$$
\begin{bmatrix} \sigma_1 \\ \frac{\sigma_2^a+\sigma_2^b}{2} \\ \frac{\sigma_3^a+\sigma_3^b}{2} \\ \frac{\tau_3^a+\tau_3^b}{2} \\ \tau_2 \\ \tau_1 \end{bmatrix} = \begin{bmatrix} d_{11}^e & d_{12}^e & d_{13}^e & d_{14}^e & d_{15}^e & d_{16}^e \\ d_{12}^e & d_{22}^e & d_{23}^e & d_{24}^e & d_{25}^e & d_{26}^e \\ d_{13}^e & d_{23}^e & d_{33}^e & d_{34}^e & d_{35}^e & d_{36}^e \\ d_{14}^e & d_{24}^e & d_{34}^e & d_{44}^e & d_{45}^e & d_{46}^e \\ d_{15}^e & d_{25}^e & d_{35}^e & d_{36}^e & d_{55}^e & d_{56}^e \\ d_{16}^e & d_{26}^e & d_{36}^e & d_{46}^e & d_{56}^e & d_{66}^e \end{bmatrix} \begin{bmatrix} \frac{\epsilon_1^a+\epsilon_1^b}{2} \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_3 \\ \frac{\gamma_2^a+\gamma_2^b}{2} \\ \frac{\gamma_1^a+\gamma_1^b}{2} \end{bmatrix}
$$

$$
\begin{bmatrix} \sigma_1 \\ \sigma_2^a \\ \sigma_3^a \\ \tau_3^a \\ \tau_2 \\ \tau_1 \end{bmatrix} = \begin{bmatrix} d_{11}^a & d_{12}^a & d_{13}^a & d_{14}^a & 0 & 0 \\ d_{12}^a & d_{22}^a & d_{23}^a & d_{24}^a & 0 & 0 \\ d_{13}^a & d_{23}^a & d_{33}^a & d_{34}^a & 0 & 0 \\ d_{14}^a & d_{24}^a & d_{34}^a & d_{44}^a & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{55}^a & d_{56}^a \\ 0 & 0 & 0 & 0 & d_{56}^a & d_{66}^a \end{bmatrix} \begin{bmatrix} \epsilon_1^a \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_3 \\ \gamma_2^a \\ \gamma_1^a \end{bmatrix}
$$

$$
\begin{bmatrix} \sigma_1 \\ \sigma_2^b \\ \sigma_3^b \\ \tau_3^b \\ \tau_2 \\ \tau_1 \end{bmatrix} = \begin{bmatrix} d_{11}^b & d_{12}^b & d_{13}^b & d_{14}^b & 0 & 0 \\ d_{12}^b & d_{22}^b & d_{23}^b & d_{24}^b & 0 & 0 \\ d_{13}^b & d_{23}^b & d_{33}^b & d_{34}^b & 0 & 0 \\ d_{14}^b & d_{24}^b & d_{34}^b & d_{44}^b & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{55}^b & d_{56}^b \\ 0 & 0 & 0 & 0 & d_{56}^b & d_{66}^b \end{bmatrix} \begin{bmatrix} \epsilon_1^b \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_3 \\ \gamma_2^b \\ \gamma_1^b \end{bmatrix}
$$

Expand out:

$$
\begin{aligned}
2\sigma_1 &= d_{11}^e(\epsilon_1^a+\epsilon_1^b) + 2d_{12}^e\epsilon_2 + 2d_{13}^e\epsilon_3 + 2d_{14}^e\gamma_3 + d_{15}^e(\gamma_2^a+\gamma_2^b) + d_{16}^e(\gamma_1^a+\gamma_1^b) && \text{(1a)} \\
\sigma_1 &= d_{11}^a\epsilon_1^a + d_{12}^a\epsilon_2 + d_{13}^a\epsilon_3 + d_{14}^a\gamma_3 && \text{(1b)} \\
\sigma_1 &= d_{11}^b\epsilon_1^b + d_{12}^b\epsilon_2 + d_{13}^b\epsilon_3 + d_{14}^b\gamma_3 && \text{(1c)} \\
(\sigma_2^a+\sigma_2^b) &= d_{12}^e(\epsilon_1^a+\epsilon_1^b) + 2d_{22}^e\epsilon_2 + 2d_{23}^e\epsilon_3 + 2d_{24}^e\gamma_3 + d_{25}^e(\gamma_2^a+\gamma_2^b) + d_{26}^e(\gamma_1^a+\gamma_1^b) && \text{(1d)} \\
\sigma_2^a &= d_{12}^a\epsilon_1^a + d_{22}^a\epsilon_2 + d_{23}^a\epsilon_3 + d_{24}^a\gamma_3 && \text{(1e)} \\
\sigma_2^b &= d_{12}^b\epsilon_1^b + d_{22}^b\epsilon_2 + d_{23}^b\epsilon_3 + d_{24}^b\gamma_3 && \text{(1f)} \\
(\sigma_3^a+\sigma_3^b) &= d_{13}^e(\epsilon_1^a+\epsilon_1^b) + 2d_{23}^e\epsilon_2 + 2d_{33}^e\epsilon_3 + 2d_{34}^e\gamma_3 + d_{35}^e(\gamma_2^a+\gamma_2^b) + d_{36}^e(\gamma_1^a+\gamma_1^b) && \text{(1g)} \\
\sigma_3^a &= d_{13}^a\epsilon_1^a + d_{23}^a\epsilon_2 + d_{33}^a\epsilon_3 + d_{34}^a\gamma_3 && \text{(1h)} \\
\sigma_3^b &= d_{13}^b\epsilon_1^b + d_{23}^b\epsilon_2 + d_{33}^b\epsilon_3 + d_{34}^b\gamma_3 && \text{(1i)} \\
(\tau_3^a+\tau_3^b) &= d_{14}^e(\epsilon_1^a+\epsilon_1^b) + 2d_{24}^e\epsilon_2 + 2d_{34}^e\epsilon_3 + 2d_{44}^e\gamma_3 + d_{45}^e(\gamma_2^a+\gamma_2^b) + d_{46}^e(\gamma_1^a+\gamma_1^b) && \text{(1j)} \\
\tau_3^a &= d_{14}^a\epsilon_1^a + d_{24}^a\epsilon_2 + d_{34}^a\epsilon_3 + d_{44}^a\gamma_3 && \text{(1k)} \\
\tau_3^b &= d_{14}^b\epsilon_1^b + d_{24}^b\epsilon_2 + d_{34}^b\epsilon_3 + d_{44}^b\gamma_3 && \text{(1l)} \\
2\tau_2 &= d_{15}^e(\epsilon_1^a+\epsilon_1^b) + 2d_{25}^e\epsilon_2 + 2d_{35}^e\epsilon_3 + 2d_{45}^e\gamma_3 + d_{55}^e(\gamma_2^a+\gamma_2^b) + d_{56}^e(\gamma_1^a+\gamma_1^b) && \text{(1m)} \\
\tau_2 &= d_{55}^a\gamma_2^a + d_{56}^a\gamma_1^a && \text{(1n)} \\
\tau_2 &= d_{55}^b\gamma_2^b + d_{56}^b\gamma_1^b && \text{(1o)} \\
2\tau_1 &= d_{16}^e(\epsilon_1^a+\epsilon_1^b) + 2d_{26}^e\epsilon_2 + 2d_{36}^e\epsilon_3 + 2d_{46}^e\gamma_3 + d_{56}^e(\gamma_2^a+\gamma_2^b) + d_{66}^e(\gamma_1^a+\gamma_1^b) && \text{(1p)} \\
\tau_1 &= d_{56}^a\gamma_2^a + d_{66}^a\gamma_1^a && \text{(1q)} \\
\tau_1 &= d_{56}^b\gamma_2^b + d_{66}^b\gamma_1^b && \text{(1r)} \\
& && \text{(1s)}
\end{aligned}
$$

By eq. 1(b,c) we have

$$
\epsilon_1^b = A\epsilon_1^a + B\epsilon_2 + C\epsilon_3 + D\gamma_3 \tag{2}
$$

where $A = d_{11}^a/d_{11}^b$, $B = (d_{12}^a - d_{12}^b)/d_{11}^b$, $C = (d_{13}^a - d_{13}^b)/d_{11}^b$, $D = (d_{14}^a - d_{14}^b)/d_{11}^b$.

