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Homework 11

September 7, 2020

V1

Find general solution to $u_x = 5u$ where $u = u(x,y)$

- $du/5u = dx \implies ln(5u) = x + C$
- $5u = e^x + C \implies u = \frac{(e^x)}{5} + C'$
- We replace C' with an arbitrary function of y to give us our final answer of $u(x,y) = \frac{(e^x)}{5} + g(y)$

V2

What is one possible advantage to using a PDE model over an ODE model to describe a real life system? Can you think of a disadvantage? A PDE allows you to have more than one independent variable, which means we can model both time and space, for example. This is really useful for real life modeling, such as diffusion and elasticity. A potential disadvantage could be that we won't be able to solve the PDE analytically, but we could solve the corresponding ODE much more easily.

Consider the following PDEs:

1. $u_t + xu_{xx} + x^2 = 0$
2. $u_t + xu_x = u^2$

Equation 1 is linear
Equation 2 is homogenous
Equation 2 is first order

V3

6. We know that $u(x,y) = f(bx - ay) = f(c) \implies u(x,y) = f(c)$
Thus, $u_x = 0$ and $u_y = 0 \implies u_x + u_y = 0 \implies au_x + bu_y = 0$ thus verifying the assertion

7. Suppose u satisfies PDE $u_x + 2u_y = 0$ and $u(0,0) = 3$ and $u(0,1) = 4$
(a) We know from equation 5 that $a = 1$ and $b = 2$ so the characteristic line that crosses $(x,y) = (0,0)$ is $2x - y = 0$
(b) We want to check if $2x - y$ is 0 for all of the examples. We also know that the characteristic line for $u(0,1)$ is $2x - y = -1$

- $u(1, 2) = 3$ is definitely true
- $u(1, 3) = 3$ is definitely false
- $u(2, 4) = 3$ is definitely true
- $u(2, 6) = 3$ is definitely false

V4

8. Solve $u_x + 2u_y = 0$ with auxiliary condition $u(x, 0) = \sin(x)$
We know that $u(x, y) = f(2x - y)$ and that $u(x, 0) = f(2x) \implies f(2x) = sin(x)$
Now, let $w = 2x \implies x = w/2$, so we get:
$f(w) = \sin(w/2)$ which implies that $u(x, y) = \sin(x - (y/2))$