Analytical solution for the HCMP stresses and energy

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Cloud computations of HCMP stresses, tractions, energy, etc.

Links for cloud computation

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https://www.wolframcloud.com/env/wenqiang_fang/HCP_test/SelectZStack
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Cloud computations of HCMP stresses, tractions, energy, etc.

Links for cloud computation

```
https://www.wolframcloud.com/env/wenqiang_fang/HCP_test/TableOut
```

Cloud computations of HCMP stresses, tractions, energy, etc.

Links for cloud computation

https://www.wolframcloud.com/obj/wenqiang_fang/HCP_test/FullVersion

Steps to perform calculations on the cloud

(Currently) Need to use Wolfram Cloud

Usage of Wolfram cloud is free for all all Brown affiliated students and faculty

However, we are working to not require the use of Wolfram Cloud in the future. In that scenario the only requirement will be a web browser.

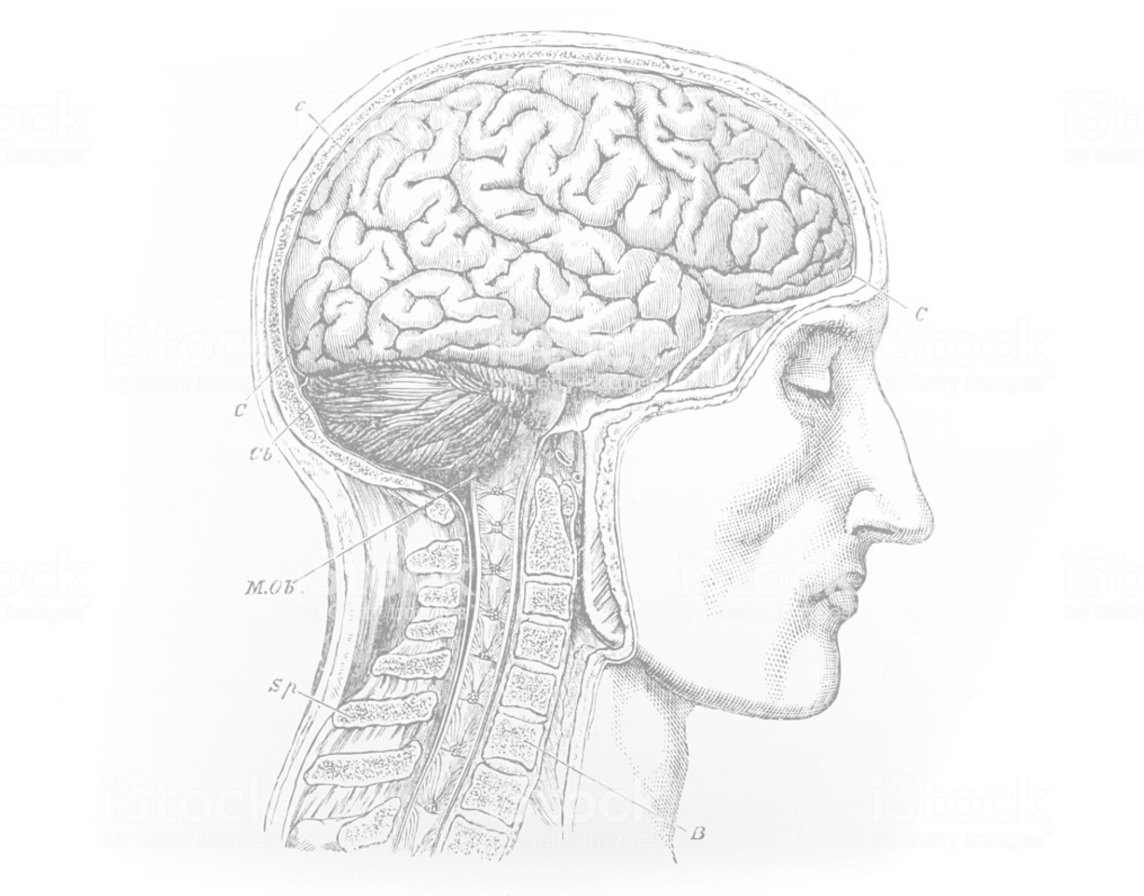
To use Wolfram Cloud you will need to create an Wolfram ID.

