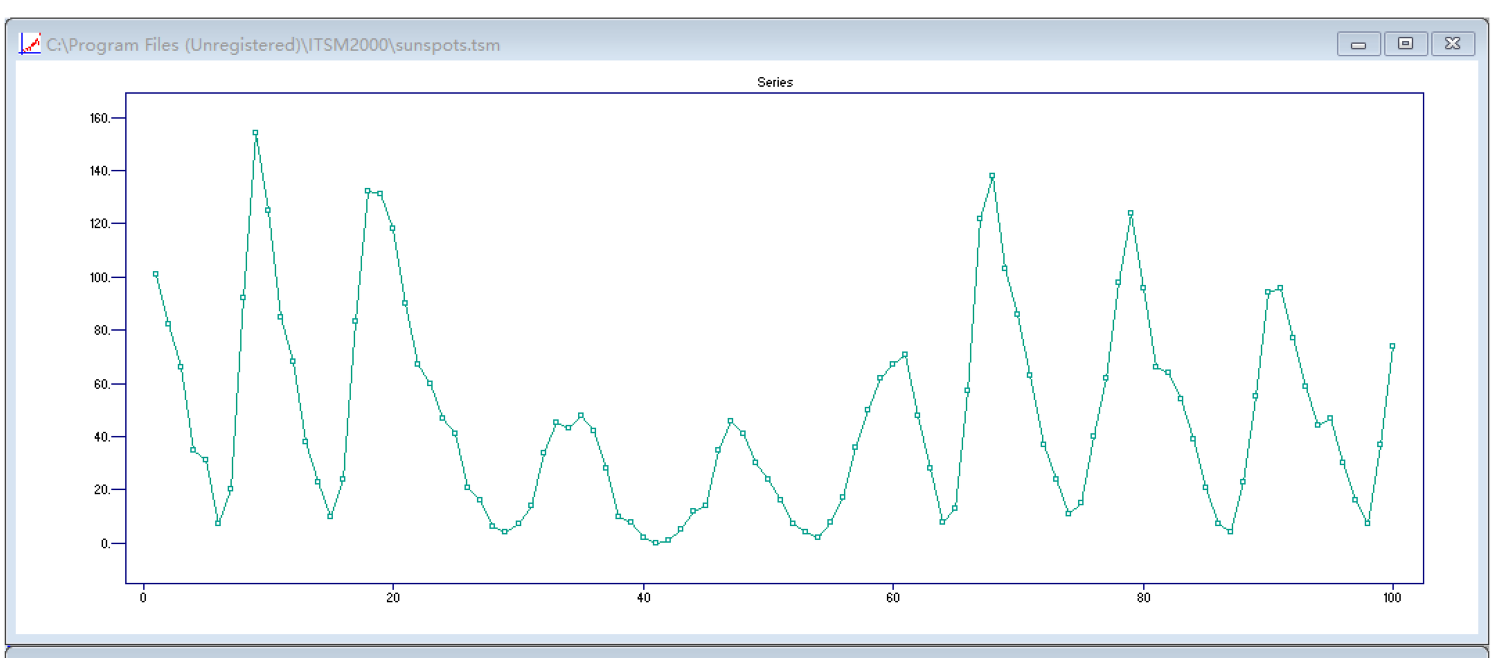
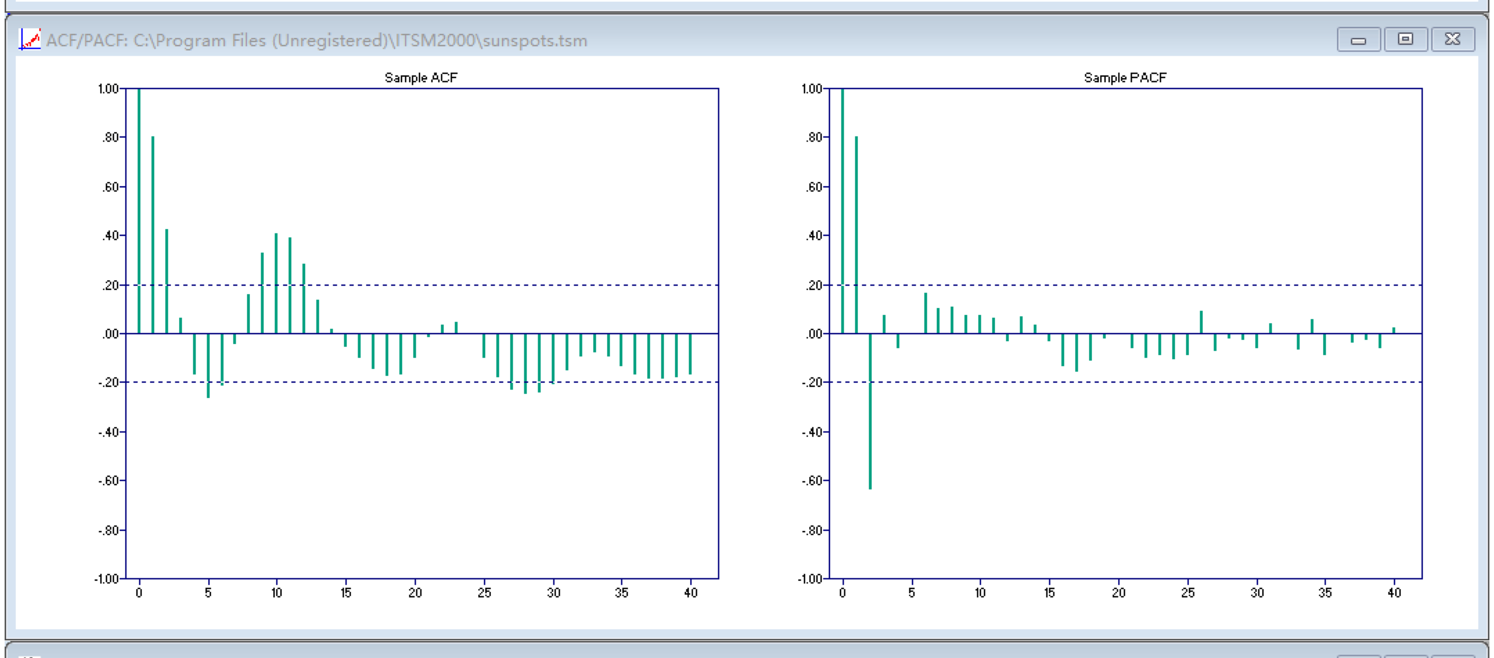
