

Introduction

Kernel Canonical Correlation Analysis (KCCA) is an efficient way to detect brain activation globally with less computational complexity. However, the current KCCA is limited to the linear kernel, and the performance for other more general types of kernels is not completely understood due to a lack of inverse mapping i.e. Back Construction (BC) methods. This study aims to expand the current KCCA method to arbitrary nonlinear kernels.

	Multi variable	Global method	Nonlinear relationship
Single voxel smoothing [1]	No	No	No
Canonical Correlation Analysis (CCA) [2]	Yes	No	Nonlinear constraints [3]
Linear Kernel Canonical Correlation Analysis (KCCA) [4]	Yes	Yes	No
Nonlinear KCCA	Yes	Yes	Nonlinear kernels

Method

Processing

$\mathbf{Y} \in \mathbb{R}^{T \times Q} \xrightarrow{\text{fMRI data}} \tilde{\mathbf{Y}} = \mathbf{Y}\mathbf{A} \xrightarrow{\text{Voxel specific activation}} K_{\mathbf{Y}} \xrightarrow{\text{Voxel specific activation}} K_{\mathbf{Y}}\mathbf{v}_{\mathbf{Y}} \xrightarrow{\text{Voxel specific activation}} \boldsymbol{\alpha} \in \mathbb{R}^{Q \times 1}$

$$s = \text{sign}[\text{corr}(\mathbf{X}_{\text{eff}}, K_{\mathbf{Y}}\mathbf{v}_{\mathbf{Y}})]$$

$$\max_{\mathbf{v}_{\mathbf{Y}}, \mathbf{v}_{\mathbf{X}}} \frac{\mathbf{v}_{\mathbf{Y}}^T K_{\mathbf{Y}} K_{\mathbf{X}} \mathbf{v}_{\mathbf{X}}}{\sqrt{\mathbf{v}_{\mathbf{Y}}^T (K_{\mathbf{Y}}^2 + \gamma K_{\mathbf{Y}}) \mathbf{v}_{\mathbf{Y}}} \sqrt{\mathbf{v}_{\mathbf{X}}^T (K_{\mathbf{X}}^2 + \gamma K_{\mathbf{X}}) \mathbf{v}_{\mathbf{X}}}}$$

$\mathbf{X} \in \mathbb{R}^{T \times D} \xrightarrow{[5]} \mathbf{X}_{\text{eff}} \xrightarrow{\text{Design signal}} K_{\mathbf{X}} = \mathbf{X}_{\text{eff}} \mathbf{X}_{\text{eff}}^T$

Design signal
 A: spatial filter
 Q: number of voxels
 T: number of time points
 D: number of design signals

Back construction algorithm

$$\mathbf{Y}\boldsymbol{\alpha} \equiv K_{\mathbf{Y}}\mathbf{v}_{\mathbf{Y}}$$

Back construction 1: $\boldsymbol{\alpha} = s \sum_t \frac{\partial (K_{\mathbf{Y}}\mathbf{v}_{\mathbf{Y}})_t}{\partial \mathbf{Y}_t}$

Back construction 2: $\boldsymbol{\alpha} = s \mathbf{A} \tilde{\mathbf{Y}}^T (\tilde{\mathbf{Y}} \tilde{\mathbf{Y}}^T)^{\dagger} K_{\mathbf{Y}}\mathbf{v}_{\mathbf{Y}}$

