In the name of God, the Compassionate, the Merciful

DIGITAL IMAGE PROCESSING

Digital Image Fundamentals

Light and the Electromagnetic Spectrum

$\lambda = \frac{c}{\nu}$ λ Wavelength, ν Frequency, c the speed of light
Electromagnetic Spectrum
p45 chromatic, lumin ...
image cordinate and axies

contrast

Neighbors and Connectivities

Neighbors of Pixel

- **4-neighbors ($N_4(p)$):** $(x-1,y)$, $(x+1,y)$, $(x,y-1)$, and $(x,y+1)$
- **8-neighbors($N_8(p)$):** $N_4(p)+$ diagonal directions $(x-1,y-1)$, $(x-1,y+1)$, $(x+1,y-1)$ and $(x+1,y+1)$

Connectivity

- **4-connected** Two pixels p and q are 4-connected if they are 4-neighbors and $p \in V$ and $q \in V$;
- **8-connected** Two pixels p and q are 8-connected if they are 8-neighbors and $p \in V$ and $q \in V$;
- **mixed-connected** Two pixels p and q are mix-connected if
  - p and q are 4-connected, **or**
  - p and q are 8-connected **and** not 4-connected through a third pixel ($N_4(p) \cap N_4(q) \not\subset V$)

Distances

- **Euclidean distance**:
  $D_E(p,q) = \sqrt{(x-u)^2 + (y-v)^2} = [|x-u|^2 + |y-v|^2]^{1/2}$
- **City-block distance**:
  $D_4(p,q) = |x-u| + |y-v| = [|x-u|^1 + |y-v|^1]^{1/1}$
- **Chess-board distance**:
  $D_8(p,q) = max\{|x-u|, |y-v|\} = [|x-u|^\infty + |y-v|^\infty]^{1/\infty}$
- **General distance definition**:
  $D(p,q) = [|x-u|^L + |y-v|^L]^{1/L}$

$$D_4 = \begin{matrix} 4 & 3 & 2 & 3 & 4 \\ 3 & 2 & 1 & 2 & 3 \\ 2 & 1 & 0 & 1 & 2 \\ 3 & 2 & 1 & 2 & 3 \\ 4 & 3 & 2 & 3 & 4 \end{matrix} \qquad D_8 = \begin{matrix} 2 & 2 & 2 & 2 & 2 \\ 2 & 1 & 1 & 1 & 2 \\ 2 & 1 & 0 & 1 & 2 \\ 2 & 1 & 1 & 1 & 2 \\ 2 & 2 & 2 & 2 & 2 \end{matrix}$$

Transformations and Spatial Filtering

??? deference between correlation and convolution (180' rotation)
Gamma correction : $y = Ax^\gamma$

Filtering in Frequency Domain

++++ mesale tasviri az aliasing charkhe machine +++++
Fourier Series of a function $f(t)$ with period T:
$f(x) = \sum_{n=-\infty}^{\infty} c_n \; e^{2\pi i (n/T)x}$
and
$c_n = \frac{1}{T} \int_{-T/2}^{T/2} f(x) \; e^{-2\pi i (n/T)x} dx.$

Fourier Transform (1D):
$F(\omega) = \int_{-\infty}^{\infty} f(x) \; e^{-2\pi i x \omega} \; dx$
Inverse Fourier Transform (1D):
$f(x) = \int_{-\infty}^{\infty} F(\omega) \; e^{2\pi i \omega x} \; d\omega$

Fourier Transform (2D):
$F(\omega_x, \omega_y) = \int_{-\infty}^{\infty} f(x,y) \; e^{-2\pi i (x\omega_x + y\omega_y)} \; dxdy$
Inverse Fourier Transform (2D):
$f(x,y) = \int_{-\infty}^{\infty} F(\omega_x, \omega_y) \; e^{-2\pi i (x\omega_x + y\omega_y)} \; d\omega_x d\omega_y$

