

# Homework8

2022-07-04

```
library(itsmr)
set.seed(999)
a = specify(ma=c(0.6))
x = matrix(NA,200,20)
acf_hat = vector(length = 20)
for (i in 1:20){
  x[,i] = sim(a,200)
  acf_hat[i] = acvf(x[,i],1)[2]/acvf(x[,i],1)[1]
}
acf_hat
```

```
## [1] 0.4443118 0.4503503 0.3817277 0.5119393 0.4353348 0.4243807 0.3256582
## [8] 0.4833978 0.4859726 0.4598280 0.4314230 0.3599058 0.4137189 0.4193485
## [15] 0.4384594 0.5086540 0.4644715 0.4834186 0.4350242 0.4280180
```

```
which(acf_hat > 0.5)
```

```
## [1] 4 16
```

```
theta_hat = vector(length = 20)
# sigma2_hat = vector(length = 20)
for (i in 1:20){
  if (i %in% which(acf_hat > 0.5)){
    theta_hat[i] = acf_hat[i]/abs(acf_hat[i])
  }else{
    theta_hat[i] = (1-sqrt(1-4*acf_hat[i]^2))/2/acf_hat[i]
  }
  # sigma2_hat[i] = acvf(x[,i],1)[1]/(1+theta_hat[i]^2)
}

theta_hat
```

```
## [1] 0.6092149 0.6279107 0.4638640 1.0000000 0.5836101 0.5551903 0.3703173
## [8] 0.7700169 0.7868680 0.6603287 0.5731417 0.4248758 0.5298799 0.5429872
## [15] 0.5922571 1.0000000 0.6779504 0.7701469 0.5827649 0.5643264
```

```
# Innovation Alfgorithm
theta_ia = vector(length=20)
theta_mle = vector(length=20)
for (i in 1:20){
  theta_ia[i] = ia(x[,i],1)$theta[1]
  theta_mle[i] = arma(x[,i],0,1)$theta[1]
}
theta_ia
```

```
## [1] 0.5393502 0.6606079 0.5077782 0.7174780 0.5615193 0.5791360 0.4729568
## [8] 0.6479518 0.6742351 0.5577074 0.6059612 0.4660173 0.5367229 0.5174971
## [15] 0.6790625 0.6606245 0.7469928 0.6496040 0.5959140 0.5988819
```

```
theta_mle
```

```
## [1] 0.5016677 0.6387145 0.5470482 0.6431216 0.6140347 0.6239795 0.5635286
## [8] 0.6059541 0.6658716 0.5227050 0.5936168 0.4863901 0.5660908 0.5247266
## [15] 0.6879111 0.6335676 0.7833396 0.6028570 0.6368909 0.5955050
```

```
Results <- matrix(NA, 3, 2)
Results[1,] <- c(mean(theta_hat), var(theta_hat))
Results[2, ] <- c(mean(theta_ia), var(theta_ia))
Results[3, ] <- c(mean(theta_mle), var(theta_mle))
rownames(Results) <- c('moment estimate', 'innovations algorithm', 'mle')
colnames(Results) <- c('mean', 'variance')
round(Results, 5)
```

```
##
## moment estimate      0.63428  0.02708
## innovations algorithm 0.59880  0.00632
## mle                  0.60188  0.00483
```

```
Results <- matrix(NA, 3, 2)
Results[1,] <- c(var(theta_hat),(1+0.6^2+4*0.6^4+0.6^6+0.6^8)/(1-0.6^2)^2/200)
Results[2, ] <- c(var(theta_ia),1/200)
Results[3, ] <- c(var(theta_mle),(1-0.6^2)/200)
rownames(Results) <- c('moment estimate', 'innovations algorithm', 'mle')
colnames(Results) <- c('variance', 'asymptotic variance')
round(Results, 5)
```

```
##
## moment estimate      0.02708      0.0237
## innovations algorithm 0.00632      0.0050
## mle                  0.00483      0.0032
```