Mathematics of Gossip

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1 Introduction

Gossip, the spread of misinformation, has become the most pressing issue of our generation. Broadly defined as a social activity, it accounts for two-thirds of speaking time and has many functions including cultural learning, indirect aggression, and social bonding. In this paper, we discussed how the compartmental model of SIR could be used to simulate the spread of gossip within a community. The organization of this paper is outlined as follows. In Section 2, we detail the SIR model for modeling the population biology of infectious diseases. In Section 3, we establish the Gossip Model using the modelling tools developed to understand infectious disease. In Section 4, we conduct *Anthropomorphized Sensitivity Analysis* to illustrate the contextual meaning of the parameters and translate social characteristics into mathematical language.

2 SIR Model

2.1 Background

Introduced by Kermack and Mckendrick in 1927, the reputational Compartmental Models (SIR model), a system of ordinary differential equations (ODEs), was initially used to model the spread of infectious disease within populations. The population is assigned to compartments with labels such as S (susceptible), I (infectious) and R (recovered). The system of ODEs describes the following population dynamics: S becomes infected at a transmission rate α when interacting with the I population. I recover (and join the R population) at a rate β. In this paper, we plan to discuss the background, assumptions, and development of a novel SIR model, a system of ODEs used to understand the spread of a lie.

2.2 The Model

According to SIR model we have learned during the class,

- $dS/dt = - \alpha SI$
- $dI/dt = \alpha SI - \beta I$
- $dR/dt = \beta I$

In the ordinary SIR model, we assume the rate of interaction between susceptible people and infected people is proportional to both susceptible and infectious populations. Also, infected individuals recover at a rate that is proportional to the infected population. .

3 Gossip Model

3.1 The Dynamics of Three Populations

Similar with SIR model used to predict the number of infected people, the new SIR model (Gossip Model) was also composed of three parts:

S: The number of **s**usceptible individuals that have not heard gossip. When a susceptible individual comes into "contact with gossip", the susceptible individual starts to spread lies.
I: The number of **i**nfectious individuals. These are individuals who have heard gossip and believe it's true.
R: The number of recovered individuals. These are individuals who have heard gossip and believe it's false.

Furthermore, gossip is only spread through direct interaction with the infected (it's not airborne), and there is a fixed amount of people in the system, $N = S + I + R$.

3.2 The Model

3.21 The ODE system

The main reason that we could use the SIR model to analyze the spread of Gossip is that there are also three different but related groups having certain functional relationships with each other in the Gossip model. Since, the original SIR model of the disease spreading could not accurately simulate the gossip spreading, a new SIR model, the Gossip Model, is created to analyze the gossip propagation. The system of ODEs for the Gossip Model is written as:

- $\partial S/\partial t = -\beta \cdot S \cdot I$
- $\partial I/\partial t = \rho \cdot \beta \cdot S \cdot I - \gamma \cdot I + \alpha \cdot R$
- $\partial R/\partial t = \gamma \cdot I - \alpha \cdot R + (1 - \rho) \cdot \beta \cdot S \cdot I.$

3.22 The Parameters

In this set up of differential equations, we assume that when the gossip is first heard, the proportion of the $(1-\rho)$ population will immediately know its falseness, and the proportion of the ρ population will believe it as true. The transmission rate, β, describes the efficiency of the spread of rumors, representing how many people are infected within a unit of time. This parameter is determined by the spreading environment, the nature of gossip and the origin. The rate of recovery, γ, describes how easily the rumors can be offset with other evidence and abandoned as untruthful by the people who are initially infected. This parameter depends on the environment and the nature of the rumors. The parameter α describes the persuasive power of the gossip followers, who act through the interaction between the I and R groups and cause the R group to be infected by the rumors again.

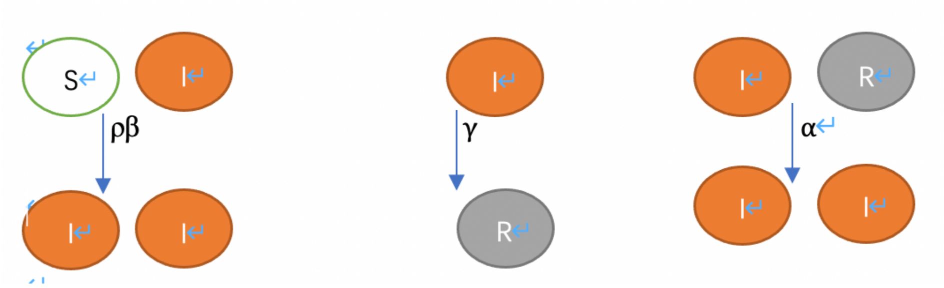
3.221 literal Summation of parameters

The parameters $\beta, \rho, \gamma, \alpha$ represent various characteristics of the rumor and/or rumor spreader.

β : the rate at which the rumor is spread
$(1 - \rho)$: the percentage of the population that rejects the rumor immediately upon hearing it
γ : the rate at which the Infected reject the rumor
α : the rate of persuasion of the Infected on the Recovered

3.222 Graphical Summation of Parameters

Suppose there is a rumor in Dickinson that student Forrester is about to win the Fields Award. $\rho\beta$ represents the proportion at which people become infected by gossip. γ is the rate at which people recover from the gossip. α represents the rate that particular believers re-convince nonbelievers of the gossip. The functions of parameters $\rho\beta, \gamma\ \alpha$ are shown in graph as follows:



4 Anthropomorphized Sensitivity Analysis

4.1Introduction of Regina George

Regina George is the main antagonist of the movie and musical Mean Girls. Though being extremely mean spirited, she has gained wide popularity in her high school. In the film, Regina isn't just controlling her own destiny, but she also controls everyone else's. She manipulates the people around her to give her exactly what she wants, including a bigger bedroom from her parents and a broke-up couple within a well-placed call. Her privileged position and such superpower in her high school are actually benefited from a flexible use of gossip, which will be further explained using the Gossip Model.

