JMTS-12: Probability and Random Processes

Fall 2020

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Orga

Textbook:

Henry Stark & John W. Woods

Probability and Random Processes with Applications to Signal Processing

Chapters 1-4 … parts of 5+6 if time permits

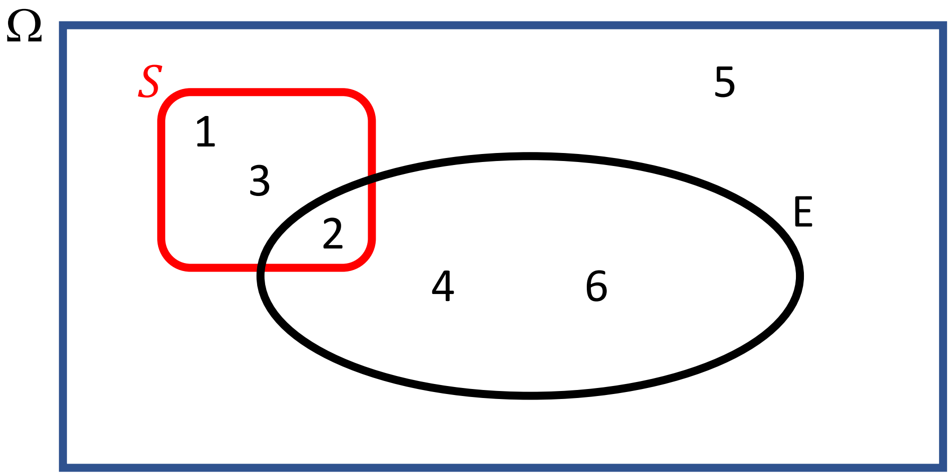
Main platform: campusnet … course page !!!

For the online meetings, use headsets if possible…

Lecture 2

Chapter 1

1.6 Joint, conditional and total probability ...
"Beauty Contest"



Roll a fair die...

Consider events ... and their favorable cases:

E = ``even´´

S = ``small´´

B = ``both´´= ES

$$P[E] = \frac{3}{6}, P[S] = \frac{3}{6}, P[B = ES] = \frac{1}{6} \text{ (joint)}$$

$$\frac{fav_{ES}}{fav_E} = \frac{fav_{ES}/all}{fav_E/all} \rightarrow \frac{P[ES]}{P[E]} =: P[S|E]$$

Probability of S given E: 1/3

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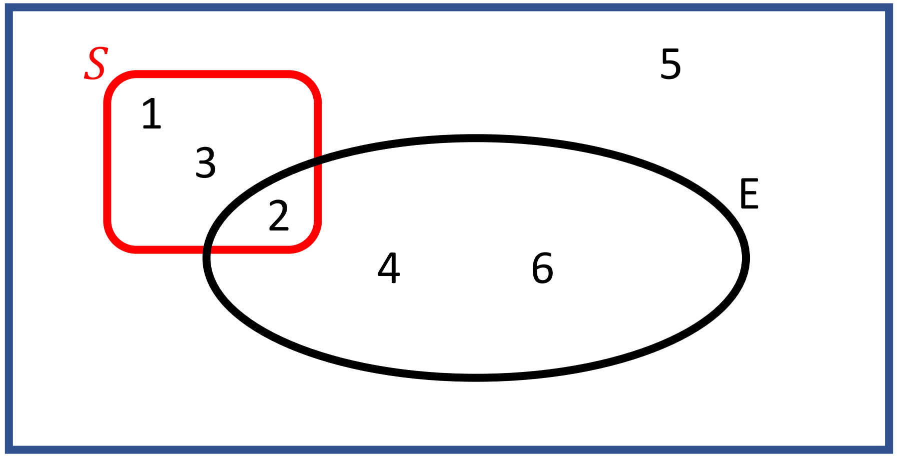

