Group C Member:

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In [33]:
```
import pandas as pd
import numpy as np
import requests
import io
import matplotlib.pyplot as plt
import statsmodels.api as sm
from statsmodels.stats.outliers_influence import summary_table
from scipy.stats import norm
from scipy.stats import t
from scipy import stats
```

I. Summary statistics

In [3]:
```
url = 'https://raw.githubusercontent.com/adufaye/GDP_expectancy/main/APM_Returns.csv'
s = requests.get(url).content
df = pd.read_csv(io.StringIO(s.decode('utf-8')))
df = df.drop("Unnamed: 0", axis = 1)
df.iloc[:,:10]
```

Out[3]:

	Date	Mkt-RF	SMB	HML	RMW	CMA...6	MMM	ABT	ATVI	ADBE
0	2000/01	-4.74	4.15	-0.29	-6.05	4.73	-1.20	-7.37	-5.35	-16.43
1	2000/02	2.45	18.32	-9.93	-18.33	-0.51	-6.24	0.91	-19.23	84.81
2	2000/03	5.20	-14.91	7.38	11.68	-1.05	-0.04	6.16	-5.40	8.66
3	2000/04	-6.40	-5.55	8.61	7.55	5.27	-2.65	8.78	-48.65	8.19
4	2000/05	-4.42	-3.68	2.56	4.63	0.74	-1.51	5.35	-1.50	-7.43
...	...	...	...	...	...	...	...	...	...	...
205	2017/02	3.57	-2.12	-1.79	0.78	-1.72	6.56	7.88	12.20	4.33
206	2017/03	0.17	0.78	-3.17	0.68	-1.00	2.64	-1.52	10.45	9.93
207	2017/04	1.09	0.49	-1.87	2.00	-1.55	2.30	-1.78	4.74	2.72
208	2017/05	1.06	-3.05	-3.78	1.21	-1.88	4.35	4.57	12.05	6.01
209	2017/06	0.78	2.48	1.35	-2.01	-0.07	1.76	6.40	-1.78	-0.36

210 rows × 10 columns

1.1 Computing monthly average return and standard deviation

In [4]:
```
df_1 = df.iloc[:,6:]
```

In [5]:
```
asset_name_list = list(df_1.columns)

# we used describe() function to generate dataframes
# of average return and standard deviation

asset_average_return_list = list(df_1.describe().iloc[1])
asset_standard_deviation_list = list(df_1.describe().iloc[2])
df_1.describe().iloc[:,:7]
```

Out[5]:

	MMM	ABT	ATVI	ADBE	AES	AET	AMG
count	210.000000	210.000000	210.000000	210.000000	210.000000	210.000000	210.000000
mean	0.734286	0.572381	2.379476	1.569095	0.488429	1.784286	1.396762
std	5.661786	5.714191	12.059177	11.942540	14.581512	9.353504	10.969253
min	-15.500000	-20.870000	-48.650000	-33.860000	-62.330000	-31.210000	-44.100000
25%	-2.592500	-2.625000	-3.460000	-4.582500	-5.452500	-3.870000	-4.190000
50%	0.955000	0.950000	2.265000	2.710000	-0.665000	2.495000	1.930000
75%	3.885000	4.520000	7.800000	7.600000	6.417500	6.905000	6.677500
max	20.150000	13.820000	59.500000	84.810000	74.290000	34.480000	49.710000

In [6]:
```
plt.rcParams['figure.figsize'] = (10.0, 7.0)
plt.scatter(asset_average_return_list, asset_standard_deviation_list, c = 'black', s = 5,
            alpha = 0.7, label = 'samples')

plt.grid()
plt.legend(fontsize = 14)

plt.xlabel('Average return')
plt.ylabel('Standard deviation')
plt.show()
```

1.2 Use OLS to estimate the model

The model is: $std_i = \beta_1 + \beta_2 \times ave_i + \epsilon_i$

In [7]:
```
ave = np.array(asset_average_return_list)
std = np.array(asset_standard_deviation_list)

x_model = sm.add_constant(ave)
y_model = std
model = sm.OLS(y_model, x_model)
result = model.fit()
result.params
```

Out[7]: array([6.82753557, 2.65556998])

Here, $\hat{\beta}_1$ = 6.82753557, and $\hat{\beta}_2$ = 2.65556998

$std_i = 6.83 + 2.66 * ave_i$

1.3 Interpret the coefficient β_2

The literal interpretation of the estimated coefficient β_2 is that a one-unit increase in ave_i (asset average return) will produce an expected increase in std_i (asset standard deviation)of β_2=2.66 units.

