T27. 矩估计:

(1) $$\frac{\hat{r}}{N}=\frac{t}{s}$$

∵ N很大

∴ $\frac{r}{N}$ 的 $1-\alpha$ 置信区间为

$$\left(\frac{t}{s}-u_{\frac{\alpha}{2}}\sqrt{\frac{\frac{t}{s}(1-\frac{t}{s})}{N}},\ \frac{t}{s}+u_{\frac{\alpha}{2}}\sqrt{\frac{\frac{t}{s}(1-\frac{t}{s})}{N}}\right)$$

(2) 矩估计:

$$\hat{N}=\frac{rs}{t}$$

置信区间 $\left(\frac{rs}{t}-\frac{S'}{\sqrt{S}}u_{\frac{\alpha}{2}},\ \frac{rs}{t}+\frac{S'}{\sqrt{S}}\right)$

S'为样本标准偏差.

T28. $$f(x,\theta)=\begin{cases}-\frac{1}{\theta}, & x\in(\theta,0)\\ 0, & 其他\end{cases}$$

$$\hat{L}(\theta)=\begin{cases}\prod_{i=1}^{n}\frac{1}{(-\theta)^n}, & x_i\in(\theta,0)\\ 0, & 其他\end{cases}$$

易得: 最大似然估计: $\hat{\theta}=\min\{x_1,x_2,\cdots x_n\}$

由自助法得

置信下限: $2\hat{\theta}-\hat{\theta}^*([(B+1)(1-\frac{\alpha}{2})])$

置信上限: $2\hat{\theta}+\hat{\theta}^*([B+1](1-\frac{\alpha}{2})]$

T3. 检验统计量 $T=\frac{\sqrt{n}(\bar{x}-\mu)}{6}=\frac{\sqrt{16}(\bar{x}-\mu)}{1}$

(1) V_1:

$\alpha(0)=P_0(4|\bar{x}|\geq u_{0.05})$

$=P_0(4|\bar{x}-0|\geq u_{0.05})$

$=0.10$

V_2:

$\alpha(0)=P_0(4|\bar{x}|\leq u_{0.45})$

$=P_0(4|\bar{x}-0|\leq u_{0.45})$

$=1-0.90=0.10$

V_3: $\alpha(0)=P_0(4\bar{x}\geq u_{0.10})$

$=0.10$

V_4: $\alpha(0)=P_0(4\bar{x}\leq -u_{0.10})$

$=0.10$

查表得

(2) V_1:

$\beta(1)=P_1(4|\bar{x}|<u_{0.05})$

$=\cancel{P_1(4(\bar{x}-1)<u_{0.05}-4)}$

$=P_1(4(\bar{x}-1)<u_{0.05}-4)$

$+P_1(4(\bar{x}-1)>-u_{0.05}-4)$

$=0.010$

V_2:

$\beta_2=P_1(4|\bar{x}|\geq u_{0.45})$

$=P_1(4(\bar{x}-1)\geq u_{0.45}-4)$

$+P_1(4(\bar{x}-1)\leq -u_{0.45}-4)$

$=1$

V_3:

$\beta(1)=P_1(4\bar{x}\leq u_{0.10})$

$=P_1(4(\bar{x}-1)\leq u_{0.10}-4)$

$=0.004$

$\beta(1)=P_1(4\bar{x}\geq -u_{0.10})$

$=P_1(4(\bar{x}-1)\geq -u_{0.10}-4)$

$=\cancel{0.84}\ 1$

故拒绝域 V_3 犯第一类错误概率最小

$W_1 =$

T5. (1) 拒绝域 $\{(x_1, x_2, \cdots x_n): \sum_{i=1}^{10} x_i \le 1 \text{ 或 } \ge 12\}$

$\sum_{i=1}^{10} x_i$ 服 $\lambda \lambda' = 10\lambda$ 的泊松分布

$\beta(\lambda) = P_\lambda\{(x_1, x_2, \cdots x_n) \in W_1\}$

$= P_\lambda\{\sum_{i=1}^{10} x_i \le 1\} + P_\lambda\{\sum_{i=1}^{10} x_i \ge 12\}$

$= F_\lambda(\sum_{i=1}^{10} x_i = 1) + \cancel{F_\lambda(\sum_{i=1}^{10} x_i \ge 12)}$ $1 - F_\lambda(\sum_{i=1}^{10} x_i = 12)$

$\cancel{= 1 - e^{10\lambda} + 1 - e^{120\lambda}}$ $e^{120\lambda}$

~~当$\lambda = 0.25$时，$\beta(\lambda) = 1 - e^{-\frac{5}{2}} + e^{-30} =$~~

~~当$\lambda = 0.5$时，$\beta(\lambda) = 1 - e^{-5} + e^{-60} =$~~

~~当$\lambda = 1$时，$\beta(\lambda) = 1 - e^{-10} + e^{-120}$~~

$\beta_\psi(\theta) = e^{-2.5} + 2.5 \times e^{-2.5} + 1 - \sum_{k=12}^{\infty} \frac{2.5^k}{k!} e^{-2.5} = 0.287$