4.2 Regina George (Initial)

In Figure 1, we assume an extremely high transmission rate (β = 0.03), which means each infected person spreads the gossip to 30 other people per day. Because Regina expressed her gossip with strong appeal, it takes longer for people to recover from this rumor (γ = 0.01, a recovery rate of 10 people per day). We also assume that Regina's use of Gossip is masterful, and that her lies are always powerful enough to make everyone believe what she says initially. This accounts that 60% of people (ρ= 60% or ($1-\rho$) = 40%) who hear the gossip automatically discount it as true.

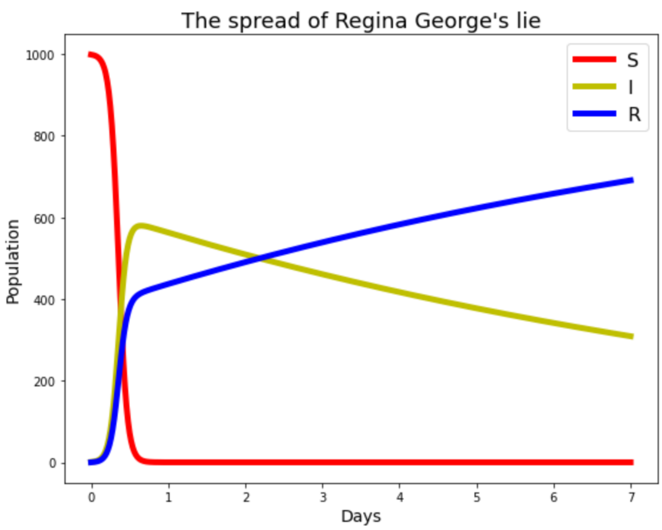


Figure 1

According to the dynamics of a petty rumor spread by Regina George, we see that by the end of the second day after the gossip started, everybody in the school has heard the rumor. However, by the end of the week, more than 60% of the school knows that the rumor is false and less than 40% of people in school are still deceived by her gossip.

4.2 Regina George (Revised)

As shown in the movie, we assume that Regina has a very strong ability to spread lies. However, in reality, teachers in school always warm students not to believe rumors. In this case, we assume a lower transmission rate (β = 0.01) due to the intervention by advisors on campus. Meanwhile, since the teachers would always publicly denounce the harmful rumors, we create a higher recovery rate (γ = 0.02, 20 people per day). Because of the warning of advisors, most people do not believe the lie initially and this unreliability is reflected in that 80% of people ($(1-\rho)$ = 0.8) who hear the gossip automatically discount it as false.

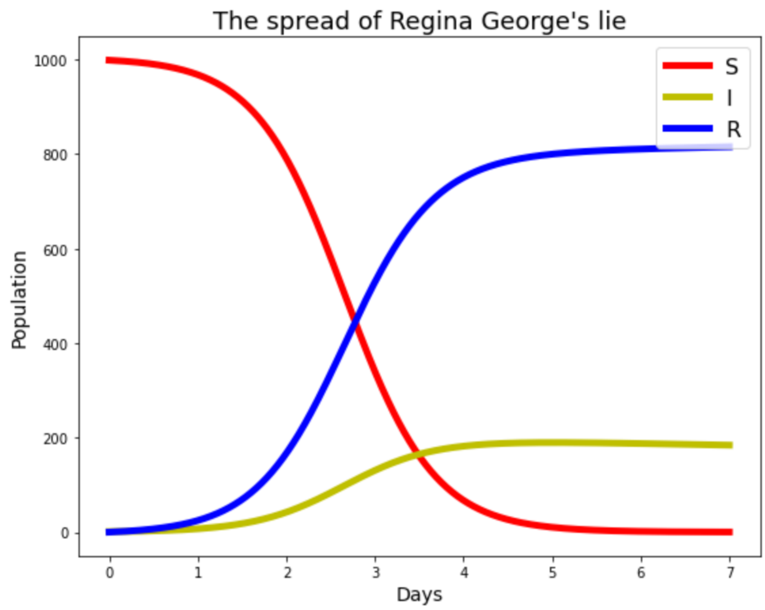


Figure 2

According to the dynamics of a petty rumor spread by Regina George, we see that by the end of the fifth day after the gossip started, everybody in the school has heard the rumor. Compared with the first case, the transmission speed is indeed much slower. The intervention of advisors has a decisive influence on the transmission efficiency of gossip. And by the end of the week, about 80% of the school knows that the rumor is false and 20% of people in school were deceived by her gossip. Recovered population is also increased due to the increase of recovery rate and unreliability of gossip.

5 Conclusion

The SIR model is a useful model, and its role is not limited to analyzing the transmission efficiency of diseases and gossip. After revision, the SIR model can be used to simulate other more complex scenarios with high realistic validity. After this research, we learn the use of ODE in modeling, understand the importance of sensitivity analysis as well as parameter dependence, and develop skills to transform social problems into mathematics. Such ability to analyze problems is not only limited to the use of SIR models but can be flexibly applied to most mathematical problems.

Bibliography

"The Mathematics of Gossip - Claremont Colleges." 2021. Accessed May 16. https://scholarship.claremont.edu/cgi/viewcontent.cgi?article=1036&context=codee.