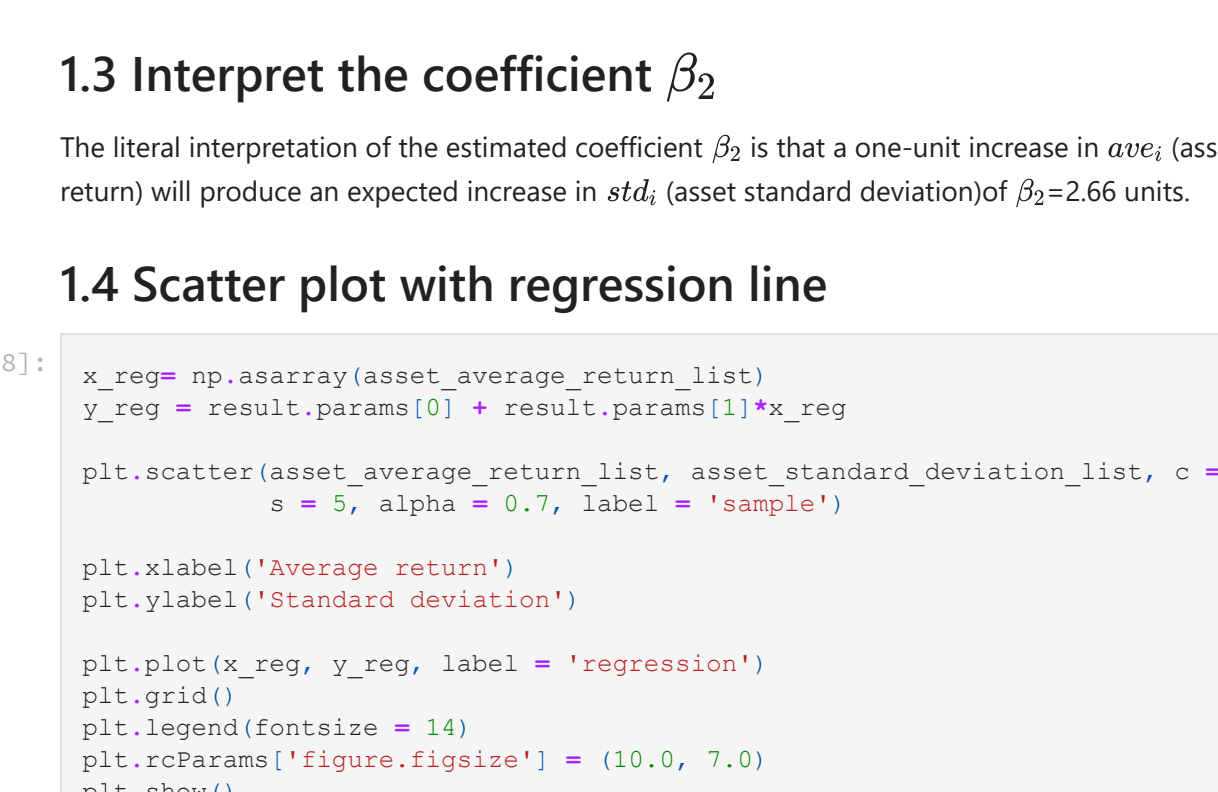
1.4 Scatter plot with regression line

In [8]:
```
x_reg= np.asarray(asset_average_return_list)
y_reg = result.params[0] + result.params[1]*x_reg

plt.scatter(asset_average_return_list, asset_standard_deviation_list, c = 'black',
            s = 5, alpha = 0.7, label = 'sample')

plt.xlabel('Average return')
plt.ylabel('Standard deviation')

plt.plot(x_reg, y_reg, label = 'regression')
plt.grid()
plt.legend(fontsize = 14)
plt.rcParams['figure.figsize'] = (10.0, 7.0)
plt.show()
```



1.5 Write explicit mathematical formula for the estimators

$\hat{\beta}_2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^{n}(x_i-\bar{x})^2} = \frac{cov(ave,std)}{var(ave)}$

$\hat{\beta}_1 = average(std) - \hat{\beta}_2 * average(ave)$

1.6 Is Beta2 statistically significant? Perform a hypothesis testing

In [9]:
```
result.summary()
```

Out[9]:

