Images as Points

Example Supplements

EECS Computer Vision

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Change of Basis into Frequency Domain

$$F(u) = \int f(x) e^{-2\jmath\pi ux} dx$$

$$e^{-2\jmath\pi ux} = \cos(2\pi ux) - \jmath \sin(2\pi ux)$$

$$f(x) = \int F(u) e^{-2\jmath\pi ux} du$$

Change of Basis into Frequency Domain

- Fourier Contributions:
  - Any periodic signal can be represented as the integral of sines and/or cosines weighted by the appropriate coefficient.
  - Even any finite aperiodic signal can be represented as the integral of sines and/or cosined weighted be the appropriate function.
- Recall the *Fourier* representation of a function
  - $$F(u) = \int f(x) e^{-2\jmath\pi ux} dx$$
  - recall that $e^{-2\jmath\pi ux} = \cos(2\pi ux) - \jmath \sin(2\pi ux)$
  - Also we have $$f(x) = \int F(u) e^{-2\jmath\pi ux} du$$
  - $F(u) = |F(u)|\, e^{i\,\Phi(u)}$
    - a decomposition into magnitude (|F(u)|) and phase $\Phi(u)$
    - If F(u) = a + i b then
    - $|F(u)| = (a^2 + b^2)^{1/2}$ and $\Phi(u) = \text{atan2}(a,b)$
  - |F(u)|^2 is the *power spectrum*
- Questions: what function takes many many many terms in the Fourier expansion?

Source: G Hager, D. Kriegman Slides

Change of Basis into Frequency Domain

Discrete Fourier Transform (DFT)

$$F[u,v] \equiv \frac{1}{N}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} I[x,y]e^{\frac{-2\pi\ j}{N}(xu+yv)}$$

Inverse DFT

$$I[x,y] \equiv \frac{1}{N}\sum_{u=0}^{N-1}\sum_{v=0}^{N-1} F[u,v]e^{\frac{+2\pi\ j}{N}(ux+vy)}$$

Implemented via the "Fast Fourier Transform" algorithm (FFT)

Source: G Hager, D. Kriegman Slides

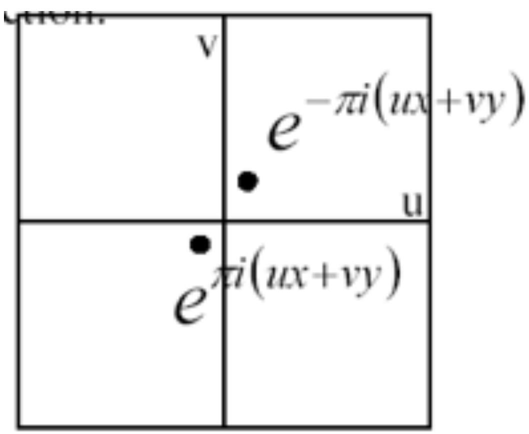
Fourier basis element

$$e^{-i2\pi(ux+vy)}$$

Transform is sum of orthogonal basis functions

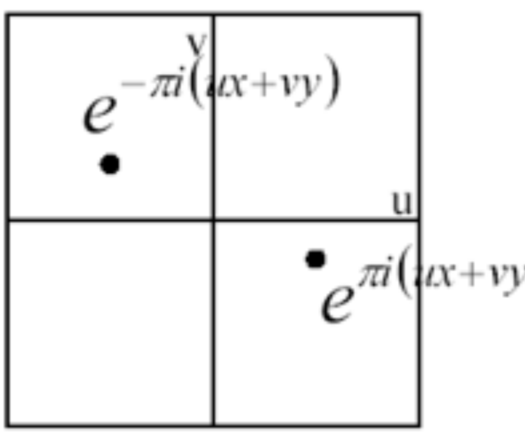
Vector (u,v)

- Magnitude gives frequency
- Direction gives orientation.



Source: G Hager, D. Kriegman Slides

Here u and v are larger than in the previous slide.



