Math 3209 - Fall 2020

Partial Differential Equations - Individual Homework i8

Be sure to show your work! And show details!! Imagine you are writing a worked example for a textbook

Easy point problems :-)

1. If you used any sources other than the book, the videos, discussions with me, or your brain, please include them here. Examples include other people you talked to in class, or web pages. If not applicable, simply write NA.

2. Include pictures of your notes from the video lectures from this week. You will get credit just for submitting these.

Problems for video lecture V30: The sound of a PDE solution

3. If you pluck the "top" string on a guitar, you will hear the note E2, which corresponds to a frequency of 82.4 Hz, roughly. Suppose someone places their finger exactly in the middle of the string, and presses down, effectively making the string length half its original length, and then plucks it once again. What frequency will the person hear? Which note that does frequency correspond to? You can use this webpage to check which frequencies correspond to which notes: `http://www.sengpielaudio.com/calculator-notenames.htm`

4. Consider a guitar string that is two feet long on a guitar. The fundamental note of the top string is E2, which corresponds to a frequency of 82.4 Hz.

   (a) Write down the solution $u(x,t)$ of the wave equation that corresponds to the fundamental note E2.

   (b) Sketch the solution at $t = 0$.

   (c) Sketch the solution that corresponds to the first overtone at $t = 0$. What frequency will you hear if the first overtone is played?

Problems for video lecture V31: Eigenvalue problems

5. Consider the eigenvalue problem

$$-\frac{d}{dx^2}X = \lambda X, \qquad X'(0) = X'(\ell) = 0$$

   where $X(x)$ is a function and λ is a scalar. In the video I found the positive $\lambda > 0$ eigenvalues and corresponding eigenfunctions, and I found the $\lambda = 0$ eigenvalue and corresponding eigenfunction. Show that there are no negative eigenvalues.

Problems for video lecture V32: Wave equation with a Neumann boundary conditions (step 1-3)

6. Consider the diffusion equation with Neumann boundary conditions,

$$u_t - ku_{xx} = 0, \qquad u(x,0) = \phi(x)$$

$$u_x(0,t) = 0, \quad u_x(\ell,t) = 0$$

   Our goal is to find a series solution using separation of variables. Some of the steps are the same as in the video for the wave equation! So you don't need to repeat that work, but you do need go find that information.

   (a) Write Down the ODE for $X(x)$ and the ODE for $T(t)$ (one of these will look different than in wave)

   (b) Write down the eigenvalues and eigenfunctions for the X ODE (will look the same as in wave)

   (c) Find the general solution to the $T(t)$ ODE. Be sure to include a random 1/2 factor in front of any arbitrary constant, which we will justify next week. (This will look different than in wave)

Problems for video lecture V33: Wave equation with a Neumann boundary conditions (step 4-5)

7. (continued from above) Consider the diffusion equation with Neumann boundary conditions,

$$u_t - ku_{xx} = 0, \qquad u(x,0) = \phi(x)$$

$$u_x(0,t) = 0, \quad u_x(\ell,t) = 0$$

   (a) Take the information you found in problem 6 and make the appropriate linear combination to obtain a series solution.

   (b) Find a formula that relates the initial value to the arbitrary constants in your general solution from part 7a. You DO NOT need to solve for the arbitrary constants.