OLS Regression Results

Dep. Variable:	y	R-squared:	0.219
Model:	OLS	Adj. R-squared:	0.217
Method:	Least Squares	F-statistic:	107.4
Date:	Fri, 28 May 2021	Prob (F-statistic):	2.47e-22
Time:	18:07:44	Log-Likelihood:	-975.32
No. Observations:	384	AIC:	1955.
Df Residuals:	382	BIC:	1963.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	6.8275	0.293	23.303	0.000	6.251	7.404
x1	2.6556	0.256	10.361	0.000	2.152	3.160

Omnibus:	108.212	Durbin-Watson:	1.901
Prob(Omnibus):	0.000	Jarque-Bera (JB):	255.712
Skew:	1.405	Prob(JB):	2.97e-56
Kurtosis:	5.844	Cond. No.	3.48

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

From the table above, we can see for $\beta 2$, the t-value is 10.361, and $P > |t|$ is nearly 0, thus we can say $\beta 2$ is statistically significant.

We can also calculate t-value by using the formula:

In [10]:
```
y = np.array(asset_standard_deviation_list)
y_reg = result.params[0] + result.params[1]*np.array(asset_average_return_list)

residual_square = (np.square(y-y_reg)).sum()/(len(y)-2)
x_variance = np.var(asset_average_return_list)
x_error = x_variance*(len(np.array(asset_average_return_list))-1)

t = 2.656/(math.sqrt(residual_square/x_error))
t
```

Out[10]: 10.349174525454748

$H_0: \beta_2 = 0$

$H_1: \beta_2 \neq 0$

Test statistic: $t_{\hat{\beta}_2} = \frac{2.656-0}{SE} = \frac{2.6556-0}{0.2563} = 10.36$

$P-value = 2[1-p(t_{382} \le t_{\hat{\beta}_2})] = 0 < 5\%$

We should reject the null hypothesis $\beta_2 = 0$.

β_2 = 2.656 is thus statistically significant.

II. Linear Regression

In [11]:
```
url = 'https://raw.githubusercontent.com/adufaye/GDP_expectancy/main/life_gdp.csv'
s = requests.get(url).content
df_2 = pd.read_csv(io.StringIO(s.decode('utf-8')))
```

