Cohesive zone (CZ) methods have proven to be very significant for modeling interfacial fracture. There are several versions of CZ methods discussed in literature. Most of those methods can be put into two primary classes. The first class is roughly based on the numeric implementation of Hillerborg et al.[17] while the second class is based on the ideas of Xu and Needleman,[29] and Camacho and Ortiz.[7] Both classes are based on the work of Dugdale[13] and Barenblatt[1] and built atop of classical finite elements technqiues. In that crack growth is modeled by allowing adjoining finite element pairs to separate along their shared boundary. As the elements seperate, tractions are applied on the separating faces of the elements as per a pre-assigned traction-separation law. The shape and parameters in the traction separation law are chosen to model different types of damage behavior (ductile, quasi-brittle, etc.). This idea is made numerically feasible by incorporating what are termed *cohesive zone (or interface) elements* between the shared boundaries of adjoning finite element pairs. The primary difference beween the two classes of CZ methods is in the selection of the finite element boundaries at which the CZ elements are included.

In the classical version of the CZ method, given by Hillerborg et al., the CZ elements are only placed along finite element boundaries which are known/expected to be close to the final crack path. Thus, the application of the classical CZM is predicated upon having some *a priori* knowlege of the crack path. However, the success of the proposed research program hinges on developing and leveraging the ability to predict the evolution of complicated crack paths of which we have no or very little *a priori knowledge*. Consequently, classical CZ methods are not suitable for the proposed research project.

In the generalized version presented by Xu and Needleman the CZ elements are placed between all shared boundaries of the finite elements. This enables the simulation of crack propagation without requiring any *a priori* knowledge of the crack path. This generalized version is also capable of modeling the evolution of complicated crack paths. However, based on our study of the computational mechanics literature and the preliminary research that we undertook for planning the proposed project we found that the generalized CZ method is not very accurate in reproducing experimentally observed crack paths.[10, 11, 27] For example, Fig. 1 shows a comparison between experimentally observed crack paths and those that were computationally predicted using the generalized CZ method. As can be seen by comparing Fig. 1(b) with either (c) or (d), the generalized CZ method predicted cracks paths are quite different from the experimentally observed ones. Furthermore, as can be seen from Figs. 1(c)–(d), the generalized CZ method predicted crack paths are *mesh dependent*. That is, they depend on the size and structure of the finite element mesh employed.

The inaccuracy and mesh sensitivity of the generalized CZ method is widely discussed in literature.[10, 11, 25, 27] For example, it is well known that the crack path predicted by CZ method is inherently biased because crack growth in it is restricted to inter-finite element boundaries. This could give rise to mesh dependent crack paths. Also, Borst[10, 11] gives the following reason for the method's poor performance. In theory, a CZ element is supposed to have infinite stiffness before the traction acting on it reaches a critical value, i.e, prior to initiation of any damage in it. In practice, however, in order to make the numerical implementation feasible, the CZ elements are provided with a finite stiffness in the simulations. However, assigning a finite stiffness to the CZ elements can artifically change the bulk elastic constitutive behavior of the material composing the solid, since recall that the CZ elements are placed between all finite element boundaries in this method. This artifact can be avoided by increasing the CZ elements' stiffness. But that has the consequence of increasing the condition number of the numerical matrices in the simulation making the calculations unstable and prone to numerical errors. Borst suggests that this no win situation in selecting an ideal stiffness for the CZ elements coupled with the CZ elements being present between all shared finite