Joint probability: $P[ES]$

Probability of *S* given *E:*

$$P[S|E] = \frac{P[ES]}{P[E]}$$

Probability of *E* given *S :*

$$P[E|S] = \frac{P[ES]}{P[S]}$$

$$P[ES] = P[S|E]P[E] = P[E|S]P[S]$$

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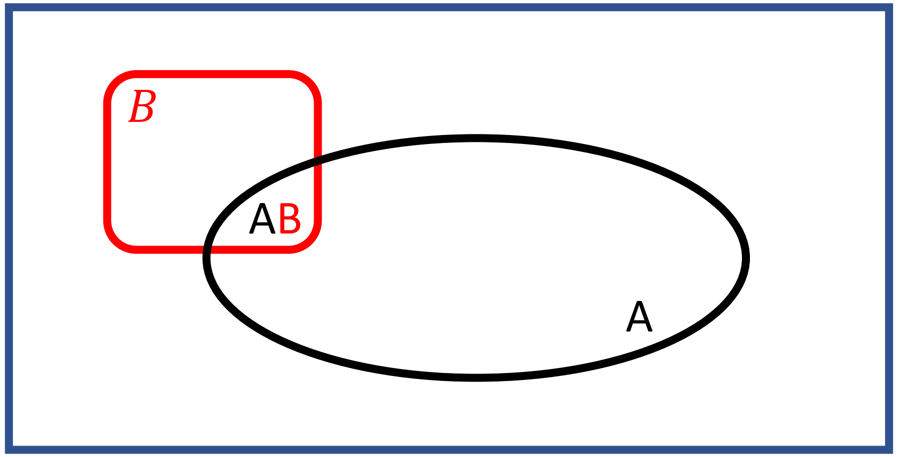

What if ...

$$P[B|A] := \frac{P[AB]}{P[A]} = P[B] \ ?$$

$$\Rightarrow P[AB] = P[A]P[B]$$

$$\Rightarrow P[A|B] := \frac{P[AB]}{P[B]} = P[A]$$

So, A does not matter for B.
Also, B does not matter for A.

→ Independence

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Beauty Contest

1) N candidates come in one by one.
2) Jury forms an opinion on the spot.
3) Stops when *they think* they found the best.

4) Cannot pick a former one.
5) Have to pick someone … possibly the last.

6) ``Win´´ = jury picked the best

P[win]=?

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Beauty Contest: Strategy

1) Observe test set of size α ... don‘t pick any!
2) Store ``beauty value´´ of *best of test*.
3) After test set, pick first candidate better than *best of test.*
4) *Can the jury win*
   = pick the overall best?

P[win]=? ... **Guess!**

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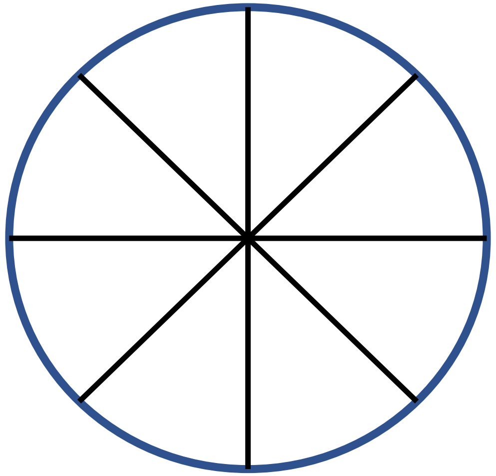
Beauty Contest: Evaluate Strategy

1) Looks difficult
2) Divide and conquer: Cut problem into simpler parts.
3) Solve per part.
4) Combine



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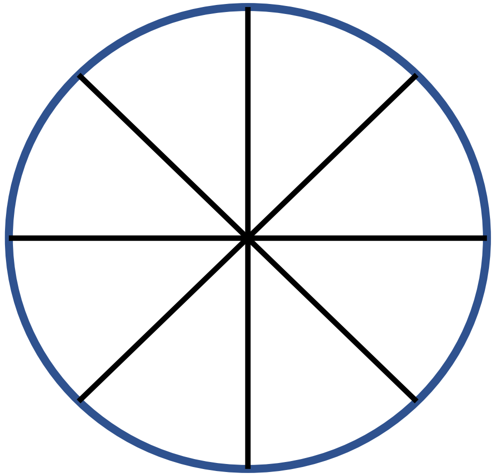

Beauty Contest: Evaluate Strategy

What is the chance experiment?

→Queue in front of the door.

Understand: Based on that queue, the result is clear.

How many different queues exist?

N!

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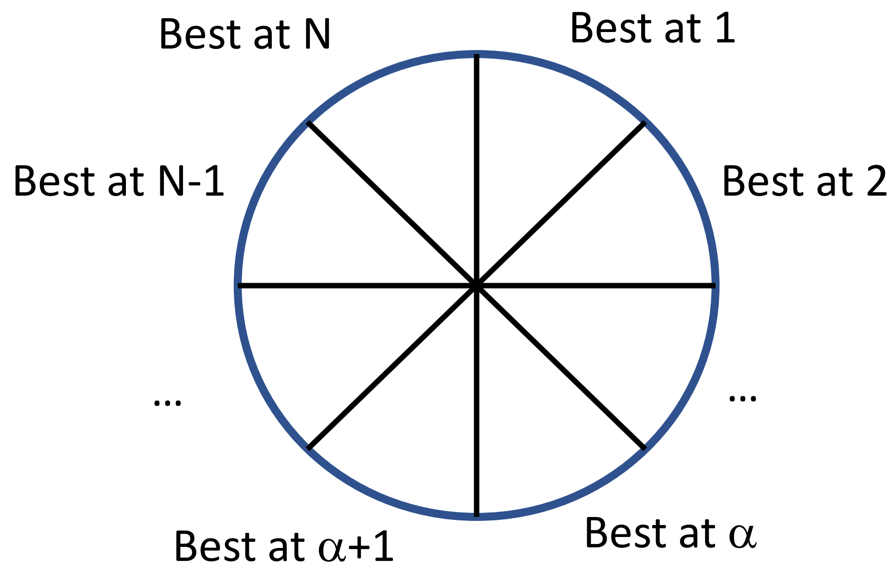