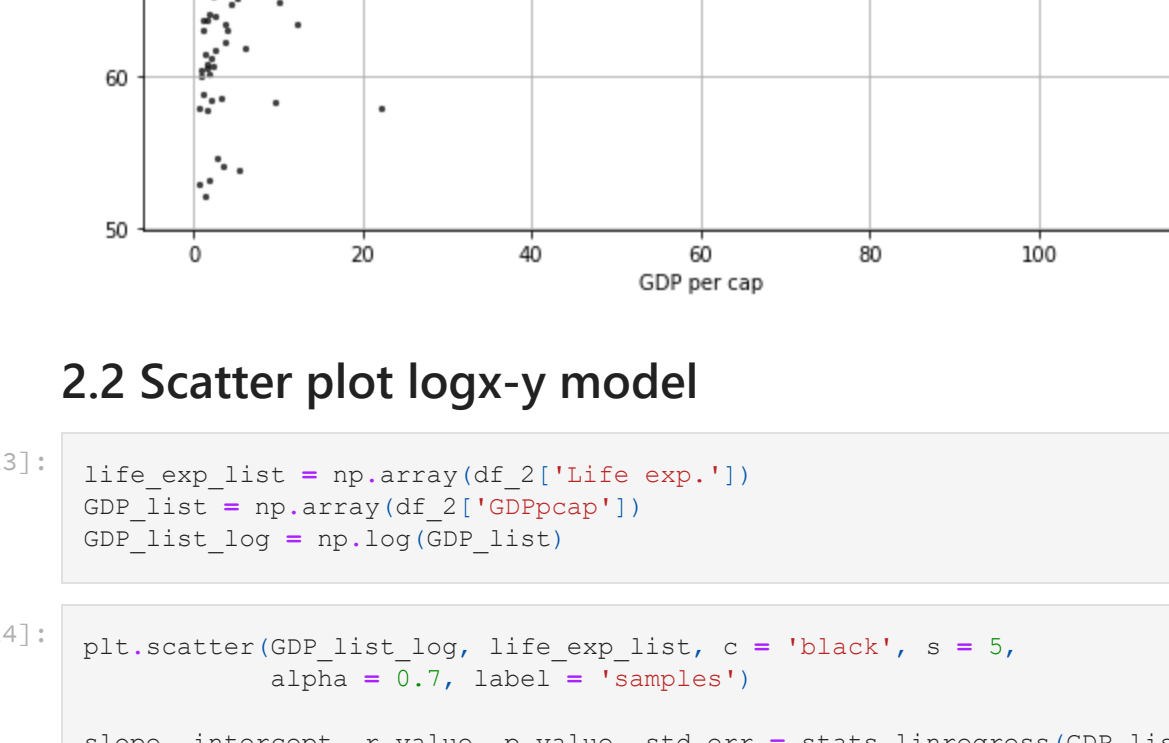
2.1 Scatter plot x-y model

In [12]:
```
plt.scatter(df_2['GDPpcap'], df_2['Life exp.'], c = 'black', s = 5,
            alpha = 0.7, label = 'samples')

slope, intercept, r_value, p_value, std_err = stats.linregress(df_2['GDPpcap'],
                                                               df_2['Life exp.'])

x_reg_1 = np.linspace(0, 120, 10)
y_reg_1 = intercept + slope*x_reg_1
plt.plot(x_reg_1, y_reg_1, label = 'regression')

plt.grid()
plt.legend(fontsize = 14)
plt.xlabel('GDP per cap')
plt.ylabel('Life expectancy')
plt.rcParams['figure.figsize'] = (10.0, 7.0)
plt.show()
```



2.2 Scatter plot logx-y model

In [13]:
```
life_exp_list = np.array(df_2['Life exp.'])
GDP_list = np.array(df_2['GDPpcap'])
GDP_list_log = np.log(GDP_list)
```

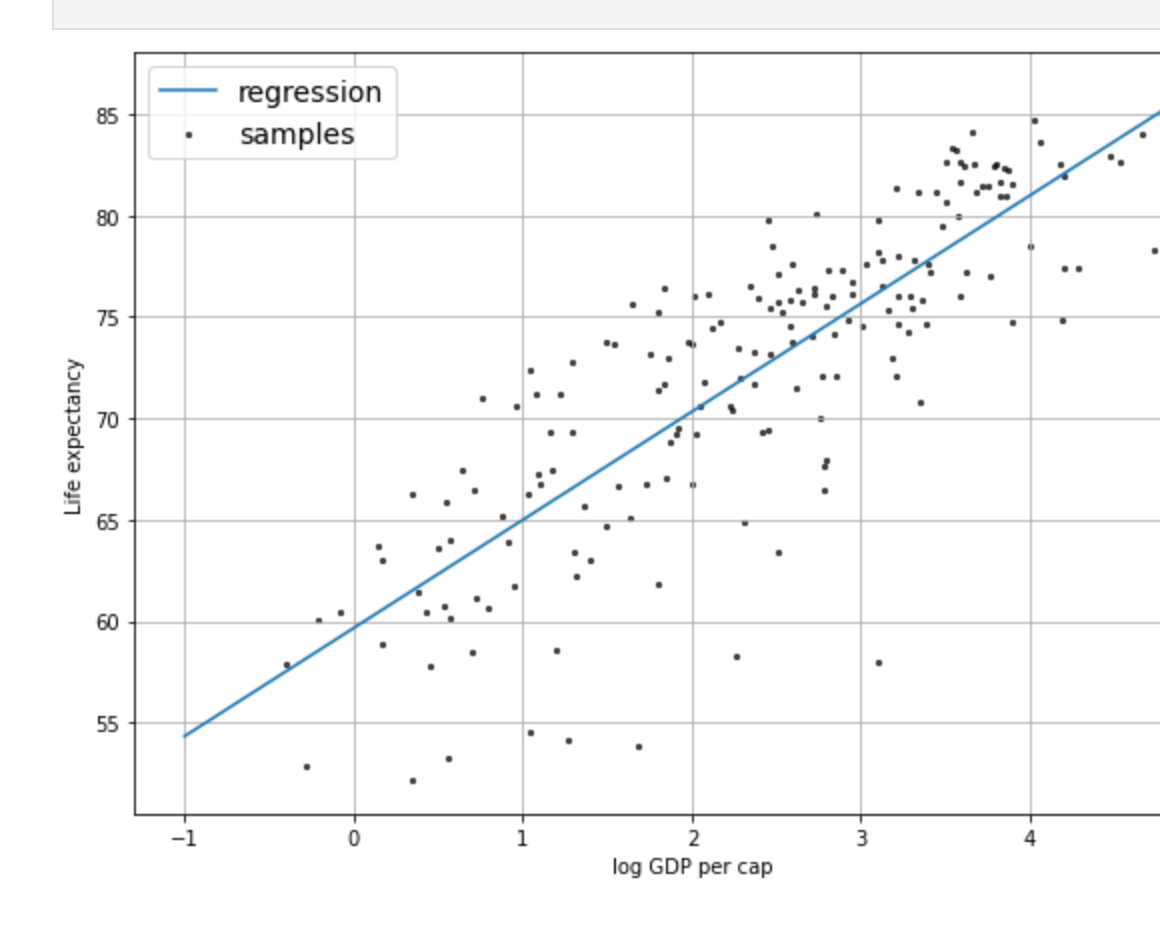
In [14]:
```
plt.scatter(GDP_list_log, life_exp_list, c = 'black', s = 5,
            alpha = 0.7, label = 'samples')

slope, intercept, r_value, p_value, std_err = stats.linregress(GDP_list_log,
                                                               life_exp_list)

x_reg_2= np.linspace(-1, 5, 10)
y_reg_2 = intercept + slope*x_reg_2
plt.plot(x_reg_2, y_reg_2, label = 'regression')

plt.grid()
plt.legend(fontsize = 14)
plt.rcParams['figure.figsize'] = (10.0, 7.0)

plt.xlabel('log GDP per cap')
plt.ylabel('Life expectancy')
plt.show()
```



