Deep-Sea Research, 1972, Vol. 19, pp. 157 to 163. Pergamon Press. Printed in Great Britain.

INSTRUMENTS AND METHODS

The calibration of thermistors over the temperature range 0–30°C*

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(*Received* 29 *April* 1971; *in revised form* 10 *September* 1971; *accepted* 14 *September* 1971)

Abstract—Least squares fitting is used to fit the function

$$1/T = \text{polynomial } (\log R)$$

to data on thermistor resistance, R, versus temperature, T, over the range 0–30°C. For an accuracy of 0·001°C, the polynomial must be of third degree, requiring a minimum of four calibration points. Within a batch of thermistors, the behaviour is sufficiently similar for a reduction in the number of necessary calibration points to be possible if the mean properties of the batch are used to determine the higher order polynomial coefficients. After eliminating one thermistor with high drift from the 21 tested, the mean value of the root mean square drift of ice-point resistance corresponds to a random walk temperature drift not exceeding 0·00125°Cyr$^{-\frac{1}{2}}$. The effect of pressure on the steel probe type of thermistor is $1{\cdot}3 \times 10^{-6} \pm 1{\cdot}5 \times 10^{-6}$ (standard deviation) °C/bar at pressures up to 700 bar (10,000 psi).

INTRODUCTION

The work of Steinhart and Hart (1968) suggests the use of the interpolation formula

$$1/T = \text{polynomial } (\log R) \tag{1}$$

where T is the Kelvin temperature, R is the thermistor resistance, and in particular the formula

$$1/T = A + B \log R + C (\log R)^3. \tag{2}$$

Arguments presented below indicate that this latter formula is not acceptable in that it omits the term in $(\log R)^2$. This term should be retained in order to preserve independence of the system of units used. However, the work presented here confirms that the former formula does give a very good fit. In order to facilitate comparisons with different types of thermistor, the fits here are presented in terms of the equivalent formula

$$1/T - 1/T_0 = \sum_{j=1}^{p} A_j \{\log (R/R_0)\} \tag{3}$$

where R_0 is a further fitted parameter, being the resistance at a specified temperature, T_0, taken as the ice point (273·15 K), and p is the order of fit.

Results for two batches of Fenwal type G427G steel probe thermistors calibrated over the range 0–30°C against Rosemount type 162C platinum resistance thermometers indicate that fitting a third order polynomial reduces systematic residuals below 0·001°C. The behaviour of all the thermistors is very similar. This enables higher order terms to be replaced by batch means: the third order coefficients may be advantageously replaced in this way.

High accuracy of laboratory calibration is of little use unless similar accuracy can be maintained under field conditions. In particular, it is necessary to know the effects of time and, in the case of

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oceanographic temperature sensors, of pressure. The long term stability of the thermistors has been tested by repeated ice point calibrations. The effectiveness of the steel probe in isolating the thermistor bead from the effects of hydrostatic pressure has also been tested. The results of these tests are presented below.

THE CHOICE OF AN INTERPOLATION FUNCTION

STEINHART and HART (1968) proposed five criteria for the choice of an interpolation function. Stated briefly, these are:

(1) a single function should cover the entire range.
(2) The function must be the 'right shape' so as to fit the data.
(3) The required variable, in this case temperature, should be an explicit function of the measured variable (resistance).
(4) The mathematical form of the interpolation function should be sufficiently simple that its properties may be readily assessed.
(5) Linear fitting procedures must be available, if necessary after transformation of variables.

To these, the present author would like to add a further criterion which may be stated:

(6) the form of the function should not be affected by a change of units. This is clearly a desirable property: one ought to be able to measure resistance in ohms or megohms without affecting the quality of the fit obtained. Equivalently, one should be able to use the same thermistor material in different shapes and sizes. This sixth criterion does impose further constraints on the choice of interpolation function, as will be seen below. Equations (1) and (3) give identical pth order least squares fits and are in accord with all six of the criteria above. Equation (3) has the advantage over equation (1) that the A_j of equation (3) depend only on the bulk properties of the thermistor material, not on its shape and size (apart from surface effects).

