2023.03.23

P55　以 x 表示光源的水平距离，y 表示高度

T1. $n(y) = n_0(1+Ay)$　　$A = 0.8\times10^{-6}\,m^{-1}$.　　$h=1.6m$.

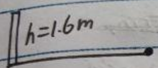
取一个 Δy 单位元，~~设距离~~设高度 y_1 处的空气折射率为 n_m

由费马定理有 $n_m\sin\theta_m = n_m'\sin\theta_m'$ 其中 $n_m' = \frac{1+A(y_1+\Delta y)}{1+Ay_1} n_m$.

故同理有 $n_0 = n_1\sin\theta_1 = n_2\sin\theta_2 = \cdots = n_K\sin\theta_K$.

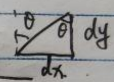
$y=0$ 时　$n(0) = n_0$

~~以公路上一点为光源，易知当其为人眼能看到的最远点时~~



以公路上一点为光源

$n_0 = n\sin\theta = n_0(1+Ay)\sin\theta$.



$\frac{dy}{dx} = \cot\theta$.　　$\sin\theta = \sqrt{\frac{1}{1+\cot^2\theta}} = \sqrt{\frac{1}{1+(\frac{dy}{dx})^2}}$

$$1+Ay = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

平方得　$1+2Ay+A^2y^2 = 1+\left(\frac{dy}{dx}\right)^2$.

$\because$ A很小，y有限　故 A^2y^2 可略.　　$\frac{dy}{dx} = \sqrt{2Ay}$

$$\frac{dy}{\sqrt{y}} = \sqrt{2A}\,dx$$

积分得　$2\sqrt{y} = \sqrt{2A}\,x + C$.

又 $\because$ $x=0$ 时，$y=0$　解得光线轨迹为 $y = \frac{A}{2}x^2$.

这是一抛物线轨迹.　　$y=h$ 时　$x = \sqrt{\frac{2h}{A}} = \sqrt{\frac{2\times1.6}{0.8\times10^{-6}}}$

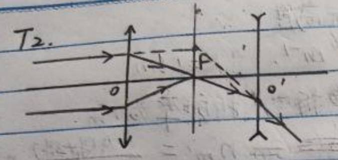
$= 2\times10^3 m$.

平行光入射

$f_1=f_2=50cm$.　$d=100cm$

$|OF|=f=50cm$　$|O'F|=f=50cm=s$

T2.



对右侧凹透镜，由高斯物像公式

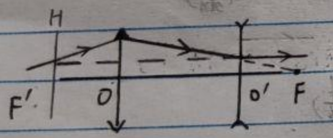
$\frac{-f_2}{s}+\frac{-f_2}{s'}=1$　$s=50cm$

解得 $s'=25cm$.　$|O'F'|=25cm$.

即 物距为50cm. 像距为25cm.

~~又∵ $|FF'|=50cm-25cm=25cm$~~

反向延长出射光线与原入射光线相交于点F所在与光轴垂直的平面，故 像方焦距 $|O'F'|=25cm$.



平行光出射

延长凹透镜的入射光与光轴交于点F.

$|O'F'|=50cm$.　$|OF|=d+|O'F'|=150cm$.

对凸透镜. 由高斯物像公式.

$\frac{f}{s}+\frac{f'}{s'}=1$　$\frac{f}{s}+\frac{50cm}{150cm}=1$

$s=75cm$.　像距$=150cm$.

延长出射光线与入射光线相交.点

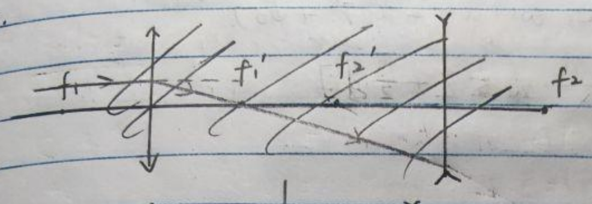
位于与光轴垂直的平面H.

