Images as Functions

Foundations of Computer Vision

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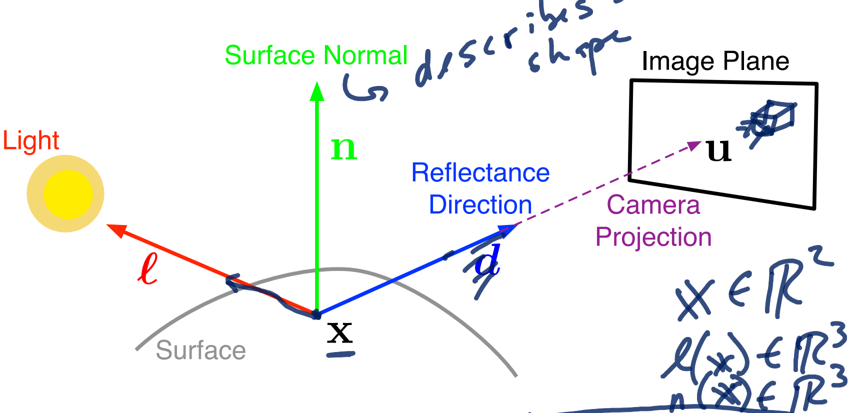
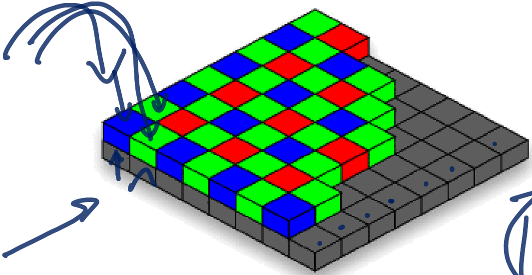
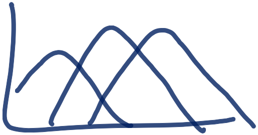
Lambertian Image Example



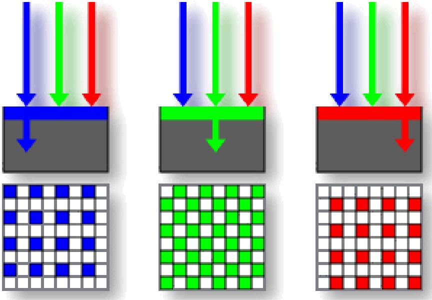
- Reflectance at the point $\mathbf{x}$ on the surface: $\boxed{R(\mathbf{x}) = \rho \boldsymbol{\ell}(\mathbf{x})^{\mathsf{T}} \mathbf{n}(\mathbf{x})}$
- This energy is projected onto the image plane by some camera function P: $\mathcal{I}(\mathbf{u}) = P(R(\mathbf{x}))$.
- The details of this camera function are not important now.

Bayer Pattern Filter Example

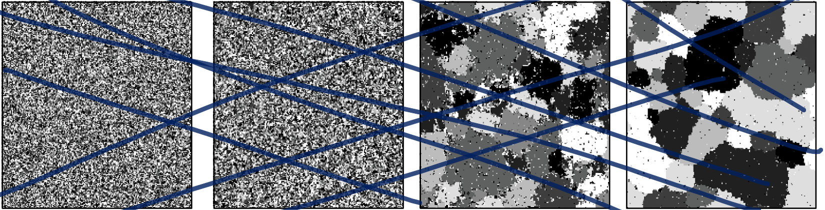




YUV 4/22



Potts Model Examples



$\beta = 0.3$ $\beta = 0.5$ $\beta = 0.7$ $\beta = 0.8$

$(s,t) \in \Lambda$

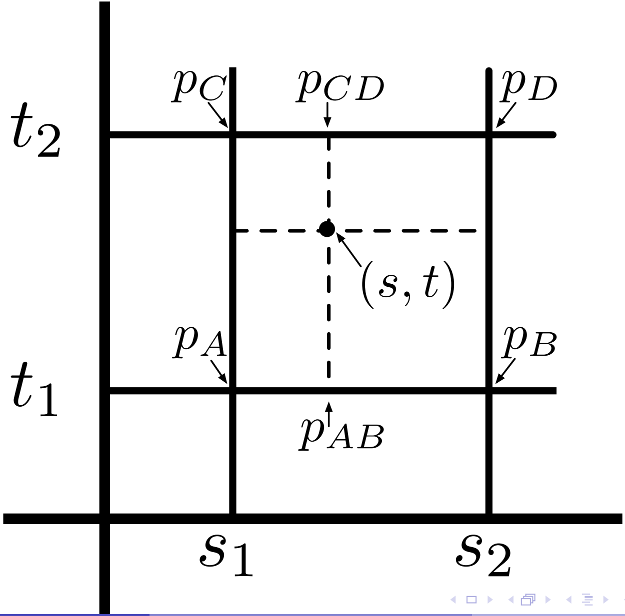
$$E(\mathbf{I}) = \beta \sum_{(s,t)} \left(\mathbb{1}\left[\mathbf{I}(s,t) \neq \mathbf{I}(s+1,t)\right] + \mathbb{1}\left[\mathbf{I}(s,t) \neq \mathbf{I}(s,t+1)\right] \right)$$

