THE STRESS INTENSITY FACTOR FOR A PENNY-SHAPED CRACK IN AN ELASTIC BODY UNDER THE ACTION OF SYMMETRIC BODY FORCES

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ABSTRACT

A formula is derived for the stress intensity factor at the rim of a penny-shaped crack in an infinite solid in which there is an axisymmetric distributing of body forces acting in a direction normal to the original crack surfaces. An expression for the surface displacement of the crack is also given. The use of these formulae is illustrated by a consideration of the special case in which the solid is deformed by the action of two point forces situated symmetrically with respect to the crack.

INTRODUCTION

In [1] a method was given of determining, by a simple process, the stress intensity factor for a Griffith crack in a solid acted upon by a symmetrical distribution of body forces. The object of the present note is to show how the same method may be applied to the problem of calculating the stress intensity factor for a penny-shaped crack $0 \leq \rho < a$, $z = 0$ in an infinite solid in which there is a distribution $Z(\rho, z)$ of body forces acting in the z-direction, the function $Z(\rho, z)$ being an odd function of z.

The general solution is derived in § 4, and formulae given for the stress intensity factor at the rim of the crack and for the normal component $u_z(\rho, 0)$ of the displacement of the crack surface. The use of these results in a special case is illustrated in § 4.

RESOLUTION OF THE PROBLEM INTO TWO COMPONENT PROBLEMS

The problem we are considering is that of solving the equations of elastic equilibrium in the presence of a distribution of body forces parallel to the z-axis $Z(\rho, z)$, subject to the boundary conditions

$$\sigma_{\rho z}(\rho, 0) = 0, \qquad 0 < \rho < \infty, \tag{2.1}$$

$$\sigma_{zz}(\rho, 0) = 0, \qquad 0 \leq \rho \leq a, \tag{2.2}$$

$$u_z(\rho, 0) = 0, \qquad \rho > a, \tag{2.3}$$

and to the condition that the components of the stress tensor tend to zero as $r = \sqrt{(\rho^2 + z^2)} \to \infty$. Here we are taking the penny-shaped crack to be given by the relations $\rho \leq a$, $z = 0$.

The solution of this problem may be obtained as the sum of the solutions of the following pair of problems: -

Problem 1: The problem here is to find the solution $u_\rho^{(1)}(\rho, z)$, $u_z^{(1)}(\rho, z)$ of the axisymmetric equations of elastic equilibrium, in the presence of the prescribed body force $Z(\rho, z)$, assumed to be *odd* in z, which satisfies the conditions at infinity and the conditions

$$\sigma_{\rho z}^{(1)}(\rho, 0) = u_z^{(1)}(\rho, 0) = 0, \qquad 0 \leq \rho \leq \infty$$

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on the z-plane. For this solution we can calculate the normal stress $p(\rho)$ on the z-plane: -

$$\sigma^{(1)}_{zz}(\rho, 0) = p(\rho) \tag{2.4}$$

Problem 2: The second problem is that of determining the displacement field $[u^{(2)}_\rho(\rho, z), u^{(2)}_z(\rho, z)]$ in the half-space $z \geq 0$ in the absence of body forces when the boundary plane is subjected to the boundary conditions

$$\sigma^{(2)}_{\rho z}(\rho, 0) = 0 \qquad 0 \leq \rho < \infty, \tag{2.5}$$

$$\sigma^{(2)}_{zz}(\rho, 0) = -p(\rho) \qquad 0 \leq \rho \leq a, \tag{2.6}$$

$$u^{(2)}_z(\rho, 0) = 0, \qquad \rho > a. \tag{2.7}$$

The solution of *Problem 2* for an arbitrary axisymmetric loading function $p(\rho)$ has been derived by Sneddon [2] - see also pp. 487 - 491 of [3] - so that the solution of the present problem can be drived merely by finding a solution of *Problem 1*. In the next section we demonstrate a formal solution of this problem.