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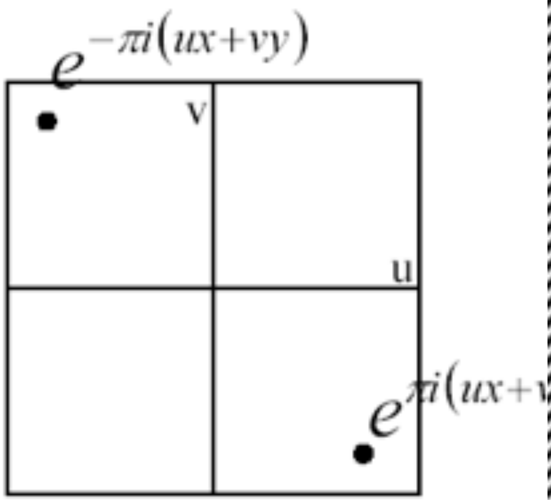
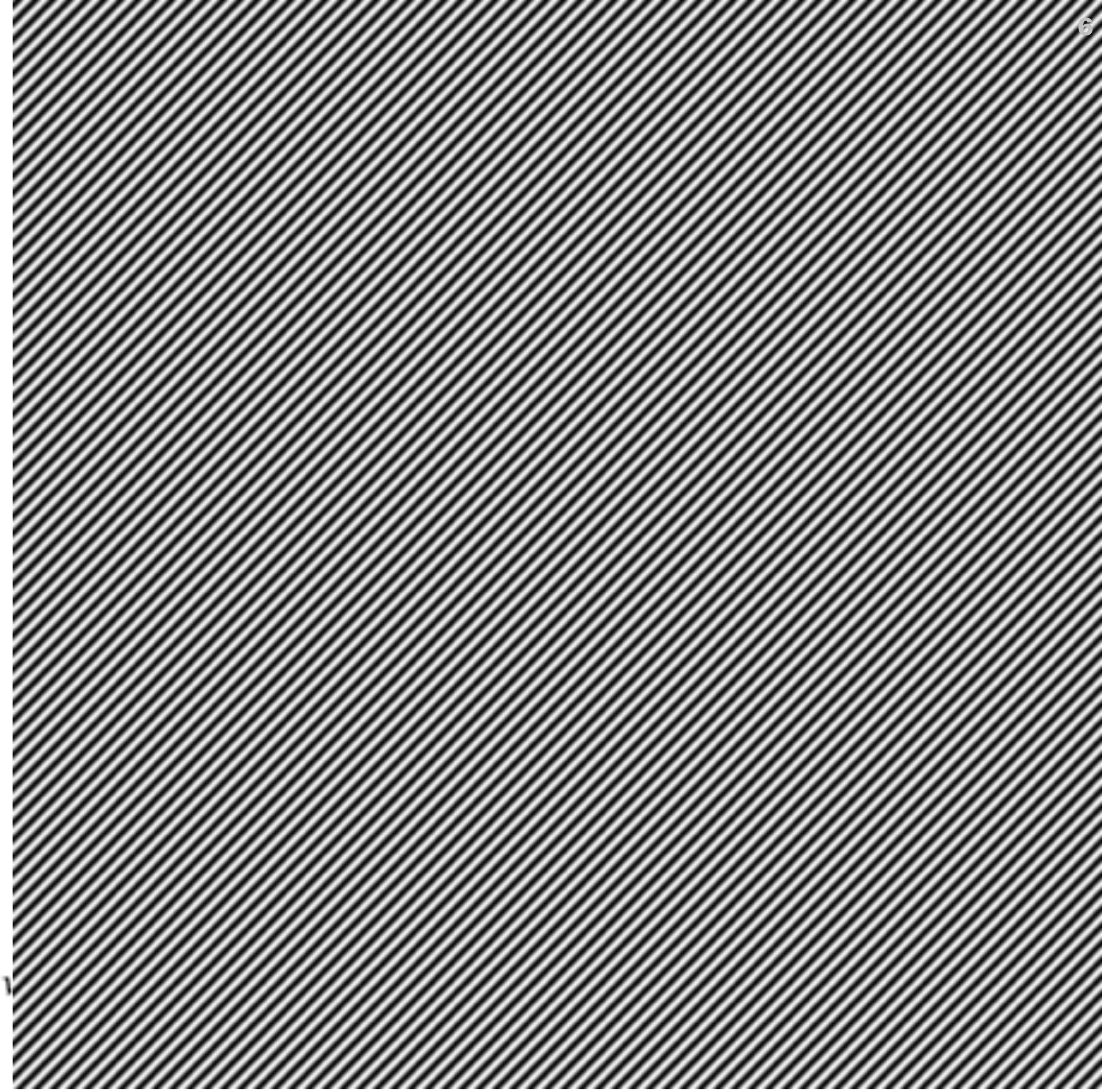
And larger still...





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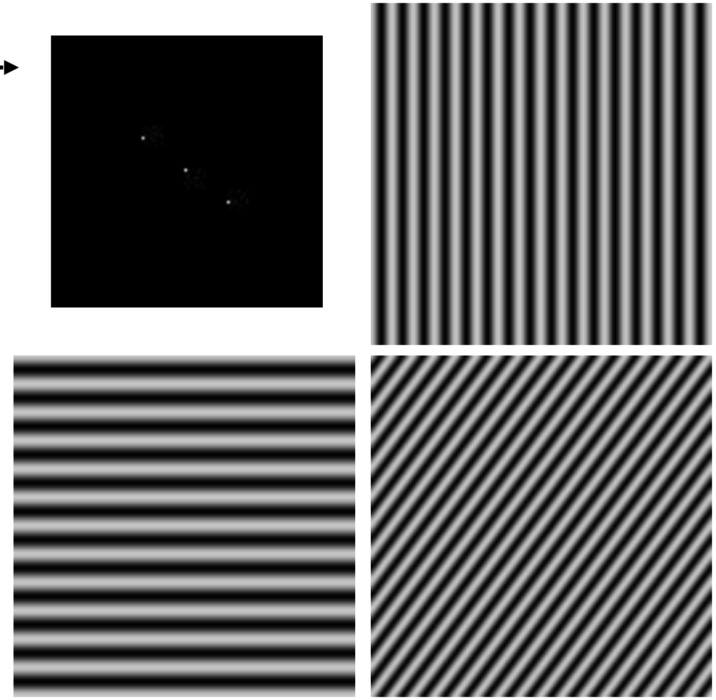
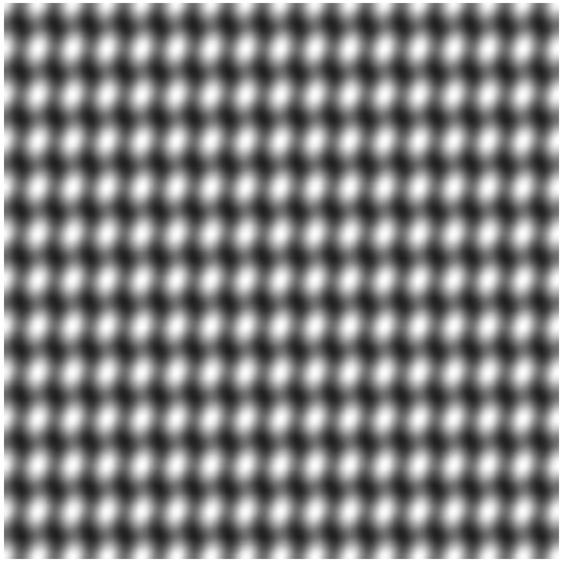
The Fourier "Hammer"

"Power Spectrum"



Linear Combination:

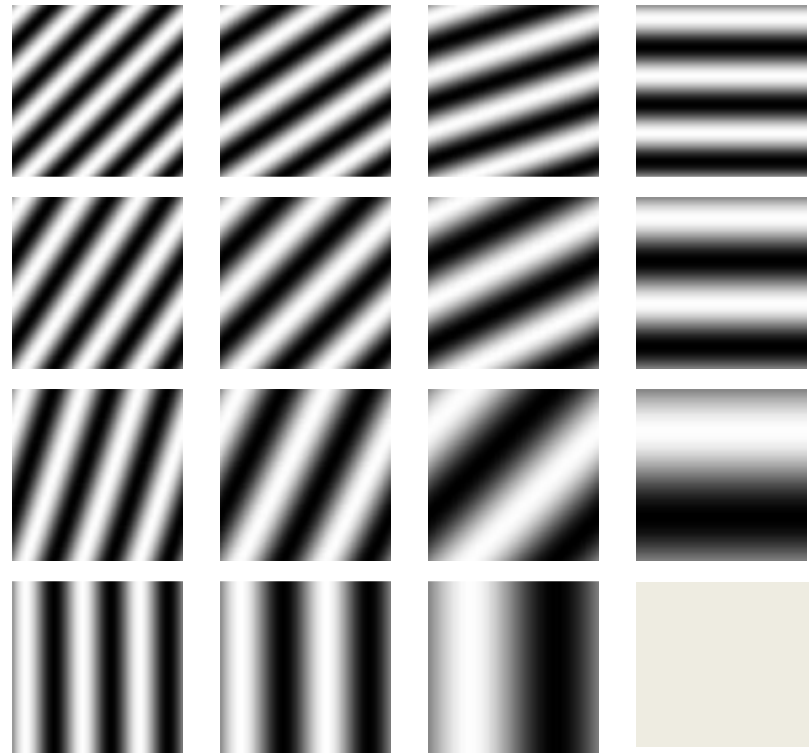


Basis vectors

Frequency Decomposition

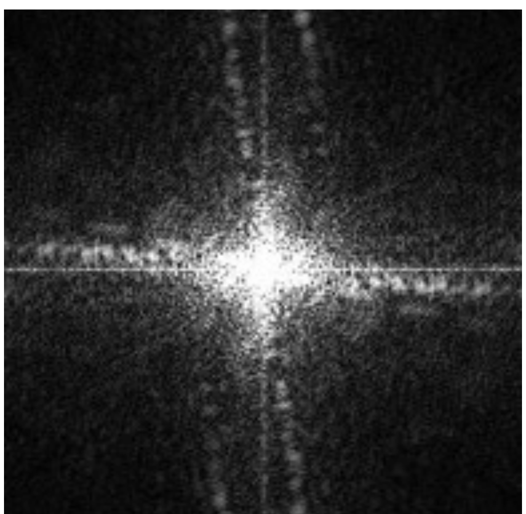
All Basis Vectors →

… … …



… … …

← Example



intensity ~ that frequency's coefficient

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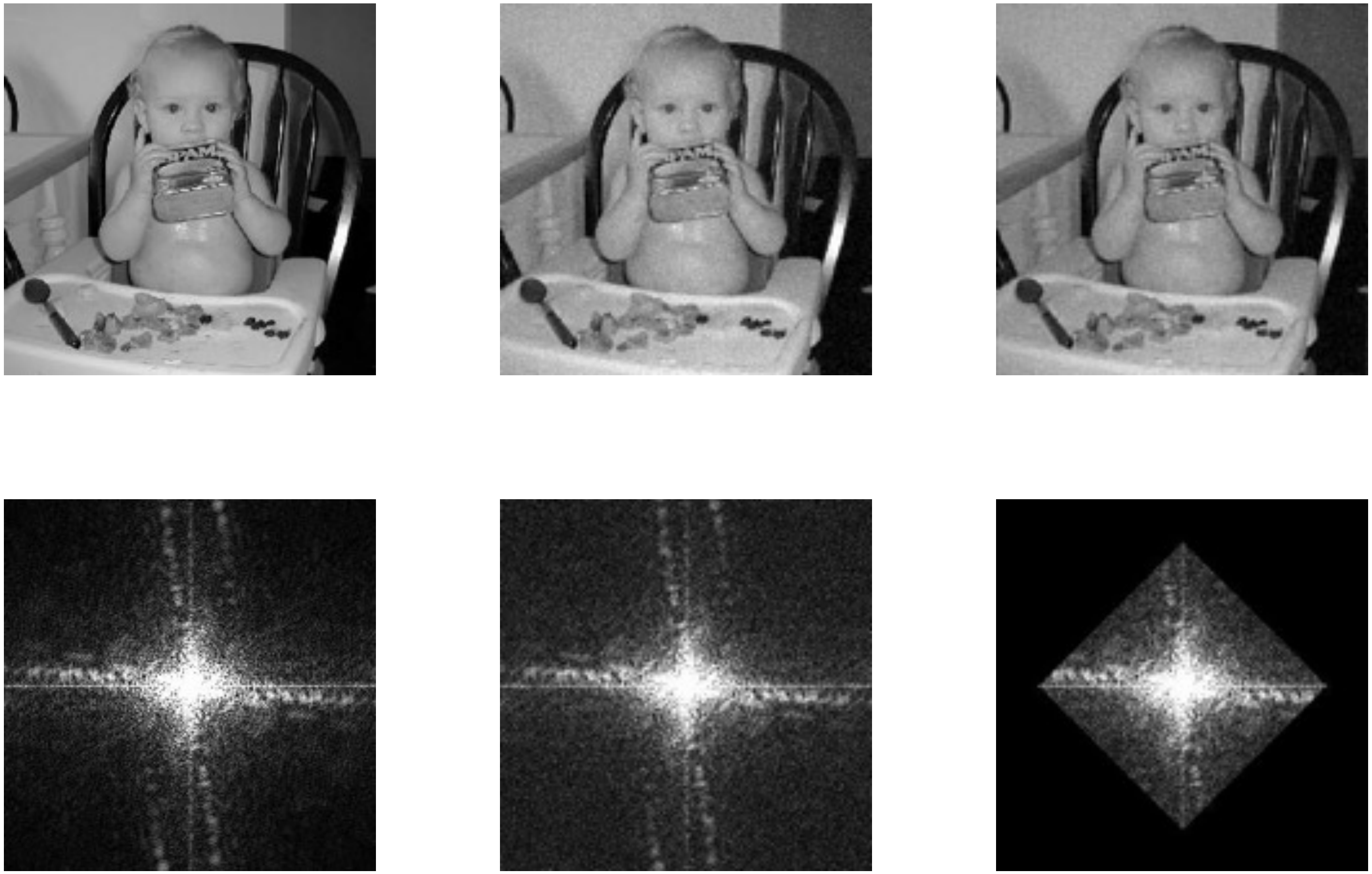
Using Fourier Representations