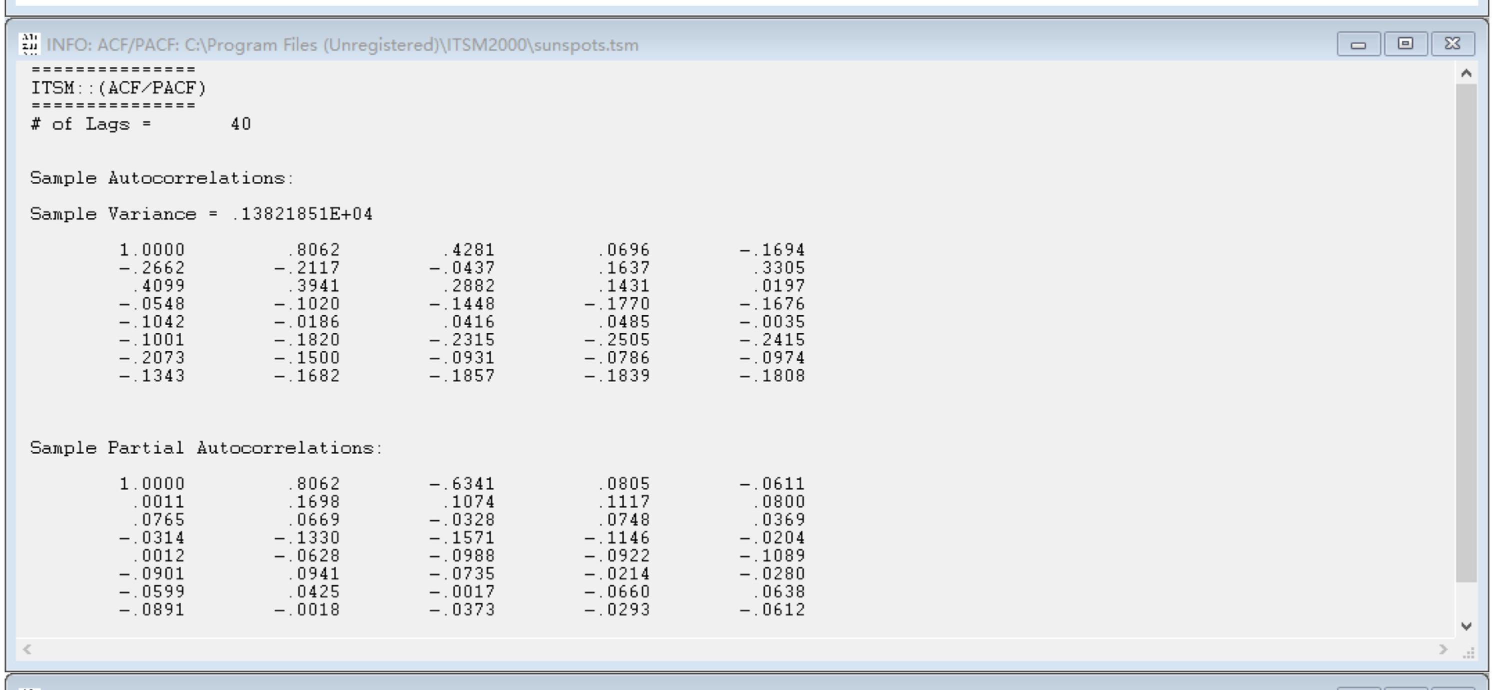
Homework 5

```
library(itsmr)
```