In the application of calculations of this kind to discussions in fracture mechanics attention is focussed not so much on the form of either the displacement field or of the stress field but on the numerical values of certain constants of physical significance. In [4] it is shown that one of these constants, the stress intensity factor defined by the equation

$$K = \lim_{\rho \to 1+} \surd(\rho - 1)\sigma_{zz}(\rho, 0),$$

can be derived directly from the form of the function $p(\rho)$ through the formula

$$K = \frac{1}{\pi}\sqrt{\frac{2}{a}} \int_0^a \frac{\rho p(\rho) d\rho}{\surd(a^2 - \rho^2)} \tag{2.8}$$

In the same paper it is also shown that the shape of the penny-shaped crack is given in terms of the function $p(\rho)$ by the double integral

$$u_z(\rho, 0) = \frac{4(1 - \eta^2)}{\pi E} \int_\rho^a \frac{du}{\surd(u^2 - \rho^2)} \int_0^u \frac{x p(x) dx}{\surd(u^2 - x^2)}, \quad 0 \leq \rho \leq a \tag{2.9}$$

To calculate the value of the constant K and the form of the normal component of the surface displacement $u(\rho, 0)$ we need only know the form of the function $p(\rho)$. In the next section we use the solution of Problem 1 to calculate the function $p(\rho)$ corresponding to a prescribed body force $Z(\rho, z)$ and hence to calculate K and $u_2(\rho, 0)$ through equations (2.8) and (2.9) respectively.

THE SOLUTION OF PROBLEM 1

In the case of a symmetrical deformation the components of the displacement vector are $[u_\rho(\rho, z), 0, u_z(\rho, z)]$, u_ϕ being identically zero and u_ρ and u_z independent of ϕ. We shall denote by $u^*_\rho(\xi, \zeta)$ the transform of the radial component

$$u^*_p(\xi, \zeta) = \mathfrak{F}\left[\mathscr{H}_1\left\{u_p(\rho, z); \rho \to \xi\right\}; z \to \zeta\right] \tag{3.1}$$

where $\mathfrak{F}$ and $\mathscr{H}_1$ denote, respectively, the operators of the Fourier transform and of the Hankel transform of order unity, so that

$$u^*_p(\xi, \zeta) = \frac{1}{\surd(2\pi)} \int_{-\infty}^{\infty} dz \int_0^\infty \rho u_p(\rho, z) J_1(\xi\rho) e^{i\zeta z} d\rho$$

In a similar way we denote by $u^*_z(\rho, \zeta)$ the transform

$$u_z^*(\xi, \zeta) = \mathfrak{F}\left[\mathcal{H}_0\left\{u_z(\rho, z); \rho \to \xi\right\}; z \to \zeta\right] \tag{3.2}$$

of the z-component of the displacement, $\mathcal{H}_0$ denoting the operator of the Hankel transform of order zero, so that

$$u_z^*(\xi, \zeta) = \frac{1}{\sqrt{(2\pi)}} \int_{-\infty}^{\infty} dz \int_0^{\infty} \rho u_z(\rho, z) J_0(\xi\rho) e^{i\zeta z}\, d\rho$$

If we apply the operator $\mathfrak{F}\mathcal{H}_1$ to both sides of the first equation of elastic equilibrium

$$\beta^2\left(\frac{\partial^2 u_\rho}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial u_\rho}{\partial\rho} - \frac{u_\rho}{\rho^2}\right) + (\beta^2 - 1)\frac{\partial^2 u_z}{\partial\rho\partial z} + \frac{\partial^2 u_\rho}{\partial z^2} = 0$$

in which β is defined in terms of Poisson's ratio by the equation

$$\beta^2 = \frac{2(1 - \eta)}{1 - 2\eta} \tag{3.3}$$

and make use of well-known properties of the Fourier and Hankel transforms we find that it is equivalent to the simple algebraic equation

$$(\beta^2\xi^2 + \zeta^2)u_\rho^* - i(\beta^2 - 1)\xi\zeta u_z^* = 0. \tag{3.4}$$