Beauty Contest: Evaluate Strategy

Divide and conquer:

Cut problem into simpler parts... different sorts of queues.

... according to only one aspect:

What is the position of the best candidate in the queue?

→**Partition**

1) Union of the parts is Ω (exhaustive)
2) Parts do not overlap (mutually exclusive)

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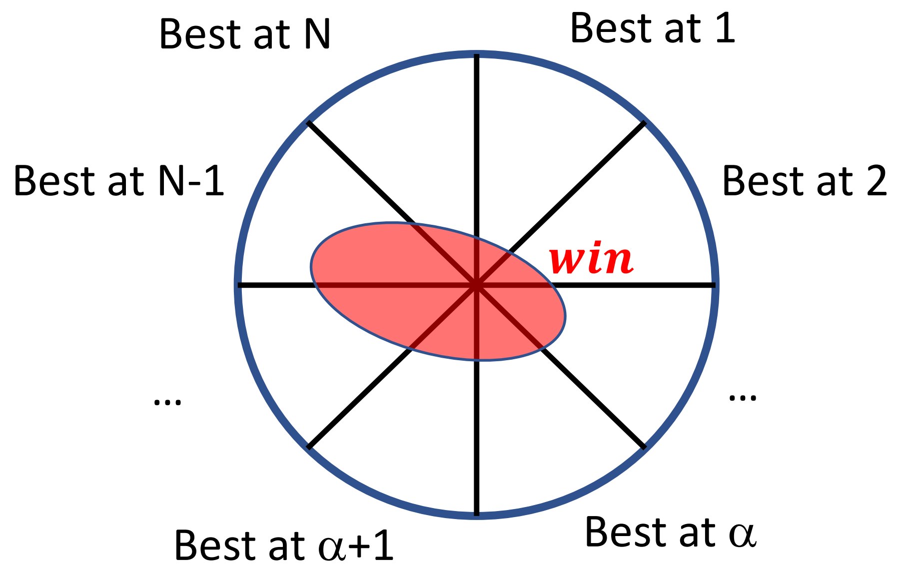

Beauty Contest: Evaluate Strategy

Solve ... use total probability:

$$P[win] = \sum_{j=1}^{N} P[win, best\ at\ j]$$

$$= \sum_{j=1}^{N} P[win|best\ at\ j]P[best\ at\ j]$$

Easy cases:

$j \leq \alpha$ (best inside test set):

$$P[win|best\ at\ j] = 0$$

Also:

$$P[best\ at\ j] = \frac{1}{N}$$

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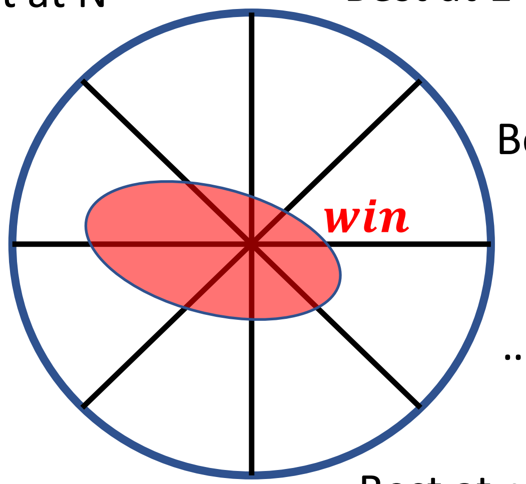