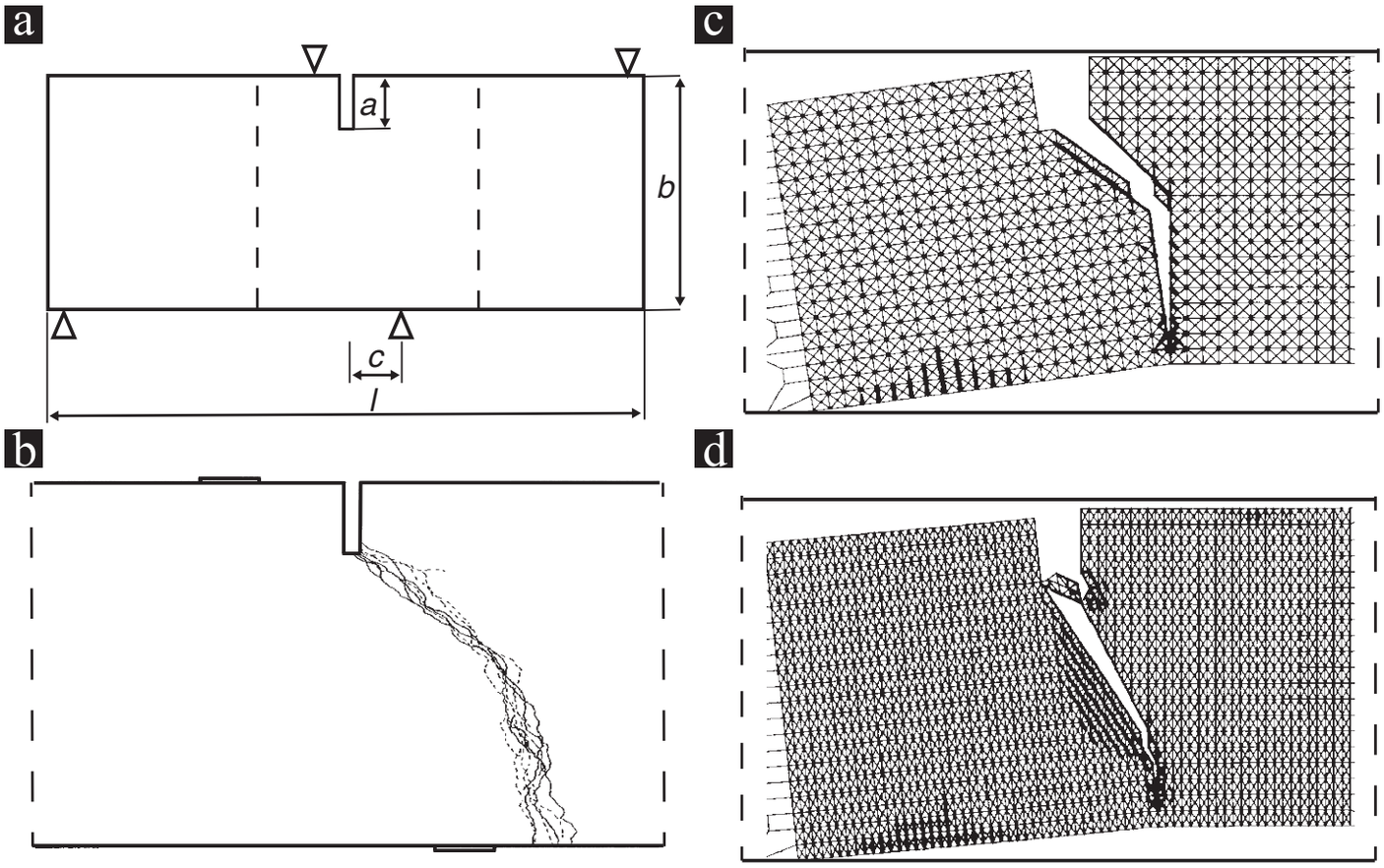

Figure 1: Comparison between experimentally observed and computationally predicted crack paths. (a) shows the set-up of the experiment, which is a single edge notch (SEN) shear load test. (b) shows the experimental crack paths observed in concrete specimens by Schlangen.[23] Each curve corresponds to a different specimen. (c) and (d) show the computationally predicted crack paths for a coarse and fine finite element mesh, respectively. These computational crack paths were predicted by Tijssens*et al.*[27] for the experiments shown in (a)–(b) using the generalized CZ method. {Fig:CZM}

element boundaries is the primary reason behind the method's poor performance.

