Econometrics_homework2

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1 CAPM versus Fama-French model

1.1 Estimate the CAPM model

[3]:
```
import pandas as pd
import statsmodels.api as sm
import matplotlib.pyplot as plt
import numpy as np
from scipy import stats


url = "https://raw.githubusercontent.com/adufays/GDP_expectancy/main/
 ↪IBM_returns.csv"
df = pd.read_csv(url, sep=",")
df = df.drop('Unnamed: 0', axis = 1)
df
```

[3]:
```
        Date  IBM - RF  Mkt - RF    SMB   HML    RMW   CMA
0    2000/01     -3.64     -4.74   4.15 -0.29  -6.05  4.73
1    2000/02     -8.89      2.45  18.32 -9.93 -18.33 -0.51
2    2000/03     14.74      5.20 -14.91  7.38  11.68 -1.05
3    2000/04     -6.27     -6.40  -5.55  8.61   7.55  5.27
4    2000/05     -4.26     -4.42  -3.68  2.56   4.63  0.74
..       …         …         …      …     …      …     …
205  2017/02      3.00      3.57  -2.12 -1.79   0.78 -1.72
206  2017/03     -3.19      0.17   0.78 -3.17   0.68 -1.00
207  2017/04     -8.00      1.09   0.49 -1.87   2.00 -1.55
208  2017/05     -4.84      1.06  -3.05 -3.78   1.21 -1.88
209  2017/06      0.73      0.78   2.48  1.35  -2.01 -0.07

[210 rows x 7 columns]
```

[ ]:
```
# CAPM parameters
y = df["IBM - RF"]
x = df["Mkt - RF"]
```

```python
x_capm = sm.add_constant(x)
y_capm = y
model_capm = sm.OLS(y_capm, x_capm).fit()
print(model_capm.summary())
```

