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Due: September 11, 2019
Assignment 1

*Elements of Probability*

In each one of the problems below, if you are asked to compute a probability, first identify the sample space and the event in question explicitly.

**(1.1)** The birthday of six random people has been checked. Find the probability that
(a) At least one of them is born in September.
(b) All six are born in the Spring. Spring here means one of the month March, April, or May.
(c) At least two of them are born in the same month.
In this problem you can assume that a year is 365 days and the probability that a person is born on a specific day of the year is exactly $1/365$.

**(1.2)** A fair die is rolled three times. We say that a match has occurred if the outcome of the first throw is 1, or the outcome of the second throw is 2, or the outcome of the third throw is 3. Find the probability of the event that a match occurs.

**(1.3)** An ordinary deck of playing cards (containing 52 standard cards, 13 of each suit) is randomly divided into two parts, each containing at least one card.
(a) What is the probability that each part contains at least one ace.
(b) Find the probability that each part contains exactly two aces.

**(1.4)** A is called a *palindrome* if it reads the same from left and right. For instance, 13631 is a palindrome, while 435734 is not. A 6-digit number $n$ is randomly chosen. Find the probability of the event that
(a) $n$ is a palindrome.
(b) $n$ is odd and a palindrome.
(c) $n$ is even and a palindrome.

**(1.5)** (a) Suppose $A$ and $B$ are two events. Let $S$ be the event that $A$ or $B$ occur, but not both. Show that

$$\mathbb{P}[S] = \mathbb{P}[A] + \mathbb{P}[B] - 2\,\mathbb{P}[A \cap B].$$

(b) Suppose $A, B$, and $C$ are three events in a sample space. Let $T$ denote the event that exactly one of these three events occur. Deduce from the axioms that

$$\mathbb{P}[T] = \mathbb{P}[A] + \mathbb{P}[B] + \mathbb{P}[C] - 2\,\mathbb{P}[A \cap B] - 2\,\mathbb{P}[A \cap C] - 2\,\mathbb{P}[B \cap C] + 3\,\mathbb{P}[A \cap B \cap C].$$

*Hint:* Draw a Venn diagram and use it to describe $S$ and $T$ as Boolean combination of the given events.

**(1.6)** (Bonus) Suppose $A_1, \dots, A_n$ are events in some probability space. Show that

$$\mathbb{P}[A_1 \cap A_2 \cdots \cap A_n] \geq \sum_{i=1}^{n} \mathbb{P}[A_i] - (n-1).$$