Using a Neural Network to Approximate the Runge Function

This experiment applies a neural network to approximate the Runge function: $f(x) = 1 / (1 + 25x^2)$, $x \in [-1, 1]$ We use a feedforward neural network with two hidden layers of 16 neurons each. The first hidden layer uses the ReLU activation and the second uses the Sigmoid activation. The output layer is linear. The network is trained with the Adam optimizer (learning rate 0.005) and mean squared error (MSE) loss. The training set has 1200 uniformly sampled points and the validation set has 300 points. Training runs for 100 epochs with batch size 64.

Results:
1. The neural network prediction closely matches the true Runge function. The curves almost completely overlap.
2. Training and validation losses both decrease smoothly and stabilize, showing good convergence.
3. Final validation performance: MSE ≈ 0.000092, Max error ≈ 0.03853.

Discussion:
The results confirm that even with a relatively small network (16 neurons per hidden layer), the model successfully approximates the Runge function. The use of mixed activations (ReLU and Sigmoid) still achieves stable convergence. The low MSE and small maximum error demonstrate that the neural network generalizes well to unseen validation data. Further improvements could involve experimenting with deeper networks, different activations, or regularization to test robustness.