Function	Fourier transform
$f(x)$	$F(\omega)$
$a.f(x) + b.g(x)$	$a.F(\omega) + b.G(\omega)$
$f(x-a)$	$e^{-2\pi i a\omega} F(\omega)$
$e^{2\pi i a x} F(x)$	$F(\omega - a)$
$f(ax)$	$\frac{1}{a} F(\frac{\omega}{a})$
$f(x) * h(x)$	$F(\omega) H(\omega)$
$f(x) h(x)$	$F(\omega) * H(\omega)$
$\delta(x)$	1
e^{iax}	$\delta(\omega - \frac{a}{2\pi})$
$u(x)$	$\frac{1}{2}(\frac{1}{i\pi\omega} + \delta(\omega))$
$rect(x)$	$sinc(\omega)$
$sinc(x)$	$rect(\omega)$
$tri(x)$	$sinc^2(\omega)$
$sinc^2(x)$	$tri(\omega)$

convolution theorem:

4.6.2 -¿ transformation and rotation

4.6.6 wraparound error

Image Restoration and Reconstruction

Noises

Gaussian: $p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\mu)^2/2\sigma^2}$

Rayleigh: $p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & z \geq a \\ 0 & z < a \end{cases}$

$\mu = a + \sqrt{\pi b/4}$ $\qquad$ $\sigma^2 = \frac{b(4-\pi)}{4}$

Erlang (Gamma): $p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$

$\mu = \frac{b}{a}$ $\qquad$ $\sigma^2 = \frac{b}{a^2}$

Exponential: $p(z) = \begin{cases} ae^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$

$\mu = \frac{1}{a}$ $\qquad$ $\sigma^2 = \frac{1}{a^2}$

Uniform: $p(z) = \begin{cases} \frac{1}{b-a} & a \leq z \leq b \\ 0 & \text{otherwise} \end{cases}$

$\mu = \frac{a+b}{2}$ $\qquad$ $\sigma^2 = \frac{(b-a)^2}{12}$

Bipolar impulse (salt-and-pepper) $p(z) = \begin{cases} P_a & z = a \\ P_b & z = b \\ 0 & \text{otherwise} \end{cases}$

Filters

Arithmetic mean $\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s,t)$

Geometric mean $\hat{f}(x,y) = \left[\prod_{(s,t) \in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$

Harmonic mean (for salt noise) $\hat{f}(x,y) = \dfrac{mn}{\displaystyle\sum_{(s,t) \in S_{xy}} \frac{1}{g(s,t)}}$

Contraharmonic mean ($Q > 0$ for pepper noise, $Q < 0$ for salt noise) $\hat{f}(x,y) = \dfrac{\displaystyle\sum_{(s,t) \in S_{xy}} g(s,t)^{Q+1}}{\displaystyle\sum_{(s,t) \in S_{xy}} g(s,t)^{Q}}$

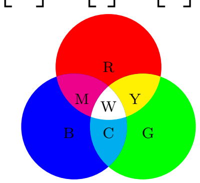
$Q = 0 \rightarrow$Arithmetic mean $\qquad$ $Q = -1 \rightarrow$Harmonic mean
image bluring due to motion - uniform linear motion:

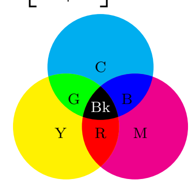
Color Processing

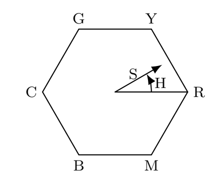
Color Models

RGB: Additive primaries
CMY: Secondary or subtractive primaries

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} B+G \\ R+B \\ R+G \end{bmatrix}$$







Color Complements

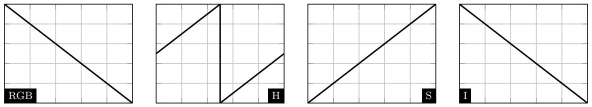


Image compression

Relative data redundancy: $R_D = 1 - \frac{1}{C_R}$

Compression ratio: $C_R = \frac{n_1}{n_2}$

1. lossless: text, file
2. lossy: multimedia data

Data Redundancies:

1. Coding (i.e., variable length coding: Entropy coding)
2. Interpixel (i.e., mapping: Run-Length coding, Predictive coding, DCT, PCA, KLT, Welsh)
3. Psychovisual (i.e., quantization:): image data that is ignored by the human visual system

Objective fidelity criterion between the original image $f(x,y)$ and the reconstructed output image $\hat{f}(x,y)$

Root-mean ($M \times N$ image):

$e_{rms} = \left[\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[\hat{f}(x,y) - f(x,y)]^2\right]^{1/2}$

mean-square SNR:

$$SNR_{ms} = \frac{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\hat{f}(x,y)^2}{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[\hat{f}(x,y)-f(x,y)]^2}$$

Entropy of a random source: $H = -\sum_{j=1}^{J} P(a_j)\log P(a_j)$

Unitary Transform???????????