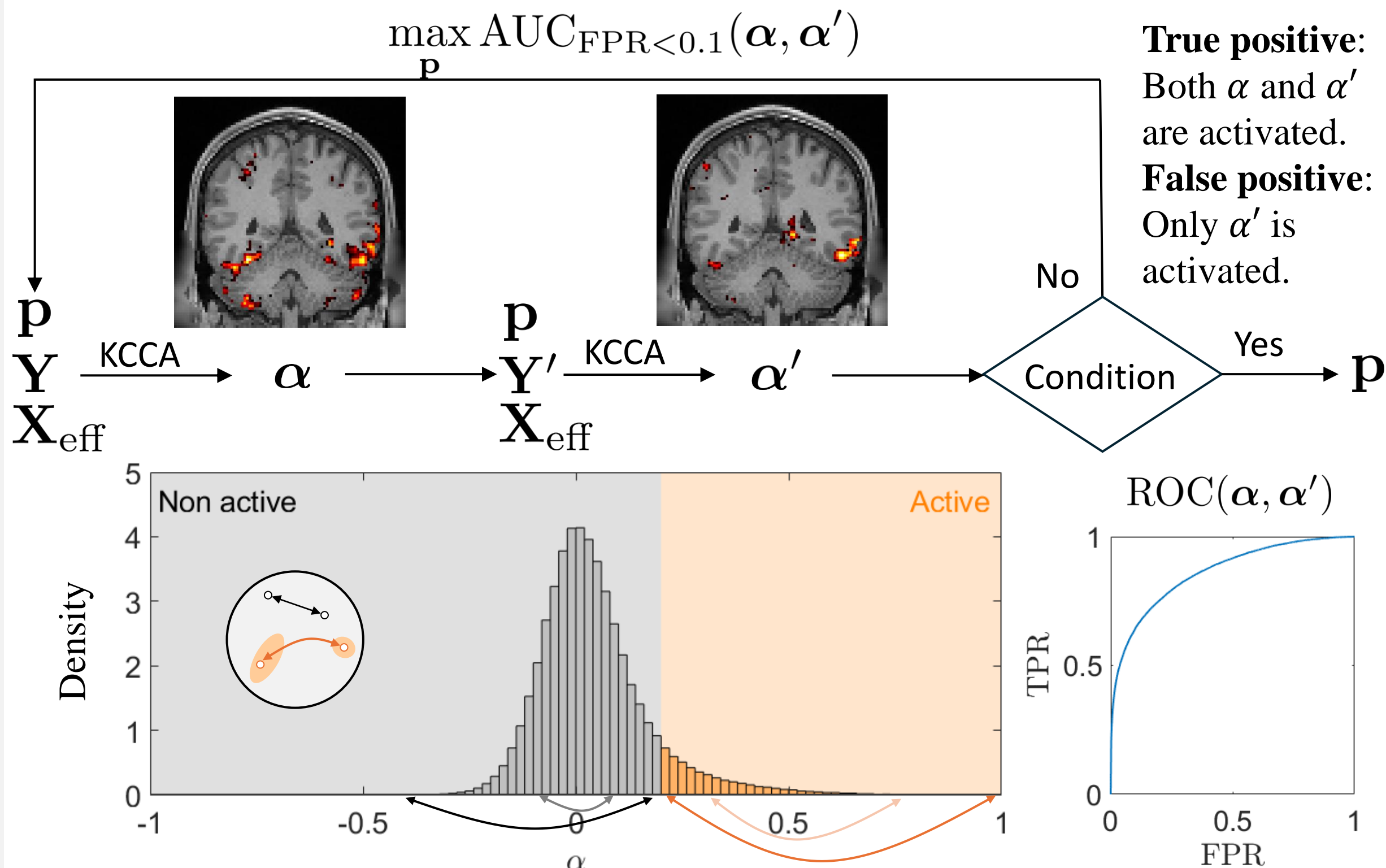
Nonlinear kernels

Name	Expression	Number of hyperparameters
Linear	$K_{\mathbf{Y}} = \tilde{\mathbf{Y}}\tilde{\mathbf{Y}}^T$	1
Parabolic	$K_{\mathbf{Y}} = (\tilde{\mathbf{Y}}\tilde{\mathbf{Y}}^T + b^2)^2$	2
Gaussian	$K_{\mathbf{Y}} = \exp(-\ \tilde{\mathbf{Y}} - \tilde{\mathbf{Y}}^T\ /\sigma^2)$	2
Inverse square	$K_{\mathbf{Y}} = 1/\sqrt{\ \tilde{\mathbf{Y}} - \tilde{\mathbf{Y}}^T\ + b^2}$	2
Bounded Linear	$K_{\mathbf{Y}} = \min(C, \tilde{\mathbf{Y}}\tilde{\mathbf{Y}}^T)$	2
Square	$K_{\mathbf{Y}} = \ \tilde{\mathbf{Y}} - \tilde{\mathbf{Y}}^T\ $	1
Tanh	$K_{\mathbf{Y}} = \tanh(b\tilde{\mathbf{Y}}\tilde{\mathbf{Y}}^T + c)$	3
Mixed Tanh	$K_{\mathbf{Y}} = \tanh(b_1\tilde{\mathbf{Y}}\tilde{\mathbf{Y}}^T + b_2\ \tilde{\mathbf{Y}} - \tilde{\mathbf{Y}}^T\ + c)$	4

Hyperparameter optimization

\mathbf{p} : unknown hyperparameters for KCCA
 $\boldsymbol{\alpha}$: Activation before shuffling
 $\boldsymbol{\alpha}'$: Activation after shuffling

Isolated activated voxels are rare. A good method will try to maintain the prediction results even after the voxel location has changed



