

T1 检验问题为

H_0 : 吸烟与患慢性气管炎无关

在列联表检验中, $n=1000$, $r=5-2$

查表 $\chi^2_{(r-1)}(5-1)(0.05) = 3.841$

检验统计量为

$$k = \frac{339(43 \times 121 - 13 \times 162)^2}{205 \times 134 \times 56 \times 283} = 7.469 > 3.841$$

故拒绝原假设, 即吸烟者患慢性气管炎的比例较高

检验问题为:

T5. (1) H_0 : 看电视的男女比率无差异

$$(2) p_1 = \frac{248}{800} = 0.31$$

$$p_2 = \frac{156}{600} = 0.26$$

(3)

$$k = \frac{1400(248 - \frac{404 \times 800}{1400})^2}{404 \times 800} + \frac{1400(156 - \frac{404 \times 600}{1400})^2}{404 \times 600} + \frac{1400(525 - \frac{996 \times 800}{1400})^2}{996 \times 800} + \frac{1400(444 - \frac{996 \times 600}{1400})^2}{996 \times 600}$$

$$= 4.175 \quad \chi^2_{1(0.05)} = 3.841$$

$$\text{拒绝域 } W = \{ \chi^2 > 3.841 \}$$

∴ 可以认为看电视的男女比例有显著差异

$$P\text{值: } P = P(\chi^2 \geq 4.175) = 0.041$$

T6. H_0 : 红球个数为5.

$$n = 1 + 31 + 55 + 25 = 112$$

$$np_0 = 112 \times \frac{\binom{8}{3}}{\binom{8}{3}} = 2$$

$$np_1 = 112 \times \frac{\binom{5}{1} \binom{3}{2}}{\binom{8}{3}} = 30$$

$$np_2 = 112 \times \frac{\binom{5}{2} \binom{3}{1}}{\binom{8}{3}} = 60$$

$$np_3 = 112 \times \frac{\binom{5}{3}}{\binom{1}{3}} = 20$$

$$\text{拟合优度: } S = 2P(X_{4-2-1}(0.05)) = 3.841$$

$$\text{检验统计量 } k = \frac{(1-2)^2}{2} + \frac{(3-30)^2}{30} + \frac{(15-60)^2}{60} + \frac{(25-20)^2}{20}$$

$$= 2.2 < 3.841$$

∴ 接受 H_0

即“ $\alpha = 0.05$ 水平下, 红球的个数为 5.”

可以接受

T9. $n=61$

$$\chi^2_{(3-1)(2-1)}(0.05) = 5.991$$

检验统计量

$$k = 61 \times \left[\frac{6^2}{28 \times 13} + \frac{8^2}{28 \times 29} + \frac{14^2}{28 \times 19} + \frac{7^2}{33 \times 13} + \frac{21^2}{33 \times 29} + \frac{5^2}{33 \times 19} - 1 \right]$$

$$= 9.823$$

$$\therefore k > 5.991$$

∴ A 和 B 两种蜗牛在 3 种珊瑚礁中分布是有差异的

T10. 原假设:

H_0 : 各工厂质量一样

$$r=3 \quad s=3 \quad (r-1)(s-1)=4$$

$$\chi^2_{(0.05)} = 9.488$$

检验统计量

$$\chi^2 = 300 \times \left[\frac{58^2}{n_{11} \cdot n_{21}} \right.$$

$$+ \frac{40^2}{n_{12} \cdot n_{21}} + \frac{11^2}{n_{13} \cdot n_{21}} + \frac{38^2}{n_{11} \cdot n_{22}} + \frac{44^2}{n_{12} \cdot n_{22}} + \frac{18^2}{n_{13} \cdot n_{22}} +$$

$$\frac{30^2}{n_{11} \cdot n_{23}} + \frac{35^2}{n_{12} \cdot n_{23}} + \frac{26^2}{n_{13} \cdot n_{23}} - 1]$$

$$= 15.41 > 9.488$$

\therefore 拒绝 H_0

即在显著性水平 0.05 下, 认为三个工厂产品质量不一致

由 ~~手算~~ 比较 3 个工厂的一等品率

$$x_{\text{甲}} = \frac{58}{109} = 0.532 \quad x_{\text{乙}} = \frac{38}{100} = 0.38$$