Similarly if we apply the operator $\mathfrak{F}\mathcal{H}_0$ to both sides of the second equation of elastic equilibrium

$$\frac{\partial^2 u_z}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial u_z}{\partial\rho} + (\beta^2 - 1)\frac{\partial}{\partial z}\left(\frac{\partial u_\rho}{\partial\rho} + \frac{u_\rho}{\rho}\right) + \beta^2\frac{\partial^2 u_z}{\partial z^2} + \frac{2(1 + \eta)\sigma}{E} Z(\rho, z) = 0,$$

in which E denotes Young's modulus and σ the density of the solid, we find that it is equivalent to the algebraic equation

$$i(\beta^2 - 1)\xi\zeta u_\rho^* + (\xi^2 + \beta^2\zeta^2)u_z^* = (\sigma/\mu)Z^*(\xi, \zeta) \tag{3.5}$$

where

$$Z^*(\xi, \zeta) = \mathfrak{F}\left[\mathcal{H}_0\left\{Z(\rho, z)\ ;\ \rho \to \xi\right\}\ ;\ z \to \zeta\right] \tag{3.6}$$

The solution of the equations (3.3) and (3.5) is easily found to be

$$u_\rho^* = \frac{(1 + \eta)(1 - 2\eta)\sigma}{(1 - \eta)E} \cdot \frac{(\beta^2 - 1)i\xi\zeta}{(\xi^2 + \zeta^2)^2} Z^*(\xi, \zeta), \tag{3.7}$$

$$u_z^* = \frac{(1 + \eta)(1 - 2\eta)\sigma}{(1 - \eta)E} \cdot \frac{\beta^2\xi^2 + \zeta^2}{(\xi^2 + \zeta^2)^2} Z^*(\xi, \zeta). \tag{3.8}$$

If we now apply the operator $\mathfrak{F}\mathcal{H}_1$ to both sides of the stress-strain equation

$$\sigma_{\rho z} = \frac{E}{2(1 + \eta)}\left(\frac{\partial u_\rho}{\partial z} + \frac{\partial u_z}{\partial\rho}\right)$$

we find that the transform

$$\sigma^*_{\rho z} = \mathfrak{F}\left[\mathcal{H}_1\{\sigma_{\rho z}(\rho, z) \,;\, \rho \to \xi\} \,;\, z \to \zeta\right] \tag{3.9}$$

is related to the transforms u^*_ρ, u^*_z through the equation

$$\sigma^*_{\rho z} = -\frac{E}{2(1+\eta)}(\xi u^*_z + i\xi u^*_\rho) \tag{3.10}$$

Similarly if we apply the operator $\mathfrak{F}\mathcal{H}_0$ to both sides of the stress-strain equation

$$\sigma_{zz} = \frac{E}{2(1+\eta)}\left[(\beta^2 - 2)\left(\frac{\partial u_\rho}{\partial \rho} + \frac{u_\rho}{\rho}\right) + \beta^2 \frac{\partial u_z}{\partial z}\right] \tag{3.11}$$

we see that it is equivalent to the equation

$$\sigma_{zz} = \frac{E}{2(1+\eta)}\left[(\beta^2 - 2)\xi u^*_\rho - i\beta^2 \zeta u^*_z\right] \tag{3.12}$$

where σ^*_{zz} denotes the transform

$$\sigma^*_{zz} = \mathfrak{F}\left[\mathcal{H}_0\left\{\sigma_{zz}(\rho, z) \,;\, \rho \to \xi\right\} \,;\, z \to \zeta\right]. \tag{3.13}$$

If we substitute from equations (3.7) and (3.8) into equations (3.10) and (3.12) we find that the transform of the stress components $\sigma_{\rho z}$, σ_{zz} are given in terms of the transform of the body force $Z(\rho, z)$ by the pair of equations

$$\sigma^*_\rho = -\frac{(1-2\eta)\sigma}{2(1-\eta)} \cdot \frac{\xi\left[\beta^2\zeta^2 - \beta^2 - 2)\zeta^2\right]}{(\xi^2 + \zeta^2)^2} Z^*(\xi, \zeta) \tag{3.14}$$

$$\sigma^*_{zz} = -\frac{(1-2\eta)\sigma}{2(1-\eta)} \cdot \frac{i\xi\left[3\beta^2 - 2)\xi^2 + \beta^2\zeta^2\right]}{(\xi^2 + \zeta^2)^2} Z^*(\xi, \zeta) \tag{3.15}$$