Smoothing



Data Reduction: only use *some* of the existing frequencies

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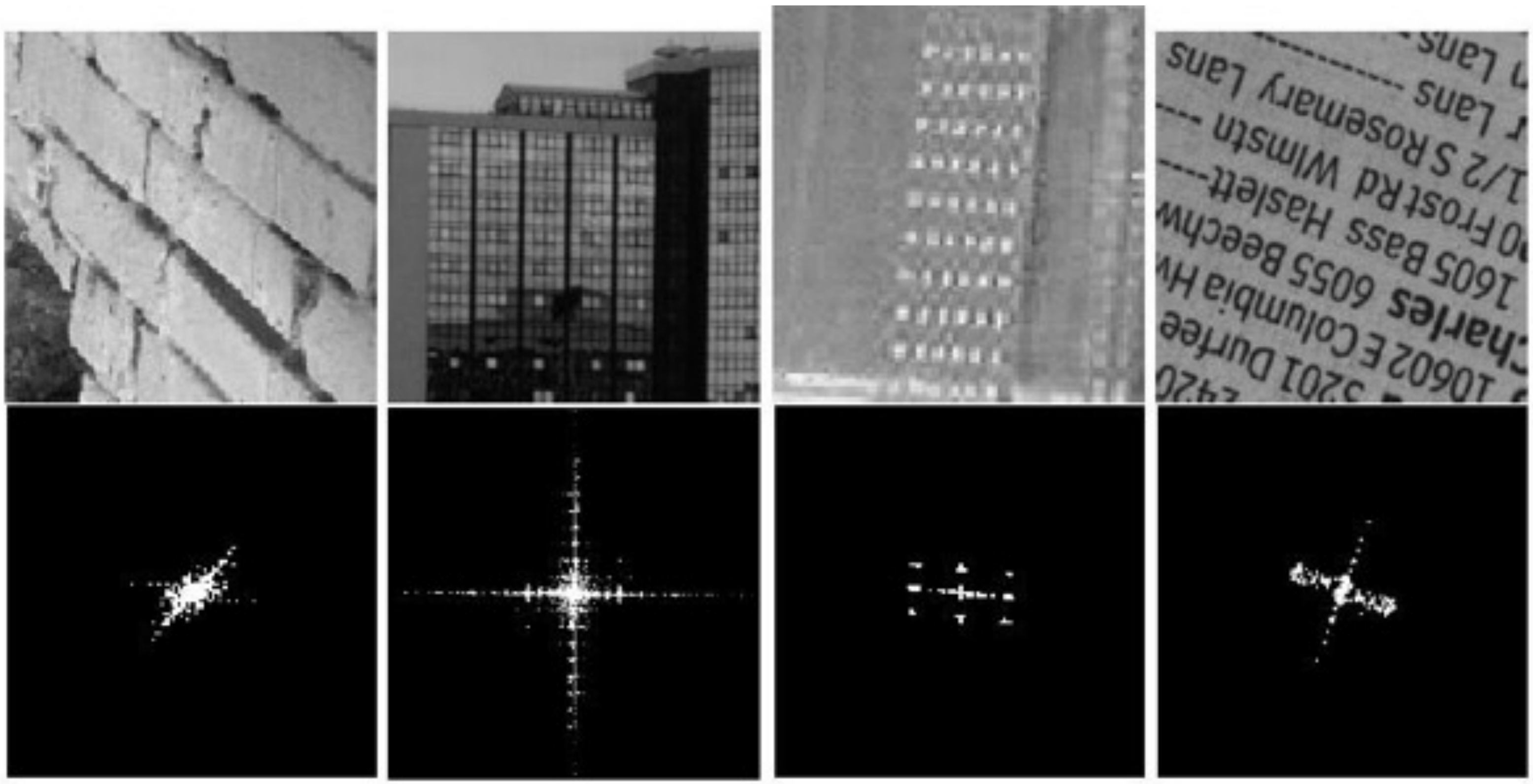
Using Fourier Representations

Dominant Orientation



<u>Limitations</u>: not useful for local segmentation

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Phase and Magnitude

$$e^{it} = \cos t + i \sin t$$

- Fourier transform of a real function is complex with real (R) and imaginary (I) components
  - difficult to plot, visualize
  - instead, we can think of the phase and magnitude of the transform
- Phase is the phase of the complex transform
  - p(u) = atan(I(u)/R(u))
- Magnitude is the magnitude of the complex transform
  - m(u) = sqrt(R^2(u) + I^2(u))
- Curious fact
  - all natural images have about the same magnitude transform
  - hence, phase seems to matter, but magnitude largely doesn't
- Demonstration
  - Take two pictures, swap the phase transforms, compute the inverse - what does the result look like?

