# 機器學習實務與應用

## Homework #5
## Due 2019 Mar 25 9:00AM

**Exercise 1**. (Linear System) Consider the following linear system of equations:

$$\mathbf{Ax} = \mathbf{b}$$

with

1. $\mathbf{A} = \begin{bmatrix} 3 & 2 & -1 \\ 6 & -1 & 3 \\ 1 & 10 & -2 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -7 \\ -4 \\ 2 \end{bmatrix}$

2. $\mathbf{A} = \begin{bmatrix} 4 & -1 & 3 \\ 21 & -4 & 18 \\ -9 & 1 & -9 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 5 \\ 7 \\ -8 \end{bmatrix}$

3. $\mathbf{A} = \begin{bmatrix} 7 & -4 & 1 \\ 3 & 2 & -1 \\ 5 & 12 & -5 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -15 \\ -5 \\ -5 \end{bmatrix}$

Determine whether each linear system is consistent or inconsistent. If the linear system is consistent, determine whether the linear system has exact one solution or an infinite number of solutions. You can use the information displayed from Module3_LinearAlgebraIII_Exercise1_template.py to complete this exercise.

**Exercise 2**. (Least Squares Problem) Consider

$$\min_{\mathbf{x}} f(\mathbf{x}) = \min_{\mathbf{x}} \|\mathbf{Ax} - \mathbf{b}\|_2^2 = \min_{\mathbf{x}} (\mathbf{Ax} - \mathbf{b})^T (\mathbf{Ax} - \mathbf{b})$$

where $\mathbf{A} = \begin{bmatrix} 1 & -2 \\ 1 & -2 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

1. Calculate the solution to the least squares problem using the normal equation.
2. Implement the pseudo code of gradient descent in p.46 to solve the least squares problem:
   I. Report the sequence $\mathbf{x}^k, \mathrm{f}(\mathbf{x}^k)$ as in p.48.
   II. Plot the sequence $\mathbf{x}^k, \mathrm{f}(\mathbf{x}^k)$ in $\mathbb{R}^3$ as in p.47. (You can modify Module3_LinearAlgebraIII_Exercise2_template.py to complete this exercise.)
   III. Discuss how to properly select the learning rate $\alpha$.