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Group Assignment 3

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(a) $\int_a^b \rho(x)dx$

(b) $\int_a^b \rho(x)u_{tt}(x,t)dx$

(c) $\int_a^b -\rho(x)gdx$

(d) The tension is pulling in opposite directions, making the tension at $x = a$ negative and the tension at $x = b$ positive. Thus, the left vertical force is $-T\sin(\theta_a)$ and the right vertical force $T\sin(\theta_b)$. We are ignoring horizontal force because we are assuming there is a small displacement in the string from $x = a$ to $x = b$, making the horizontal force negligable. We see this by noting that when θ is small, $\cos(\theta) \approx 1$, so $T_x \approx 0$

(e) When the angle between the tangent line and the string is small at a given point, it means that $\cos\theta \approx \tan\theta$ (since the hypotenuse and adjacent side are similar in length, while the opposite side is very small).

(f) We know that $\tan(\theta)$ is equal to the length of the opposite side divided by the adjacent side of a right triangle. In this scenario, θ is the angle between the line formed by the string and the horizontal force. Therefore, there is a right triangle between the horizontal and the vertical forces, so $\tan(\theta)$ is equal to the magnitude of the vertical force divided by the horizontal force. We can note, therefore, that $\tan(\theta)$ is now also equivalent to the slope of the line segment of our string (rise over run!). We also know that $u_x(x,t)$ is equivalent to the slope of the line at a certain point x. Therefore, $u_x(a,t)$ is the slope of the line at $x = a$, which we explained is also the same as $\tan(\theta_a)$. Logically, $u_x(b,t)$ is the slope of the line at $x = b$, which is also the same as $\tan(\theta_b)$. So, $u_x(a,t) = \tan(\theta_a)$ and $u_x(b,t) = \tan(\theta_b)$

(g) $-Tu_x(a,t) + Tu_x(b,t)$

(h) $\int_a^b -Tu_{xx}(x,t)dx$

(i) $\int_a^b \rho(x)u_{tt}(x,t)dx = \int_a^b -\rho(x)gdx + \int_a^b Tu_{xx}(x,t)dx$
$\int_a^b \rho(x)u_{tt}(x,t)dx = \int_a^b Tu_{xx}(x,t) - \rho(x)gdx$

(j) Since we know a and b are arbitrary, the only way (i) can be true is if the elements inside the integrals on both sides are equal, so $\rho(x)u_{tt} = Tu_{xx} - \rho(x)g$
Now, we are assuming density is constant along the string, so therefore the string tension can be treated as a constant independent of x. Thus, we can divide everything by $\rho(x) = \rho$ and end up with $u_{tt} = \frac{T}{\rho}u_{xx} - g$
Now, we can let a constant $c = \sqrt{\frac{T}{\rho}}$ so that $c^2 = \frac{T}{\rho}$ (since we know that both T and ρ

are unchanging constants)
Finally, this gives us the desired expression:

$$u_{tt} = c^2 u_{xx} - g$$

(k) Since $c = \sqrt{\frac{T}{\rho}}$, c is of the units $\sqrt{(kgm/s^2) \cdot (m/kg)} = \sqrt{(m^2/s^2)} = m/s$
The units of c are thus m/s. In plain english, this is the velocity of the string!