We do not have any ideas as how to remedy the generalized CZ method's poor accuracy and high mesh sensitivity. Therefore, we decided not to select it for further development for the purpose of our pursing the project's overarching research objective.

Extended Finite Element methods (XFEM). The XFEM is a remarkable method. It has been used to simulate quasi-static crack growth in both 2D[3,6,12] and 3D,[16,21,26] and dynamic crack growth in 2D.[4,24] It is currently widely used to predict crack paths[2,15,22] and is even a feature in the commercial finite element software package Abaqus.[8] It was introduced by Belytschko and Black[3] for modeling cracks in solids without the need for remeshing. The numerical ideas underlying XFEM are based on the partition of unity concept of Melenk and Babuška.[19] Currently, there are several versions of the XFEM discussed in its literature. So, it is not unlikely that our comments will not apply to all of them. Also, we have sincerely tried not to cherry pick which features of the XFEM we discuss so as not to give an unfair appraisal of its

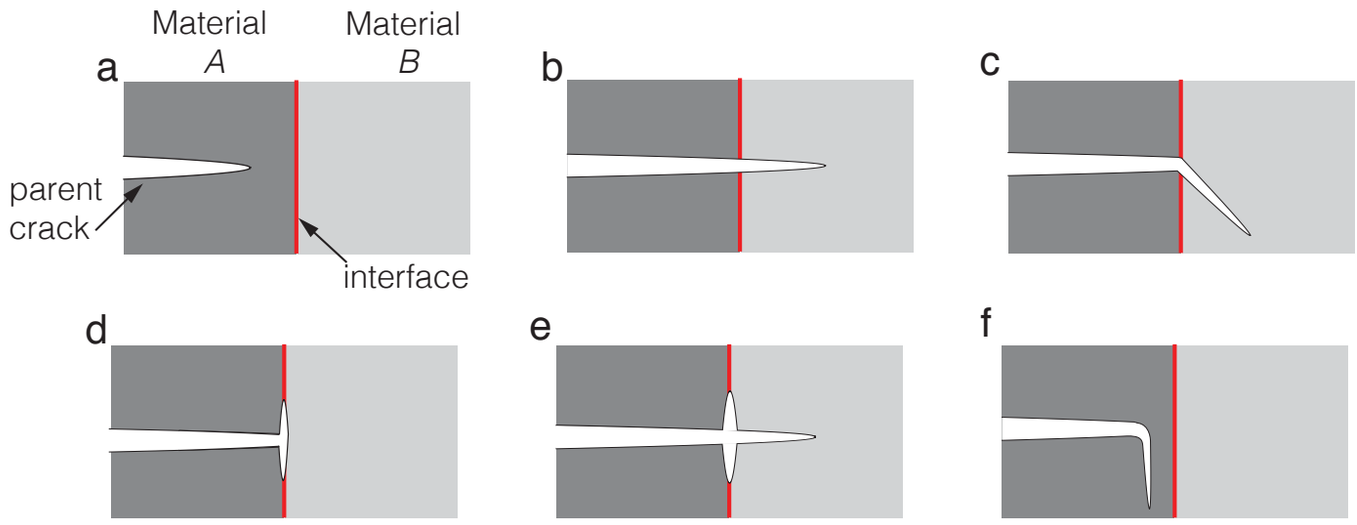

Figure 2

{Fig:Topolo

capabilities.