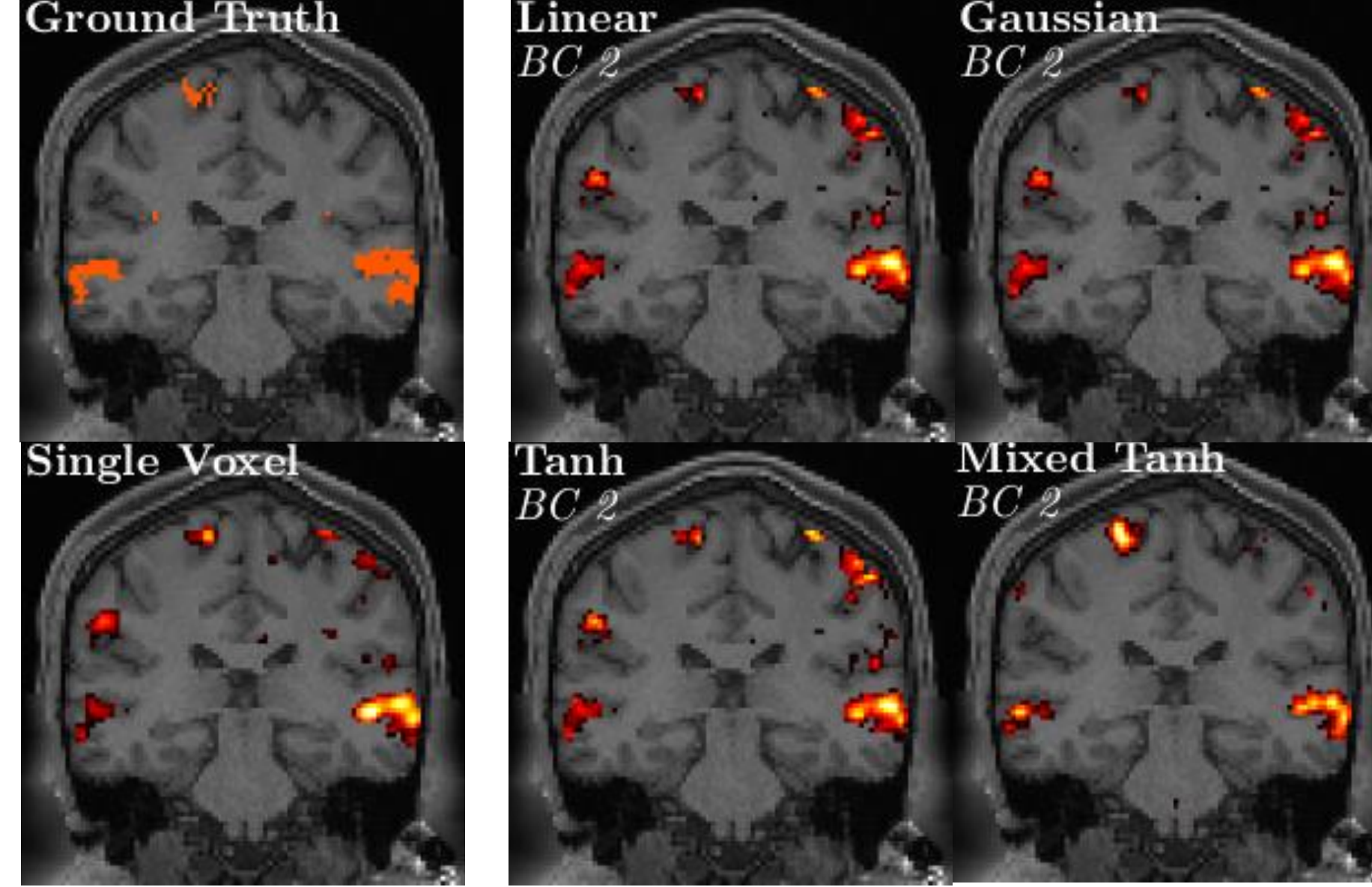
Simulated fMRI

Generate simulation

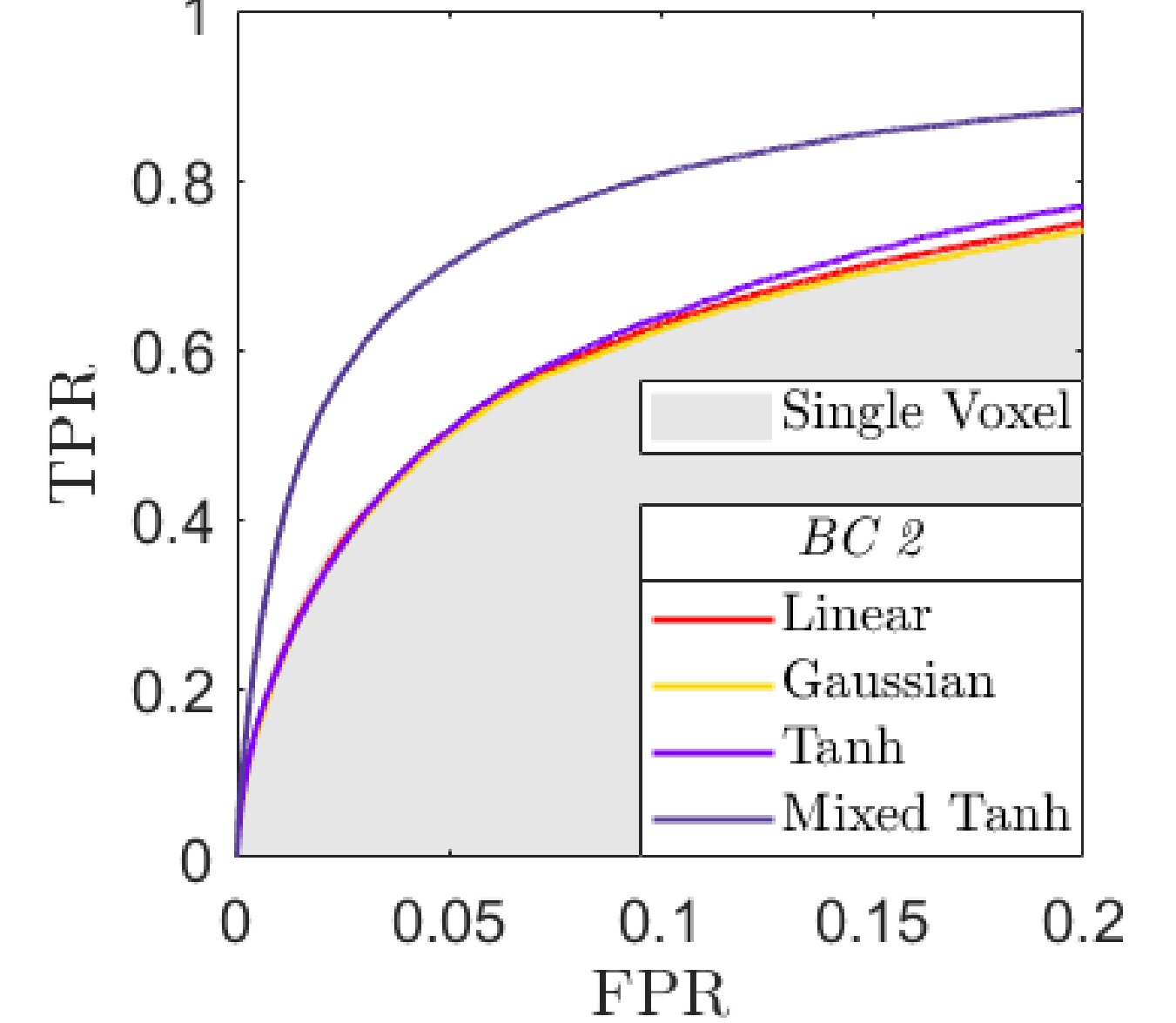
Choose 6 AAL regions as activated, adding signal linearly with contrast β [6]