Equation (2), with which Steinhart and Hart achieved their best fits to thermistor data, violates criterion (6), as can be seen by transforming to a different unit of resistance, which introduces a term in $(\log R)^2$. Omission of the squared term is not, therefore, permissible in this case if criterion (6) is to be retained. Omission of terms would, however, be permissible in the case of a polynomial in R rather than $\log R$, or for a polynomial in $\log (R/R_1)$ with R_1 a further fitted parameter. Changing Steinhart and Hart's equation to the latter form

$$1/T = A+B\log(R/R_1)+C\{\log(R/R_1)\}^3, \tag{4}$$

it can be seen by comparison with equation (3) with the order $p = 3$ that the two equations will give identical least squares fits with

$$\begin{aligned} 1/273{\cdot}15 &= A+B\log(R_0/R_1)+C\{\log(R_0/R_1)\}^3 \\ A_1 &= B+3C\{\log(R_0/R_1)\}^2 \\ A_2 &= 3C\log(R_0/R_1) \\ A_3 &= C, \end{aligned}$$

so that R_1 may be easily determined from fits to equation (3) as

$$\log(R_0/R_1) = A_2/3A_3.$$

The various sets of data give values of R_1 as follows:

G427G thermistors (Table 2)	0·019 Ω to 1·9 Ω
G427G from mean coefficients	0·46 Ω
BOSSON, GUTMANN and SIMMONS (1950), No. 4756–12, quoted by Steinhart and Hart	1·4 Ω
S4 thermistor (Steinhart and Hart)	4·7 Ω

It will be seen that R_1 is indeed of order 1 Ω, as required for agreement with the Steinhart and Hart results. This agreement must, however, be accidental—there is no reason to suppose that the ohm has special significance in thermistor measurements.

The surprising result of Steinhart and Hart that including the squared term with the cubed term

Table 1. *Standard relative error, defined by* STEINHART *and* HART (1968) *as* $\left[\frac{\Sigma(T_{est}-T_{obs}{}^2/T_{obs}{}^2}{N}\right]^{\frac{1}{2}}$ *for N observations, for the fit of several equations involving powers of log R to* BOSSON, GUTMANN and SIMMONS (1950) *data: comparison of Steinhart and Hart results with recalculated values. The computer program used for recalculation could not be used for polynomials with omitted terms.*

Equation	*Steinhart and Hart*	*Recalculated*
$T^{-1} = A+B \log R$	0·006012	0·00601
$T^{-1} = A+B \log R+C (\log R)^2$	0·000469	0·000427
$T^{-1} = A+B \log R+C (\log R)^3$	0·000162	—
$T^{-1} = A+B \log R+C (\log R)^2+D (\log R)^3$	0·000472	0·000139
$T^{-1} = A+B \log R+C (\log R)^3+D (\log R)^5$	0·000091	—

degraded the fit must now be re-examined. The 'standard relative error' as defined by Steinhart and Hart in their paper is recomputed in Table 1 for fits of equation (1) to the data of Bosson, Gutmann and Simmons for their thermistor No. 4756–12. The results presented in Table 4 of Steinhart and Hart are included for comparison. Excellent agreement is obtained for the first order fit but the agreement deteriorates rapidly with increase in the number of fitted parameters, Steinhart and Hart's figures being too large. This may indicate a computational problem, possibly stemming from rounding errors. The recalculated figures show that inclusion of the squared term does improve the fit slightly compared to Steinhart and Hart's figure for the fit without the squared term, as is to be expected.

METHODS OF MEASUREMENT

Ice point observations were made in an ice bath prepared according to the method used by the National Research Council, Ottawa (Dauphinée, private communication). Measurements at other temperatures were made in a circulating water bath (Fig. 1) whose temperature was measured with one of two Rosemount type 162C platinum resistance thermometers. The thermometers were cali-