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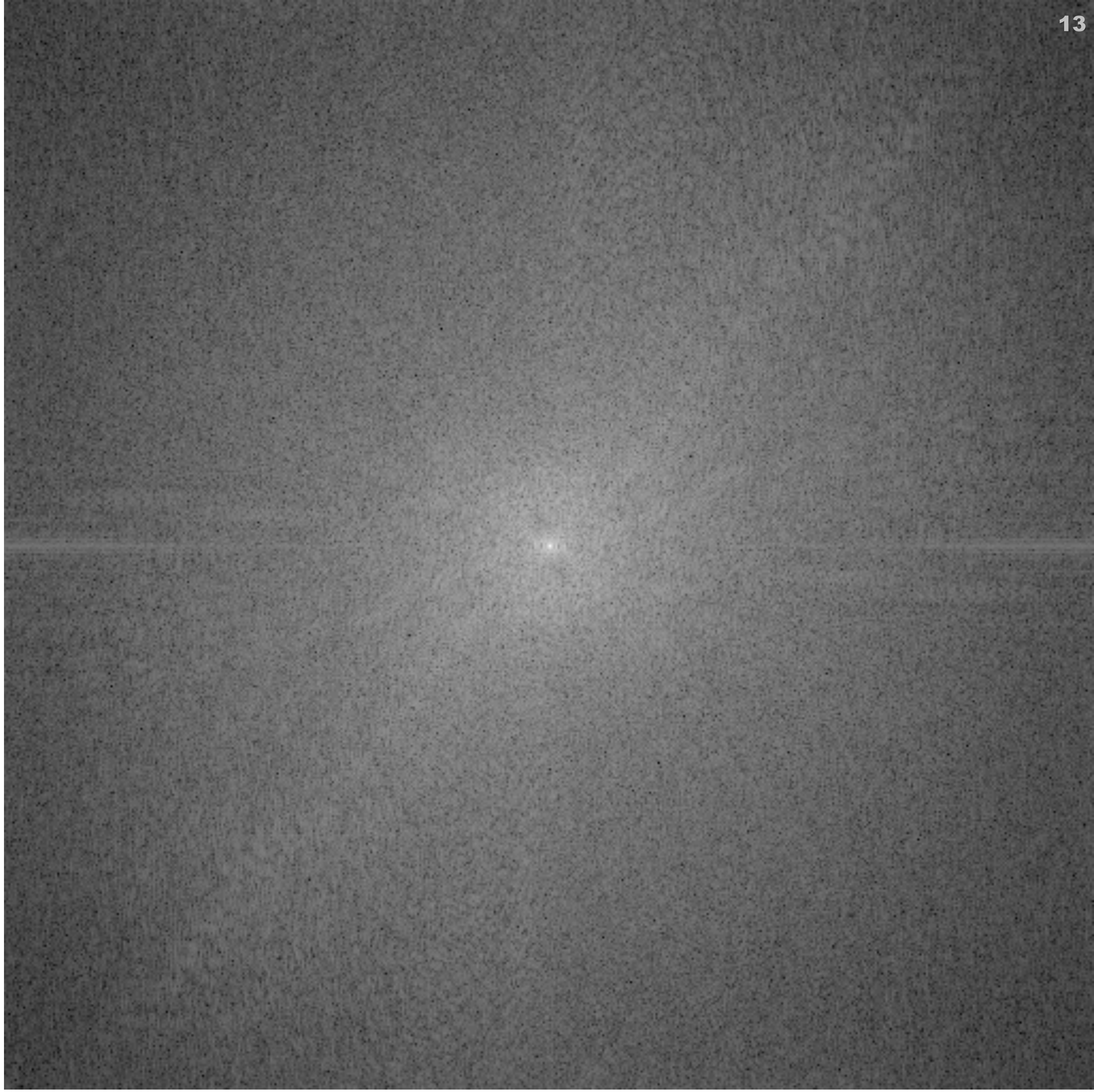
This is the magnitude transform of the cheetah pic



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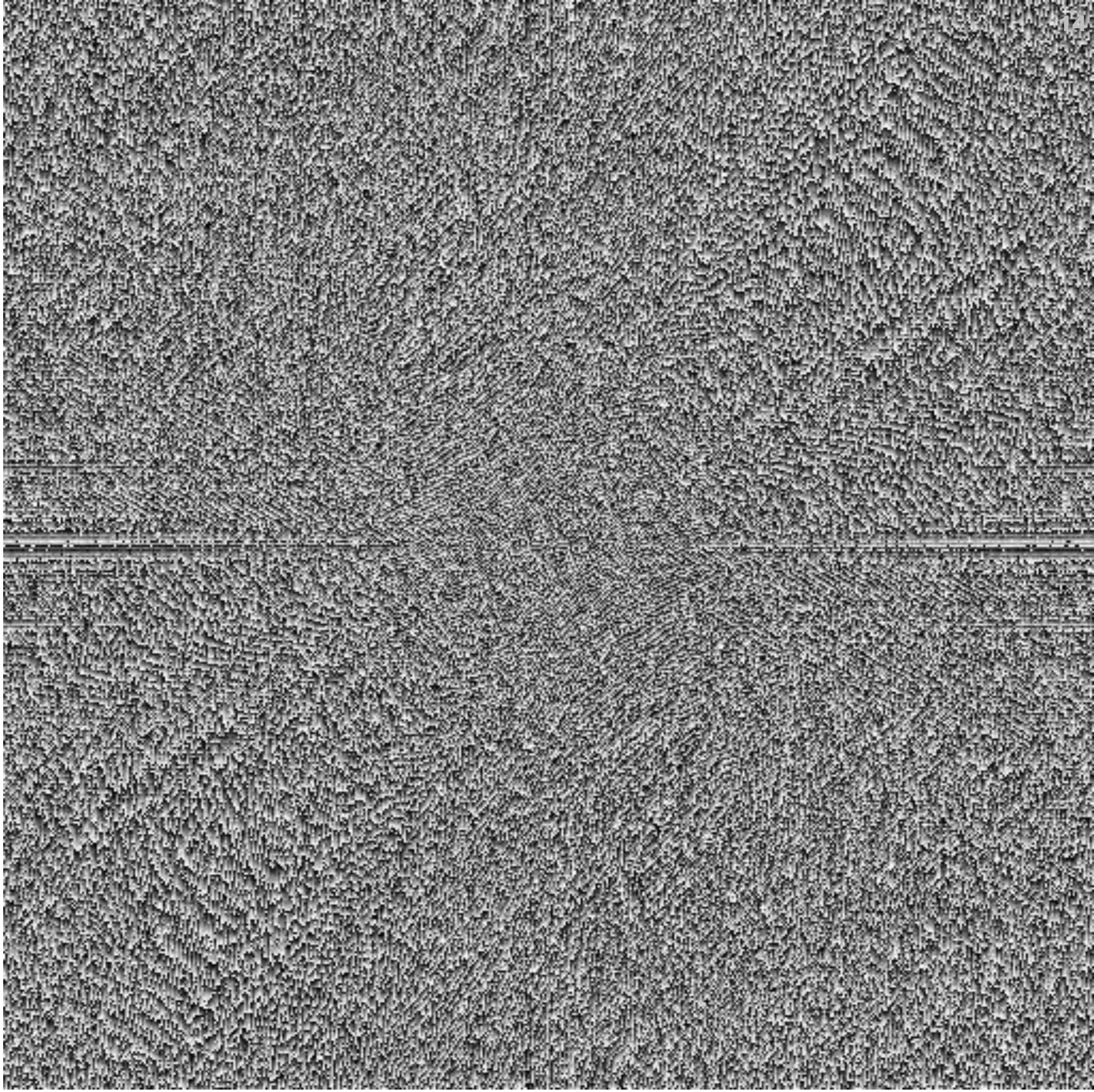
This is the phase transform of the cheetah pic