$$\mathbf{Y}_{\text{simulated}} = \mathbf{Y}_{\text{resting state}} + \rho \sum_{i=1}^6 \mathbf{X}\beta_i \mathbf{M}_i \quad \mathbf{M}_i \in \mathbb{R}^{1 \times Q} : \text{mask}$$

Activation pattern



AUC Curve



Group level analysis

Using result from single voxel smoothing as reference

$$r = \frac{\text{AUC}_{\text{FPR} < 0.1}(\text{KCCA})}{\text{AUC}_{\text{FPR} < 0.1}(\text{Single Voxel Smoothing})}$$

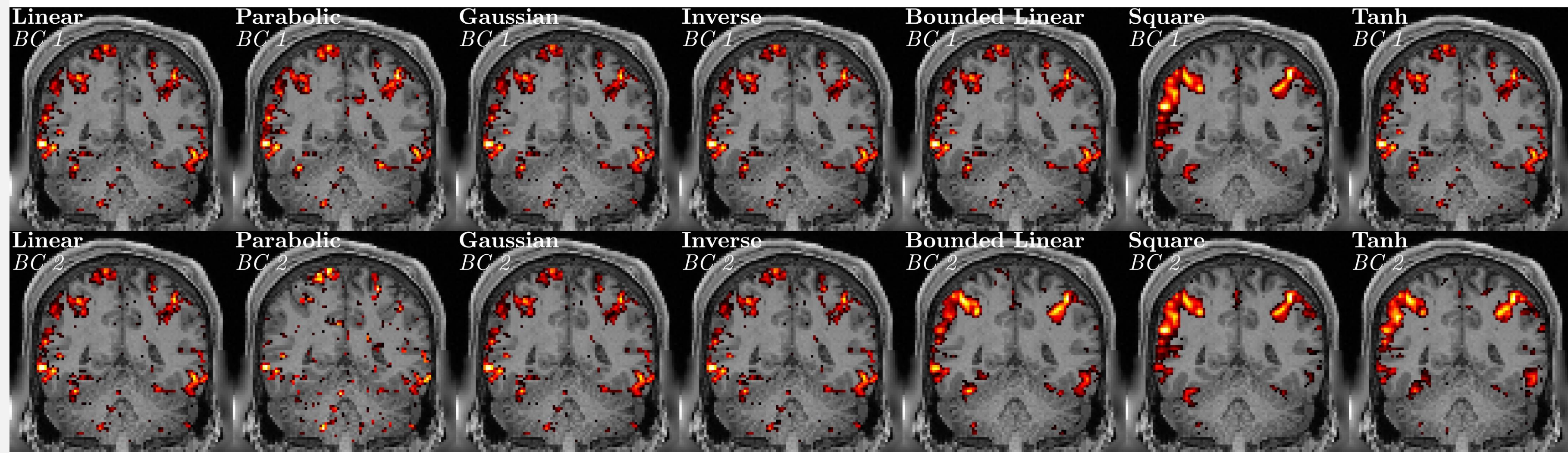
Mean and standard deviation for 87 subjects