Morphological Image Processing

Translation	$(B)_z = \{w \mid w = b + z, \text{for} b \in B\}$
Reflection	$\hat{B} = \{w \mid w = -b, \text{for} b \in B\}$
Complement	$A^c = \{w \mid w \notin A\}$
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$
Dilation	$A \oplus B = \{z \mid (\hat{B}_z) \cap A \neq \varnothing\}$
Erosion	$A \ominus B = \{z \mid (B)_z \subseteq A\}$
Opening	$A \circ B = (A \ominus B) \oplus B$
Closing	$A \bullet B = (A \oplus B) \ominus B$
Hit-or-Miss	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$
	$(A \ominus B_1) - (A \oplus \hat{B}_2)$
Boundary Extraction	$\beta(A) = A - (A \ominus B)$
Hole Filling	$X_k = (X_{k-1} \oplus B) \cap A^c$
Connected Components	$X_k = (X_{k-1} \oplus B) \cap A$
Geodesic Dilation	$D_G^{(1)}(F) = (F \oplus B) \cap G$
	$D_G^{(n)}(F) = D_G^{(1)}[D_G^{(n-1)}(F)]$
	$D_G^{(0)}(F) = F$
Geodesic Erosion	$E_G^{(1)}(F) = (F \ominus B) \cup G$
	$E_G^{(n)}(F) = E_G^{(1)}[E_G^{(n-1)}(F)]$
	$E_G^{(0)}(F) = F$
Recon by Dilation	$R_G^D(F) = D_G^{(k)}(F)$
Recon by Erosion	$R_G^E(F) = E_G^{(k)}(F)$

Dualities:

- Erosion & Dilation: $(A \ominus B)^c = A^c \oplus \hat{B}$,$(A \oplus B)^c = A^c \ominus \hat{B}$
- Opening & Closing: $(A \bullet B)^c = (A^c \circ \hat{B})$, $(A \circ B)^c = (A^c \bullet \hat{B})$

gray: b=$b(-x,-y)$ $f^c = -f(x,y)$

Image Segmentation

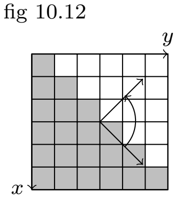
monochrom image segmentation
edge-based
region-based

1. First-order derivatives generally produce thicker edges in an image.
2. Second-order derivatives have a stronger response to fine details, such as thin lines and isolated points.
3. First order derivatives generally have a stronger response to a gray-level step.
4. Second-order derivative produce a double response at step changes in gray level.
5. Second-order derivative is more sensitive to noise.

edge models:

- Step
- Ramp
- Roof

fig 10.12



Line Detection

-1	-1	-1
2	2	2
-1	-1	-1

Horizontal

-1	-1	2
-1	2	-1
2	-1	-1

$+45°$

-1	2	-1
-1	2	-1
-1	2	-1

Vertical

2	-1	-1
-1	2	-1
-1	-1	2

$-45°$

$|R_i| > |R_j|$ for all $j \neq i$: point associated with line in direction of mask i

0	1	0
1	-4	1
0	1	0

Laplacian

1	1	1
1	-8	1
1	1	1

Diagonal Laplacian

-1	-2	-1
0	0	0
1	2	1

Sobel

-1	-1	-1
0	0	0
1	1	1

Prewitt

LoG: $\Delta^2 G(x,y) = -\left[\frac{x^2+y^2-2\sigma^2}{\sigma^4}\right] e^{-\frac{x^2+y^2}{2\sigma^2}}$

Marr-Hildreth

1. Smooth image with an nxn Gaussian lowpass filter. n is the smallest odd integer greater than or equal to 6σ.
2. Compute the Laplacian of smoothed image.
3. Find the zero crossing.

Canny Edge Detector

1. Image smoothed by Gaussian filter
2. Local gradient computed
3. Apply nonmaxima suppression to gradient magnitude image
4. Use double thresholding and connectivity analysis to *detect* and *link* edges.

Representation and Description

Representation Boundary Descriptors Regional Descriptors: simple:area, perimeter, compactness, circularity ratio topological: texture: GLCM

References:

[1] Digital Image Processing, by Gonzalez and Woods

Made by ma.mehralian using LaTeX