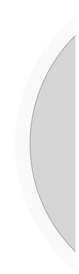
To create Wolfram ID visit the following website

`https://www.wolframcloud.com`

For Brown affiliates the Wolfram ID (a.k.a cloud ID) will simply be their brown email address, mine is “haneesh_kesari@brown.edu” and Wenqiang’s is “haneesh_kesari@brown.edu”

Visit

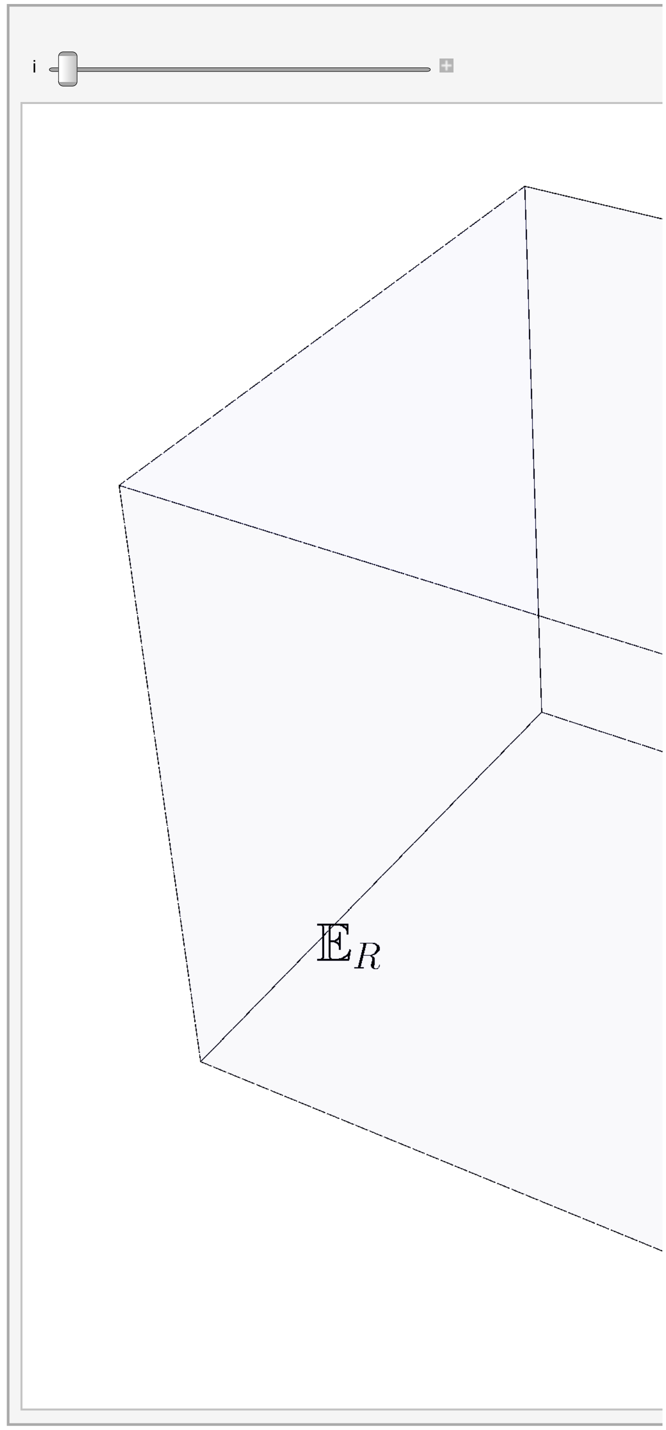




Left image from https://www.istockphoto.com/photo/human-head-brain-spine-medical-diagram-gm623931190-109575361

