Math 3209 - Fall 2020

Partial Differential Equations - Individual Homework i3

Be sure to show your work!

Easy point problems :-)

1. Include pictures of your notes from the video lectures from week 3. You will get credit just for submitting these.

Problems for video lecture V9: The advection equation

2. Consider the advection equation describing particles moving though a tube

$$u_t + cu_x = 0$$

where $u = u(x,t)$ is the density of particles in grams/cm^3. Let x have the unit of cm and let t have the unit of seconds.

(a) What are the units of c? How did you arrive at that conclusion? What does c represent in plain words?

(b) For the remaining problems, let $c = 2$. Solve the PDE subject to the auxiliary condition

$$u(x,0) = \begin{cases} x & 0 < x < \frac{1}{2} \\ 1 - x & \frac{1}{2} \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

(c) Sketch the graph of $u(x,t)$ in the u vs. x plane for the values of $t = 0$, $t = 1$ and $t = 5$.

(d) How far did the "peak" of the solution travel after 5 seconds? Is this consistent with your answer from part 2a

Problems for video lecture V10: The diffusion and heat equation

3. For the diffusion process, the density $u(x,t)$ and flux $\phi(x,t)$ are related through,

$$\phi = -ku_x$$

where $k > 0$ is a constant. Explain in your own words why there is a negative sign in front of the k.

4. Suppose that the density of particles $u = u(x,t)$ satisfies the diffusion equation

$$u_t - u_{xx} = 0$$

The solution at $t = 0$ is shown in Fig. 1. Without solving the PDE, draw a graph (u vs x) estimating what the solution $u(x,t)$ might look like at some later time, say $t = 5$. To answer this question, use what you know about the derivation of the diffusion equation.

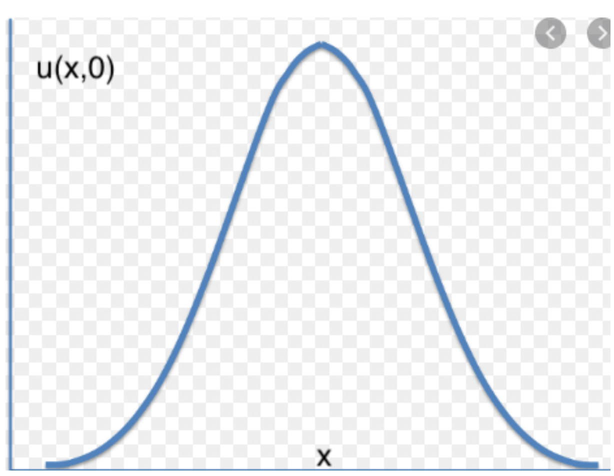

Figure 1: Graph of $u(x,0)$

Problems for video lecture V11: The Laplace equation

5. Write down the equation that describes the steady state of the diffusion equation.

$$u_t - u_{xx} = 0$$

6. Find the general solution of the steady state PDE.

Problems for video lecture V12: Summary of models

7. Number 4 in Section 1.6 in the book

8. Can you think of a situation where being able to classify a second order PDE as parabolic, elliptic or hyperbolic would be useful?