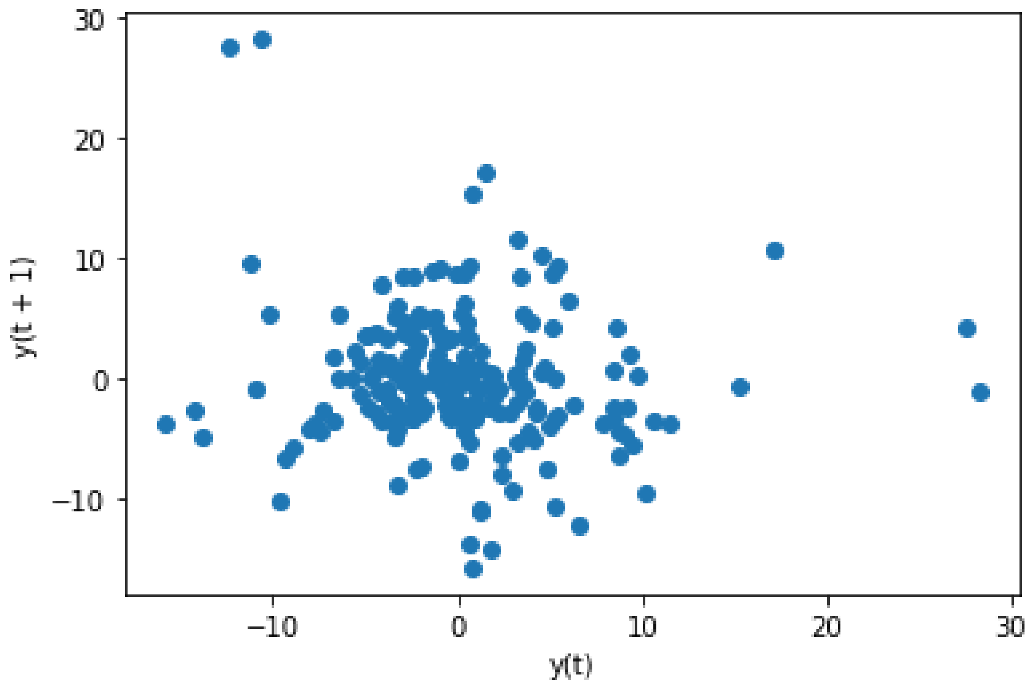
```
                            OLS Regression Results
==============================================================================
Dep. Variable:               IBM - RF   R-squared:                       0.372
Model:                            OLS   Adj. R-squared:                  0.369
Method:                 Least Squares   F-statistic:                     123.1
Date:                Tue, 15 Jun 2021   Prob (F-statistic):           9.13e-23
Time:                        09:58:11   Log-Likelihood:                -663.24
No. Observations:                 210   AIC:                             1330.
Df Residuals:                     208   BIC:                             1337.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.1379      0.396     -0.348      0.728      -0.919       0.644
Mkt - RF       0.9983      0.090     11.094      0.000       0.821       1.176
==============================================================================
Omnibus:                       62.284   Durbin-Watson:                   2.199
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              258.673
Skew:                           1.104   Prob(JB):                     6.76e-57
Kurtosis:                       7.969   Cond. No.                         4.42
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

So the model now is: $P[K_t = 1|x_t] = \Phi(-0.1379 + 0.9983 * x_{t,2})$

[26]:
```python
# scatter plot of residuals
resid = y - model_capm.fittedvalues
pd.plotting.lag_plot(resid, lag=1)
```

[26]:
```
<matplotlib.axes._subplots.AxesSubplot at 0x7f9af26c2990>
```



The lag plot shows that there exists no linear relation between error terms. The white noise assumption thus holds.

$H_0 : \beta_1 = 0$ VS $H_1 : \beta_1 \neq 0$ at significance level of 5%

The absolute value of t-statistic is 0.348, below the critical value 1.96. The p-value of beta1 is 0.728, way higher than 5%. We do not reject the null hypothesis. β_1 is **not** statiscally significant.

$H_0 : \beta_2 = 0$ VS $H_1 : \beta_2 \neq 0$ at significance level of 5%.

The absolute value of t-statistic is 0.0.090, below the critical value 1.96. The p-value of beta1 is 0.000, lower than 5%. We reject the null hypothesis. β_2 is statiscally significant.

1.2 Estimate the FF model

[ ]:
```
Y = df["IBM - RF"]
T = np.size(Y)
X = np.zeros([T,6])
X[:,0] = 1 #intercept
X[:,1:6] = df.iloc[:,2:7]

X = sm.add_constant(X)
model_ff = sm.OLS(Y, X).fit()
print(model_ff.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                IBM - RF   R-squared:                       0.425
Model:                             OLS   Adj. R-squared:                  0.411
Method:                  Least Squares   F-statistic:                     30.17
Date:                 Tue, 15 Jun 2021   Prob (F-statistic):           6.68e-23
Time:                         09:58:11   Log-Likelihood:                -653.92
No. Observations:                  210   AIC:                             1320.
Df Residuals:                      204   BIC:                             1340.
Df Model:                            5
Covariance Type:             nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.2760      0.408      0.676      0.500      -0.529       1.081
x1             0.9670      0.108      8.955      0.000       0.754       1.180
x2            -0.3988      0.147     -2.713      0.007      -0.689      -0.109
x3            -0.2093      0.169     -1.237      0.218      -0.543       0.124
x4            -0.1820      0.188     -0.968      0.334      -0.553       0.189
x5            -0.2687      0.243     -1.104      0.271      -0.748       0.211
==============================================================================
Omnibus:                       49.207   Durbin-Watson:                   2.093
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              161.488
Skew:                           0.925   Prob(JB):                     8.58e-36
Kurtosis:                       6.877   Cond. No.                         5.60
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

[ ]:

```python
content = round(model_ff.pvalues,3)
content = pd.DataFrame(content)
content = content.reset_index()
content.columns = ['param', 'p-value']
content
```

[ ]:

	param	p-value
0	const	0.500
1	x1	0.000
2	x2	0.007
3	x3	0.218
4	x4	0.334
5	x5	0.271

Among 5 parameters, only β_2 and β_3 have p-values smaller than 5%. So β_2 and β_3 are statistically different from zero.

