x_1x_2x_3 \\ f &= ((x_1'x_2'x_3 + x_1'x_2x_3' + x_1x_2'x_3' + x_1x_2x_3)')' \\ f &= ((x_1'x_2'x_3)'(x_1'x_2x_3')'(x_1x_2'x_3')'(x_1x_2x_3)')' \end{aligned}$$



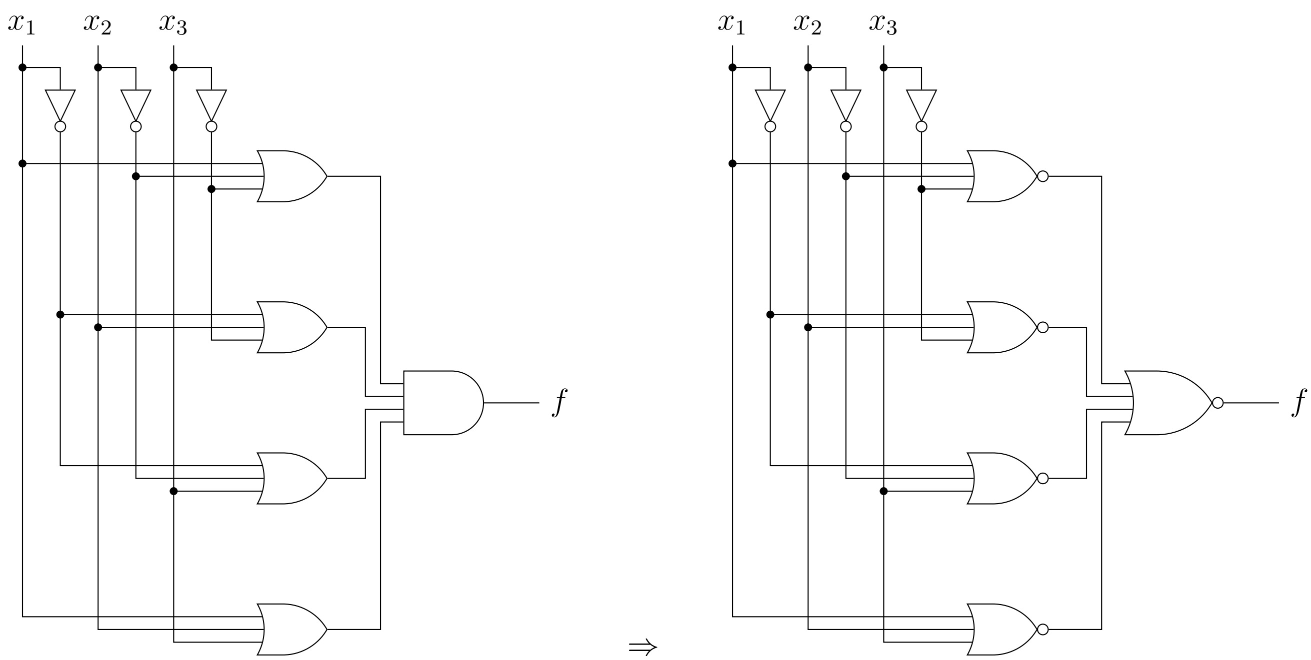
$\Rightarrow$



Q5: Consider the following product-of-sums expression $f = (x_1 + x_2' + x_3')(x_1' + x_2 + x_3')(x_1' + x_2' + x_3)(x_1 + x_2 + x_3)$. Implement f as a 2-level circuit using only NOR gates.

Solution:
This can be done algebraically (double inversion) or graphically since we are starting from a POS.

$$\begin{aligned} f &= (x_1 + x_2' + x_3')(x_1' + x_2 + x_3')(x_1' + x_2' + x_3)(x_1 + x_2 + x_3) \\ f &= (((x_1 + x_2' + x_3')(x_1' + x_2 + x_3')(x_1' + x_2' + x_3)(x_1 + x_2 + x_3))')' \\ f &= ((x_1 + x_2' + x_3')' + (x_1' + x_2 + x_3')' + (x_1' + x_2' + x_3)' + (x_1 + x_2 + x_3)')' \end{aligned}$$



Q6: Consider the logic function $f = (((x_1x_2') + (x_1'x_2))x_3) + (((x_1x_2') + (x_1'x_2))'x_4)$ which is shown below (note that inverters are shown explicitly) in Figure 1.

(a) Implement/convert this circuit to one that uses only NAND gates and NOT gates.

(b) Implement/convert this circuit to one that uses only NOR gates and NOT gates.

