機器學習實務與應用

Homework #8
Due 2019 April 22 9:00AM

Exercise 1. (**Point Set Registration**) Let $\mathbf{P}$ and $\mathbf{Q}$ be two given matrices in $\mathbb{R}^{3\times 49}$. Each column of $\mathbf{P}$ (or $\mathbf{Q}$) represents a point in $\mathbb{R}^3$. Thus, column vectors of $\mathbf{P}$ (or $\mathbf{Q}$) represent 49 points in $\mathbb{R}^3$.

In Figure 1, the blue points are column vectors of $\mathbf{P}$, while the red points are column vectors of $\mathbf{Q}$. Our goal is to search for a scaling factor $s \in \mathbb{R}$ and a translation vector $\mathbf{t} \in \mathbb{R}^3$ such that $s\mathbf{q}_i + \mathbf{t}$ is as close as possible to $\mathbf{p}_i$ for $i = 1,2,\ldots,49$, as shown in Figure 2. To be precise, we aim to solve s and $\mathbf{t}$ that minimize the following optimization problem:

$$\min_{s,\mathbf{t}} \sum_{i=1}^{49} \|\mathbf{p}_i - s\mathbf{q}_i - \mathbf{t}\|_2^2 = \min_{s,\mathbf{t}} \sum_{i=1}^{49} (\mathbf{p}_i - s\mathbf{q}_i - \mathbf{t})^T (\mathbf{p}_i - s\mathbf{q}_i - \mathbf{t}) = \min_{s,\mathbf{t}} f(s)$$

where $\mathbf{p}_\mathrm{i} \in \mathbb{R}^3$ is the ith column of $\mathbf{P}$, and $\mathbf{q}_\mathrm{i} \in \mathbb{R}^3$ is the ith column of $\mathbf{Q}$.

You can follow the instructions in the code template Module5_Part2_Template.py to complete this exercise (Please see the comments that begin with **TODO**). Report the optimal solution $(s, \mathbf{t})$ and plot the curve in which the x-axis denotes the iteration number and the y-axis denotes the loss.

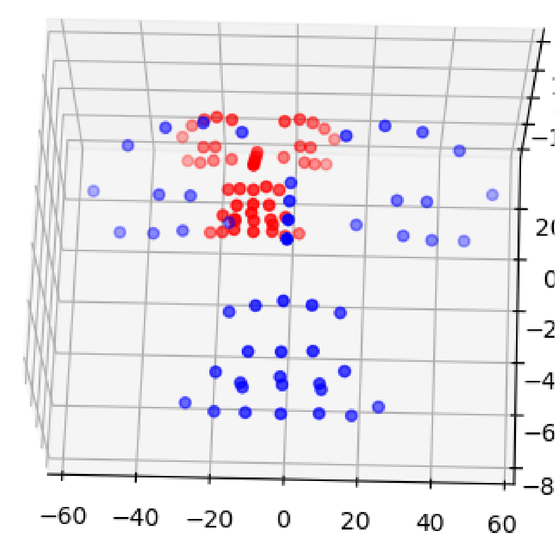

Figure 1

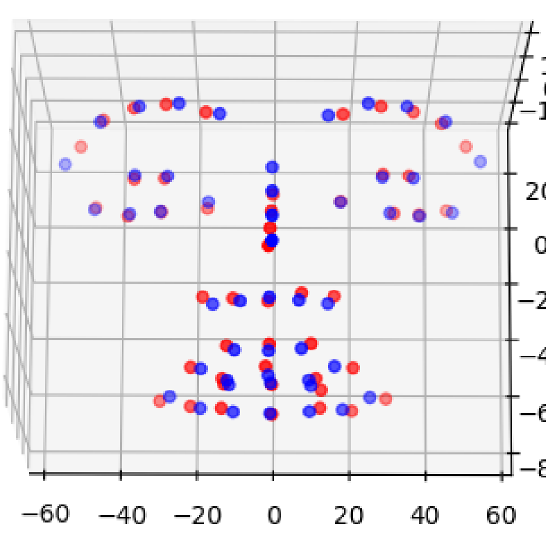

Figure 2