Linear	1.12 \pm 0.27	Bounded Linear	1.39 \pm 0.37
Parabolic	1.09 \pm 0.31	Square	0.99 \pm 0.40
Gaussian	1.21 \pm 0.27	Tanh	1.39 \pm 0.37
Inverse	1.14 \pm 0.30	Mixed Tanh	1.42 \pm 0.36

Real fMRI

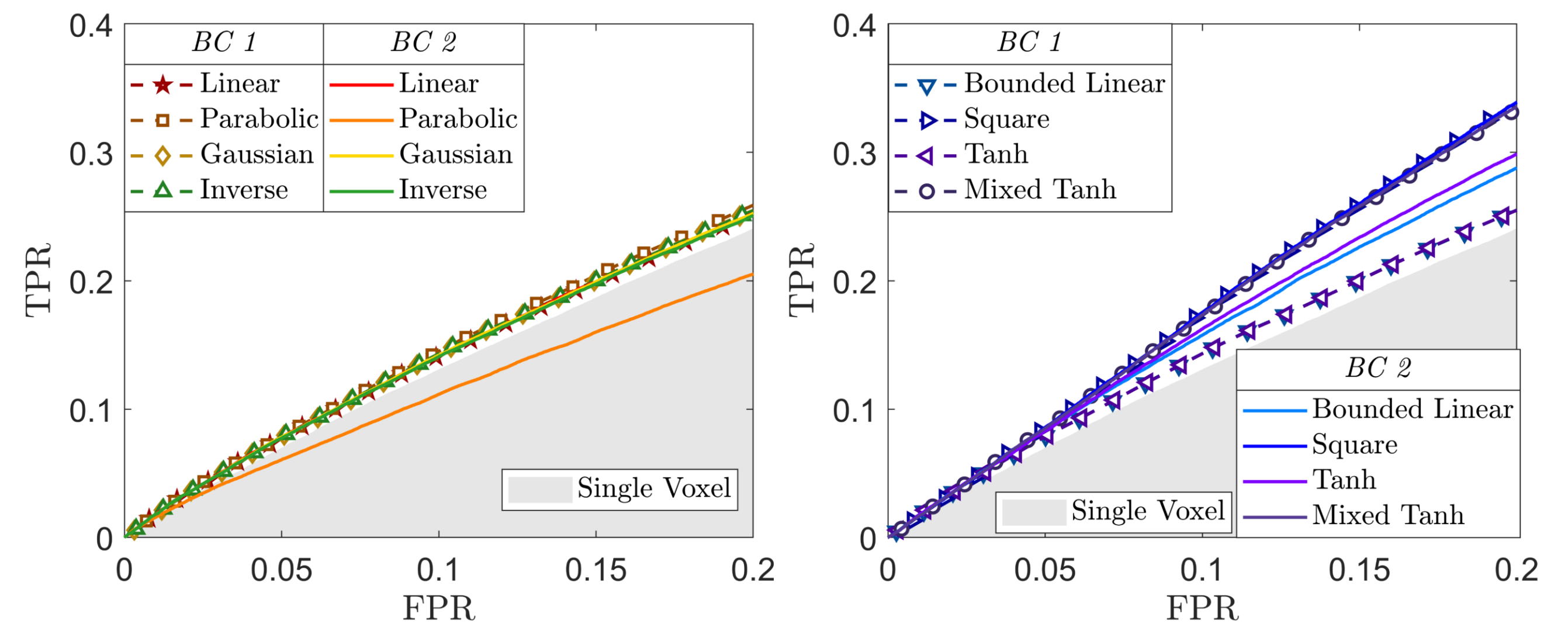
Human Connectome Project (HCP) [7]