1.3 Fisher test

$H_0 : \beta_3^{(FF)} = \beta_4^{(FF)} = \beta_5^{(FF)} = \beta_6^{(FF)} = 0 \quad VS \quad H_1 : at\ least\ one\ \beta_i^{(FF)} \neq 0, i \in [3, 6]$ at significance level of 5%

```
[ ]: # restricted model
     Y_tilde = Y
     X_2 = X[:,0:2]
     model_ff2 = sm.OS(Y_tilde, X_2).fit()
     resid_R = Y - model_ff2.fittedvalues
     SSR_R = np.sum(resid_R**2)
```

```
[ ]: # unrestriced model
     beta_U = [model_ff.params[i] for i in range(6)]
     resid_U = Y - X@beta_U
     SSR_U = np.sum(resid_U**2)
```

```
[ ]: K = np.size(X,1)
     T = np.size(Y)
     R_sq = 1-SSR_U/SSR_R
     df1 = 4 #number of constraints
     df2 = T-K
     F_test = (df2/df1)*(SSR_R-SSR_U)/SSR_U
     F_p_val = (1-stats.f.cdf(F_test,df1,df2))
     print("F statistic:", F_test)
     print("p-value:", F_p_val)
```

```
F statistic: 4.736029153392673
p-value: 0.0011261179333424964
```

Here T = 210 >100, the F-statistic is proportional to a chi-square realization. F-statistics is 4.74 > 1. We reject the null hypothesis.

The p-value of 1.13% also shows that under the null hypothesis, it is unlikely to observe such a sample. Therefore we reject the null hypothesis.

We conclude that CAPM does NOT fit comparably to the FF model and FF includes more statistically significant variables.

2 Limited dependent variables

2.1 Estimate the probit model

The probit model is: $P[K_t = 1|x_t] = \Phi(\beta_1 + \beta_2 x_{t,2} + \beta_3 x_{t,3} + \beta_4 x_{t,4})$, now we can use the Probit function to calculate.

```
[4]: from statsmodels.discrete.discrete_model import Probit

     url_2 = "https://raw.githubusercontent.com/adufays/GDP_expectancy/main/
      ↪AER_App_data.csv"
     data = pd.read_csv(url_2, sep = ",")
```

```python
game = np.array(data['cat5'])
data_gameonly=data[game==1]
killer=data_gameonly["killerappgros"]
X_killer = data_gameonly[["scoreapp","avprice","avsize"]]
X_killer = sm.add_constant(X_killer)

# fit the model using maximum likelihood
probit_model = Probit(killer, X_killer).fit()
print(probit_model.summary())
```

```
Optimization terminated successfully.
         Current function value: 0.207425
         Iterations 8
                          Probit Regression Results
==============================================================================
Dep. Variable:          killerappgros   No. Observations:                 7683
Model:                         Probit   Df Residuals:                     7679
Method:                           MLE   Df Model:                            3
Date:                Tue, 15 Jun 2021   Pseudo R-squ.:                  0.1791
Time:                        12:25:03   Log-Likelihood:                -1593.6
converged:                       True   LL-Null:                       -1941.4
Covariance Type:            nonrobust   LLR p-value:                1.916e-150
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -2.6547      0.072    -36.849      0.000      -2.796      -2.513
scoreapp       0.2636      0.016     16.139      0.000       0.232       0.296
avprice        0.1294      0.012     11.062      0.000       0.106       0.152
avsize         0.0063      0.001     10.342      0.000       0.005       0.008
==============================================================================
```

2.2 Significance of parameters

[5]:
```python
print(probit_model.pvalues)
```

```
const       3.071704e-297
scoreapp     1.347346e-58
avprice      1.913249e-28
avsize       4.530415e-25
dtype: float64
```

From the result we can see, all parameters are significant, because their p-values are almost equal to zero.

