JMTS-12: Probability and Random Processes

Fall 2020

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Orga

Textbook:

Henry Stark & John W. Woods

Probability and Random Processes with Applications to Signal Processing

Chapters 1-4 … parts of 5+6 if time permits

Main platform: campusnet … course page !!!

In online meetings, use headsets if possible…

TAs:

Chhandosee Bhattacharya, Abhieshree Dhami

Lecture 1

Chapter 1

Why study probability?

- Gambling(coins, dice, cards)
- Forecasts

Think of examples …

Lecture 1

Chapter 1

1.1 Why study probability?
1.2 The different kinds of probability
1.3 Misuses, miscalculations, and paradoxes in probability
1.4 Sets, fields, events
1.5 Axiomatic definition of probability

Gambling

Want to know your chances to gain or lose …

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Chapter 1

1.1 Why study probability?
1.2 The different kinds of probability
1.3 Misuses, miscalculations, and paradoxes in probability
1.4 Sets, fields, events
1.5 Axiomatic definition of probability

Forecasts

Is it going to rain later, today?

… or tomorrow .. Or next week?

How can we tell?

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Chapter 1

1.2 The different kinds of probability

- Favorable cases/ all
- Observe frequencies
- Intuitive
- Axiomatic approach

Lecture 1

Chapter 1

1.1 Why study probability?
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Favorable cases/ all

- Dice
- Coins
- Rain/no rain?

Imagine those cases …

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Chapter 1

Observe frequencies

Think of an unfair die … and roll it many times.

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Chapter 1

Intuitive

What about rain, today?

… tomorrow… next week?

Aggregate similar situations…

Think of humidity, clouds in the sky, or the day of the week …

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Chapter 1

Axiomatic approach

Have a solid ground for your reasoning … the math part

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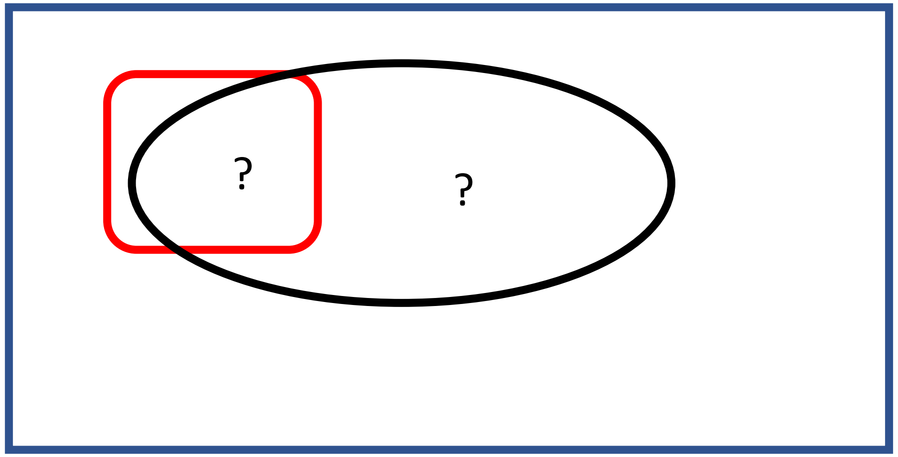
Chapter 1



1.3 Misuses, miscalculations, and paradoxes in probability

Example: Medical sreening of a population

All people (suppose 100 Mio)

Sick people (suppose 1 Mio)

People with symptom … (suppose
hospital says: 95% of ``sick´´ show symptom

also suppose, overall, 20 Mio show the symptom.

How many are sick and show the symptom?

How many are healthy and show the symptom?

Is this a ``useful´´ symptom?