2.3 Coefficient of determination

x-y model

In [15]:
```
x = np.array(df_2['GDPpcap'])
y = np.array(df_2['Life exp.'])

x_model = sm.add_constant(x)
y_model = y
model_2 = sm.OLS(y_model, x_model)
result_2 = model_2.fit()
```

In [16]:
```
result_2.params
```

Out[16]: array([67.51367284, 0.24932456])

In [17]:
```
result_2.summary()
```

Out[17]:

OLS Regression Results

Dep. Variable:	y	R-squared:	0.442
Model:	OLS	Adj. R-squared:	0.439
Method:	Least Squares	F-statistic:	142.4
Date:	Fri, 28 May 2021	Prob (F-statistic):	1.47e-24
Time:	18:07:45	Log-Likelihood:	-574.97
No. Observations:	182	AIC:	1154.
Df Residuals:	180	BIC:	1160.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	67.5137	0.585	115.478	0.000	66.360	68.667
x1	0.2493	0.021	11.933	0.000	0.208	0.291

Omnibus:	22.094	Durbin-Watson:	2.006
Prob(Omnibus):	0.000	Jarque-Bera (JB):	26.566
Skew:	-0.925	Prob(JB):	1.70e-06
Kurtosis:	3.285	Cond. No.	38.5

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

The adjusted coefficient of determination for x-y model R^2_{xy} is 0.439

lnx-y model

In [18]:
```
x_model = sm.add_constant(GDP_list_log)
y_model = life_exp_list
model_3 = sm.OLS(y_model, x_model)
result_3 = model_3.fit()
```

In [19]:
```
result_3.params
```

Out[19]: array([59.65932948, 5.33364812])

In [20]:
```
result_3.summary()
```

Out[20]:

OLS Regression Results

Dep. Variable:	y	R-squared:	0.680
Model:	OLS	Adj. R-squared:	0.678
Method:	Least Squares	F-statistic:	382.8
Date:	Fri, 28 May 2021	Prob (F-statistic):	2.00e-46
Time:	18:07:45	Log-Likelihood:	-524.28
No. Observations:	182	AIC:	1053.
Df Residuals:	180	BIC:	1059.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	59.6593	0.722	82.628	0.000	58.235	61.084
x1	5.3336	0.273	19.565	0.000	4.796	5.872

Omnibus:	40.029	Durbin-Watson:	2.238
Prob(Omnibus):	0.000	Jarque-Bera (JB):	68.168
Skew:	-1.116	Prob(JB):	1.58e-15
Kurtosis:	5.002	Cond. No.	6.65

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

The adjusted coefficient of determination for lnx-y model R^2_{lnx} is 0.678

Conclusion

Here we have $R^2_{xy} = 0.439$ and $R^2_{lnx} = 0.678$. With x-y model, we improve the fit by 43.9% compared to the sample average, and we improve the fit by 67.8% with the lnx-y model.

It's clear that the lnx-y model performs better. Since GDP has a long-tailed distribution (right skewed), the logarithmic transformation on GDP(x variable) reduces or removes the skewness of our original data, creating the desired asymptotic association between input (X variable)and target (Y variable) on the original input scale.

Besides, we can infer from the scatter plot that the lnx-y model has smaller error.

