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Due: October 18, 2019
Assignment 4

Elements of Probability

(4.1) Suppose X is a random variable with normal distribution with $\mu = 2$ and $\sigma = 2$. Compute the following probabilities in terms of the function Φ (the distribution function of a standard normal distribution).
(a) $\mathbb{P}[0 \leq X \leq 3]$.
(b) $\mathbb{P}[X > 2]$.
(c) $\mathbb{P}[X < 1]$.

(4.2) A continuous random variables has the density function given by

$$f_X(x) = \begin{cases} k(2-x) & \text{if } 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine the value of k.
(b) Compute the probabilities $\mathbb{P}\left[X > \frac{3}{2}\right]$ and $\mathbb{P}\left[\frac{3}{2} < X \leq \frac{7}{4}\right]$.
(c) Compute $\mathbb{E}[X]$.
(d) Compute $\mathrm{Var}[X]$.

(4.3) The probability density function of a continuous random variable is given by

$$f_X(t) = \begin{cases} 3t^2 & \text{if } 0 < t < 1 \\ 0 & \text{otherwise} \end{cases}$$

Compute $\mathbb{E}\left[X + \frac{1}{X}\right]$.

(4.4) A die has been rolled twice. Let X denote the outcome of the first throw and Y denote the smaller of the two outcomes. For instance, if the outcomes are $2, 3$ then $X = 2$ and $Y = 2$ and if the outcomes are $4, 3$ then $X = 4$ and $Y = 3$.
(a) Describe the joint probability mass function of X and Y by drawing a table.
(b) Compute the marginal probability mass functions of X and Y.
(c) What are the possible values of $X - Y$? Compute the probability mass function of $Z = X - Y$ and use it to find $\mathbb{E}[Z]$.

(4.5) A commercial airplane used for a flight from Frankfurt to New York has 590 seats. For this flight 625 tickets have been sold. Assume further that the probability that a passenger does not show up for the flight is 0.04. Denote by N the random variable that counts the number of passengers who show up for the flight.
(a) What are possible values for N? Describe the probability mass function for N.
(b) Show that $\mu := \mathbb{E}[N] = 600$ and $\sigma := \sqrt{\mathrm{Var}[N]} = \sqrt{24} \approx 5$.
(c) Use the Central limit theorem to approximately compute the probability that the flight is full or overbooked.