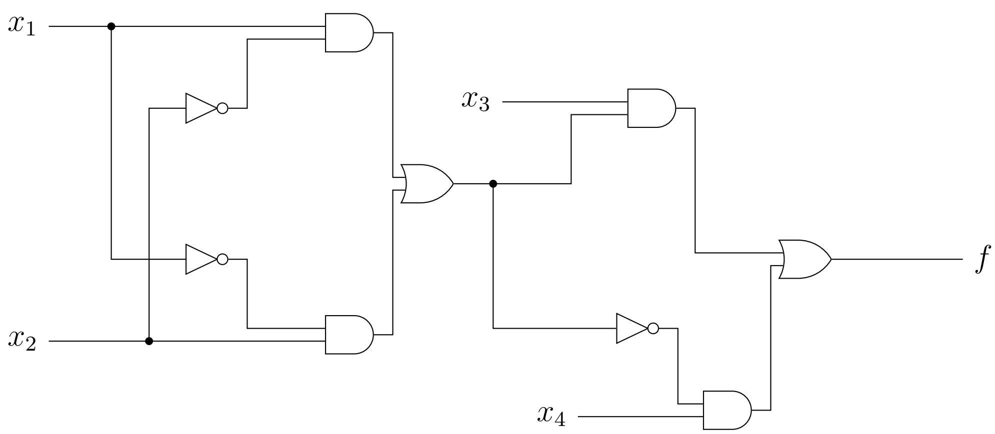
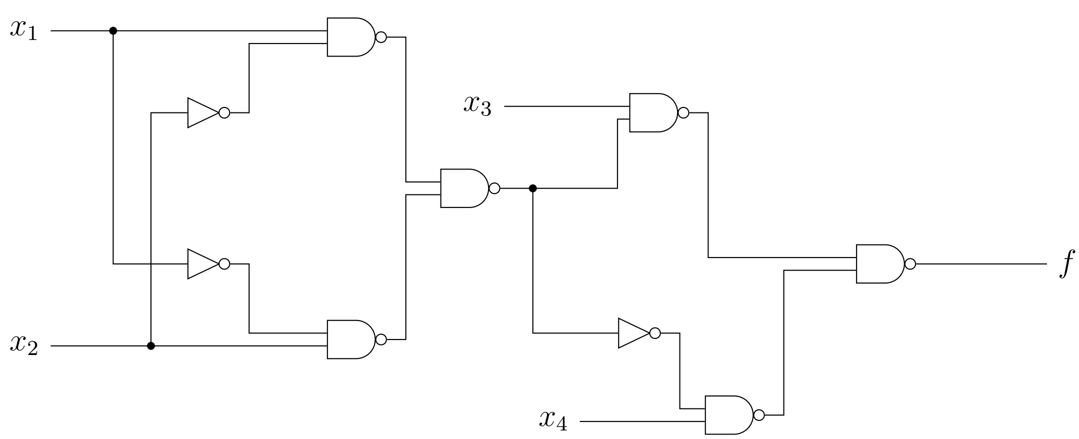


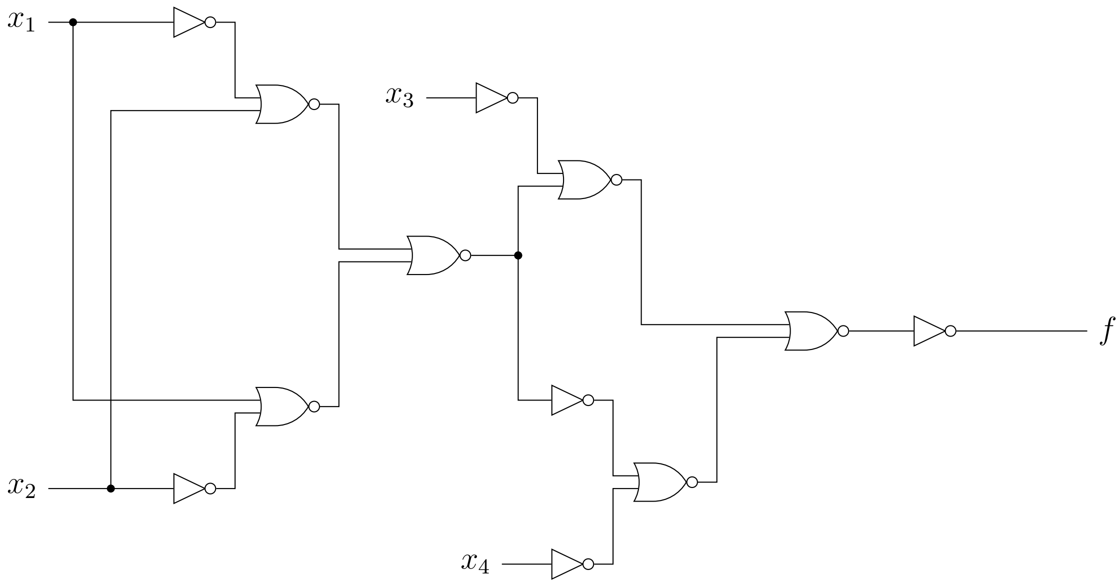
Figure 1: Circuit for Q5.