2.16





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ITSM::(ACF/PACF)
===============
# of Lags =       40

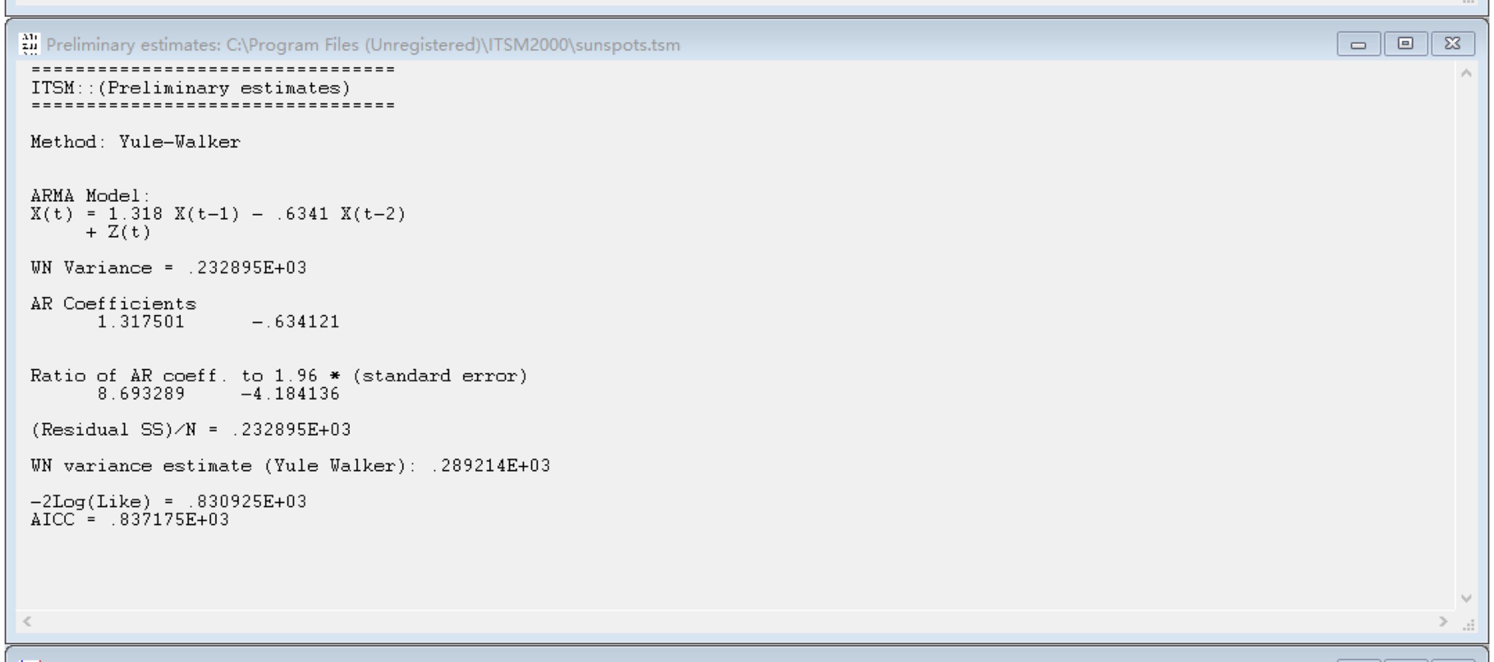
Sample Autocorrelations:

Sample Variance = .13821851E+04

        1.0000        .8062        .4281        .0696       -.1694
        -.2662       -.2117       -.0437        .1637        .3305
         .4099        .3941        .2882        .1431        .0197
        -.0548       -.1020       -.1448       -.1770       -.1676
        -.1042       -.0186        .0416        .0485       -.0035
        -.1001       -.1820       -.2315       -.2505       -.2415
        -.2073       -.1500       -.0931       -.0786       -.0974
        -.1343       -.1682       -.1857       -.1839       -.1808


Sample Partial Autocorrelations:

        1.0000        .8062       -.6341        .0805       -.0611
         .0011        .1698        .1074        .1117        .0800
         .0765        .0669       -.0328        .0748        .0369
        -.0314       -.1330       -.1571       -.1146       -.0204
         .0012       -.0628       -.0988       -.0922       -.1089
        -.0901        .0941       -.0735       -.0214       -.0280
        -.0599        .0425       -.0017       -.0660        .0638
        -.0891       -.0018       -.0373       -.0293       -.0612
```

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ITSM::(Preliminary estimates)
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Method: Yule-Walker


ARMA Model:
X(t) = 1.318 X(t-1) - .6341 X(t-2)
     + Z(t)

WN Variance = .232895E+03

AR Coefficients
      1.317501      -.634121


Ratio of AR coeff. to 1.96 * (standard error)
      8.693289     -4.184136

(Residual SS)/N = .232895E+03

WN variance estimate (Yule Walker): .289214E+03

-2Log(Like) = .830925E+03
AICC = .837175E+03
```
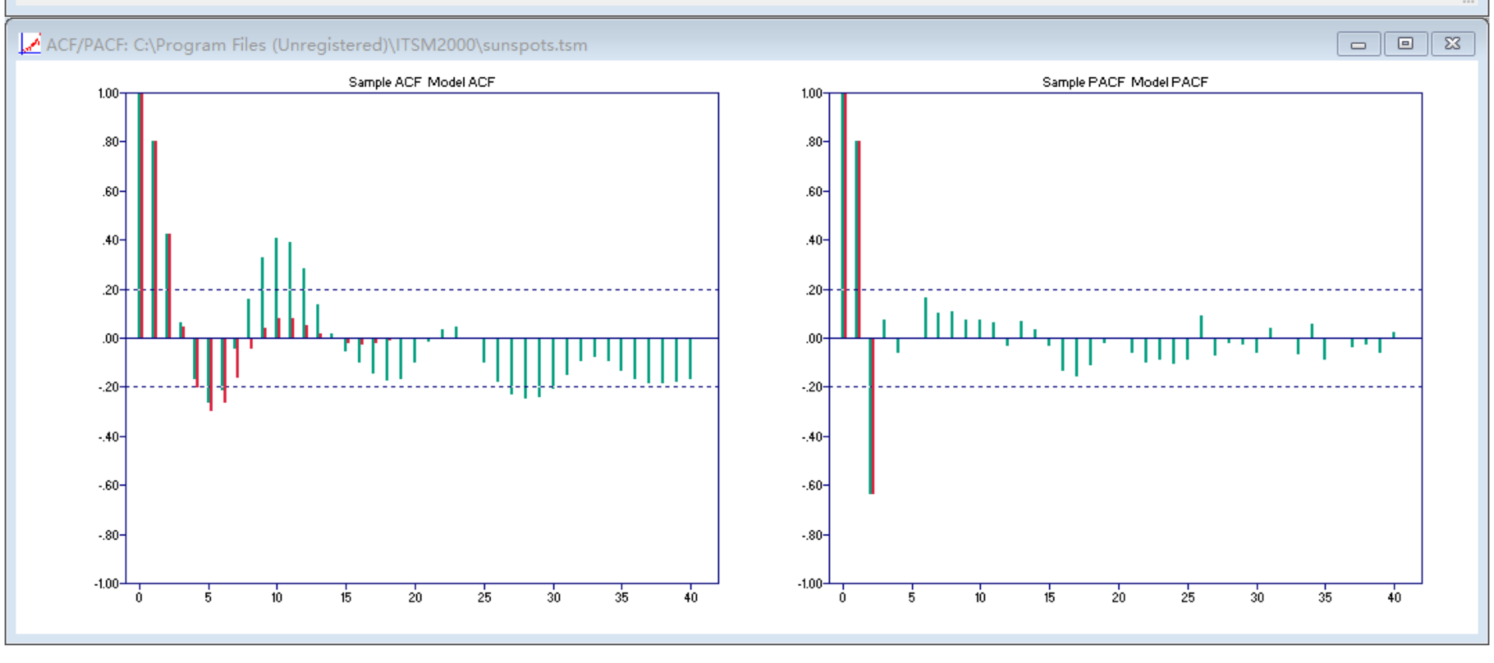