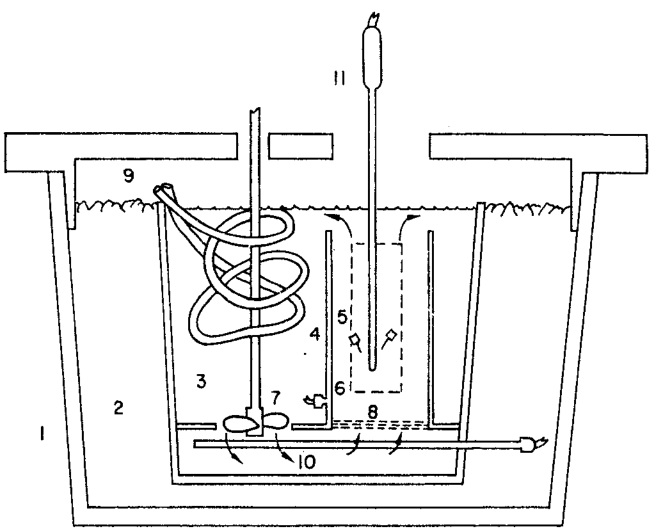

Fig. 1. The constant temperature bath used for the temperature calibrations. (1) Thermally insulating outer box. (2) Expanded plastic insulation. (3) Bath. (4) Plastic pipe, 10 cm inside diameter. (5) Inner bath containing thermistors being calibrated. The usable measuring volume is shown by the dashed lines and is 5 cm in diameter and 15 cm deep. (6) Control thermistor. (7) Stirrer. (8) Three layers of mesh to break up flow and promote mixing. (9) Plastic pipe carrying ice water for cooling. (10) Immersion heater supplying controlled heat. (11) Platinum thermometer to measure bath temperature.

brated to IPTS–68 (International Practical Temperature Scale of 1968: Comité International des Poids et Mesures, 1969) by the National Research Council, Ottawa. Day to day recalibration used a triple-point-of-water apparatus. All resistance measurements were made with a pair of a.c. Kelvin bridges at 159 Hz. The bridges have inductive voltage divider ratio arms and no further lead compensation (Hill and Miller, 1963). The measured thermistor resistances were corrected for the heating effect of the measuring current by extrapolation of measurements at different currents.

THE RESULTS OF THE CURVE FITTING

Twenty type G427G thermistors were measured at ten temperatures over the range 0–30°C. The residuals from the third order fit are plotted in Fig. 2 and the fitted coefficients are tabulated in Table 2. The root mean square value of all the residuals is 0·00032°C, corresponding, after allowance for the degrees of freedom used up in the fitting process, to a standard deviation for an individual observation of 0·00041°C. This figure includes the random errors of measurement, that part of the systematic error that varied from temperature to temperature or from time to time and any real higher order contributions to the calibration curves. Since the temperature gradients in the bath (the largest source of systematic error) and the effect of bridge errors and variation of resistance standards are estimated to total 0·00032°C r.m.s., it is concluded that higher order contributions to the calibration curves are too small to be measured with this system accuracy and that fitting should therefore be carried out only to third order despite the apparent systematic trend of the residuals.

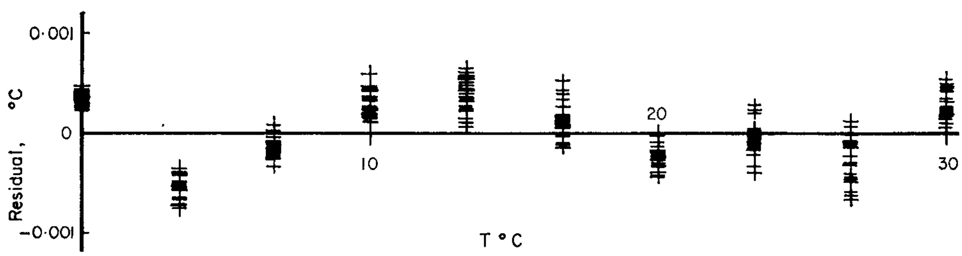

Fig. 2. Residuals (observed temperature minus calculated temperature) for the third order fit of the relation

$$1/T - 1/273{\cdot}15 = \sum_{-1}^{p} A_j \{\log(R/R_0)\}^j$$

to calibration data for the G427G thermistors.