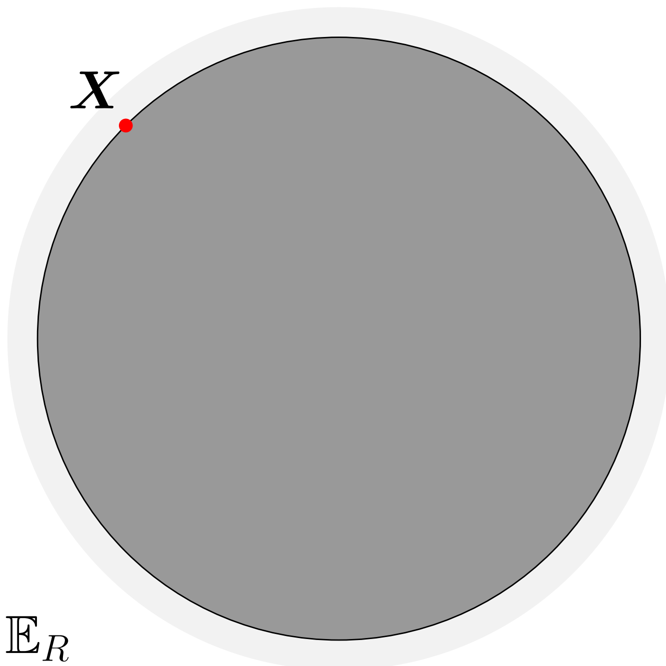
Geometry and notation: reference space

Reference configuration


i
$\mathbb{E}_R$

Position of a material particle in the reference space ($\mathbb{E}_{\mathbf{R}}$) is taken as the particle’s name

Reference configuration



$$\boldsymbol{X} = X_1 \boldsymbol{E}_1 + X_2 \boldsymbol{E}_2 + X_3 \boldsymbol{E}_3$$

Reference space ($\mathbb{E}_R$) and physical space ($\mathbb{E}$)

... Remove : There are no symbols matching "Global`objs2".

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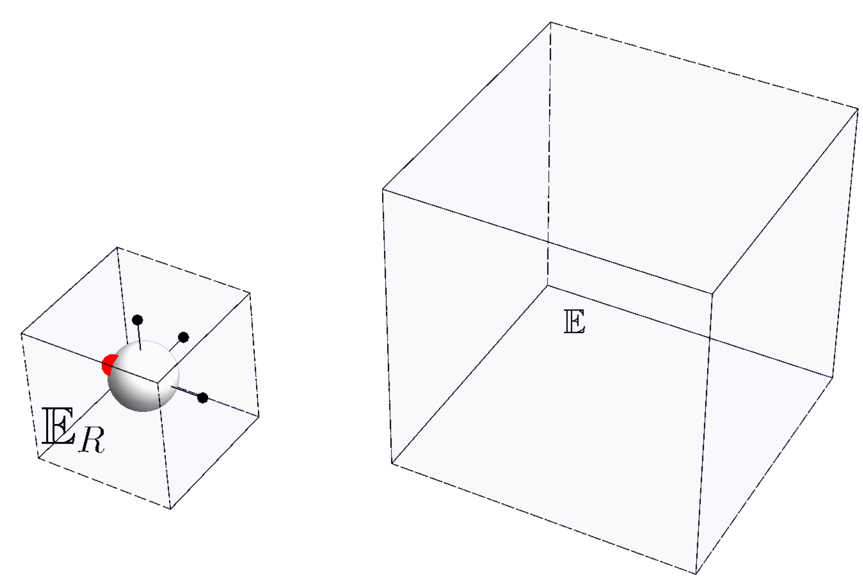


$$\boldsymbol{x}_\tau(\boldsymbol{X}) = \text{position of the material particle } \boldsymbol{X} \text{ in } \mathbb{E} \text{ at the time instance } \tau$$

Deformation mapping

$$\boldsymbol{x}_\tau(\boldsymbol{X}) = \text{position of the material particle in } \mathbb{E} \text{ at the time instance } \tau$$