We now make the assumption that $Z(\rho, z)$ is an *odd* function of z, so that

$$Z^*(\xi, \zeta) = iZ^*_s(\xi, \zeta) \tag{3.16}$$

where in terms of $\mathfrak{F}_s$, the operator of the Fourier sine transform,

$$Z^*_s(\xi, \zeta) = \mathcal{H}_0\left[\mathfrak{F}_s\left\{Z^+(\rho, z) \,;\, z \to \zeta\right\} \,;\, \rho \to \xi\right] \tag{3.17}$$

with $Z^+(\rho, z) = Z(\rho, z)H(z)$,

$$Z_s(\xi, \zeta) = \sqrt{\frac{2}{\pi}} \int_0^\infty \int_0^\infty \rho Z^+(\rho, z) J_0(\xi\rho)\sin(\zeta z)\, d\rho\, dz \tag{3.18}$$

We deduce immediately that $Z^+_s(\xi, \zeta)$ is an *odd* function of ζ.

If now we substitute from equation (3.16) into equations (3.8), (3.14), (3.15) and invert the resulting expressions by use of the Fourier inversion theorem and the Hankel inversion theorem we obtain the formulae

$$u_z(\rho, z) = \frac{(1+\eta)(1-2\eta)\sigma}{(1-\eta)E}\sqrt{\frac{2}{\pi}}\int_0^\infty \xi J_0(\xi\rho)d\xi \int_0^\infty \frac{(\beta^2\xi^2+\zeta^2)}{(\xi^2+\zeta^2)^2} Z_s^*(\xi,\zeta)\sin(\zeta z)d\zeta$$

$$\sigma_{\rho z}(\rho, z) = -\frac{(1-2\eta)\sigma}{2(1-\eta)}\sqrt{\frac{2}{\pi}}\int_0^\infty \xi^2 J_1(\xi\rho)d\xi\int_0^\infty \frac{\left\{\beta^2\xi^2 - (\beta^2-2)\xi^2\right\}}{(\xi^2+\zeta^2)^2} Z_s^*(\xi,\zeta)\sin(\zeta z)d\zeta$$

$$\sigma_{zz}(\rho, z) = \frac{(1-2\eta)\sigma}{2(1-\eta)}\sqrt{\frac{2}{\pi}}\int_0^\infty \xi J_0(\xi\rho)d\xi \int_0^\infty \frac{\zeta\left\{(3\beta^2-2)\xi^2+\beta^2\zeta^2\right\}}{(\xi^2+\zeta^2)^2} Z_s(\xi,\zeta)d\zeta$$

from which we deduce immediately that

$$u_z(\rho, 0) = 0, \qquad \sigma_{\rho z}(\rho, 0) = 0, \qquad \sigma_{zz}(\rho, 0) = p(\rho)$$

where

$$p(\rho) = \frac{(1-2\eta)\sigma}{2(1-\eta)}\sqrt{\frac{2}{\pi}}\int_0^\infty \xi J_0(\xi\rho)d\xi \int_0^\infty \frac{\zeta\left\{(3\beta^2-2)\xi^2+\beta^2\zeta^2\right\}}{(\xi^2+\zeta^2)^2} Z_s^*(\xi,\zeta)d\zeta \tag{3.19}$$

since

$$\int_0^t \frac{\rho J_0(\xi\rho)d\rho}{\surd(t^2-\rho^2)} = \frac{\sin(\xi t)}{\xi}$$

it follows at once that

$$\int_0^t \frac{\rho p(\rho)d\rho}{\surd(t^2-\rho^2)} = \frac{(1-2\eta)\sigma}{2(1-\eta)} P(t) \tag{3.20}$$

where

$$P(t) = \sqrt{\frac{2}{\pi}}\int_0^\infty \sin(\xi t)d\xi \int_0^\infty \frac{\zeta\left\{(3\beta^2-2)\xi^2+\beta^2\zeta^2\right\}}{(\xi^2+\zeta^2)^2} Z_s^*(\xi,\zeta)d\zeta \tag{3.21}$$

