EECS 442 10/12

Announcements

1. NO DISCUSSION NEXT WEEK
2. NO QUIZ NEXT WEEK

---

→ Basis that has certain desirable properties?

compact support?

orthonormal basis?

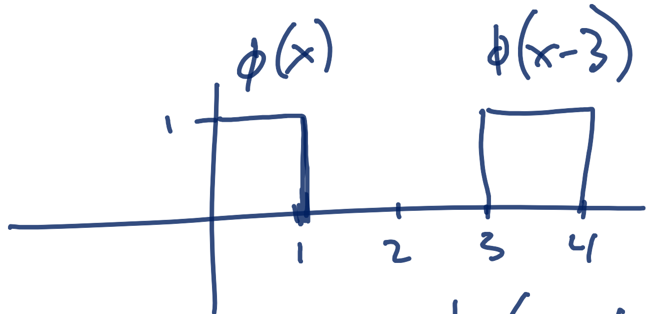
$$0 = \int V_i(x) V_j(x)\, dx \quad \text{for all } i \neq j$$

Defn Haar basis is an orthonormal basis with compact support.

Consider the 1D case

$$\phi(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

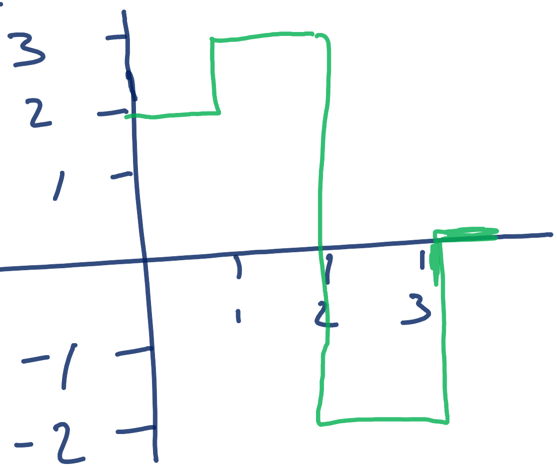
We can construct many signals from $\phi(x)$



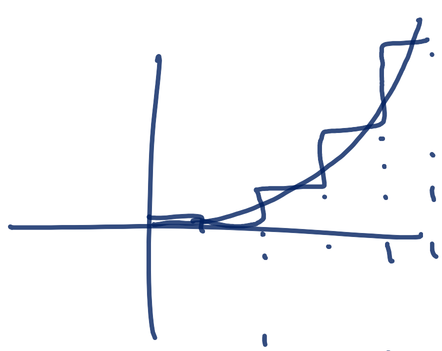

translation: $\phi(x-\underline{k})$

take linear combinations of $\phi(x-k)$

$$f(x) = 2\phi(\underline{x}) + 3\phi(x-1) - 2\phi(x-2)$$




Can approximate continuous functions

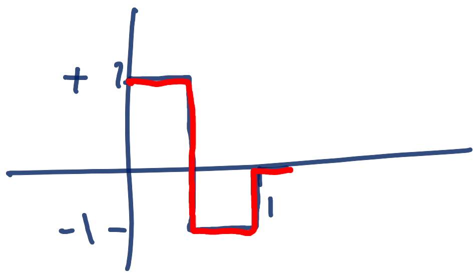


Better approximations by shrinking

$$\phi(x) \to \phi(2x)$$

Defn Haar Wavelet in 1D

$$\psi(x) = \begin{cases} 1 & \text{if } 0 \le x \le 1/2 \\ -1 & \text{if } 1/2 < x \le 1 \\ 0 & \text{otherwise} \end{cases}$$




Family of Haar Wavelets $\Psi$

① Translation $\psi(x-t) \doteq \psi_{0,t}$

② Stretching $2^{j/2}\psi(2^j x - t) \doteq \psi_{j,t}$

Remark 1: Any two wavelets of the same stretch, are orthogonal

Remark 2: Can show that <u>any</u> function can be represented as a combination of wavelets.

? [ (bounded) $\updownarrow$ continuous ]