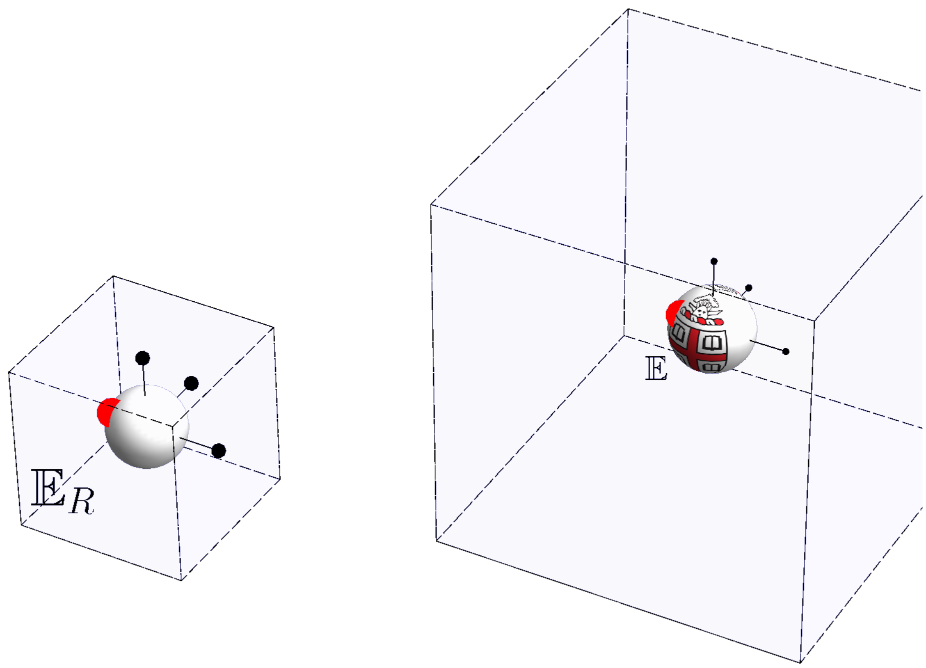
$$X_1, X_2, X_3) = X_1 + U_{1\tau}(X_1, X_2, X_3),$$

$$X_1, X_2, X_3) = X_2 + U_{2\tau}(X_1, X_2, X_3),$$

$$X_1, X_2, X_3) = X_3 + U_{3\tau}(X_1, X_2, X_3).$$

Deformation mapping for non-rotatory motion

τ



Deformation mapping for non-rotatory motion

Deformation map of material particles in the shell/skull

$$x_{1\tau}(X_1, X_2, X_3) = X_1 - c_1(\tau),$$
$$x_{2\tau}(X_1, X_2, X_3) = X_2 - c_2(\tau),$$
$$x_{3\tau}(X_1, X_2, X_3) = X_3 - c_3(\tau).$$

Deformation mapping for non-rotatory motion

Deformation map of material particles in the elastic sphere (brain)

$$
\begin{aligned}
{}_{1\tau}(X_1, X_2, X_3) &= X_1 - c_1(\tau) + u_{\tau 1}(X_1, X_2, X_3 \\
{}_{2\tau}(X_1, X_2, X_3) &= X_2 - c_2(\tau) + u_{\tau 2}(X_1, X_2, X_3 \\
{}_{3\tau}(X_1, X_2, X_3) &= X_3 - c_3(\tau) + u_{\tau 3}(X_1, X_2, X_3
\end{aligned}
$$

