The recursive equations of *K*n is



We can show that (in *NoSlip\_MultiMaterial-Sept30.nb* file)



Therefore,



and

.

(I)+(II) gives:

 (1)

(I)+(III) gives:

 (2)

(I)-(II) gives:

 (3)

(I)-(III) gives:

 (4)

Furthermore,

(1)+(2) gives:



Neglect the  terms, it can be simplified as:

 (5)

(1)-(2) gives:

 (6)

(3)+(4) gives:

 (7)

(3)-(4) gives:

 (8)

We see that eqs.(6) and (8) are the same. Basically we want to find the relationship between  and . To do this, multiply eq.(7) by  we get

 (9)

Then (6)-(9) we have

 (10)

From (5) we know

 (11)

Substitute eq.(11) into eq.(10) we obtain

 (12)

Equivalently,

 (13)

Therefore we obtain

 (14a)

**Suppose that** (which will be verified soon)

 (14b)

Then from eq.(14) we can get the following form between between  and 

 (14c)

From eqs.(I) and (II), we can obtain

 (15)

And we also show that (in *MultiMaterial\_Noslip-Sept30.nb* file)

 (16)

Substitute eq.(16) into eq.(15),

 (17)

Substitute eq.(14c) into eq.(17), neglecting the  terms, we have

 (18)

From eq.(18), we can solve for  asymptotically.

We can show that



where *M*ab only depends on *M*a and *M*b. By this property, we can approximate *M*1~*M*4 as following.

 (19)

Therefore,

 (20)

and

 (21)

Comparing eqs.(21) and (14b), we get

 (22)

Substitute eq.(22) into eq.(18),

 (23)

Since (see *NoSlip\_MultiMaterial-Sept30.nb* file)







Hence eq.(23) can be simplified by taking  and keeping the first order of 

 (24)

Since

 (25)

Hence

 (26)

Let



We find that

 (27)

Therefore, we have



which can simplified as

 (28)

The eq.(28) can be written as

 (29)

where

 (30)

From eq.(16), we have

 (31)

and

 (32)

Therefore,

 (33)

Then eq.(30) can be written as

(34)

Finally, we arrive at

 (35)

