The recursive equations of *K*n is



Then we have

 (1)

We can show that (in *NoSlip\_MultiMaterial-Oct09.nb* file)



where



Let



We find that (in *NoSlip\_MultiMaterial-Oct01.nb* file)









Substitute into eq. (1)



Suppose



then



Simplified as

then

(2a, b)

Then eliminate 



which is trivial since , indicating that eqs. 2(a) and (b) are the same to the first order.

Since, let  we have



where

 .

Taking  to the first order of 









Then we have



Let , then

 (3)

which has the form



Since ,



which is a linear ODE system of , with boundary conditions



Re-write eq. (3) as



Let



and



then we have

 (4)

which has the form

 (5)

for *n* = 2, 3, …, *N*. This is a linear ODE system of ,  with boundary conditions



As a reminder,



which are only depend on the material properties and constant for each layer. If the material properties are the same for the two layers, then  and , thus from eq. (4) we have , i.e., , indicating that  are the same for all the layers, which is true for homogeneous materials.

Since  is constant for each layer, it can be diagonal zed as , where and  and  are the eigenvalues and eigenvectors matrixes, respectively. Then eq. (4) becomes

 (6)

Let , , then

 (7)

The solution of eq. (7) is

 (7)

Then  and  for *n* = 2, 3, …, *N*. The 4 constants *c*i is determined by the boundary conditions. Recall , , and  . Substitute  and  into the boundary conditions to determine *c*i.

For the 1, 3, 5,…, 2*N*-1 layers,  and  are the same; and for the 2, 4, 6,…, 2*N* layers,  and  are also the same.

Solving eq. (3) or (4) gives . Then we can compute the effective bending stiffness. Recall that the effective bending stiffness is

 (5)

Taking the Taylor expansion of (*II*) and (*III*) around  gives



Then eq. (5) becomes

 (6)

Since the layer arrangement is periodic, then we can re-write eq. (6) as



Let , we obtain EI as

 (7)