If we use the fact that

$$\sqrt{\frac{2}{\pi}}\frac{\zeta\left\{(3\beta^2-2)\xi^2+\beta^2\zeta^2\right\}}{(\xi^2+\zeta^2)^2} = \mathfrak{F}_s\left[\left\{\beta^2+(\beta^2-1)\xi z\right\}e^{-\xi z};\ z\to\zeta\right]$$

and that if $Z^+(\rho, z) = Z(\rho, z)H(z)$,

$$Z_s^*(\xi,\zeta) = \mathfrak{F}_s\left[\mathcal{H}_0\left\{Z^+(\rho,z);\ \rho\to\xi\right\};\ z\to\zeta\right]$$

we see that the Parseval relation for sine transform gives the relation

$$\sqrt{\frac{2}{\pi}}\int_0^\infty \frac{\zeta\left\{(3\beta^2 - 2)\xi^2 + \beta^2\zeta^2\right\}}{(\xi^2 + \zeta^2)^2} Z_s^*(\xi,\zeta)d\zeta$$

$$= \int_0^\infty \left\{\beta^2 + (\beta^2 - 1)\xi z\right\} e^{-\xi z}\mathcal{H}_0\left\{Z^+(\rho,z)\, ;\, \rho \to \xi\right\}dz$$

$$= \int_0^\infty\int_0^\infty \left\{\beta^2 + (\beta^2 - 1)\xi z\right\} e^{-\xi z}\ Z^+(\rho,z)J_0(\xi\rho)\rho d\rho dz$$

so that the equation (3.21) may be written in the form

$$P(t) = \int_0^\infty\int_0^\infty \rho Z^+(\rho,z)\Pi(\rho,z,t)d\rho dz \tag{3.22}$$

where

$$\Pi(\rho,z,t) = \int_0^\infty \left\{\beta^2 + (\beta^2 - 1)\xi z\right\} e^{-\xi z} J_0(\xi\rho)\sin(\xi t)d\xi \tag{3.23}$$

The integral (3.23) is easily evaluated.

$$\Pi(\rho,z,t) = \beta^2\frac{\sin\theta}{R} + (\beta^2 - 1)(2\sin 3\theta - t\cos 3\theta)\frac{z}{R^3} \tag{3.24}$$

where

$$R^4 = (\rho^2 + z^2 - t^2)^2 + 4z^2t^2$$

$$\tan 2\theta = 2zt/(\rho^2 + z^2 - t^2)$$

(cf. [5] where, in addition, numerical tables are given).

It should also be noted that

$$\Pi(0,z,t) = \int_0^\infty \left\{\beta^2 + (\beta^2 - 1)\xi z\right\} e^{-\xi z}\ \sin(\xi t)d\xi$$

The integrations are elementary and we find that

$$\Pi(0,z,t) = \left\{(3\beta^2 - 2)z^2 + \beta^2t^2\right\}\frac{t}{(z^2 + t^2)^2} \tag{3.25}$$

THE EFFECT OF TWO POINT FORCES SYMMETRICALLY PLACED

As an example of the use of these formulae we consider the effect on the crack $0 \le \rho \le a$, $z = 0$ of two point forces of magnitude F acting at the points $(0,0,\pm h)$ in the positive and negative $\underline{z}$-directions respectively (cf. Fig. 1). In this case we have

$$Z(\rho,z) = \frac{F}{2\pi\rho\sigma}\delta(\rho)\left\{\delta(z - h) - \delta(z + h)\right\},\ h > 0$$