Cauchy momentum equations inside the sphere

Governing equation inside the sphere

$$+\mu)u_{\tau k,ki}[(X_j)] + \mu u_{\tau i,kk}[(X_j)] = \rho c_i''(\tau)$$

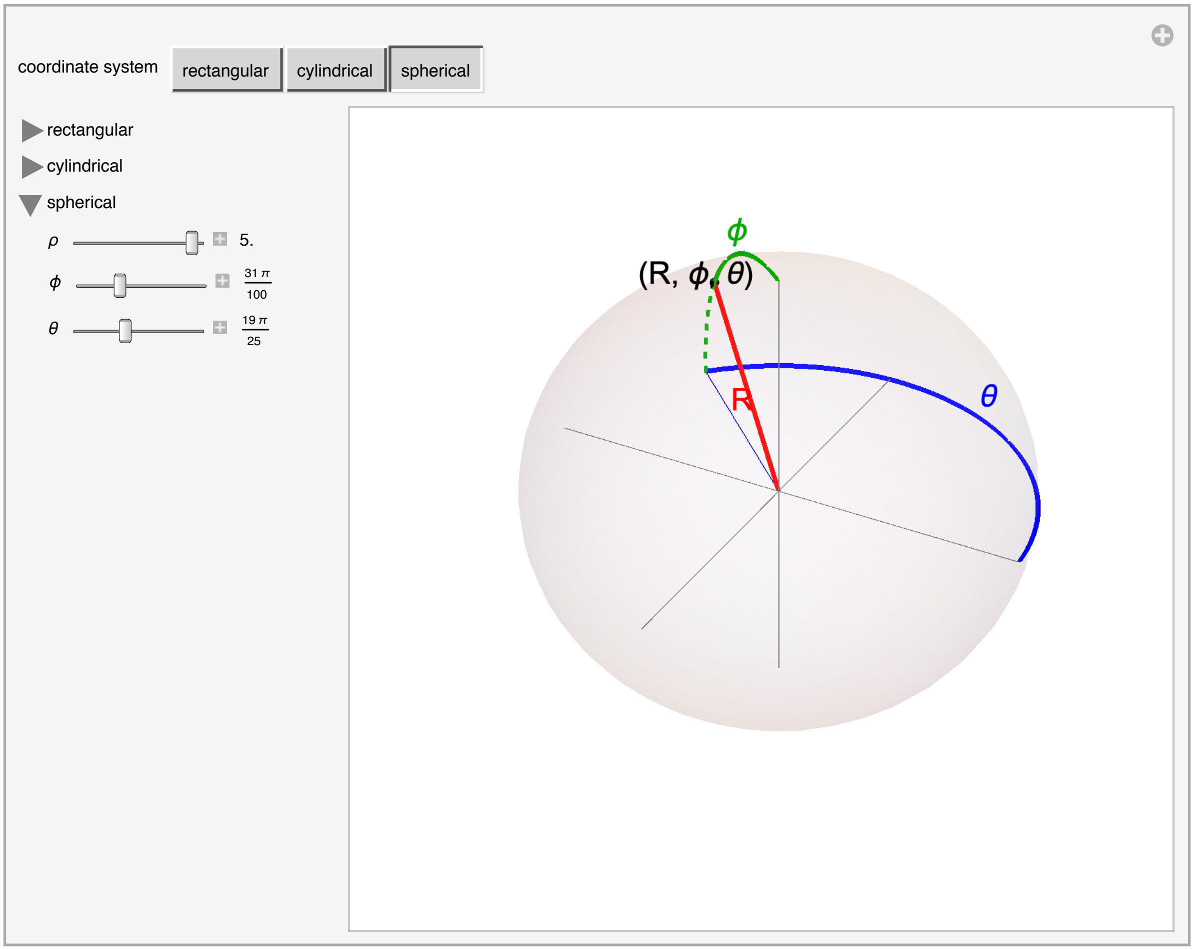
$$\forall\ (X_j) \in$$

Boundary conditions

$$u_{\tau i}[(X_j)] = 0 \qquad\qquad \forall\ (X_j) \in \partial\mathcal{B}_0$$

Solution

Spherical co-ordinates



Solution

Stress Field

$$\left(\begin{array}{ccc} -\frac{\rho R(\lambda+2\mu)\cos(\phi)c''(\tau)}{\lambda+4\mu} & \frac{\mu\rho R\sin(\phi)c''(\tau)}{\lambda+4\mu} & 0 \\ \frac{\mu\rho R\sin(\phi)c''(\tau)}{\lambda+4\mu} & -\frac{\lambda\rho R\cos(\phi)c''(\tau)}{\lambda+4\mu} & 0 \\ 0 & 0 & -\frac{\lambda\rho R\cos}{\lambda+} \end{array}\right.$$

Solution

Strain Field

$$\begin{pmatrix} -\frac{\rho R\cos(\phi)c''(\tau)}{\lambda+4\mu} & \frac{\rho R\sin(\phi)c''(\tau)}{2\lambda+8\mu} & 0 \\ \frac{\rho R\sin(\phi)c''(\tau)}{2\lambda+8\mu} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$