Jacobs University Bremen
Keivan Mallahi-Karai

Due: September 18, 2017
Assignment 1

*Elements of Probability*

Solve only 5 out of the following 7 problems.

**(1.1)** Five students have been randomly chosen from a large class. Find the probability that
(a) At least one of them is born on Sunday.
(b) At least two of them are born on the same day of the week.
(c) All five are born on the weekend.

**(1.2)** A die has been thrown 6 times.
(a) Find the probability that the number 2 appears at least once.
(b) Suppose that it is given that the number 3 has appeared at least once. Find the conditional probability that the number 2 has also appeared at least once.
(c) Compare the results of part (a) and (b). Do you find the result reasonable? Why?

**(1.3)** Let $A$ and $B$ be two events. Let $Z$ describe the event that exactly one of these two events occurs.
(a) By using a Venn diagram or otherwise, prove that

$$Z = (A \cup B) - (A \cap B).$$

(b) Deduce from part (a) that

$$\mathbb{P}[Z] = \mathbb{P}[A] + \mathbb{P}[B] - 2\mathbb{P}[A \cap B].$$

(c) A random number $n$ is chosen from the set $\{1, 2, \dots, 100\}$. Find the probability of the event that $n$ is divisible by 3 or 5, but is not divisible by both of them.

**(1.4)** A three-element subset of the set $\{1, 2, \dots, 10\}$ is randomly chosen.
(a) Let $A$ be the event that the largest element of $A$ is 6. Find $\mathbb{P}[A]$.
(b) Let $B$ denote the event the smallest element of $A$ is 2. Find $\mathbb{P}[B]$.
(c) Find the conditional probabilities $\mathbb{P}[A|B]$ and $\mathbb{P}[B|A]$.

**(1.5)** Alex goes a the bus stop at Vegesack at some random time between noon and 1 pm, and waits for 24 minutes for the bus. The bus is also supposed to arrive at a random time between noon and 1 pm, and wait there for 6 minutes before leaving.
(a) What is the probability that Alex succeeds in catching the bus?
(b) Assuming that Alex has caught the bus, find the probability that his waiting time was less than 12 minutes.
*Hint:* Find a formulation of the problem similar to the Alice and Bob example discussed in class.

**(1.6)** Three points $M$, $N$, and $L$ are randomly chosen on a circle centered at $O$. Find the probability of the event that
(a) $O$ is inside the triangle $MNP$.
(b) $O$ on *on* one of the sides of the triangle $MNP$.
(c) $O$ is outside the triangle $MNP$.

*Hint:* Due to the rotational symmetry of the circle, one can fix one of the points.

**(1.7)** Suppose $A_1, \ldots, A_n$ are events in a sample space. Show that

$$\sum_{1 \leq i \leq n} \mathbb{P}\left[A_i\right] - \sum_{1 \leq i < j \leq n} \mathbb{P}\left[A_i \cap A_j\right] \leq \mathbb{P}\left[\bigcup_{i=1}^{n} A_i\right] \leq \sum_{1 \leq i \leq n} \mathbb{P}\left[A_i\right].$$