

### Problem 10.1

a)

double hashing function is  $h(k, i) = (h_1(k) + i \cdot h_2(k)) \bmod m$

In this sequence of  $\langle 3, 10, 2, 4 \rangle$ , no collision has taken place, which is just  $h_1(k) \bmod m$ .

Also,  $h_1(k) = k \bmod 5$  which is  $m$ , when collision happen,  $i$  is 0, hash function equals  $k \bmod 5$ .

Handwritten notes on a grid background:

Hash table structure (array of 5 slots):

10
2
3
4

Calculations:

$$h_1(k) = k \bmod 5$$

$$h_2(k) = 7k \bmod 8$$

$$h(k, i) = (h_1(k) + i \cdot h_2(k)) \bmod m$$

$$h(3, 0) = 3 \bmod 5 = 3$$

$$h(10, 0) = 0 \bmod 5 = 0$$

$$h(2, 0) = 2 \bmod 5 = 2$$

$$h(4, 0) = 4 \bmod 5 = 4$$

~~$h(3, 0) =$~~

b)

hash\_table.cpp

$h'$  function =  $\text{key} \% \text{maxSize}$ ;

Idea of linear probing is to increment 1 if the current Hash-index is occupied. This means when  $h'$  function =  $\text{key} \% (\text{a number smaller than maxSize})$ , the number of collisions will not be minimal, because this function can only return a Hash-index at most to the number being moduled. Take  $h' = \text{function} = \text{key} \% 7$  as an example, this function will only return up to index 6, and the index between 6 and  $\text{maxSize}$  will have to start at most from 6, which will eventually cost a lot of collision.

To prove the above statement, I have an int "Collison" in HashTable class which records how many collisions happen when a HashTable is full inserted. To be fair I used  $\text{rand}()$  with time as seed in the beginning for the keys. Below are collisions recorded when  $m=10$ .

num											average
3	35	38	38	34	35	38	35	35	34	36	35.8
7	22	11	24	25	22	24	18	18	16	14	19.4
10	12	17	16	13	8	12	17	10	4	13	12.2

This proves set  $h'$  function as  $\text{key} \% \text{maxSize}$  is well-suited.

### Problem 10.2

a)

counterexample:

activity:	A1	A2	A3	sort with shortest duration	A2	A3	A1
start:	1	4	5		4	5	1

finish:	5	6	8		6	8	5
duration:	5	2	3		2	3	5

greedy choice of selecting the activity with shortest duration: **A2** one activity.

globally optimal solution: **A1->A3** two activities.

In this counterexample, if the shortest activity is in between and crosses the boundaries of the neighbor activities, it will fail at producing the globally optimal solution.