2.4 Empirical correlation between the log of GDP and life expectancy

The correlation coefficient is:

In [21]:
```
fanx = np.array(df_2['GDPpcap'])
x_1 = np.log(x)
y = np.array(df_2['Life exp.'])
cor = np.vstack((x_1, y))
print('The  empirical correlation is:', (np.corrcoef(cor)[0][1]))
print('The squared empirical correlation is:', (np.corrcoef(cor)[0][1])**2)
print('The coefficient of determination for lnx-y model is: ', result_3.rsquared)
```

```
The  empirical correlation is: 0.8247155249526711
The squared empirical correlation is: 0.6801556970979599
The coefficient of determination for lnx-y model is:  0.6801556970979601
```

The coefficient of determination is equal to the square of the empirical correlation.

III. Generating spurious correlation

In [22]:
```
url = 'https://raw.githubusercontent.com/adufaye/GDP_expectancy/main/Appl_Russell.csv'
s = requests.get(url).content
df_3 = pd.read_csv(io.StringIO(s.decode('utf-8')))
df_3 = df_3.drop('Unnamed: 0', axis = 1)
df_3
```

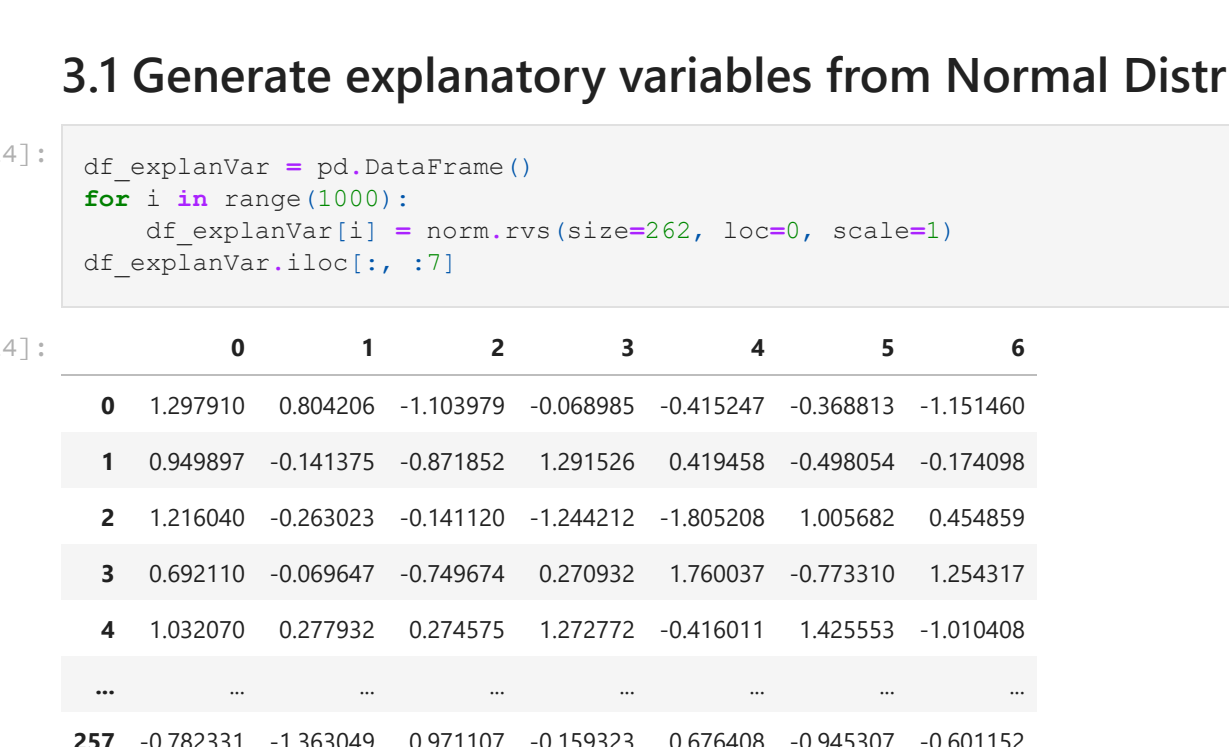
Out[22]:

	Date	Apple returns	Russell returns
0	2016-04-25	-11.989084	-1.260337
1	2016-05-02	-1.094085	-0.561795
2	2016-05-09	-1.794334	-0.581591
3	2016-05-16	5.061906	0.377336
4	2016-05-23	5.247411	2.334956
...	...	...	...
257	2021-03-29	1.465979	1.245510
258	2021-04-05	7.816477	2.372223
259	2021-04-12	0.868402	1.428772
260	2021-04-19	0.505571	-0.641943
261	2021-04-19	0.074139	-0.000118