The variation in the first and second order coefficient is large compared with the variation to be expected as a result of the errors of measurement. The variation in the measured third order coefficient A_3, however, is consistent with its being entirely due to measurement errors, the true value being the same for all the thermistors. The mean third order coefficients for the two batches of thermistors are respectively $1{\cdot}42 \times 10^{-7}$ and $1{\cdot}75 \times 10^{-7}$, corresponding to peak temperature contributions (at 30°C) of 0·00125°C and 0·00154°C. The difference is considered to be insignificant compared to the possible systematic measurement errors. It is felt that the observations are best represented by the mean third order coefficient of $1{\cdot}62 \times 10^{-7}$ with a standard deviation, representing an estimate of the systematic error (excluding any platinum thermometer calibration errors) of $0{\cdot}34 \times 10^{-7}$. This value is also similar to the values obtained by Steinhart and Hart for their coefficient C which should approximately equal A_3. This is notably so for their thermistor S4, Fenwal type K824A, for which they obtained the value $1{\cdot}58816 \times 10^{-7}$ for calibration over a similar temperature range (0–35°C). Thus, while it is not suggested that the value obtained for the third order coefficient is a universal value for all thermistors, it is likely that the same value may apply to all thermistors made of similar materials, at least from one manufacturer. It is advantageous to replace the measured third order coefficient with a well determined batch mean value since this reduces the statistical uncertainty of the fit or enables a reduction in the number of calibration points without sacrificing accuracy or reliability. Over a smaller span it may be possible to replace the second order coefficients with batch means: the variation in the contributions of the various orders is plotted as a function of span in Fig. 3. The values plotted are the 99·9% confidence limits (3·29 standard deviations) of the peak variation from the batch mean assuming Gaussian distributions of coefficients. The values are valid over the range 0–30°C.

Table 2. Fitted coefficients of equation (3) *for two batches of* G427G *thermistors.*

Thermistor no.	R_0	A_1	A_2	A_3
Batch 1				
2	1430·139	$2{\cdot}97594 \times 10^{-4}$	$3{\cdot}908 \times 10^{-6}$	$1{\cdot}62 \times 10^{-7}$
3	1426·229	2·97630	3·859	1·37
4	1429·110	2·97585	3·894	1·53
5	1424·117	2·97349	3·856	1·37
6	1429·077	2·97081	3·869	1·48
9	1428·613	2·97716	3·860	1·34
10	1423·657	2·97518	3·811	1·13
11	1426·607	2·97073	3·865	1·55
Batch 2				
12	1426·666	2·97629	3·918	1·52
13	1423·863	2·97944	3·929	1·46
14*	1423·968	2·97898	3·956	1·48
15	1423·976	2·98235	4·021	1·73
16	1428·324	2·97377	3·925	1·72
17	1424·365	2·97363	3·906	1·67
18	1424·496	2·98276	3·991	1·72
19	1430·133	2·97638	3·969	1·90
20	1424·997	2·98039	4·050	1·98
21	1424·008	2·98175	4·024	1·75
22	1420·014	2·97999	4·048	2·04
23	1420·380	2·97989	4·051	2·02

*This thermistor has a high drift rate and was omitted from the calculations.

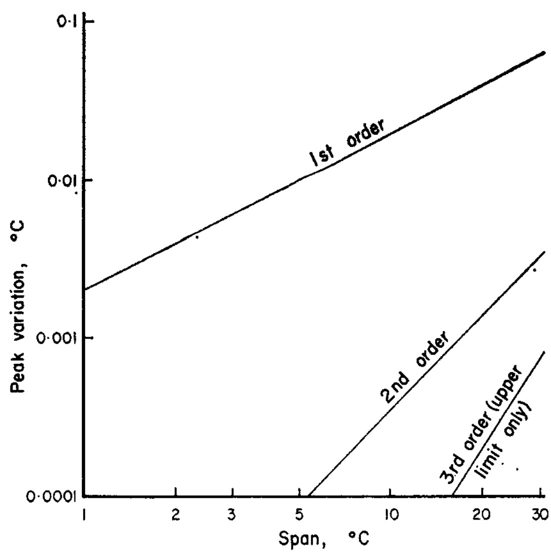

Fig. 3. Peak contribution of the variation in the first, second and third order coefficients. The values plotted are 99·9 % confidence limits on the peak value of the error incurred in replacing a measured coefficient with a batch mean value.