$$x_{\text{丙}} = \frac{30}{91} = 0.330$$

$$x_{\text{甲}} > x_{\text{乙}} > x_{\text{丙}}$$

$$n = 300$$

$$n_{11} = 58 + 38 + 30 = 126$$

$$n_{12} = 40 + 44 + 35 = 119$$

$$n_{13} = 11 + 18 + 26 = 55$$

$$n_{21} = 58 + 40 + 11 = 109$$

$$n_{22} = 38 + 44 + 18 = 100$$

$$n_{23} = 30 + 35 + 26 = 91$$

比较三等品率

$$y_甲 = \frac{11}{109} = 0.101 \quad y_乙 = \frac{18}{100} = 0.18 \quad y_丙 = \frac{26}{91} = 0.286$$

$$y_丙 > y_乙 > y_甲$$

故甲厂产品质量较优; 丙厂产品质量较差.

T12. ~~χ^2 test~~ H_0 : 男性和女性对这三种类型的啤酒的偏好无显著差异

$$n = 180 + 120 = 300$$

$$\chi^2_{(r-1)(c-1)}(0.05) = \chi^2_2(0.05) = 5.991$$

$$\begin{aligned} \text{检验统计量 } k &= 300 \times \left[\frac{49^2}{180 \times 100} + \frac{31^2}{180 \times 51} + \frac{100^2}{180 \times 149} + \right. \\ &\quad \left. + \frac{51^2}{100 \times 100} + \frac{20^2}{100 \times 51} + \frac{49^2}{100 \times 149} \right] \\ &= 8.1968 \end{aligned}$$

$$8.1968 > 5.991$$

\therefore 拒绝 H_0 .

即在显著性水平为 0.05 的条件下, 男性和女性对这三种类型的啤酒的偏好有显著差异.

T13. H_0 : 每页上印刷错误的个数服从泊松分布

将数据重新分组, 使得每组的错页个数不少于5.

整理后如下:

错误的个数 f_i	0	1	2	≥ 3
含有 i 个错误的页数	86	40	19	5

$$\hat{\lambda} = \bar{x} = \frac{100}{150} = \frac{2}{3}$$

$$H_0: X \sim \text{Poisson 分布 } P(\lambda = \frac{2}{3})$$

$$n = 150$$

$$np_0 = 150 \times \frac{(\frac{2}{3})^0 e^{-\frac{2}{3}}}{0!} = 77.01$$

$$np_1 = 150 \times \frac{(\frac{2}{3})^1 e^{-\frac{2}{3}}}{1!} = 51.34$$

$$np_2 = 150 \times \frac{(\frac{2}{3})^2 e^{-\frac{2}{3}}}{2!} = 17.11$$

$$np_3 = 4.53$$

$$\text{检验统计量 } K = \frac{(86-77.01)^2}{77.01} + \frac{(40-51.34)^2}{51.34} + \frac{(19-17.11)^2}{17.11} + \frac{(5-4.53)^2}{4.53}$$
$$= 3.812$$

$$\therefore \chi_{4-2}^2(0.05) = 5.991$$

$\therefore H_0$ 成立

即在显著性水平为 0.05 的条件下,
每页上的印刷错误个数服从泊松分布

T14. $H_0: \text{r.v. } X \sim N(60, 15^2)$

$$r=8 \quad \hat{p}_i = P(a_{i-1} \leq X \leq a_i) \\ = \Phi(u_i) - \Phi(u_{i-1}) \quad i=1, \dots, 8$$

$$u_i = \frac{(a_i - 60)}{15} \quad a_0 = -\infty \quad a_8 = +\infty$$

$$\text{故有 } \hat{p}_1 = P(-\infty < X < 30) = \Phi(-2) = 0.023$$

$$\hat{p}_2 = P(30 < X < 40) = \Phi(-\frac{4}{3}) - \Phi(-2) = 0.068$$

$$\text{同理有 } \hat{p}_3 = 0.161 \quad \hat{p}_4 = 0.248$$

$$\hat{p}_5 = 0.248 \quad \hat{p}_6 = 0.161$$

$$\hat{p}_7 = 0.068 \quad \hat{p}_8 = 0.023$$

$$k_n^* = 6.557 \quad \chi_{8-2-1}^2(0.05) = 11.071$$

故可认为原假设成立, 符合正态分布