

Problem 1:

A	B	C_{in}	$A \vee B \vee C$	$A \wedge B$	$C_{in} \wedge (A \vee B)$	C_{out}
0	0	0	0	0	0	0
1	0	0	1	0	0	0
0	1	0	1	0	0	0
1	1	0	0	1	0	1
0	0	1	1	0	0	0
1	0	1	0	0	1	1
0	1	1	0	0	1	1
1	1	1	1	1	0	1

$$a) S = (A \wedge \neg B \wedge \neg C_{in}) \vee (\neg A \wedge B \wedge \neg C_{in}) \vee (\neg A \wedge \neg B \wedge C_{in}) \vee (A \wedge B \wedge C_{in})$$

$$! C_{out} = (A \wedge B \wedge \neg C_{in}) \vee (A \wedge \neg B \wedge C_{in}) \vee (\neg A \wedge B \wedge C_{in}) \vee (A \wedge B \wedge C_{in})$$

$$b) S = (A \vee B \vee C_{in}) \wedge (\neg A \vee \neg B \vee C_{in}) \wedge (\neg A \vee B \vee \neg C_{in}) \wedge (\overline{A \uparrow B})$$

$$A \vee \neg B \vee \neg C_{in})$$

$$C_{out} = (A \vee B \vee C_{in}) \wedge (\neg A \vee B \vee C_{in}) \wedge (A \vee \neg B \vee C_{in}) \wedge (A \vee B \vee \neg C_{in})$$

$$c) A \vee B = \neg A \uparrow \neg B$$

$$A \wedge B = \neg(A \uparrow B)$$

$$A \vee B = (A \uparrow (A \uparrow B)) \uparrow (B \uparrow (A \uparrow B))$$

$$S = [(A \vee B) \uparrow ((A \vee B) \uparrow C_{in})] \uparrow [C_{in} \uparrow ((A \vee B) \uparrow C_{in})]$$

$$C_{out} = \neg(\neg(A \uparrow B)) \uparrow \neg(\neg(C_{in} \uparrow (A \vee B)))$$

$$= (A \uparrow B) \uparrow (C_{in} \uparrow (A \vee B))$$

Problem2:

a) set $xs = [x, y, z]$

$$\text{foldl } op \ e \ xs = (((e \ op \ x) \ op \ y) \ op \ z)$$

because $e \ op \ x = x$ therefore $\text{result} = (x \ op \ y) \ op \ z$

because $x \ op \ e = x$ therefore $\text{foldr } op \ e \ xs = x \ op (y \ op \ z)$

because op is associative

Therefore $\text{foldr } op \ e \ xs = \text{foldl } op \ e \ xs$.

b) if $xs = [x, y, z]$

$$\text{foldl } op \ z \ e \ xs = ((e \ op' z) \ op' y) \ op' z$$

$$\text{foldr } op \ e \ xs = x \ op' (y \ op' (z \ op' e))$$

$$= x \ op' (y \ op' (e \ op' z))$$

$$= x \ op' ((y \ op' e) \ op' z)$$

$$= (x \ op' (e \ op' y)) \ op' z$$

$$= ((x \ op' e) \ op' y) \ op' z$$

$$= ((e \ op' x) \ op' y) \ op' z$$

$$\text{foldr } op \ e \ xs = \text{foldl } op \ z \ e \ xs$$

c) if $xs = [x, y, z]$ then $\text{reverse } xs = [z, y, x]$

$$\text{foldl } op' \ a \ (\text{reverse } xs) = ((a \ op' z) \ op' y) \ op' x$$

$$\text{foldr } op \ a \ xs = x \ op (y \ op (z \ op a))$$

$$= x \ op (y \ op (a \ op' z))$$

$$= x \ op ((a \ op' z) \ op' y)$$

$$= ((a \ op' z) \ op' y) \ op' x$$

$$\text{foldr } op \ a \ xs = \text{foldl } op' \ a \ (\text{reverse } xs)$$