3.2

(b)

$\psi(z) = 1 + 1.9z + 0.88z^2 = 0$ has roots $z_1 = -\frac{10}{11}, z_2 = -\frac{5}{4}$, in which the first one is not outside the unit circle. So the process is not causal.

(d)

$\psi(z) = 1 + 1.8z + 0.81z^2 = 0$ has roots $z_1 = z_2 = -\frac{10}{9}$, which are both outside the unit circle. So the process is causal.

$\therefore \gamma(h) = (\frac{10}{9})^{-h}P(h)$, where $P(h)$ is a polynomial in h of degree 1. Thus $\gamma(h) = (\frac{10}{9})^{-h}(ah + b)$.

$\therefore \gamma(0) = b, \gamma(1) = \frac{9}{10}(a + b), \gamma(2) = \frac{81}{100}(2a + b)$.

Combining with Y-W equations, which include

$$\gamma(0) - \phi_1\gamma(1) - \phi_1\gamma(2) = \sigma^2, \gamma(1) - \phi_1\gamma(0) - \phi_2\gamma(1) = 0, \gamma(2) - \phi_1\gamma(1) - \phi_2\gamma(0) = 0$$

We can get

$$a = -3.077\sigma^2, b = 1.462\sigma^2$$

$\therefore \gamma(h) = (\frac{10}{9})^{-h}(-3.077h + 1.462)\sigma^2$.

$\therefore \rho(h) = (\frac{10}{9})^{-h}\frac{-3.077h+1.462}{1.462}$

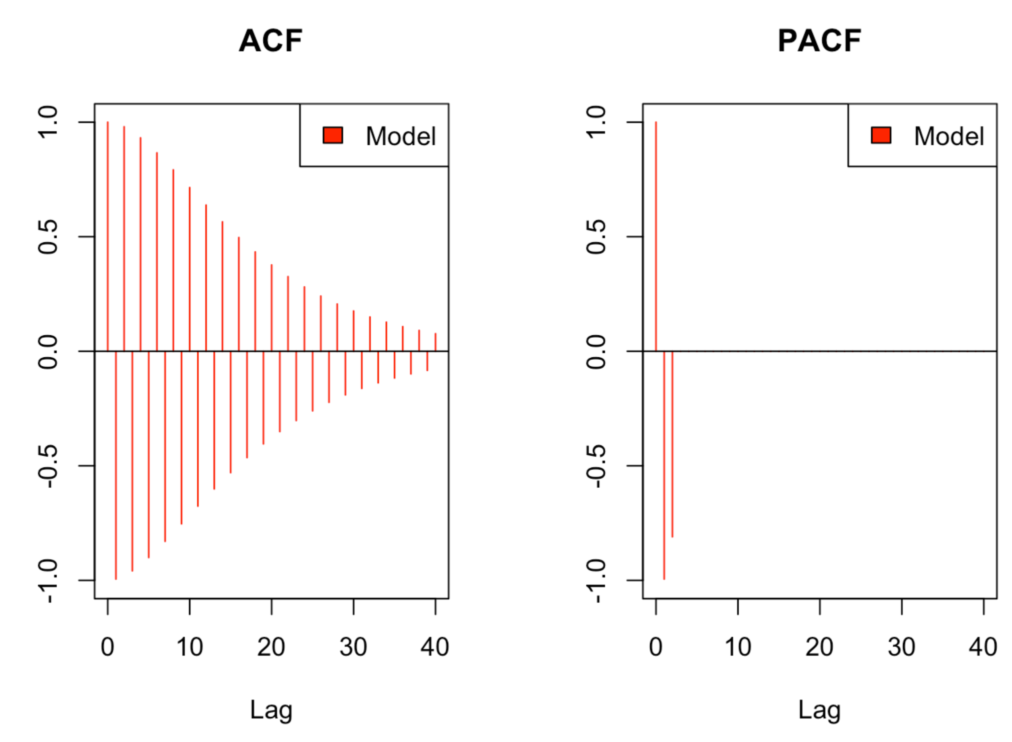
$\because X_t = -1.8X_{t-1} - 0.81X_{t-2} + Z_t$