262 rows × 3 columns

In [23]:
```
df_4 = df_3.set_index('Date')
df_4.plot()
```

Out[23]: <AxesSubplot:xlabel='Date'>



3.1 Generate explanatory variables from Normal Distribution

In [24]:
```
df_explanVar = pd.DataFrame()
for i in range(1000):
    df_explanVar[i] = norm.rvs(size=262, loc=0, scale=1)
df_explanVar.iloc[:, :7]
```

Out[24]:

	0	1	2	3	4	5	6
0	1.297910	0.804206	-1.103979	-0.068985	-0.415247	-0.368813	-1.151460
1	0.949897	-0.141375	-0.871852	1.291526	0.419458	-0.498054	-0.174098
2	1.216040	-0.263023	-0.141120	-1.244212	-1.805208	1.005682	0.454859
3	0.692110	-0.069647	-0.749674	0.270932	1.760037	-0.773310	1.254317
4	1.032070	0.277932	0.274575	1.272772	-0.416011	1.425553	-1.010408
...	...	...	...	...	...	...	...
257	-0.782331	-1.363049	0.971107	-0.159323	0.676408	-0.945307	-0.601152
258	-0.557416	0.606416	1.174797	0.120092	-0.875927	0.722186	-0.542585
259	-1.240483	0.434995	0.636910	1.106097	-1.190532	-0.411126	0.029605
260	-2.497227	-1.754366	-0.786905	-0.505423	0.352118	-1.304097	0.417685
261	0.639011	0.809549	1.469021	-1.084726	-0.934225	1.164064	-0.631074

262 rows × 7 columns

Now we have 1000 explanatory variables of size 262

3.2 Test statistics

In [25]:
```
y_model_1 = df_3['Apple returns']
test_statistics = []
x_for_plot = []
beta1 = []
beta2 = []
p_value_lst = []
std_err_lst = []
```

In [26]:
```
for i in range(1000):
    x_model = df_explanVar.iloc[:,i]
    slope, intercept, r_value, p_value, std_err = stats.linregress(x_model,
                                                                   y_model_1)

    x_for_plot.append(i)
    test_statistics.append(slope/std_err)
    beta1.append(intercept)
    beta2.append(slope)
    p_value_lst.append(p_value)
    std_err_lst.append(std_err)
```

In [27]:
```
df_lm = pd.DataFrame(list(zip(beta1, beta2,
                              test_statistics,
                              p_value_lst,
                              std_err_lst)),
                     columns=["intercept", "slope",
                              "test-statistics",
                              "p-value",
                              "std err"])
df_lm
```

Out[27]:

	intercept	slope	test-statistics	p-value	std err
0	0.647768	-0.021644	-0.089326	0.928892	0.242300
1	0.646012	-0.192805	-0.740348	0.459757	0.260425
2	0.650820	-0.230201	-0.897561	0.370250	0.256474
3	0.650115	-0.004412	-0.017935	0.985705	0.246016
4	0.661927	0.203463	0.837105	0.403302	0.243056
...	...	...	...	...	...
995	0.651106	-0.011481	-0.050252	0.959961	0.228480
996	0.630077	-0.238862	-1.019512	0.308907	0.234291
997	0.638431	-0.188627	-0.732820	0.464329	0.257399
998	0.650794	-0.068434	-0.272627	0.785356	0.251017
999	0.670594	-0.309140	-1.298278	0.195342	0.238115