2.3 Interpret signs of coe icient

[6]:
```
a = list(probit_model.params)
print('Coefficient for Constant is {}, \nCoefficient for scoreapp is {}, \
    \nCoefficient for avprice is {}, \nCoefficient for avsize is \
    {}'.format(a[0],a[1],a[2],a[3]))
```

```
Coefficient for Constant is -2.654660819428224,
Coefficient for scoreapp is 0.2636077537641145,
Coefficient for avprice is 0.12935353364239707,
Coefficient for avsize is     0.006336470446564775
```

From the result, we can see all of the coefficients are positive (except the one for constant),indicating that all factors exert a positive effect on the app popularity.

2.4 Probit model with ratings only

Here we use a second model, $P[K_t = 1|x_t] = \Phi(\beta_1 + \beta_2 x_{t,2})$.

[ ]:
```
score=data_gameonly["scoreapp"]
score = sm.add_constant(score)
probit_model_score = Probit(killer, score).fit()
print(probit_model_score.summary())
```

```
Optimization terminated successfully.
         Current function value: 0.224877
         Iterations 8
                          Probit Regression Results
==============================================================================
Dep. Variable:          killerappgros   No. Observations:                 7683
Model:                         Probit   Df Residuals:                     7681
Method:                           MLE   Df Model:                            1
Date:                Tue, 15 Jun 2021   Pseudo R-squ.:                  0.1101
Time:                        09:58:12   Log-Likelihood:                -1727.7
converged:                       True   LL-Null:                       -1941.4
Covariance Type:            nonrobust   LLR p-value:                 5.965e-95
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -2.3512      0.064    -37.003      0.000      -2.476      -2.227
scoreapp       0.2632      0.015     17.211      0.000       0.233       0.293
==============================================================================
```

2.5 Likelihood ratio test

$H_0 : \beta_3 = \beta_4 = 0 \quad VS \quad H_1 : at\ least\ one\ \beta_i \neq 0, i \in [3, 4]$ at significance level of 95%

- **Under the null**

  The nested model (restricted model): $P[K_t = 1 \mid x_t] = \Phi(\beta_1 + \beta_2 x_{t,2})$

- **Under the alternative**

  The full model (unrestricted model): $P\left[K_t=1 \mid x_t\right]=\Phi\left(\beta_1+\beta_2 x_{t,2}+\beta_3 x_{t,3}+\beta_4 x_{t,4}\right)$

- **Likelihood ratio test**

$2\left[\ln f\left(c_1,\ldots,c_T \mid \hat{\beta}, x_{1:T}\right)-\ln f\left(c_1,\ldots,c_T \mid \tilde{\beta}_1, \tilde{\beta}_2\right)\right] \sim \chi^2(m)$ under H_0

[ ]:
```
def lrtest(likelihood_R, likelihood_U, df):
    """
      Compute the likelihood ratio test and the p-value.
    """
    lr = 2 * (likelihood_U - likelihood_R)
    pvalue = 1-stats.chi2.cdf(lr, df)
    return lr, pvalue
```

[ ]:
```
ll_U = probit_model.llf #Log-likelihood of model
ll_R = probit_model_score.llf
m = 2 # number of restrictions
print(ll_U, ll_R)
```

```
-1593.6458689504368 -1727.7288714316323
```

[ ]:
```
lr, p = lrtest(ll_R, ll_U, m)
print('Likelihood ratio test retult is {}, p value is: {}'.format(lr, p))
```

```
Likelihood ratio test retult is 268.16600496239107, p value is: 0.0
```

[ ]:
```
# To determine the threshold
threshold = stats.chi2.ppf(0.95,2)
print('The threshold is: {}'.format(threshold))
```

```
The threshold is: 5.991464547107979
```

Here the p-value is almost 0 and the likelihood test result (268.17) is far larger than the critical value(5.99).

Therefore, we reject the null hypothesis with very strong level of confidence. Full model better predicts whether an app will become a 'killer app'.

Apart from average rating, price and size are also significant factors.