indicator function

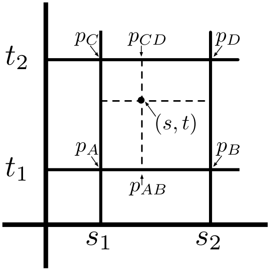
$\mathbb{1} : \text{BOOL} \rightarrow \{0,1\}$

$\beta \in \mathbb{R}$

Bilinear Interpolation



Bilinear Interpolation



$$\mathbf{I}(p_{AB}) = \mathbf{I}(s, t_1) \approx \frac{s_2 - s}{s_2 - s_1}\mathbf{I}(p_A) + \frac{s - s_1}{s_2 - s_1}\mathbf{I}(p_B)$$

$$\mathbf{I}(p_{CD}) = \mathbf{I}(s, t_2) \approx \frac{s_2 - s}{s_2 - s_1}\mathbf{I}(p_C) + \frac{s - s_1}{s_2 - s_1}\mathbf{I}(p_D)$$

$$\mathbf{I}(s, t) \approx \frac{t_2 - t}{t_2 - t_1}\mathbf{I}(p_{AB}) + \frac{t - t_1}{t_2 - t_1}\mathbf{I}(p_{CD})$$

Spatial Range Operation Example: Sum a Window

5	8	10	10	12
4	6	8	10	20
4	4	5	5	7
7	8	10	11	11
10	10	8	8	7

Spatial Range Operation Example: Sum a Window

5	8	10	10	12
4	6	8	10	20
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Spatial Range Operation Example: Sum a Window

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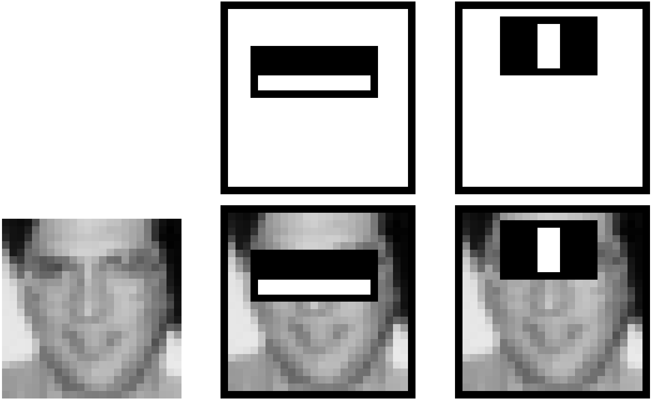
Haar Operator-based Features for Face Detection

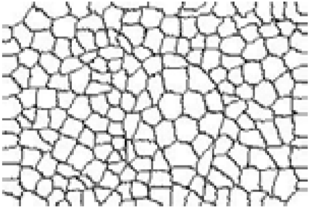


Proposed by Viola and Jones CVPR 2001.

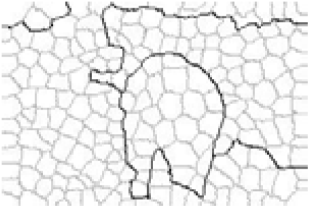
Superpixel Example – Arbitrarily-shaped Windows

The Image

A Human Segmentation



Superpixel Map



Reconstruction of Human Segmentation with Superpixels

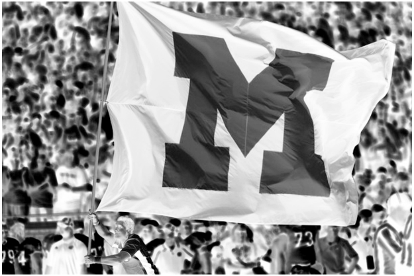
Oversegmentation as a preprocessing step was codified by X. Ren and J. Malik. *Learning a classification model for segmentation.* ICCV 2003.