By eq. 1(n,o,r,s) we have

$$
\begin{bmatrix} \gamma_2^a \\ \gamma_1^a \end{bmatrix} = [N_1]^{-1}[N_2] \begin{bmatrix} \gamma_2^b \\ \gamma_1^b \end{bmatrix} = [N] \begin{bmatrix} \gamma_2^b \\ \gamma_1^b \end{bmatrix} \tag{3}
$$

where

$$
[N_1] = \begin{bmatrix} d_{55}^a & d_{56}^a \\ d_{56}^a & d_{66}^a \end{bmatrix} \qquad [N_2] = \begin{bmatrix} d_{55}^b & d_{56}^b \\ d_{56}^b & d_{66}^b \end{bmatrix} \qquad [N] = \frac{1}{d_{55}^a d_{66}^a - (d_{56}^a)^2} \begin{bmatrix} d_{55}^b d_{66}^a - d_{56}^a d_{56}^b & d_{56}^b d_{66}^a - d_{56}^a d_{66}^b \\ d_{55}^a d_{56}^b - d_{55}^b d_{56}^a & d_{55}^a d_{66}^b - d_{56}^a d_{56}^b \end{bmatrix}
$$

Substitute eq. 2, eq. 3, and eqs. 1(b,c,e,f,h,i,k,l) into eqs. 1(a,d,g,j,m,p), we have

$$\begin{aligned}
&(d^a_{11} + Ad^b_{11})\epsilon^a_1 + (d^a_{12} + d^b_{12} + Bd^b_{11})\epsilon_2 + (d^a_{13} + d^b_{13} + Cd^b_{11})\epsilon_3 + (d^a_{14} + d^b_{14} + Dd^b_{11})\gamma_3 \\
&= (1 + A)d^e_{11}\epsilon^a_1 + (Bd^e_{11} + 2d^e_{12})\epsilon_2 + (Cd^e_{11} + 2d^e_{13})\epsilon_3 + (Dd^e_{11} + 2d^e_{14})\gamma_3 \\
&+ [d^e_{15}(N_{11} + 1) + N_{21}d^e_{16}]\gamma^b_2 + [d^e_{16}(N_{22} + 1) + N_{12}d^e_{15}]\gamma^b_2
\end{aligned}$$

$$\begin{aligned}
&(d^a_{12} + Ad^b_{12})\epsilon^a_1 + (d^a_{22} + d^b_{22} + Bd^b_{12})\epsilon_2 + (d^a_{23} + d^b_{23} + Cd^b_{12})\epsilon_3 + (d^a_{24} + d^b_{24} + Dd^b_{12})\gamma_3 \\
&= (1 + A)d^e_{12}\epsilon^a_1 + (Bd^e_{12} + 2d^e_{22})\epsilon_2 + (Cd^e_{12} + 2d^e_{23})\epsilon_3 + (Dd^e_{12} + 2d^e_{24})\gamma_3 \\
&+ [d^e_{25}(N_{11} + 1) + N_{21}d^e_{26}]\gamma^b_2 + [d^e_{26}(N_{22} + 1) + N_{12}d^e_{25}]\gamma^b_2
\end{aligned}$$

$$\begin{aligned}
&(d^a_{13} + Ad^b_{13})\epsilon^a_1 + (d^a_{23} + d^b_{23} + Bd^b_{13})\epsilon_2 + (d^a_{33} + d^b_{33} + Cd^b_{13})\epsilon_3 + (d^a_{34} + d^b_{34} + Dd^b_{13})\gamma_3 \\
&= (1 + A)d^e_{13}\epsilon^a_1 + (Bd^e_{13} + 2d^e_{23})\epsilon_2 + (Cd^e_{13} + 2d^e_{33})\epsilon_3 + (Dd^e_{13} + 2d^e_{34})\gamma_3 \\
&+ [d^e_{35}(N_{11} + 1) + N_{21}d^e_{36}]\gamma^b_2 + [d^e_{36}(N_{22} + 1) + N_{12}d^e_{35}]\gamma^b_2
\end{aligned}$$