Beauty Contest: Evaluate Strategy

$$P[win] = \frac{1}{N}\sum_{j=\alpha+1}^{N} P[win|best\ at\ j]$$

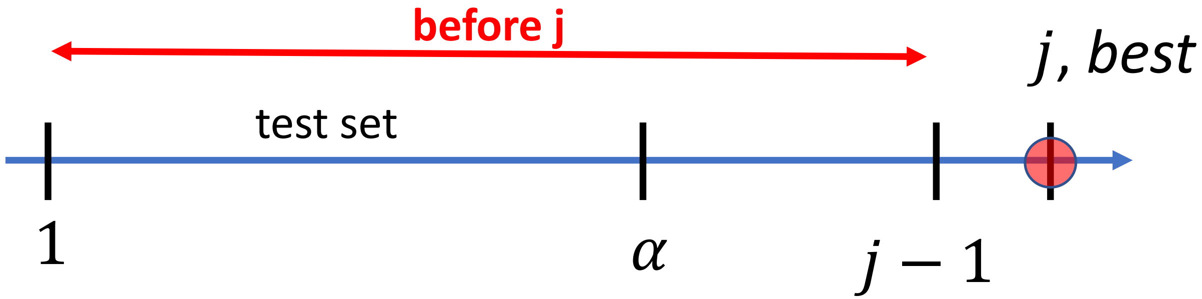
Not so easy cases:

$$j = \alpha + 1 : P[win|best\ at\ j] = 1$$

$$j = \alpha + 2 : P[win|best\ at\ j] = ?$$

$$\ldots$$

$$= P[best\ before\ j\ is\ in\ test\ set|best\ at\ j] = \frac{\alpha}{j-1}$$



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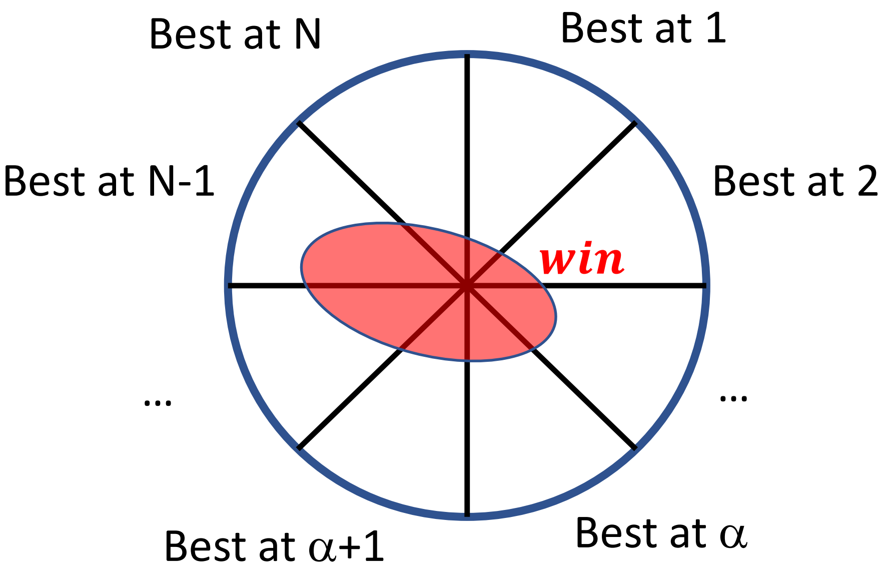


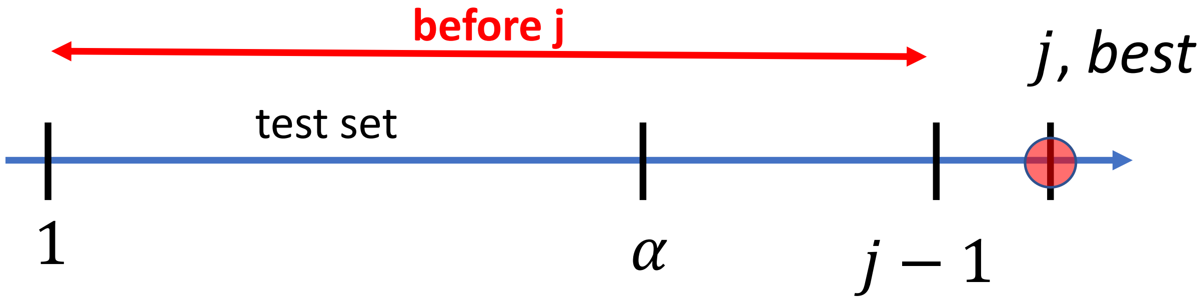
Beauty Contest: Evaluate Strategy

$$P[win] = \frac{1}{N}\sum_{j=\alpha+1}^{N}\frac{\alpha}{j-1}$$

Approximate for large N and α:

$$P[win] = \frac{1}{N}\sum_{j=\alpha+1}^{N}\frac{\alpha}{j-1} \approx \frac{\alpha}{N}\int_{\alpha}^{N}\frac{dx}{x} = -\frac{\alpha}{N}\ln\frac{\alpha}{N}$$

... max at $\alpha_{opt} = \frac{N}{e}$, with $P[win\ with\ \alpha_{opt}] = \frac{1}{e}$



The End

Next time: cont. chp. 1