The finite element implementation of regularized variational fracture model is benchmarked for a single edge notch tension geometry subjected to far-field loading σ^∞ as shown in figure. 1(A). For a prescribed value of energy release rate G_c, the critical stress σ_{cr}^∞ at which crack initiates is given by,

$$\sigma_{cr}^\infty = \sqrt{\frac{E^* G_c}{2b \tan\left(\frac{\pi a}{2b}\right)}} C\left(\frac{a}{b}\right)^{-1},$$

where E^* is the plane strain Youngs modulus, a is the length of the notch, b is the width of the geometry and $C\left(\frac{a}{b}\right)$ is the compliance. The above equation is non-dimensionalized by introducing non-dimensional variables $\Pi_1 = G_c/E^* b$ and $\Pi_2 = a/b$ given as,

$$\frac{\sigma_{cr}^\infty}{E^*} = \sqrt{\frac{\Pi_1}{2 \tan\left(\frac{\pi \Pi_2}{2}\right)}} C\left(\Pi_2\right)^{-1}. \tag{1} \quad \{\texttt{sig\_cr}\}$$

Numerical calculations are performed for the geometry shown in figure. 1(A) by developing a finite element mesh with a mesh size of $h = 0.01$ close to the notch tip. Crack initiation is characterized when the value of the damage parameter ϕ reaches 0.99 at the first node in front of the notch tip. The numerical results for σ_{cr}^∞/E^* is plotted against Π_2 for different values of Π_1 and are compared with the analytical solution given in eq. 1. The solid lines correspond to the analytical solution and markers to the numerically obtained data. In figure. 1(B) σ_{cr}^∞/E^* (left hand side of eq. 1) is plotted against $\left(\Pi_1/2 \tan\left(\pi \Pi_2/2\right)\right)^{1/2} C\left(\Pi_2\right)^{-1}$ (right hand side of eq. 1) and the identical numerical data from figure. 1(A) are compared against the reference line of slope 1. The results in both the plots compare accurately within an error of 5%. From these figures we can conclude that the implementation is correct and accurate to numerical precision.
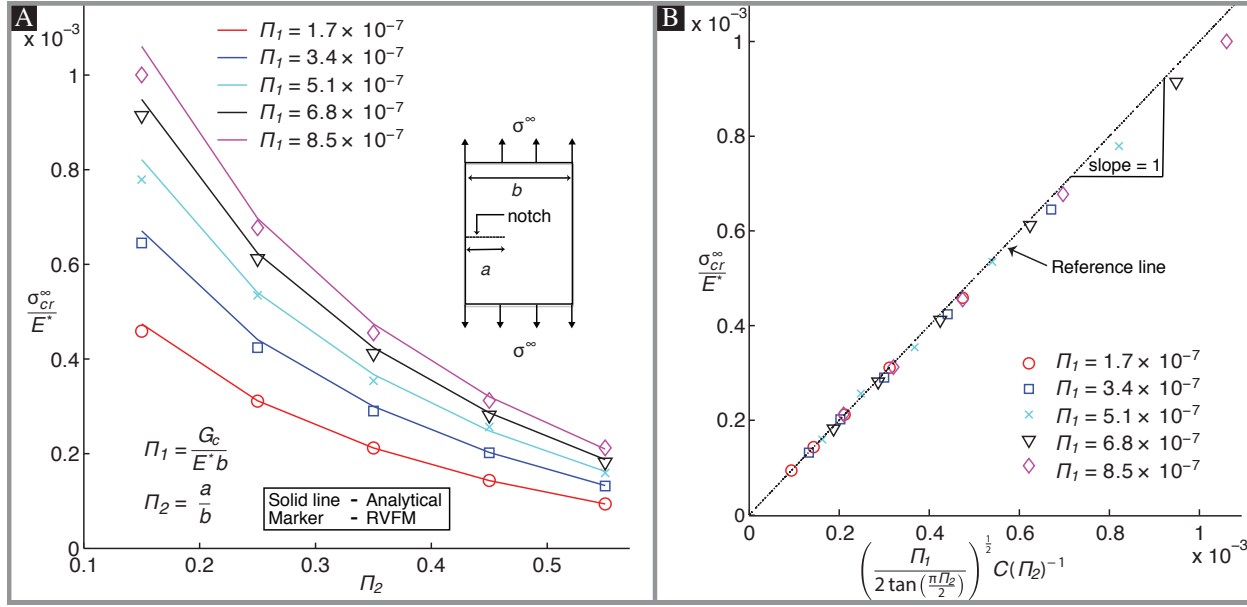
Figure 1: (A)In this figure the results of crack initiation for a single edge notch geometry of notch length a and width b, subjected to far-field stress σ^∞ is shown. The crack is assumed to have initiated when the value of ϕ at the first node in front of the notch tip reaches 0.99 and the far-field stress at that instant is σ_{cr}^∞. The non-dimensionalized stress (σ_{cr}^∞/E^*) is plotted against Π_2. The solid lines correspond to the analytical solution given in eq. 1 and markers to the numerical data points obtained for various values of Π_1 and Π_2. (B) Identical data points from fig. 1(A) are now plotted against the right hand side of the analytical formula given in eq. 1 and the deviation from the reference line is less than 5%. fig1