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Due: October 19, 2018
Assignment 4

Elements of Probability

(**4.1**) Consider a discrete random variable X with the probability mass function given by

$$p_X(x) = \begin{cases} k|x| & \text{if } x = -1, 1, -2, 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Compute the value of k.
(b) Compute $\mathbb{E}[X]$ and $\mathrm{Var}[X]$.

(**4.2**) A continuous random variables has the density function given by

$$f_X(x) = \begin{cases} k(1-x) & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine the value of k.
(b) Compute the probabilities $\mathbb{P}\left[X > \frac{1}{2}\right]$ and $\mathbb{P}\left[\frac{1}{2} < X \leq \frac{2}{3}\right]$.
(c) Compute $\mathbb{E}[X]$.
(d) Compute $\mathrm{Var}[X]$.

(**4.3**) Suppose X is a continuous random variable with the uniform distribution over the interval $[1, 2]$ and $Y = X^2$.
(a) Compute $\mathbb{P}[Y \leq t]$ as a function of t. You need to distinguish three different cases.
(b) Find the probability density function of Y and use it to compute $\mathbb{E}[Y]$.

(**4.4**) Let X be a random variable with the density function

$$f(x) = \begin{cases} \lambda x^{-3} & \text{if } x > 1 \\ 0 & \text{otherwise} \end{cases}$$

where $\lambda > 0$.
(a) Compute the value of λ.
(b) Find $\mathbb{P}[-1 < X < 2]$.
(c) Compute $\mathbb{E}[X]$.

(**4.5**) The joint probability mass function of discrete random variables X and Y is given by

$$p_{X,Y}(x, y) = \begin{cases} kxy & \text{if } 1 \leq x, y \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine the value of constant k.

(b) Determine the probability mass functions of X and Y.

(c) Find $\mathbb{P}[X \geq Y]$.