$\therefore P_nX_{n+1} = -1.8X_n - 0.81X_{n-1}$ for all $n \geq 2$. So $\alpha(h) = 0$ for all $h \geq 3, \alpha(2) = -0.81$.

By D-L algorithm, $\alpha(1) = \frac{1}{\gamma(0)}\gamma(1) = -\frac{171}{172}$.

As convention $\alpha(0) = 1$.

```
model = list()
model$phi = c(-1.8, -0.81)
model$theta = c(0)
# model$sigma2 = c()
plota(model, h = 40)
```



3.3

(d)

Start with $(1 - \phi_1 z - \phi_2 z^2)(\psi_0 + \psi_1 z + \ldots) = 1$, where $\phi_1 = -1.8, \phi_2 = -0.81$.

Coefficient of z^0: $\psi_0 = 1$.

Coefficient of z^1: $\psi_1 = \phi_1\psi_0 = \phi_1 = -1.8$.

Coefficient of z^2: $\psi_2 = \phi_1\psi_1 + \phi_2\psi_0 = -1.8\phi_1 + \phi_2 = 2.43$.

Coefficient of z^3: $\psi_3 = \phi_1\psi_2 + \phi_2\psi_1 = 2.43\phi_1 - 1.8\phi_2 = -2.916$.

Coefficient of z^4: $\psi_4 = \phi_1\psi_3 + \phi_2\psi_2 = -2.916\phi_1 + 2.43\phi_2 = 3.2805$.

Coefficient of z^5: $\psi_5 = \phi_1\psi_4 + \phi_2\psi_3 = 3.2805\phi_1 - 2.916\phi_2 = -3.54294$.

3.4

$$X_t = Z_t + 0.8Z_{t-2} + 0.8^2Z_{t-4} + \ldots = \sum_{i=0}^{\infty} 0.8^i Z_{t-2i}$$

$\therefore \gamma(h) = \frac{0.8^{\frac{h}{2}}}{1-0.8^2}$ if $2|h$. $\gamma(h) = 0$ otherwise.

$\therefore \alpha(2) = 0.8, \alpha(h) = 0$ otherwise.

3.8

The stationary solution is $X_t = -\sum_{i=1}^{\infty} \phi^{-i}Z_{t+j}$ since $|\phi| > 1$. It follows that $E(X_t) = 0, \gamma_X(h) = \frac{\sigma^2}{\phi^2-1}\phi^{-|h|}$.

Now with $W_t = X_t - \frac{1}{\phi}X_{t-1}, E(W_t) = 0$,
$\gamma_W(h) = Cov(X_{t+h} - \phi^{-1}X_{t+h-1}, X_t - \phi^{-1}X_{t-1}) = \gamma_X(h) - \phi^{-1}\gamma_X(h-1) - \phi^{-1}\gamma_X(h+1) + -\phi^{-2}\gamma_X(h)$.

$\therefore \gamma_W(0) = \frac{\sigma^2}{\phi^2}, \gamma_W(h) = 0$ otherwise.

Thus $\{W_t\} \sim \mathrm{WN}(0, \sigma_W^2)$ where $\sigma_W^2 = \frac{\sigma^2}{\phi^2}$.

Thus X_t is the unique stationary solution of $X_t = \frac{1}{\phi}X_{t-1} + W_t$.