$$\begin{aligned}
&(d^a_{14} + Ad^b_{14})\epsilon^a_1 + (d^a_{24} + d^b_{24} + Bd^b_{14})\epsilon_2 + (d^a_{34} + d^b_{34} + Cd^b_{14})\epsilon_3 + (d^a_{44} + d^b_{44} + Dd^b_{43})\gamma_3 \\
&= (1 + A)d^e_{14}\epsilon^a_1 + (Bd^e_{14} + 2d^e_{24})\epsilon_2 + (Cd^e_{14} + 2d^e_{34})\epsilon_3 + (Dd^e_{14} + 2d^e_{44})\gamma_3 \\
&+ [d^e_{45}(N_{11} + 1) + N_{21}d^e_{46}]\gamma^b_2 + [d^e_{46}(N_{22} + 1) + N_{12}d^e_{45}]\gamma^b_2
\end{aligned}$$

$$\begin{aligned}
&[2d^b_{55} - d^e_{55}(N_{11} + 1) - N_{21}d^e_{56}]\gamma^b_2 + [2d^b_{56} - d^e_{56}(N_{22} + 1) - N_{12}d^e_{55}]\gamma^b_1 \\
&= d^e_{15}(1 + A)\epsilon^a_1 + (Bd^e_{15} + 2d^e_{25})\epsilon_2 + (Cd^e_{15} + 2d^e_{35})\epsilon_3 + (Dd^e_{15} + 2d^e_{45})\gamma_3
\end{aligned}$$

$$\begin{aligned}
&[2d^b_{56} - d^e_{56}(N_{11} + 1) - N_{21}d^e_{66}]\gamma^b_2 + [2d^b_{66} - d^e_{66}(N_{22} + 1) - N_{12}d^e_{56}]\gamma^b_1 \\
&= d^e_{16}(1 + A)\epsilon^a_1 + (Bd^e_{16} + 2d^e_{26})\epsilon_2 + (Cd^e_{16} + 2d^e_{36})\epsilon_3 + (Dd^e_{16} + 2d^e_{46})\gamma_3
\end{aligned}$$

Compare the coefficients of $\epsilon^a_1, \epsilon_2, \epsilon_3, \gamma_3, \gamma^b_2, \gamma^b_1$, we obtain

$$d^e_{11} = \frac{d^a_{11} + Ad^b_{11}}{1 + A} \tag{5a}$$

$$d^e_{12} = \frac{d^a_{12} + Ad^b_{12}}{1 + A} = \frac{d^a_{12} + d^b_{12} + B(d^b_{11} - d^e_{11})}{2} \tag{5b}$$

$$d^e_{13} = \frac{d^a_{13} + Ad^b_{13}}{1 + A} = \frac{d^a_{13} + d^b_{13} + C(d^b_{11} - d^e_{11})}{2} \tag{5c}$$

$$d^e_{14} = \frac{d^a_{14} + Ad^b_{14}}{1 + A} = \frac{d^a_{14} + d^b_{14} + D(d^b_{11} - d^e_{11})}{2} \tag{5d}$$

$$d^e_{22} = \frac{d^a_{22} + d^b_{22} + B(d^b_{12} - d^e_{12})}{2} \tag{5e}$$

$$d^e_{23} = \frac{d^a_{23} + d^b_{23} + C(d^b_{12} - d^e_{12})}{2} = \frac{d^a_{23} + d^b_{23} + B(d^b_{13} - d^e_{13})}{2} \tag{5f}$$

$$d^e_{24} = \frac{d^a_{24} + d^b_{24} + D(d^b_{12} - d^e_{12})}{2} = \frac{d^a_{24} + d^b_{24} + B(d^b_{14} - d^e_{14})}{2} \tag{5g}$$

$$d^e_{33} = \frac{d^a_{33} + d^b_{33} + C(d^b_{13} - d^e_{13})}{2} \tag{5h}$$

$$d^e_{34} = \frac{d^a_{34} + d^b_{34} + D(d^b_{13} - d^e_{13})}{2} = \frac{d^a_{34} + d^b_{34} + C(d^b_{14} - d^e_{14})}{2} \tag{5i}$$