… change the numbers…

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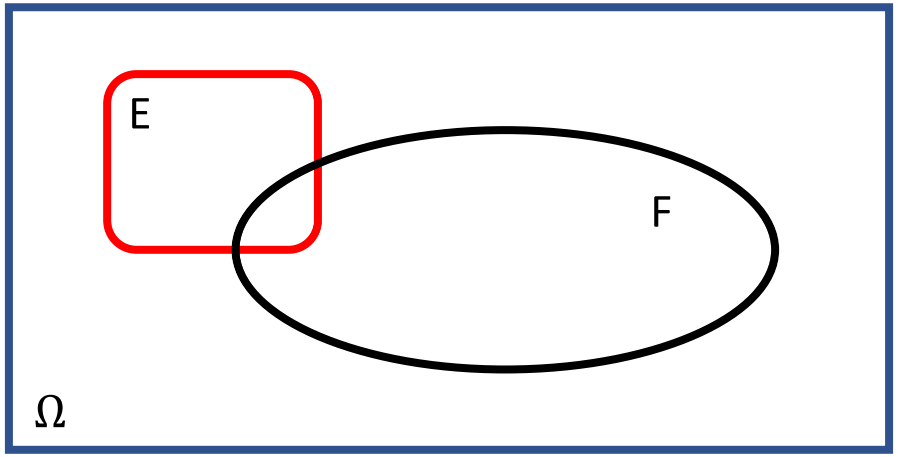
Chapter 1



1.4 Sets, fields, events

Ω: whole set (universal set)
$\emptyset$: empty set
E, F: subsets of Ω

Field (algebra): family $\mathcal{M}$ of subsets of Ω

- Including Ω and $\emptyset$
- $E, F \in \mathcal{M} \Rightarrow EF = E \cap F \in \mathcal{M}$ and $E \cup F \in \mathcal{M}$
- $E \in \mathcal{M} \Rightarrow E^c \in \mathcal{M}$

σ-Field: A field $\mathcal{M}$ where also
$E_i \in \mathcal{M} (i = 1,2, \dots) \Rightarrow \bigcup_{i=1}^{\infty} E_i \in \mathcal{M}$ and $\bigcap_{i=1}^{\infty} E_i \in \mathcal{M}$

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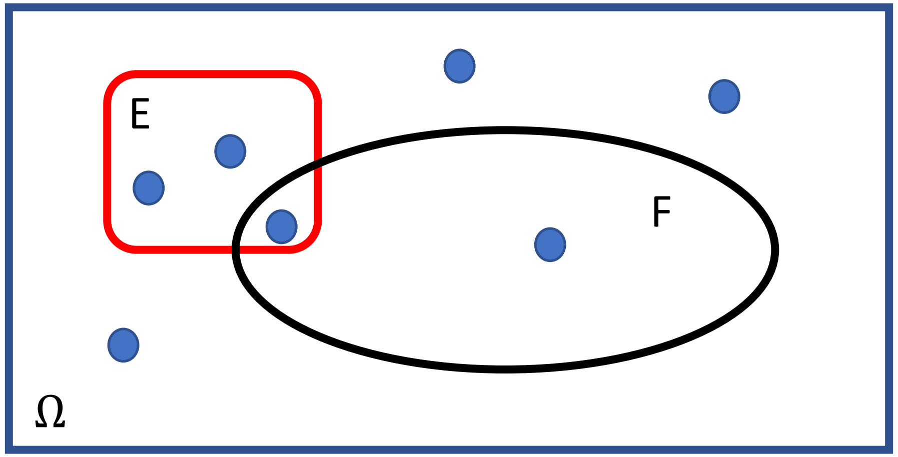
Chapter 1



1.5 Axiomatic definition of probability

Ω: Sample description set (all your possible outcomes ●)

$\mathcal{F}$: σ-field of events (on Ω)

$E, F \in \mathcal{F}$: events

P: Probability measure … maps an event $E \in \mathcal{F}$ to its probability P[E].

Together, (Ω, $\mathcal{F}$, P) are called a probability space.