THE LONG TERM ICE POINT MEASUREMENTS

A number of determinations of ice point resistance have been made on the G427G thermistors over the period September 1968 to December 1970. The changes relative to the most recent measurements are plotted in Fig. 4. The measurements were made using a variety of techniques. The ice points were set up in several different ways, using different means to ensure air saturation and using variously coarse crushed ice, fine crushed ice and shaved ice, the latter being the preferred current form. Three different resistors were used as standards and two different bridge techniques were employed. None of the measurements were corrected for barometric pressure and immersion depth: when the measurements were started, such corrections, typically up to 0·0003°C (0·02 Ω for the ice point resistances) were considered insignificant and the barometric pressure was not recorded. For these reasons, the apparent mean changes in the ice point resistance, amounting to a maximum of 0·093 Ω (65 ppm in resistance or about 0·0014°C), are not considered significant. Nevertheless, the results enable a useful upper limit to be set to the real variation in thermistor ice point resistance. If the drift of the thermistors were a random walk process, the mean square changes of ice point resistance would increase linearly with the time interval between the sets of observations. This effect is obscured by the observational errors. However, making the reasonable assumption that the observational errors and drifts are statistically independent, the expectation value of the observed mean

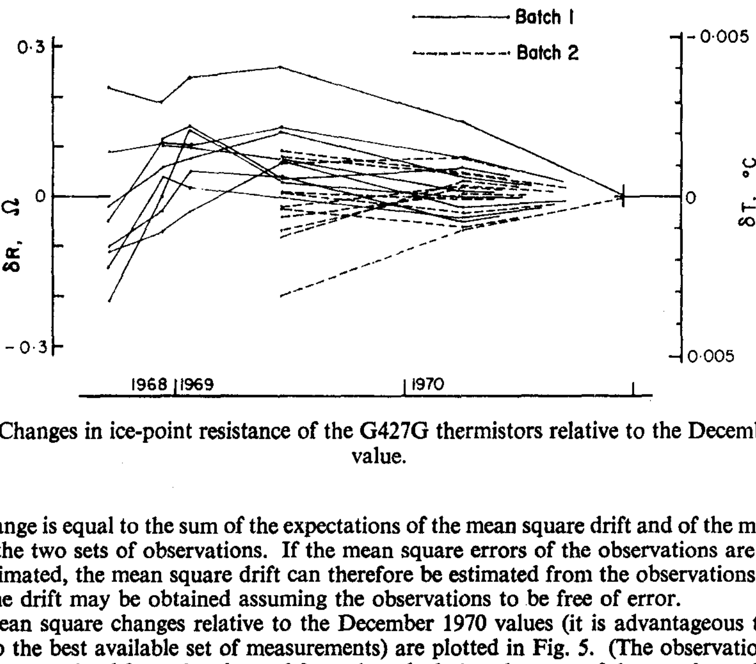

Fig. 4. Changes in ice-point resistance of the G427G thermistors relative to the December 1970 value.

square change is equal to the sum of the expectations of the mean square drift and of the mean square errors of the two sets of observations. If the mean square errors of the observations are known or can be estimated, the mean square drift can therefore be estimated from the observations: an upper limit to the drift may be obtained assuming the observations to be free of error.

The mean square changes relative to the December 1970 values (it is advantageous to refer all changes to the best available set of measurements) are plotted in Fig. 5. (The observations for one thermistor are omitted from the plot and from the calculations because of the very large drift rate of

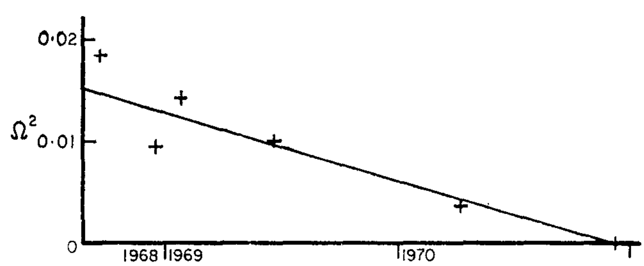

Fig. 5. Mean-square changes in the ice-point resistance of the G427G thermistors as a function of time. The fitted straight line is constrained to pass through zero in December 1970. The slope of the straight line represents a mean square or random walk type drift rate.