$$d^e_{44} = \frac{d^a_{44} + d^b_{44} + D(d^b_{14} - d^e_{14})}{2} \tag{5j}$$

$$d^e_{55} = 2\frac{d^a_{55}(d^b_{56})^2 + d^b_{55}[(d^a_{56})^2 - d^a_{55}(d^a_{66} + d^b_{66})]}{(d^a_{56} + d^b_{56})^2 - (d^a_{55} + d^b_{55})(d^a_{66} + d^b_{66})} \tag{5k}$$

$$d^e_{56} = \frac{2d^a_{56}d^b_{56}(d^a_{56} + d^b_{56}) - 2d^a_{55}d^b_{56}d^a_{66} - 2d^b_{55}d^a_{56}d^b_{66}}{(d^a_{56} + d^b_{56})^2 - (d^a_{55} + d^b_{55})(d^a_{66} + d^b_{66})} \tag{5l}$$

$$d^e_{66} = 2\frac{d^a_{66}(d^b_{56})^2 + d^b_{66}[(d^a_{56})^2 - d^a_{66}(d^a_{55} + d^b_{55})]}{(d^a_{56} + d^b_{56})^2 - (d^a_{55} + d^b_{55})(d^a_{66} + d^b_{66})} \tag{5m}$$

The other terms are zero. By simplification, we have

$$d_{11}^e = \frac{2d_{11}^a d_{11}^b}{d_{11}^a + d_{11}^b} \tag{6a}$$

$$d_{12}^e = \frac{d_{11}^a d_{12}^b + d_{11}^b d_{12}^a}{d_{11}^a + d_{11}^b} \tag{6b}$$

$$d_{13}^e = \frac{d_{11}^a d_{13}^b + d_{11}^b d_{13}^a}{d_{11}^a + d_{11}^b} \tag{6c}$$

$$d_{14}^e = \frac{d_{11}^a d_{14}^b + d_{11}^b d_{14}^a}{d_{11}^a + d_{11}^b} \tag{6d}$$

$$d_{22}^e = \frac{d_{22}^a + d_{22}^b}{2} \tag{6e}$$

$$d_{23}^e = \frac{d_{12}^a(d_{13}^b - d_{13}^a) + d_{12}^b(d_{13}^a - d_{13}^b) + (d_{11}^a + d_{11}^b)(d_{23}^a + d_{23}^b)}{2(d_{11}^a + d_{11}^b)} \tag{6f}$$

$$d_{24}^e = \frac{d_{12}^a(d_{14}^b - d_{14}^a) + d_{12}^b(d_{14}^a - d_{14}^b) + (d_{11}^a + d_{11}^b)(d_{24}^a + d_{24}^b)}{2(d_{11}^a + d_{11}^b)} \tag{6g}$$

$$d_{33}^e = \frac{d_{33}^a + d_{33}^b}{2} \tag{6h}$$

$$d_{34}^e = \frac{d_{13}^a(d_{14}^b - d_{14}^a) + d_{13}^b(d_{14}^a - d_{14}^b) + (d_{11}^a + d_{11}^b)(d_{34}^a + d_{34}^b)}{2(d_{11}^a + d_{11}^b)} \tag{6i}$$

$$d_{44}^e = \frac{d_{44}^a + d_{44}^b}{2} \tag{6j}$$

$$d_{55}^e = 2\frac{d_{55}^a(d_{56}^b)^2 + d_{55}^b[(d_{56}^a)^2 - d_{55}^a(d_{66}^a + d_{66}^b)]}{(d_{56}^a + d_{56}^b)^2 - (d_{55}^a + d_{55}^b)(d_{66}^a + d_{66}^b)} \tag{6k}$$

$$d_{56}^e = \frac{2d_{56}^a d_{56}^b(d_{56}^a + d_{56}^b) - 2d_{55}^a d_{56}^b d_{66}^a - 2d_{55}^b d_{56}^a d_{66}^b}{(d_{56}^a + d_{56}^b)^2 - (d_{55}^a + d_{55}^b)(d_{66}^a + d_{66}^b)} \tag{6l}$$

$$d_{66}^e = 2\frac{d_{66}^a(d_{56}^b)^2 + d_{66}^b[(d_{56}^a)^2 - d_{66}^a(d_{55}^a + d_{55}^b)]}{(d_{56}^a + d_{56}^b)^2 - (d_{55}^a + d_{55}^b)(d_{66}^a + d_{66}^b)} \tag{6m}$$