

Elements of Probability

- (2.1) Alice and Bob are supposed to meet in the cafeteria. Alice arrives at a random time between noon and 1pm, and wait for 15 minutes upon her arrival and then leaves. Bob also arrives at a random time between noon and 1 pm, but waits up to 20 minutes and then leaves.
- (a) What is the probability that Bob arrives before 12:20?
 - (b) What is the probability that Alice and Bob meet?
 - (c) If Bob arrives later than Alice, what is the probability that they meet?
 - (d) Suppose that Alice and Bob have managed to meet. What is the probability that Bob has arrived before 12:20?
- (2.2) A bias coin has the probability $2/3$ of turning up heads. The coin is thrown 4 times.
- (a) What is the probability that the total number of heads shown is 3?
 - (b) Suppose that we know that outcome of the first throw is a head. Find the probability that the total number of heads shown is 3.
 - (c) If we know that the total number of heads shown is 3, find the probability that the outcome of the first throw was heads.
- (2.3) Suppose that 15 percent of the messages arriving at a mailbox are spam and that 20 percent of spam messages arriving there contain the word "winner". Suppose also that the probability that the word "winner" appears in a non-spam message is 5 percent.
- (a) What percentage of the received emails contain the word "winner"?
 - (b) Suppose that a message is tagged as spam based on containing the word "winner". Find the probability that the message is indeed a spam.
- (2.4) Suppose M is an integer randomly chosen from the set $\{1, 2, \dots, 10\}$. Once M is chosen, the integer N is chosen from the set $\{1, 2, \dots, M\}$. For instance if it turns out that $M = 7$, then N can take one of the values $1, \dots, 7$, each with probability $1/7$.
- (a) Find the probability that $N = 7$.
 - (b) Find the probability of the event $M = N$.
- (2.5) A defect switch turns the light on automatically at a random time between noon and 1 pm. Charlotte checks the light at a random time between noon and 1 pm, and if the light is on, switches it off and leaves immediately.
- (a) Describe the sample space for this probabilistic situation by a two-dimensional region.
 - (b) Find the probability that the light is on at 12 : 45.
 - (c) Find the probability that the light is on at 12 : 30.