3.9

(a)

$\gamma(h) = Cov(\mu + Z_{t+h} + \theta_1 Z_{t+h-1} + \theta_{12}Z_{t+h-12}, \mu + Z_t + \theta_1 Z_{t-1} + \theta_{12}Z_{t-12}) = (1 + \theta_1^2 + \theta_{12}^2)\gamma_Z(h) + \theta_1(\gamma_Z(h+1) + \gamma_Z(h-1)) + \theta_{12}(\gamma_Z(h+12) + \gamma_Z(h-12)) + \theta_1\theta_{12}(\gamma_Z(h+11) + \gamma_Z(h-11))$

$\therefore \gamma(0) = (1 + \theta_1^2 + \theta_{12}^2)\sigma^2, \gamma(-1) = \gamma(1) = \theta_1\sigma^2, \gamma(-12) = \gamma(12) = \theta_{12}\sigma^2, \gamma(-11) = \gamma(11) = \theta_1\theta_{12}\sigma^2, \gamma(h) = 0$ otherwise.

(b)

$$\nabla\nabla_{12}X_t = \nabla(X_t - X_{t-12}) = X_t - X_{t-1} - X_{t-12} + X_{t-13}$$

```
library(itsmr)
nab = c()
for (x in 14:72){
  nab[x-13] = deaths[x]-deaths[x-1]-deaths[x-12]+deaths[x-13]
}
mean_s = mean(nab)
acvf_s = acvf(nab, 20)
paste0('The sample mean is ', mean_s, '.')
```

```
## [1] "The sample mean is 28.8305084745763."
```

```
paste0('The sample acvf with lag ', c(0:20),' is ', acvf_s, '.')
```

```
##  [1] "The sample acvf with lag 0 is 152669.632289572."
##  [2] "The sample acvf with lag 1 is -54326.5279215499."
##  [3] "The sample acvf with lag 2 is -15071.6823871964."
##  [4] "The sample acvf with lag 3 is 14584.5679159018."
##  [5] "The sample acvf with lag 4 is -17177.6942774091."
##  [6] "The sample acvf with lag 5 is 6340.2512282171."
##  [7] "The sample acvf with lag 6 is 17420.9080237025."
##  [8] "The sample acvf with lag 7 is -31164.4601736302."
##  [9] "The sample acvf with lag 8 is -1087.51323163517."
## [10] "The sample acvf with lag 9 is 15277.1754512389."
## [11] "The sample acvf with lag 10 is -12434.6704823765."
## [12] "The sample acvf with lag 11 is 29801.9685021351."
## [13] "The sample acvf with lag 12 is -50866.8975406444."
## [14] "The sample acvf with lag 13 is 13767.9425355075."
## [15] "The sample acvf with lag 14 is 17757.6610948539."
## [16] "The sample acvf with lag 15 is -6199.56720015192."
## [17] "The sample acvf with lag 16 is -9656.4134210411."
## [18] "The sample acvf with lag 17 is 27981.3878731516."
## [19] "The sample acvf with lag 18 is -29455.8134181197."
## [20] "The sample acvf with lag 19 is 3692.78017713593."
## [21] "The sample acvf with lag 20 is 7569.42950350328."
```

(c)

Now we have

$$\theta_1\sigma^2 = \hat{\gamma}(1), \theta_1\theta_{12}\sigma^2 = \hat{\gamma}(11), \theta_{12}\sigma^2 = \hat{\gamma}(12)$$

```
theta_1 = acvf_s[12]/acvf_s[13]
theta_12 = acvf_s[12]/acvf_s[2]
sigma_2 = acvf_s[13]*acvf_s[2]/acvf_s[12]
paste0('θ_1 is ', theta_1, '.')
```

```
## [1] "θ_1 is -0.585881387366357."
```

```
paste0('θ_12 is ', theta_12, '.')
```

```
## [1] "θ_12 is -0.54857119794533."
```

```
paste0('σ^2 is ', sigma_2, '.')
```

```
## [1] "σ^2 is 92726.154291669."
```

So the model should be $Y_t = 28.8305084745763 + Z_t - -0.585881387366357Z_{t-1} - 0.54857119794533Z_{t-12}$, where $\{Z_t\} \sim \mathrm{WN}(0, 92726.154291669)$.