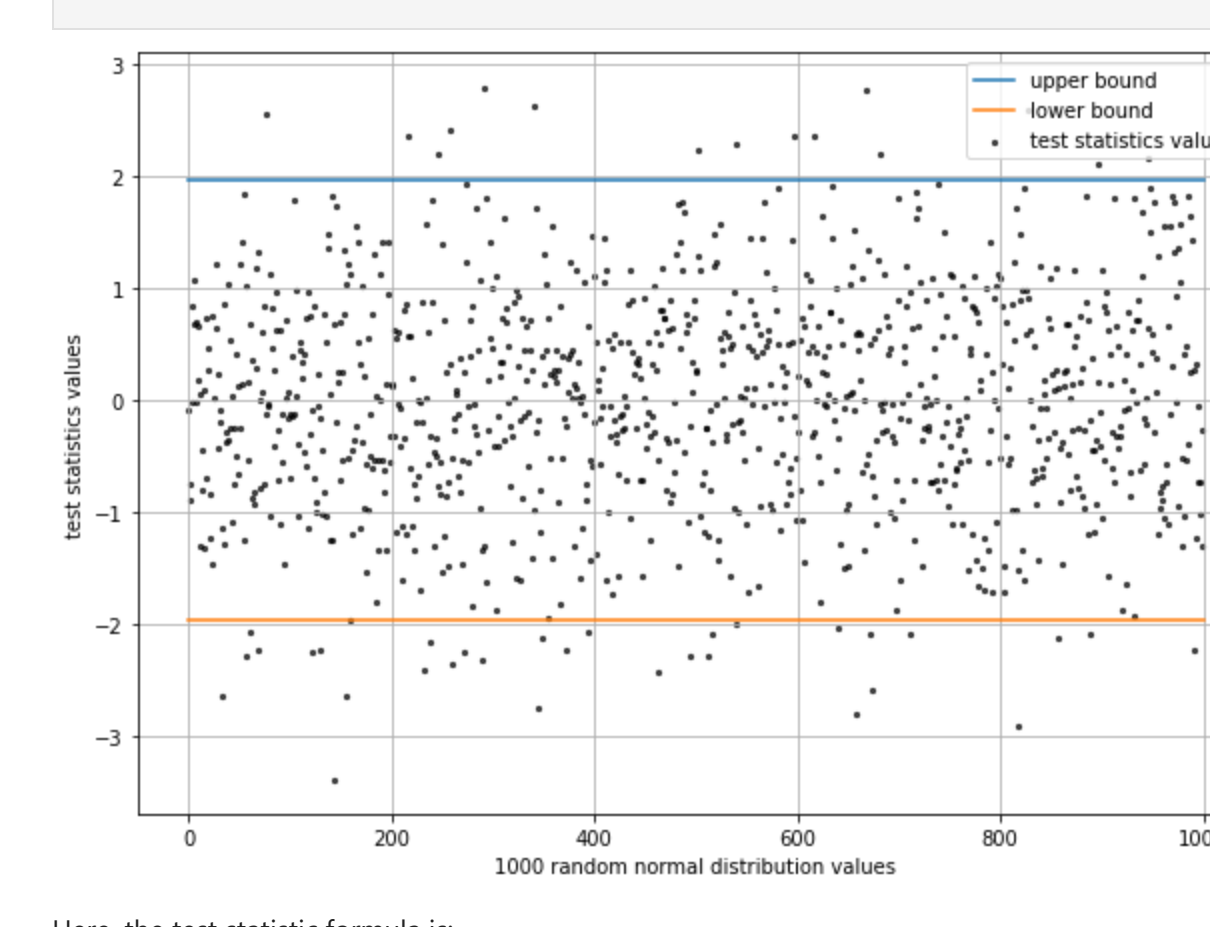
1000 rows × 5 columns

In [28]:
```
plt.rcParams['figure.figsize'] = (10.0, 7.0)
plt.scatter(x_for_plot, test_statistics, c = 'black', s = 5,
            alpha = 0.7, label = 'test statistics values')

plt.grid()
plt.legend(fontsize = 14)

one_line = np.linspace(0, 1000, 1000)
y_value = np.array([1.96]*1000)
plt.plot(one_line, y_value, label = 'upper bound')
y_value_2 = np.array([-1.96]*1000)
plt.plot(one_line, y_value_2, label = 'lower bound')

plt.legend(loc= 'upper right')
plt.xlabel('1000 random normal distribution values')
plt.ylabel('test statistics values')
plt.show()
```



Here, the test statistic formula is:

$t = \frac{\hat{\beta}_2 - 0}{SE(\hat{\beta}_2)}$

where $SE(\hat{\beta}_2) = \sqrt{\frac{\hat{\sigma}^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2}}$

3.3 Hypothesis test

In [29]:
```
maximum_t = np.absolute(np.array(test_statistics)).max()
print("So, the maximum value of test statistics is:",  maximum_t)
```

```
So, the maximum value of test statistics is: 3.378476711517529
```

Since $P-value = 2(1 - P(t_{T-2} \le |t|))$, we have:

In [34]:
```
p_value_1 = 2*(1- t.cdf(maximum_t,260))
print('P-value is: ', p_value_1)
```

```
P-value is:  0.0008406438969521535
```

With the maximum value of test statistics, we got a p-value far smaller than 0.05, which indicates that we can confidently reject the Null Hypothesis that $\beta_2 = 0$ at a significant level of 95%.

This experiment shows that, even if we generate some random numbers, it is still possible that the result can reject the Null Hypothesis, so we cannot blindly trust the result we got and write down the conclusion without thinking.