Problem 2.1:

Proof. We prove the contraposive: If n is divisible by 15, then n is also divisible by 3.

Assume is divisible by 15, then n is a multiple of 15. Multiples of 15 are also multiples of 3, therefoer is also a multiple of 3. This finally leads to n is divisible by 3.

Problem 2.2:

Proof. We prove $\sum_{k=1}^{n} (2k-1)^2 = \frac{2n(2n-1)(2n+1)}{6}$ by induction.

Base case: We show that $P(1)$ is true.

$$P(1) = \sum_{k=1}^{1} (2\times1-1)^2 = \frac{2\times1\,(2\times1-1)(2\times1+1)}{6} = 1$$

Induction step: Assume $P(n)$ is true.

$$\text{Then, } P(n+1) = \frac{2(n+1)(2(n+1)-1)(2(n+1)+1)}{6} = \frac{(2n+1)(2n+2)(2n+3)}{6}$$

$$P(n+1) \text{ also } = \frac{2n(2n-1)(2n+1)}{6} + (2(n+1)-1)^2$$

$$= \frac{2n(2n-1)(2n+1)}{6} + \frac{6(2n+1)^2}{6}$$

$$= \frac{(2n+1)(4n^2+10n+6)}{6}$$

$$= \frac{(2n+1)(2n+2)(2n+3)}{6}$$

This proves that $P(n+1)$ holds.

Problem 2.3

a)

```
isLeapYear :: Int -> Bool
isLeapYear x = if x `mod`4 == 0 && ( not(x `mod` 100 == 0) || x `mod` 400 == 0 )
                    then True
                    else False
```

b)

```
isLeapYear' :: Int -> Bool
isLeapYear' y    | y `mod` 400 == 0 = True
                 | y `mod` 100 == 0 = False
                 | y `mod` 4   == 0 = True
                 | otherwise        = False
```

Problem 2.4

a)

```
rotate :: Int -> [a] -> [a]
rotate 0 list = list
rotate 1 list = tail list ++ [head list]
rotate x list = rotate (x-1) (tail list ++ [head list])
```

b)