$|F'H|=\left[1-\frac{|OO'|}{|OF|}\right]|OF'|=\frac{1}{3}\times75cm=25cm$

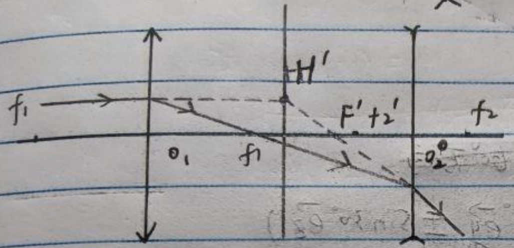
物方焦距为25cm.

T4.

平行光入射



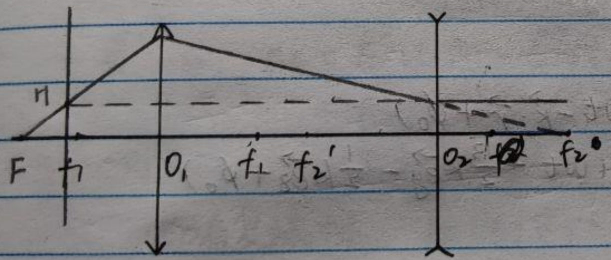
像方焦点F'，像方主点H'



$\cancel{\frac{f_1}{|O_1F|} + \frac{f_1'}{|O_1F|}}$

$$\frac{-f_2}{|O_2F'|} + \frac{-f_2}{|O_2f_1'|} = 1$$

物方焦点F，物方主点H



$$\frac{f_1}{|O_1F|} + \frac{f_1}{|O_1f_2'|} = 1.$$

T5. ① 沿~~与z轴夹角为60°方向~~.

波矢量 $\vec{k} = k(-\cos 30^\circ \vec{e_y} \pm \sin 30^\circ \vec{e_z})$

$= -\frac{\sqrt{3}}{2}k\vec{e_y} \pm \frac{1}{2}k\vec{e_z}$

① 标量波函数

$E = E_0 \cos(\omega t - \vec{k}\cdot\vec{r} + \phi_0)$

$= E_0 \cos(\omega t + \frac{\sqrt{3}}{2}k\vec{e_y} - \frac{1}{2}k\vec{e_z} + \phi_0)$

② 复波函数.

$\cancel{E = E_0(\vec{r})\cos(\omega t + \phi(\vec{r})) - iE_0}$

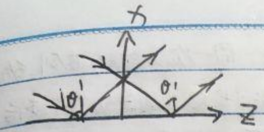
$E = E_0(\vec{r})\cos(\omega t + \frac{\sqrt{3}}{2}k\vec{e_y} - \frac{1}{2}k\vec{e_z} + \phi_0)$

$- i E_0(\vec{r}) \sin(\omega t + \frac{\sqrt{3}}{2}k\vec{e_y} - \frac{1}{2}k\vec{e_z} + \phi_0)$

$= E_0(\vec{r}) e^{-i(\frac{\sqrt{3}}{2}k\vec{e_y} - \frac{1}{2}k\vec{e_z} + \phi_0)} e^{-i\omega t}$.

③ 复振幅. $\tilde{E_0}(r) = E_0(\vec{r}) e^{-i(\frac{\sqrt{3}}{2}k\vec{e_y} - \frac{1}{2}k\vec{e_z} + \phi_0)}$

T6.



~~波矢量 $\vec{k} = k(-\sin\theta\vec{e_x}$~~

~~原~~ 波矢量 $\vec{k_1} = k(-\cos\theta\,\vec{e_x} + \sin\theta\,\vec{e_z})$

标量波函数 及复波函数

$E_1(t,\vec{r}) = E_0\cos(\omega t - \vec{k_1}\cdot\vec{r} + \phi_0)$

$= E_0\cos(\omega t + k\cos\theta\vec{e_x} - k\sin\theta\vec{e_z} + \phi_0)$

$= E_0(\vec{r}) e^{-i(k\cos\theta\vec{e_x} - k\sin\theta\vec{e_z} + \phi_0)} e^{-i\omega t}$