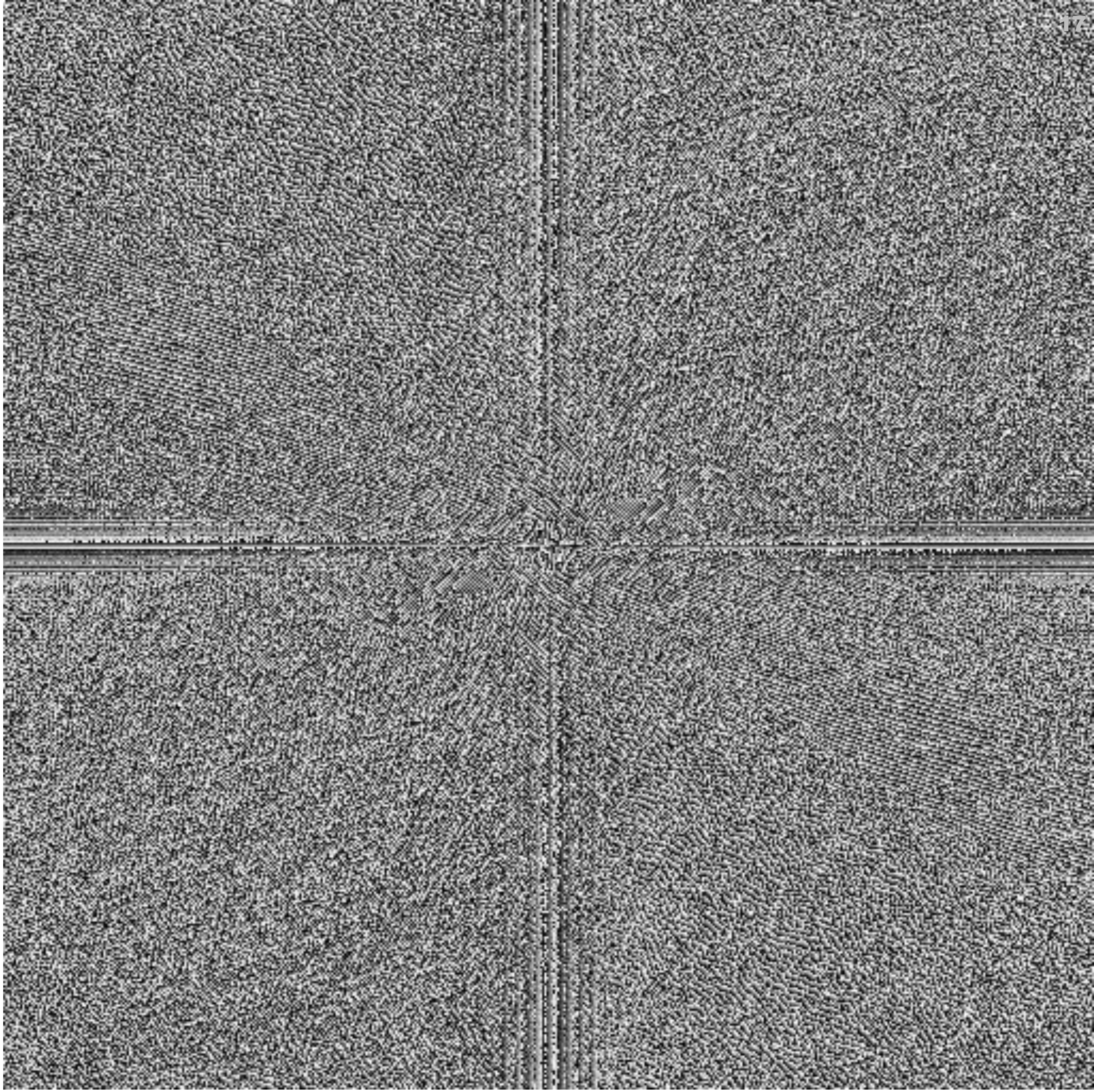
Source: G Hager, D. Kriegman Slides

Source: G Hager, D. Kriegman Slides

This is the magnitude transform of the zebra pic

Source: G Hager, D. Kriegman Slides

This is the phase transform of the zebra pic