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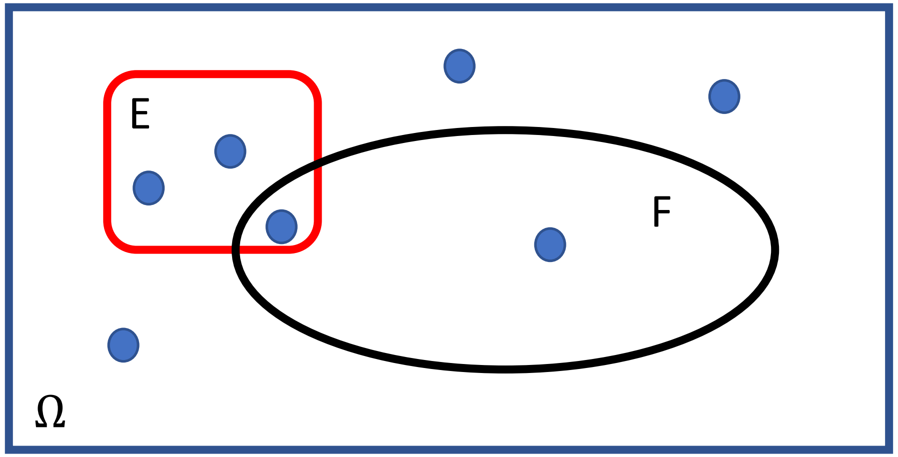

Kolmogorov‘s Axioms:

1) $P[E] \geq 0$
2) $P[\Omega] = 1$
3) $P[E \cup F] = P[E] + P[F]$

   if $EF = \emptyset$ (E,F are disjoint events)
4) Similarly, $\mathrm{P}[\cup_{i=1}^{\infty} E_i] = \sum_{i=1}^{\infty} P[E_i]$

   if $E_i E_j = \emptyset$ for all i $\neq j$

This implies

a) $P[\emptyset] = 0$
b) $P[EF^c] = P[E] - P[EF]$

Why? Find those events in the picture.

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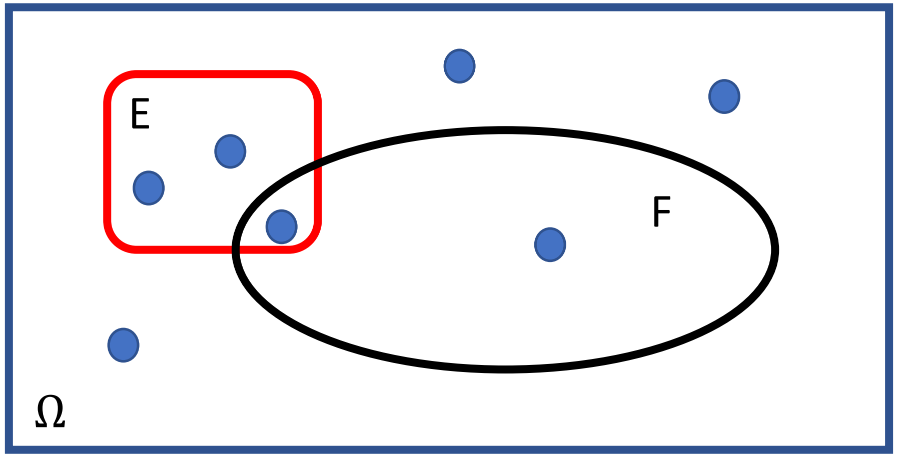
Chapter 1



This implies

a) $P[\emptyset] = 0$
b) $P[EF^c] = P[E] - P[EF]$

Why?

a) $1 = \mathrm{P}[\Omega] = P[\Omega \cup \emptyset] = P[\Omega] + P[\emptyset]$

b) $\mathrm{P}[\mathrm{E}] = P[E \cap \Omega] = P[E \cap (F \cup F^c)] =$

$$P[EF] + P[EF^c]$$

Which axioms have been used in the steps above?

The End

Next time: cont. chp. 1