In the name of God, the Compassionate, the Merciful

DIGITAL SIGNAL PROCESSING

Discrete-Time Signals and Systems

Basic sequences
- $\delta(n)$: Unit sample sequence
- Any discrete-time signal can be represented as a sum of scaled and shifted unit-impulses: $x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$
- $u(n)$: Unit step sequence
- $x(n) = A\alpha^n$: Exponential sequences
- $x(n) = A\cos(\omega_0 n + \phi)$: Sinusoidal sequences

Discrete-time Signal
- $x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$

General system

$x(n) \rightarrow \boxed{T[\quad]} \rightarrow y(n)$ system response $x(n) \rightarrow y(n)$
unit sample response $\delta(n) \rightarrow h(n)$

Linear Time-Invariant (LTI or LSI) Systems

if $x_1(n) \rightarrow y_1(n)$ and $x_2(n) \rightarrow y_2(n)$
- **linearity**: $T[ax_1(n) + bx_2(n)] = ay_1(n) + by_2(n)$
- **time-invariant**: $T[x(n-n_0)] = y(n-n_0)$
- A linear system is completely characterised by its impulse response

Convolution Sum
- $y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$
- $y(n) = x(n) * h(n) = h(n) * x(n)$
- $x(n) * (h_1(n) + h_2(n)) = x(n) * h_1(n) + x(n) * h_2(n)$
- $(x(n) * h_1(n)) * h_2(n) = x(n) * (h_1(n) * h_2(n))$

Stability
- General: every bounded input sequence produces a bounded output sequence.
- LSI: $\sum_{k=-\infty}^{\infty} |h(n)| < \infty$
- e.g., unstable: $h(n) = 2^n u(n)$ stable: $h(n) = \frac{1}{2}^n u(n)$

Causality
- General: $y(n)$ for $n = n_i$ depends on $x(n)$ only for $n \leq n_i$
- LSI: $h(n) = 0 \quad n < 0$
- e.g., non-causal stable: $h(n) = 2^n u(-n)$

System impulse response(??)
- Finite-duration impulse response (FIR) system: The impulse response has only a finite number of nonzero samples.
- Infinite-duration impulse response (IIR) system: The impulse response is infinitive in duration.
- FIR systems always are stable, if each of $h[n]$ values is finite in magnitude.
- IIR systems can be stable

Difference Equation
- An important class of LTI systems
- N^{th} order: $\sum_{k=0}^{N} a_k y(n-k) = \sum_{r=0}^{M} b_r x(n-r)$
- 1^{st} order: $y(n) + ay(n-1) = x(n)$

Frequency Response of LSI system
- $y(n) = \sum_{k=-\infty}^{+\infty} h(k)x(n-k)$ let $x(n) = e^{j\omega n}$
  $y(n) = \sum_{k=-\infty}^{\infty} h(k)e^{j\omega(n-k)}$
  $y(n) = e^{j\omega n}\sum_{k=-\infty}^{\infty} h(k)e^{-j\omega k}$
  $y(n) = H(e^{j\omega})e^{j\omega n}$
- The frequency response of discrete-time LTI systems is always a periodic function of the frequency variable ω with period 2π. Only specify over the interval $-\pi < \omega < \pi$

expressing a signal in terms of sinusoids will lead us to the Fourier transform
$g[x] = e^{i\omega x} * h[x]$
$Ae^{i\omega x} = A\cos(\omega x) + iA\sin(\omega x)$

$$\begin{aligned} g[x] &= e^{i\omega x} \star h[x] \\ &= \sum_{k=-\infty}^{\infty} h[k]e^{i\omega(x-k)} \\ &= e^{i\omega x}\sum_{k=-\infty}^{\infty} h[k]e^{-i\omega k} \end{aligned}$$

References:

[1] Discrete Time Signal Processing 2nd Ed, by Alan V. Oppenheim, Ronald W. Schafer

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