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T1. $(2,1,0)$ $(2,1,1)$ $(2,1,-1)$

$(2,0,0)$

T2. $\langle \hat{V} \rangle = \int \psi_{1s}^* \hat{V} \psi_{1s} \, d\tau$

$$= \int_0^\pi \sin\theta \int_0^\infty \frac{1}{\sqrt{\pi}} \left(\frac{1}{a}\right)^{\frac{3}{2}} e^{-\frac{r}{a}} \cdot \left(-\frac{e^2}{r}\right) \cdot \frac{1}{\sqrt{\pi}} \left(\frac{1}{a}\right)^{\frac{3}{2}} e^{-\frac{r}{a}} r^2 dr \, d\theta \int_0^{2\pi} d\varphi$$

$$= -4\pi \cdot \frac{1}{\pi} \cdot \left(\frac{1}{a}\right)^3 \cdot e^2 \cdot \int_0^\infty r e^{-\frac{2r}{a}} dr$$

$$= -\frac{e^2}{a}$$

T3. 在坐标表象中氢原子归一化的基态波函数为

$$\psi(r,\theta,\varphi) = \frac{1}{\sqrt{\pi \cdot a^3}} e^{-\frac{r}{a}} \quad 其中 \ a = \frac{\hbar^2}{\mu e^2}$$

$$\bar{x} = \iiint |\psi(r)|^2 x \, d^3 r = \frac{1}{\pi a^3} \iiint e^{-\frac{2r}{a}} (r\sin\theta\cos\varphi) \cdot r^2 \sin\theta \, dr \, d\theta \, d\varphi$$

$$\bar{x} = 0$$

$$\overline{x^2} = \iiint |\psi(r)|^2 x^2 d^3 r = \frac{1}{\pi a^3} \iiint e^{-\frac{2r}{a}} (r\sin\theta\cos\varphi)^2 r^2 \sin\theta \, dr \, d\theta \, d\varphi$$

$$= \frac{1}{\pi a^3} \int e^{-\frac{2r}{a}} r^4 dr \int_0^\pi \sin^3\theta \, d\theta \int_0^{2\pi} \cos^2\varphi \, d\varphi$$

$$= \frac{1}{\pi a^3} \int e^{-\frac{2r}{a}} r^4 dr \int_0^\pi \left(\frac{3}{4}\sin\theta - \frac{1}{4}\sin 3\theta\right) d\theta \int_0^{2\pi} \left(\frac{1}{2} + \frac{1}{2}\cos 2\varphi\right) d\varphi$$

$$= \frac{1}{\pi a^3} \cdot \frac{4!}{(\frac{2}{a})^5} \cdot \frac{8}{6} \cdot \pi = a^2 \quad (\Delta x)^2 = \overline{(x-\bar{x})^2} = \overline{x^2} - \bar{x}^2 = a^2 \quad \Delta x = a$$

$$\bar{p}_x = \iiint |\varphi(p)|^2 p_x d^3p = \frac{(2a)^3 \hbar^5}{\pi^2} \int_0^\infty \frac{p^3 dp}{(a^2p^2+\hbar^2)^4} \int_0^\pi \sin^2\theta' d\theta' \underbrace{\int_0^{2\pi} \cos\varphi' d\varphi'}_{=0}$$

$$\overline{p_x^2} = \iiint |\varphi(p)|^2 p_x^2 d^3p = \frac{8a^3\hbar^5}{\pi^2} \int_0^\infty \frac{p^4 dp}{(a^2p^2+\hbar^2)^4} \int_0^\pi \sin^3\theta' d\theta' \int_0^{2\pi} \cos^2\varphi' d\varphi'$$

$$= \frac{\hbar^2}{3a^2} \quad \Delta p_x = \frac{\hbar}{\sqrt{3}a}$$

$\Delta x \Delta p_x = a \cdot \frac{\hbar}{\sqrt{3}a} = \frac{\hbar}{\sqrt{3}} > \frac{\hbar}{2}$ 故 $\Delta x \Delta p_x$ 满足不确定度关系.

T4. 氢原子基态波函数为 $\psi_{100} = (\frac{1}{\pi a^3})^{\frac{1}{2}} e^{-\frac{r}{a}}$ $a = \frac{\hbar^2}{\mu e^2}$

相应的能量为 $E_1 = -\frac{\mu e^4}{2\hbar^2} = -\frac{e^2}{2a}$

$$T(r) = E_1 - V = -\frac{e^2}{2a} + \frac{e^2}{r}$$

由 $T<0$ 解得 $r>2a$.

电子处于经典不允许区的几率.

$$P = \frac{1}{\pi a^3} \int_{2a}^\infty \int_0^\pi \int_0^{2\pi} e^{-\frac{2r}{a}} r^2 dr \sin\theta d\theta d\varphi$$

$$\overset{\xi=\frac{2r}{a}}{=} \frac{4}{a^3} \cdot \left(\frac{a}{2}\right)^3 \cdot \int_4^\infty e^{-\xi} \xi^2 d\xi$$

$$= 0.2381$$

T5. 由选择定则，存在跃迁 $n=3, l=2 \to n=2, l=1$

及 $n=2, l=1 \to n=1, l=0$

$$\Delta E_{32} = -13.6\,eV \times (\frac{1}{3^2} - \frac{1}{2^2}) = 1.89\,eV$$

$$\Delta E_{21} = -13.6\,eV \times (\frac{1}{2^2} - \frac{1}{1^2}) = 10.2\,eV.$$

T8. 对于本征态 $Y_l^l(\theta,\varphi)$，由球谐函数性质可知

$$\langle \hat{L}^2\rangle_{(l,l)} = l(l+1)\hbar^2, \quad \langle \hat{L}_z\rangle_{(l,l)} = l^2\hbar^2$$

$$\langle \hat{L}_x^2+\hat{L}_y^2\rangle_{(l,l)} = \langle \hat{L}^2-\hat{L}_z^2\rangle_{(l,l)}$$

$$= [l(l+1)-l^2]\hbar^2 = l\hbar^2$$

由x、y方向的等价性得

$$\langle \hat{L}_x^2\rangle_{(l,l)} = \langle \hat{L}_y^2\rangle_{(l,l)} = \frac{1}{2}\langle \hat{L}^2-\hat{L}_z^2\rangle_{(l,l)}$$

$$= \frac{1}{2}l\hbar^2.$$