Generic Range Map Operator Pseudo-Code

```
procedure GENERIC RANGE MAP OPERATOR
    for each pixel s ∈ Λ_J do
        let W_s be the window into Λ at centered at s
        J(s) = f(I, W_s))
    end for
end procedure
```

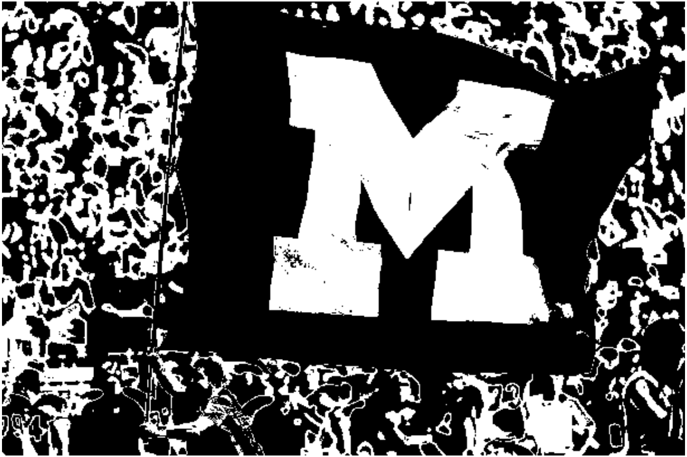
Single Pixel Range Map: Negative Image

Input Image



Negative Image

Range Map of Binary Functions: Thresholding Example



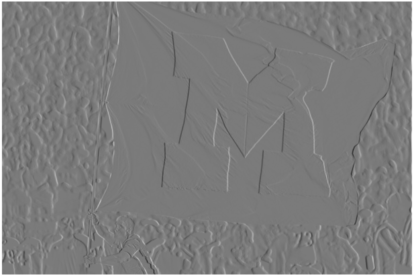
$$f_b(\mathbf{I}[W]; 128, 230) = \begin{cases} 1 & 128 \leq \mathbf{I}[W] \leq 230 \\ 0 & \text{otherwise} \end{cases}$$

Windowed Spatial Range Map: Smoothing an Image

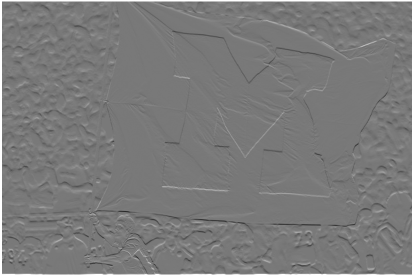
Input Image

Smoothed Image 15×15

Discrete Image Derivative Example



$\nabla_x \mathbf{I}$



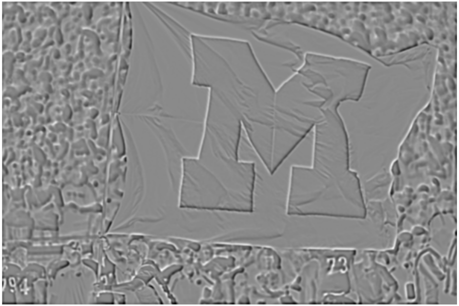
$\nabla_y \mathbf{I}$

Approximating an Image Laplacian

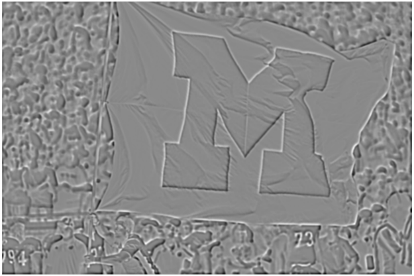
-

=

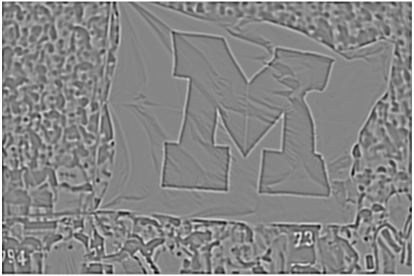
Gaussian ($\sigma = 2$) minus Gaussian ($\sigma = 1$) equals



Approximating an Image Laplacian



$\mathbf{G}_2 - \mathbf{G}_1$



$\nabla^2 \mathbf{I}$

Not exactly: $\nabla^2 \boldsymbol{\kappa}_1 \mathbf{I}$