$\beta_\psi(\theta) = e^{-5} + 5 \times e^{-5} + 1 - \sum_{k=12}^{\infty} \frac{5^k}{k!} e^{-5} = 0.046$

$\beta_\psi(\theta) = e^{-10} + 5 \times e^{-10} + 1 - \sum_{k=12} \frac{10^k}{k!} e^{-10} = 0.304$

~~水平 $\alpha = 0.046$~~

(2) $P_{\lambda=\frac{1}{2}}(\sum_{i=1}^{10} x_i \le 1 \text{ 或 } \ge 12) = \beta_{0.5}(\lambda) = 0.046$

$P_{\lambda=0.25}(1 < \sum_{i=1}^{10} x_i < 12) = 1 - \beta_{0.25}(\lambda) = 0.713$

$P_{\lambda=0.75}(1 < \sum_{i=1}^{10} x_i < 12) = 1 - \beta_{0.75}(\lambda) = 0.916$

T7. (1) $H_0: \mu=7 \leftrightarrow H_1: \mu\neq 7$

$$U=\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}=\frac{\bar{x}-7}{\frac{4}{\sqrt{10}}}=\frac{\sqrt{10}}{4}(\bar{x}-7)$$

拒绝域：$\{|U|>u_{\frac{\alpha}{2}}\}$ $u_{0.025}=1.96$

又∵ $\left|\frac{\sqrt{10}}{4}(4.8-7)\right|=1.738<1.96$

∴ 在显著性水平 0.05时，可认为H_0正确

(2) $H_0': \mu\geq 7 \leftrightarrow H_1': \mu<7$

$$U=\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}=\frac{\bar{x}-7}{\frac{4}{\sqrt{10}}}=\frac{\sqrt{10}}{4}(\bar{x}-7)$$

拒绝域：$\{U\leq -u_{\alpha}\}$ $u_{0.05}=1.65$

又∵ $\frac{\sqrt{10}}{4}(\bar{x}-7)=-1.738<-1.65$

∴ 在显著性水平 0.05时，可认~~为H0正确~~ 拒绝 H_0'

(3) $H_0'': \mu \le 2 \Leftrightarrow H_1'': \mu > 2$

$$U = \frac{\bar{x}-\mu}{\frac{6}{\sqrt{n}}} = \frac{\bar{x}-2}{\frac{4}{\sqrt{10}}} = \frac{\sqrt{10}}{4}(\bar{x}-2)$$

拒绝域：$\{U \ge u_{\alpha}\}$　$u_{0.05}=1.645$

又∵ $\frac{\sqrt{10}}{4}(\bar{x}-2) = 2.212 > 1.645$

∴在显著性水平0.05时，可拒绝H_0''

T10. 设全市会考成绩为随机变量X_1.

$\mu_1 = 80$　　$\frac{6}{\sqrt{n}} = 9$

$\mu_2 = 82$

H_0: $\cancel{\mu_1 \; \mu_2 \Leftrightarrow H_1: \mu_1 \ge \mu_2}$

$H_0: \mu_1 < \mu_2 \Leftrightarrow H_1: \mu_1 \ge \mu_2$

$$U = \frac{\bar{x}-\mu}{\frac{6}{\sqrt{n}}} = \frac{82-80}{9}$$

拒绝域：$\{U < -u_{\alpha}\}$　$u_{0.05}=1.645$

∵ $U = -\frac{2}{9} > -1.645$

∴在显著性水平0.05下，

可认为此次会考该校仍然显著高于全市平均成绩

T12. $n=10$ $S_n=0.3cm$ $\overline{x}=5.3$ $\mu_0=5.0cm$

要检验机器是否工作良好，即要作双侧检验.

$H_0: \mu_1=\mu_0 \Leftrightarrow H_1: \mu_1\neq\mu_0$

∵ ~~肥皂~~ 检验统计量 $U=\dfrac{\overline{x}-\mu}{S_n}=\dfrac{5.3-5.0}{0.3}=1$

拒绝域 $\{|U|>u_{\frac{\alpha}{2}}\}$

1° $\alpha=0.05$ $u_{\frac{\alpha}{2}}=1.96$

∵ $|U|=1<1.96$

∴ 在显著性水平 0.05 下，机器工作良好

2° $\alpha=0.01$ $u_{\frac{\alpha}{2}}=2.58$

∵ $|U|=1<2.58$

∴ 在显著性水平 0.01 下，机器工作良好.