the thermistor.) The upper limit referred to above is represented by the straight line which is a least squares fit constrained to pass through zero in December 1970, the data being weighted in accordance with the number of observations. The slope of this line is 0·0064 Ω^2/yr corresponding to an upper limit of the root mean square temperature drift of 0·00125°Cyr$^{-\frac{1}{2}}$. Assuming standard deviations based on internal evidence and replicate determinations, the estimated root mean square drift is 0·00089°Cyr$^{-\frac{1}{2}}$. This value depends quite strongly on the assumed standard deviations: on the strength of these observations, the author would hesitate to propose any lower limit greater than zero for the root mean square drift. This result contrasts strongly with the high drift rates found for earlier types of thermistors. For example, MISENER and BECK (1960) measured a drift of 1·54°C in 11 months. While all the measurements reported here refer to thermistors made by Fenwal Electronics Inc., Framingham, Massachusetts, this is not intended as an endorsement of a specific product: other manufacturers may well be producing thermistors of similar stability.

THE MEASUREMENTS OF PRESSURE COEFFICIENT

The G427G thermistor probes have the thermistor bead mounted in a 0·25 cm diameter stainless steel probe which should isolate it from outside pressure. The extent to which this was true was checked using a small pressure bomb in the constant temperature bath. Problems were encountered with the very simple means used to apply pressure: leaks caused the pressure to fall slowly during a measurement causing temperature errors by adiabatic cooling of the hydraulic fluid. This effect was corrected using calibration figures obtained from more rapid changes of pressure but, because of the uncertainty of the correction, the overall accuracy is considered to be not better than 0·001°C (estimated standard deviation). Three thermistors were tested to a maximum pressure of 700 bar (10,000 psi). The largest resistance change observed corresponded to a temperature measurement error of +0·0017°C while the mean change corresponded to +0·0009°C at 700 bar: in view of the uncertainty of the correction, these changes are not considered to differ significantly from zero. The observations are consistent with a change linear with pressure over the range 0–700 bar with a pressure coefficient of (1·3 ±1·5) × 10^{-6}°C/bar. The error figure (standard deviation) includes both the estimated correction uncertainty and the observed scatter of the results for the three thermistors.

CONCLUSIONS

The resistance–temperature relationship of thermistors can be described with an accuracy of better than 0·001°C over the range 0–30°C by a formula having four parameters. One of the four parameters may be taken as constant for a particular type of thermistor leaving only three parameters to be obtained by individual calibration. For reduced accuracy or over a narrower temperature span, only two parameters need to be obtained by individual calibration. If rapidly drifting thermistors are eliminated, the mean value of the root mean square drift rate of the remainder is likely to be below 0·00125°Cyr$^{-\frac{1}{2}}$ for the type of thermistor used in these tests. The steel probe provides adequate protection against ambient pressure. This type of thermistor is thus suitable for long-term temperature recording in the ocean.

Acknowledgements—Thanks are due to Dr. T. M. DAUPHINÉE of the National Research Council, Ottawa, for helpful advice on temperature calibration techniques. The author would like also to thank his colleagues at the Atlantic Oceanographic Laboratory who constructed apparatus, helped make measurements and analyze data, and all those who provided constructive criticism of drafts of this paper.

REFERENCES

BOSSON G., F. GUTMANN and L. M. SIMMONS (1950) A relationship between resistance and temperature of thermistors. *J. appl. Phys.*, **21**, 1267–1268.

COMITÉ INTERNATIONAL DES POIDS ET MESURES (1969) The International Practical Temperature Scale of 1968. *Metrologia*, **5**, 2, 35–44.

HILL J. J. and A. P. MILLER (1963) An a.c. double bridge with inductively coupled ratio arms for precision platinum-resistance thermometry. *Proc. Instn elect. Engrs*, **110**, 453–458.

MISENER A. D. and A. E. BECK (1960) The measurement of heat flow over land, pp. 10–61. In: *Methods and techniques in geophysics*, S. K. RUNCORN, editor, Interscience, **1**.

STEINHART J. S. and S. R. HART (1968) Calibration curves for thermistors. *Deep-Sea Res.*, **15**, 497–503.