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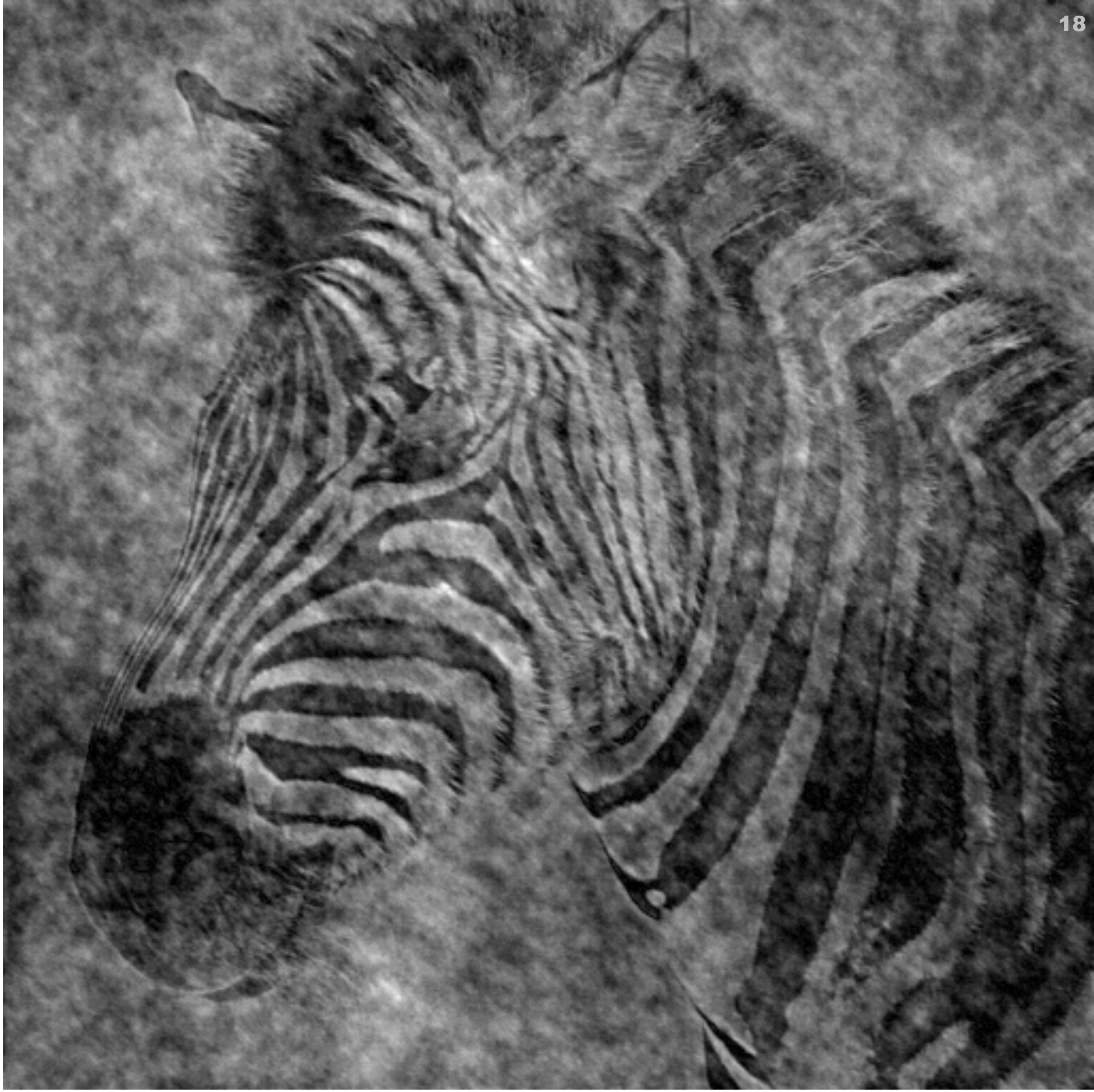
Reconstruction with zebra phase, cheetah magnitude



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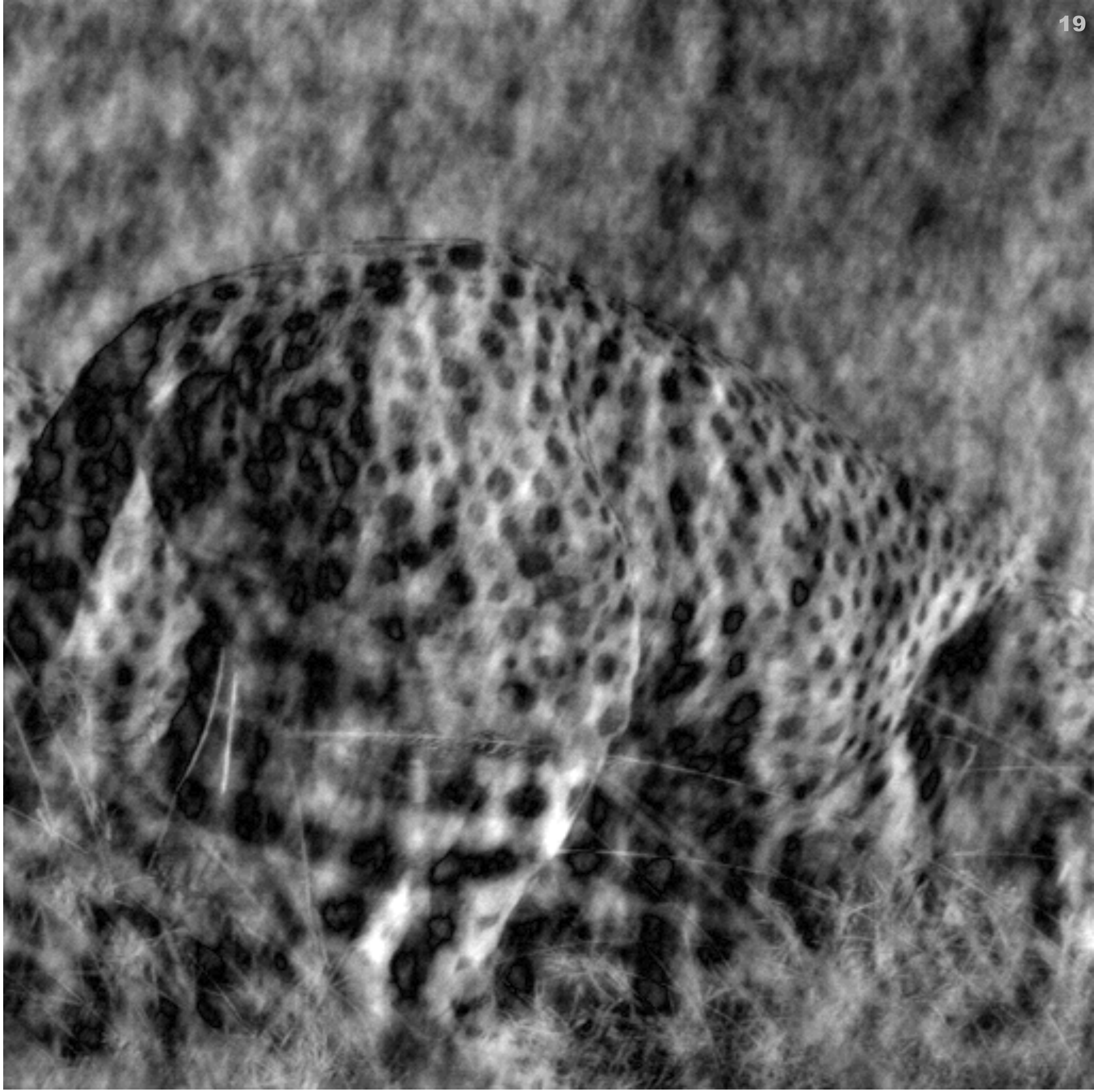
Reconstruction with cheetah phase, zebra magnitude