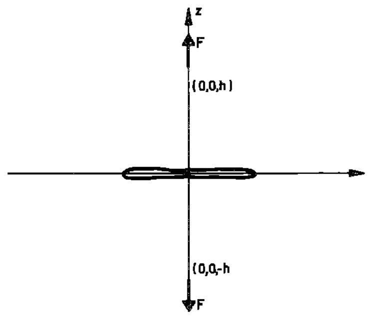

Fig. 1

so that we may write

$$Z^{+}(\rho, z) = \frac{F}{2\pi\rho\sigma}\,\delta(\rho)\delta(z - h), \quad (h > 0), \tag{4.1}$$

and hence from equation (3.22) we deduce that

$$P(t) = \frac{F}{2\pi\sigma}\,\Pi(0, h, t).$$

Making use of the result (3.25) we see that in this instance

$$P(t) = \frac{F}{2\pi\sigma}\left\{(3\beta^2 - 2)h^2 + \beta^2 t^2\right\}\frac{t}{(h^2 + t^2)^2} \tag{4.2}$$

From equations (2.8), (3.20) we deduce that for this distribution of body forces the stress intensity factor K is given by the formulae

$$K = \frac{F}{2\pi^2}\sqrt{(2a)}\cdot\frac{\beta^2 a^2 + (3\beta^2 - 2)h^2}{(a^2 + h^2)^2} \tag{4.3}$$

The variation of K with h/a in the case in which Poisson's ratio η has the value 0.25 is shown in Fig. 2.

Also from equations (2.9) and (4.2) we deduce that

$$u_z(\rho, 0) = \frac{F}{\pi^2 E}(1 + \eta)(1 - 2\eta)\int_\rho^a \frac{\left\{(3\beta^2 - 2)h^2 + \beta^2 u^2\right\}u\,du}{(h^2 + u^2)^2\sqrt{(u^2 - \rho^2)}}, \quad 0 \le \rho \le a$$

Now it is easily shown that

$$\int_\rho^a \frac{u\,du}{(h^2 + u^2)^2\sqrt{(u^2 - \rho^2)}} = \frac{1}{\sqrt{(h^2 + \rho^2)}}\tan^{-1}\left[\frac{a^2 - \rho^2}{H^2 + \rho^2}\right]^{\frac{1}{2}}$$

$$2\int_\rho^a \frac{u\,du}{(h^2 + u^2)^2\sqrt{(u^2 - \rho^2)}} = \frac{1}{(h^2 + \rho^2)^{3/2}}\tan^{-1}\left[\frac{a^2 - \rho^2}{h^2 + \rho^2}\right]^{\frac{1}{2}} + \frac{(a^2 - \rho^2)^{\frac{1}{2}}}{(h^2 + a^2)(h^2 + \rho^2)}$$

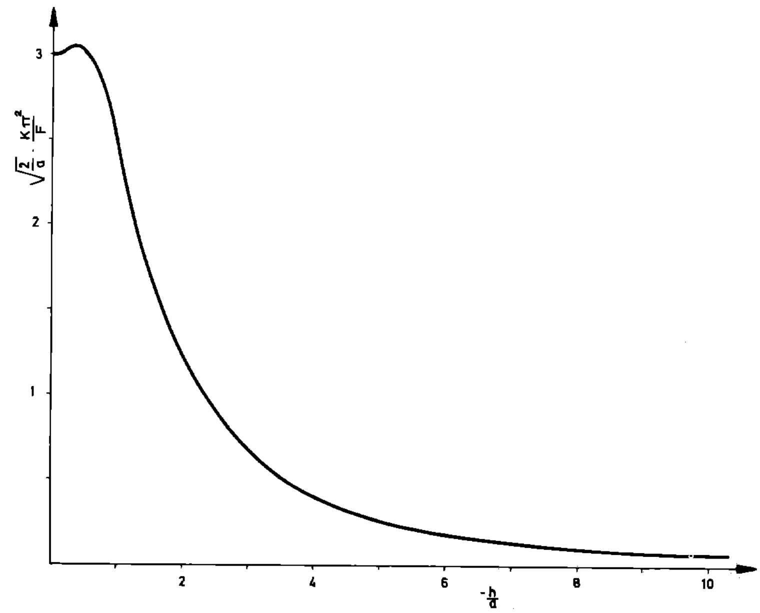

Fig. 2.