反射波的波矢

$\vec{k_2} = k(\cos\theta\,\vec{e_x} + \sin\theta\,\vec{e_z})$

反射波的频率、波长不变，相位相等

即标量波函数为 $\vec{E_2}(t,\vec{r}) = E_0\cos(\omega t - k\cos\theta\vec{e_x} - k\sin\vec{e_z} + \phi_0)$

~~叠加区~~ 复波函数: ~~$\vec{E_2}(t,\vec{r}) = E_0(\vec{r}) e^{-i(k\cos\theta}$~~

$\vec{E_2}(t,\vec{r}) = E_0(\vec{r}) e^{-i(k\cos\theta\vec{e_x} - k\sin\theta\vec{e_z} + \phi_0)} e^{-i\omega t}$

叠加区: $E_{合} = \vec{E_1}(t,\vec{r}) + \vec{E_2}(t,\vec{r})$

~~$= E_0(\vec{r}) e^{-i\omega t} e^{-i(-2k\sin\theta\vec{e_z} + 2\phi_0)}$~~

$= E_0(\vec{r}) e^{-i(\omega t + \phi_0)} [e^{-i(k\cos\theta\vec{e_x} - k\sin\theta\vec{e_z})} + e^{-i(k\cos\theta\vec{e_x} - k\sin\theta\vec{e_z})}]$

即有复振幅: $E_{合}(\vec{r}) = 2E_0 e^{-i(\omega t + \phi_0)}$

T8. $n=1.58$ $\lambda=550nm$.

设云母片厚度为 h，通过云母片光程 nh.

故云母片前后光程差 $\Delta L = nh-h$.

$\therefore nh-h = m\lambda$. 其中 $m=9$.

即 $1.58h - h = 9\times 550\,nm$.

$$h = \frac{9\times 550}{1.58-1} = 8.534\times 10^3\,nm$$

T13. 单缝衍射中，衍射光的传播方向为 θ.

第二极小到图样中心:

$$\sin\theta_1 = k_1\cdot\frac{\lambda_1}{b} \quad \lambda_1 = 589.3\,nm,\quad k_1=2$$

$$y_1 = f'\sin\theta_1 = 2f'\cdot\frac{\lambda_1}{b} = 0.30\,cm$$

第三极小到图样中心:

$$\sin\theta_2 = k_2\frac{\lambda_2}{b} \quad k_2=3 \quad \lambda_2\text{未知}.$$

$$y_2 = f'\sin\theta_2 = 3f'\cdot\frac{\lambda_2}{b} = 0.42\,cm$$

$$\frac{\lambda_2}{\lambda_1} = \frac{2}{3}\frac{y_2}{y_1} = \frac{0.42\,cm}{0.30\,cm}\cdot\frac{2}{3} = \frac{14}{15}$$

$$\lambda_2 = \frac{14}{15}\lambda \approx 550.01\,nm$$

T14. $\sin\theta = \frac{1.22\lambda}{D}$

当θ很小时 $\sin\theta \approx \tan\theta = \frac{d}{f}$

$$\frac{d}{f} = \frac{1.22\lambda}{D}$$

$$\frac{f}{D} = \frac{d}{1.22\lambda}$$ 其中 $\frac{f}{D}$ 即为 $\frac{\text{焦距}}{\text{光圈直径}}$

当 d = 像素点尺寸 p 时.

$\frac{d}{1.22\lambda}$ 即为 衍射极限光圈.

$DLA = \frac{p}{1.22\lambda}$ 衍射极限光圈 $= \frac{\text{像素尺寸}}{1.22\lambda}$.

以手机小米13为例 主摄 5000万像素 $f/1.8$ 光圈. $\Rightarrow$ CMOS 7.18mm × 8.93

1/1.49英寸 23 mm 等效焦距.

代入数据得 没有超过 光学衍射极限

T15. 由分辨率公式: $\delta\phi = \frac{1.22\lambda}{D}$ $\lambda = 5.5\times10^{-5}$ cm

$D = \frac{1.22\lambda}{\delta\phi} = \frac{1.22\times5.5\times10^{-5}}{4.8\times10^{-6}} = 13.98$ cm.