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The Fourier Transform and Convolution

- If H and G are images, and F(.) represents Fourier transform, then

$$F(H*G) = F(H)F(G)$$

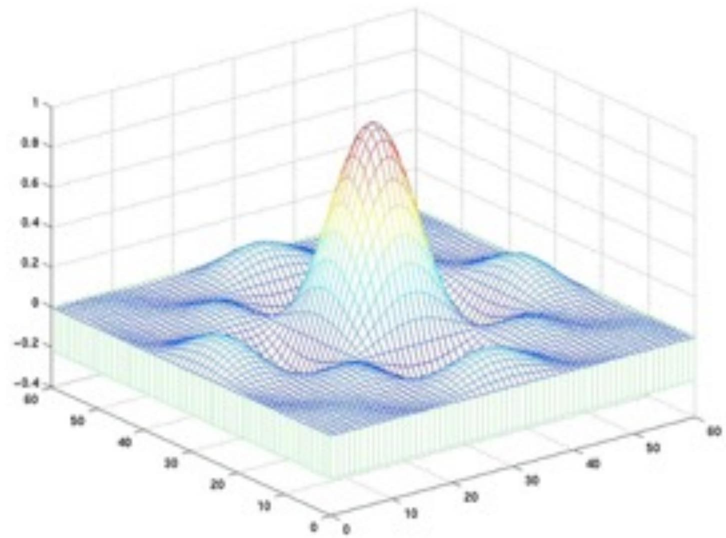
- Thus, one way of thinking about the properties of a convolution is by thinking of how it modifies the frequencies of the image to which it is applied.

- In particular, if we look at the power spectrum, then we see that convolving image H by G attenuates frequencies where G has low power, and amplifies those which have high power.

- This is referred to as the **Convolution Theorem**

The Properties of the Box Filter

F(mean filter) =



Thus, the mean filter enhances low frequencies
but also has “side lobes” that admit higher frequencies

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The Gaussian Filter: A Better Noise Reducer

- Ideally, we would like an averaging filter that removes (or at least attenuates) high frequencies beyond a given range

- It is not hard to show that the FT of a Gaussian is again a Gaussian.
  - What does this imply? FT($e^{-\alpha x^2}$) = $\sqrt{\frac{\pi}{\alpha}} \cdot e^{-\frac{(\pi\xi)^2}{\alpha}}$

- Note that in general, we truncate --- a good general rule is that the width (w) of the filter is at least such that w > 5 σ. Alternatively we can just stipulate that the width of the filter determines σ (or vice-versa).

- Note that in the discrete domain, we truncate the Gaussian, thus we are still subject to ringing like the box filter.

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Smoothing by Averaging

Kernel: 

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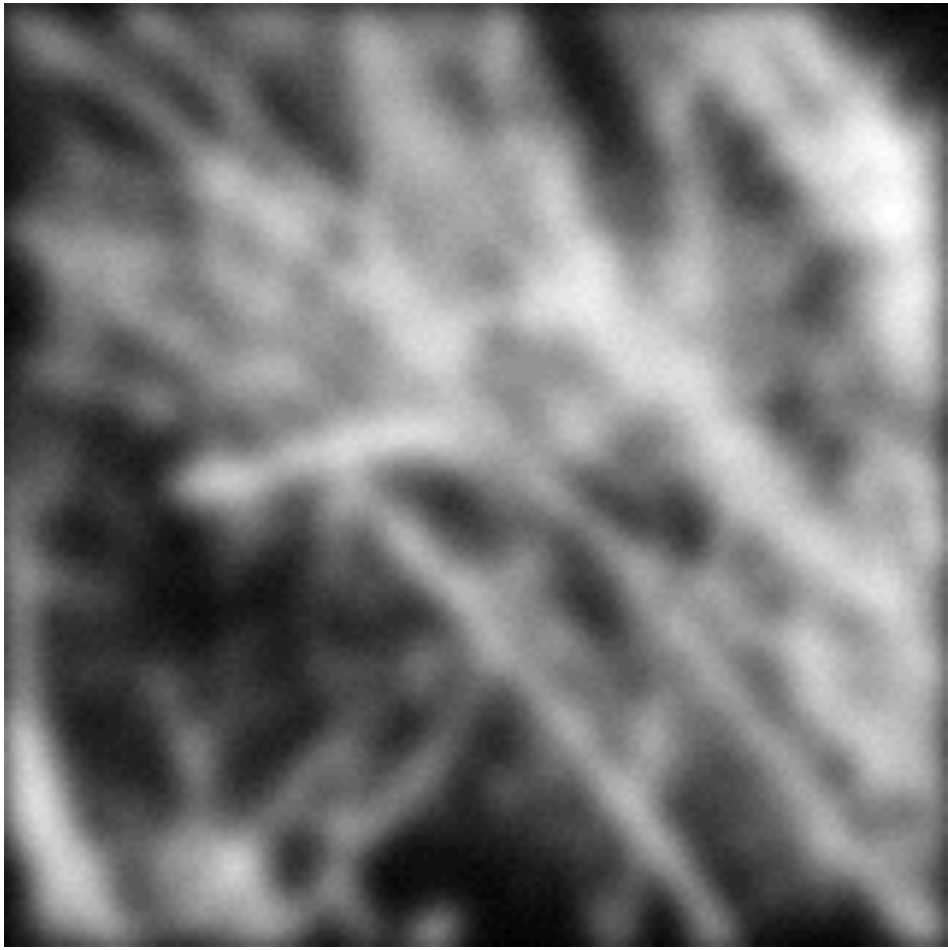
Smoothing with a Gaussian

Kernel: 



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Why Not a Frequency Domain Filter?