so that

$$u_z(\rho, 0) = \frac{F(1 + \eta)}{\pi^2 Ea} U(k/a, \ \rho/a)$$

where the function $U(\theta, x)$ is defined by the equation

$$U(\theta, x) = \frac{(3 - 2\eta)\theta^2 + 2(1 - \eta)x^2}{(\theta^2 + x^2)^{3/2}} \tan^{-1}\left[\frac{1 - x}{\theta^2 + x^2}\right]^{\frac{1}{2}} + \frac{\theta^2(1 - x^2)^{\frac{1}{2}}}{(\theta^2 + 1)(\theta^2 + x^2)} \tag{4.4}$$

or alternatively by the equation

$$U(\theta, x) = \frac{1}{x_1}\left[\left\{2(1 - \eta) + \frac{\theta^2}{x_{\frac{1}{2}}}\right\}\alpha + \frac{\theta^2}{\theta^2 + 1} \tan \alpha\right], \tag{4.5}$$

with x_1 and α defined by the relations

$$x_1 = \sqrt{(\theta^2 + x^2)}, \qquad \alpha = \tan^{-1}\left[\frac{\sqrt{(1 - x^2)}}{x_1}\right] \tag{4.6}$$

The variation of $u_z(\rho, 0)$ with ρ for three values of h and for the values 0.25 of Poisson's ratio is shown in Fig. 3.

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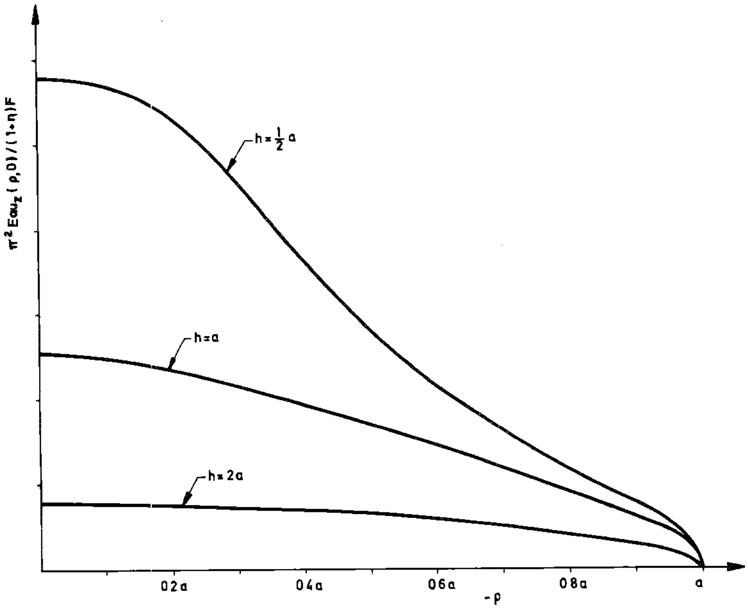

Fig. 3

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RÉSUMÉ - On a établi une formule donnant le facteur de concentration de tension aux extrémités d'une fissure fermée disposée dans un solide infini au sein duquel une distribution de forces internes à symétrie axiale agit dans une direction normale par rapport aus surfaces de la fissure. On fournit également une expression du déplacement de ces surfaces. L'utilisation de ces formules est appliquée, à titre d'exemple, au cas spécial d'un solide soumis à l'action de deux forces concentrées symétriques par rapport à la fissure.

ZUSAMMENFASSUNG - Eine Formel für den Spannungsintensitätfaktor am Rande eines pfenninggeformten Risses in einem unendlichen Festkörper ist gewonnen. Ein achsensymmetrisches Verteilen der Körperkräfte fand statt, welches in einer Richtung, normal zu der originalen Rissoberfläche wirkt. Es ist auch Ausdruck für den Oberflächenverschiebung des Risses gegeben. Die Benutzung dieser Gleichungen wird verdeutlicht durch die Betrachtung eines Spezialfalles bei dem der Festkörper durch die Wirkung zweier Punktkräfte deformiert wird, die symmetrisch zum Riss angebracht sind.