Solution:

(a)



(b)



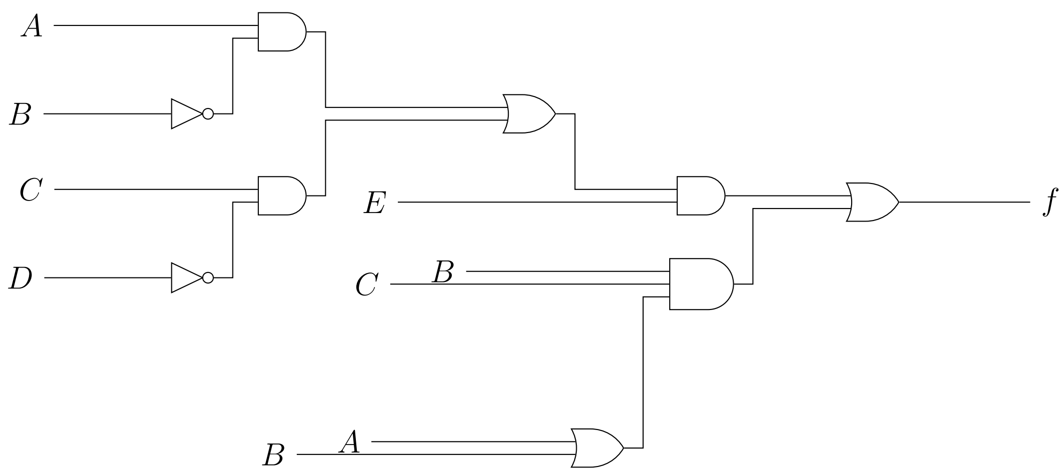
Q7: Consider the logic function $f = (AB' + CD')E + BC(A + B)$.

(a) Draw this multi-level circuit using AND, OR and NOT gates.

(b) Convert and draw a multi-level circuit implementation that uses only NAND gates and NOT gates.

Solution:

(a)



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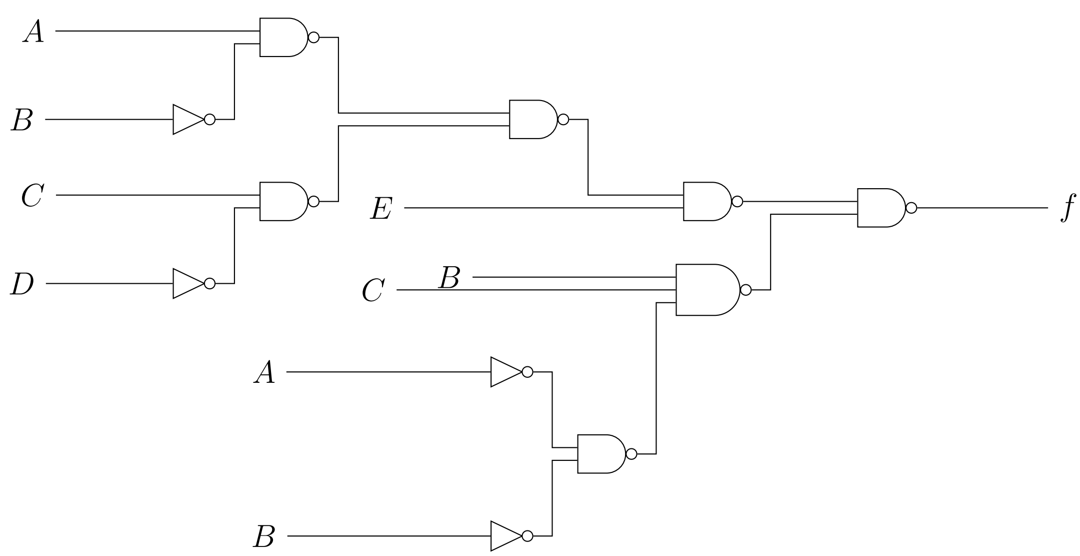


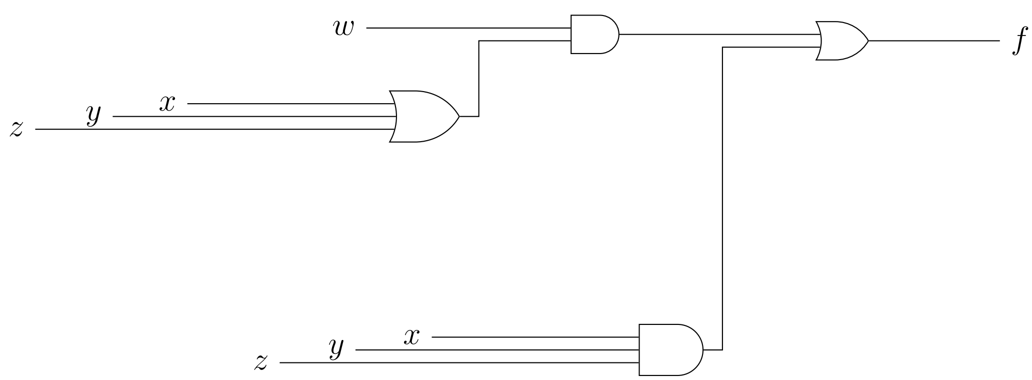
Q8: Consider the logic function $f = w(x + y + z) + xyz$.

(a) Draw this multi-level circuit using AND, OR and NOT gates.

(b) Convert and draw a multi-level circuit implementation that uses only NOR gates and NOT gates.

Solution:

(a)



(b)