Activation pattern



AUC Curve

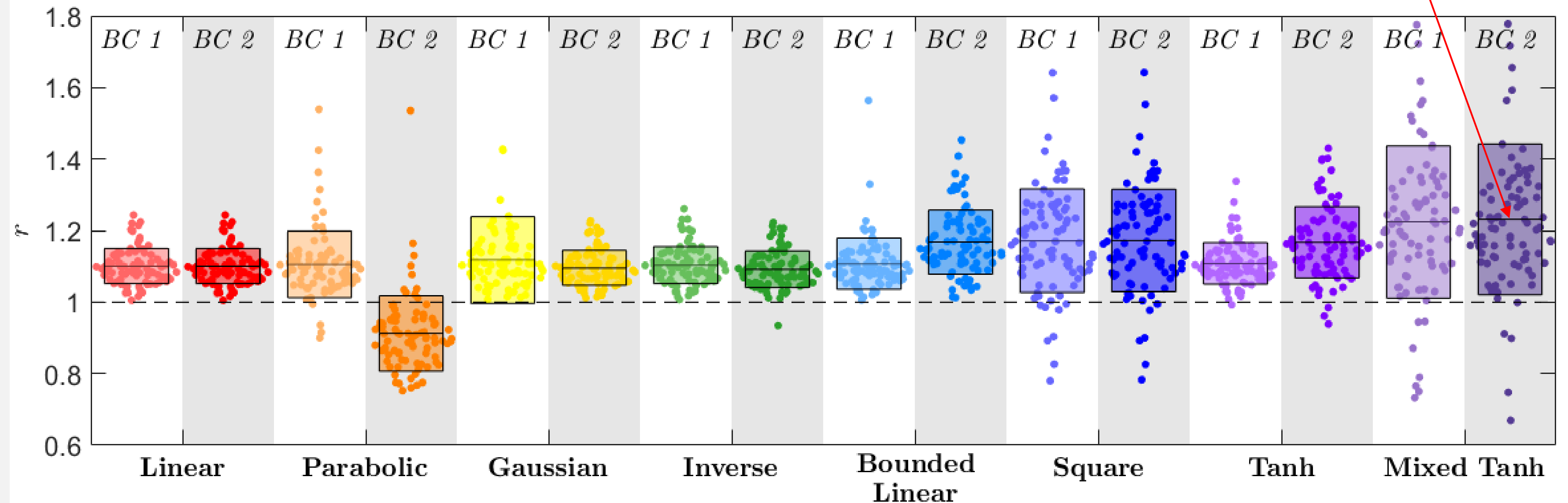
True positive: Activation appears in the Gray matter.
False positive: Activation not in the Gray matter.



Group analysis

$$r = \frac{\text{AUC}_{\text{FPR} < 0.1}(\text{KCCA})}{\text{AUC}_{\text{FPR} < 0.1}(\text{Single Voxel Smoothing})}$$

Mean and standard deviation for 87 subjects



Acknowledge

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Reference

1. Friman, O., et al. NeuroImage, **19**, 837-845. (2003)
2. Friman, O., et al. Magnetic Resonance in Medicine: An Official Journal of the International Society for Magnetic Resonance in Medicine, **45**, 323-330. (2001)
3. Xiaowei, Z., et al. NeuroImage, **149**, 63-84. (2017)
4. Hardoon, D., et al. Neural computation, **16**, 2639-2664. (2004)
5. Cordes, D., et al. Human Brain Mapping **33**, 2611-2626. (2012)
6. Yang, Z., et al. Medical Image Analysis, **60**, 101622. (2020)
7. Barch, D., et al. Neuroimage, **80**, 169-189. (2013)