Problem 7.1:

$(\neg X \to \neg Y) \longrightarrow (\neg(X \to Y))$

and: $\wedge(X, Y) :=$ $\cancel{(X \to Y) \to (\neg X \to \neg Y)}$

or: $\vee(X, Y) :=$ $(X \to Y) \longrightarrow Y$

equivalence: $\leftrightarrow(X, Y) :=$ $(X \to Y) \to (\neg(X \to Y))$

exclusive or: $\dot{\vee}(X, Y) :=$ $(\neg X \to \neg Y) \to (\neg(X \to Y))$

not and: $\uparrow(X, Y) := \neg((X \to Y) \to \neg X)$

not or: $\downarrow(X, Y) :=$ $(X \to Y) \to Y$

Problem 7.2:

a)

P	Q	R	S	$\neg P \vee Q$	$\neg Q \vee R$	$\neg R \vee S$	$\neg S \vee P$	φ
0	0	0	0	1	1	1	1	1
0	0	0	1	1	1	1	0	0
0	0	1	0	1	1	0	1	0
0	0	1	1	1	1	1	0	0
0	1	0	0	1	0	1	1	0
0	1	0	1	1	0	1	0	0
0	1	1	0	1	1	0	1	0
0	1	1	1	1	1	1	0	0
1	0	0	0	0	1	1	1	0
1	0	0	1	0	1	1	1	0
1	0	1	0	0	1	0	1	0
1	0	1	1	0	1	1	1	0
1	1	0	0	1	0	1	1	0
1	1	0	1	1	0	1	1	0
1	1	1	0	1	1	0	1	0
1	1	1	1	1	1	1	1	1

There are only two interpretations satisfy ϕ.

b)

$(\bar{P} \wedge \bar{Q} \wedge \bar{R} \wedge \bar{S}) \vee (P \wedge Q \wedge R \wedge S)$

or $(\neg P \wedge \neg Q \wedge \neg R \wedge \neg S) \vee (P \wedge Q \wedge R \wedge S)$

c)

$(\neg P \vee Q) \wedge (\neg Q \vee R) \wedge (\neg R \vee S) \wedge (\neg S \vee P)$

$= \left[((\neg P \vee Q) \wedge \neg Q) \vee ((\neg P \vee Q) \wedge R)\right] \wedge \left[((\neg R \vee S) \wedge \neg S) \vee ((\neg R \vee S) \wedge P)\right]$ distributivity

~~$= \left[\left[((\neg P \vee Q) \wedge \neg Q) \vee ((\neg P \vee Q) \wedge R)\right] \wedge ((\neg R \vee S) \wedge \neg S)\right] \vee \left[\left[((\neg P \vee Q) \wedge \neg Q) \vee ((\neg P \vee Q) \wedge R)\right] \wedge ((\neg R \vee S) \wedge P)\right]$~~

~~$\neg P \vee Q$~~

$\because (\neg P \vee Q) \wedge \neg Q \equiv (\neg P \vee \neg\neg Q) \wedge \neg Q$ double negation

$\because (\neg P \vee \neg\neg Q) \wedge \neg Q \equiv \neg(P \vee \neg Q) \wedge \neg Q$ de Morgan's laws

$\because \neg(P \vee \neg Q) \wedge \neg Q \equiv \neg Q$ absorption laws

$\therefore (\neg P \vee Q) \wedge \neg Q \equiv \neg Q$ and $(\neg R \vee S) \wedge \neg S \equiv \neg S$

$= \left[\neg Q \vee ((\neg P \vee Q) \wedge R)\right] \wedge \left[\neg S \vee ((\neg R \vee S) \wedge P)\right]$

$\because \neg Q \vee ((\neg P \vee Q) \wedge R) \equiv \left[\neg Q \vee (\neg P \vee Q)\right] \wedge (\neg Q \vee R)$ distributivity

$\because \neg Q \vee (\neg P \vee Q) \equiv 1$

$\therefore \neg Q \vee ((\neg P \vee Q) \wedge R) \equiv \neg Q \vee R$ and $\neg S \vee ((\neg R \vee S) \wedge P) \equiv \neg S \vee P$

$= (\neg Q \vee R) \wedge (\neg S \vee P)$

$= \left[(\neg Q \vee R) \wedge \neg S\right] \vee \left[(\neg Q \vee R) \wedge P\right]$ distributivity

$= (\neg Q \wedge \neg S) \vee (R \wedge \neg S) \vee (\neg Q \wedge P) \vee (R \wedge P)$