Problem 4.1

a) p is a prefix of w $\Rightarrow$ p can equal to w

$\because$ $p \preceq w$ = $w = pq$ where $q = \emptyset$ when $p = w$

$\therefore$ $w \preceq p$ also = $p \leq w$ Therefore antisymmetric ~~asym~~

$\because$ $p = w$ Thus reflexive

$\because$ $p \preceq w$ is reflexive Thus transitive

Therefore $\preceq$ is a partial order.

b) $\because$ p is a proper prefix of w $\therefore$ $p \neq w$

$\therefore$ irreflexive

$\because$ $p \neq w$ $\therefore$ when $w = pq$, q is not $\emptyset$

Thus $q \neq wq$ $\therefore$ asymmetric

$\because$ p is a proper prefix of w $\therefore$ in $p \precneqq q$

q will be a proper prefix of p, which is also a proper prefix of w.

Therefore $q \precneqq w$ also equals $p \precneqq w$.

$\prec$ is a strict partial order.

c) YES.

Problem 4.2

a) if $g(f(x))$ is bijective, every $f(x)$ must be distinct, which implies the outputs of f are all different (injective), otherwise there could be two same $f(x)$ which $g(f(x))$ bijective will not be a case. If $g(f(x))$ is bijective, then every $f(x)$ has to have a corresponding $g(x)$ which makes every y-value of $g(x)$ has at least one x-value. That proves g is surjective.

b) $f(x) = x$ $\Rightarrow$ injective

$g(x) = \sin(x)$ $\Rightarrow$ Surjective

$g(f(x)) = \sin(x)$ $\Rightarrow$ Surjective $\neq$ bijective

c) $f(x) = x$ when $\{x \mid x \geq 0\}$ $\Rightarrow$ not surjective

$g(x) = x^2$ ~~when~~ $\{y \mid y \geq 0\}$ $\Rightarrow$ not injective

$g(f(x)) = x^2$ $\{x, y \mid x \geq 0, y \geq 0\}$ $\Rightarrow$ bijective.