Like CZ methods, the XFEM too is built atop of finite elements methods. In standard finite element method (FEM), the displacements, strains, etc. are approximated as linear combinations of a select set of functions called the *basis functions*. Choosing a finite element mesh and the finite element type(s) (linear tetrahedron, etc.) in a problem is tantamount to choosing the set of basis functions. In standard FEMs, the basis functions are usually continuous or smooth. The coefficients of the basis functions in the linear combinations are related to the degrees of feedom at the finite element nodes. In simulations based on standard FEMs, the set of basis functions remains fixed during the simulations. The ingenuity of the XFEM lies in the following two modifications: (i) In addition to the usual continuous/smooth basis functions, discontinuous and irregular basis functions are allowed in the set of basis functions. This process is called *enrichment*. The choice of these discontinuous/irregular basis functions is based on the asymptotic solutions of fracture problems in the linear theory of elasticity and the total deformation theory of plasticity. These new basis functions enable the XFEM to capture the discontinuties across the crack's faces and the singular behavior at its tip. (ii) The crack growth is modeled by allowing the set of basis functions to change as the crack grows.

Despite its tremedous utility, the XFEM is not suitable for pursuing the objectives of the current porposal. This is because the XFEM in its classical form cannot be used to model topology changes (see, e.g., Fig. 2). That is, it cannot be used to capture phenomena such as the merging of two (or more) cracks to form a single crack (Fig. 2(a).i–ii), the splitting of a crack into two (or more) daughter branches (Fig. 2(b).i–iii), and the nucleation of new cracks (Fig. 2(c).i–iii). This is not to say that XFEM cannot apply to a crack that contains one of more branches. Indeed, special enrichement functions have been developed through which XFEM can yield the displacements, stresses, etc. in problems that contain a crack with multiple branches.[5,9] However, even with those special enrichments the XFEM on its own—i.e., without any modeler input during the simulation, or *a priori* knowledge about any forthcoming topolology changes—will not be able to predict whether or not a crack will split into mutiple branches, and if does split, how many branches it will split into.

Topology changes are not very common in quasi-static crack growth problems in homogenous materials. However, as we discussed in §XX, topology changes are very common in heterogenous materials, such as structural biomaterials. In fact, topology changes are a key feature of the mechanisms through which hypothesize that the structural biomaterials gain their remarkable toughness. It is possible to augument the XFEM with features from CZ methods,[18,20,28] or using older failure theories, such as the *critical principle stress failure criteria*, to give XFEMs some capability for capturing toplology changes. However, we are not aware of any rational/scientific theory that can guide such an augenmentation. Without a systematic means to guide the augumentation, we are afraid that any such augementations can only be hueristic or *ad-hoc* at best and therefore cannot be the primary agenda of a scientific project. The RVF theory, on the other hand, can readily model topology changes without any heuristic/*ad-hoc* augumentations, see Fig. 3 and Fig. **??**, e.g. For these reasons, we decided not to select the XFEM for pursuing the project's research objective. STOPPED HERE

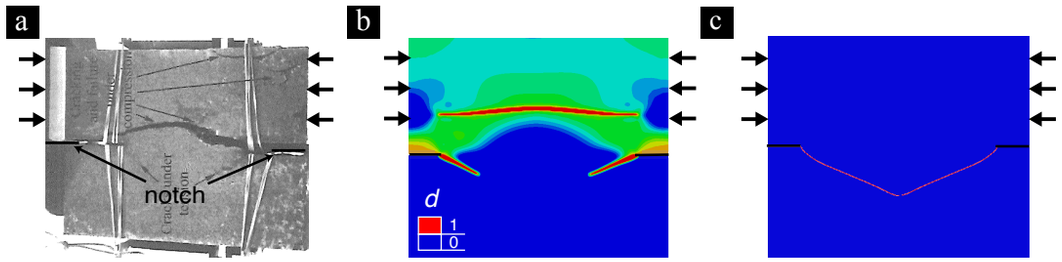

Figure 3: Comparison between experimentally observed and computationally predicted crack paths. (a) shows a photograph of a failed concrete specimen. This is from the expeiments reported by Gálvez *et al.*[14] The specimen was prepared by cutting two notches, one on each of its left and righ edges. The only loading on the specimen was on its top portion. That is, the region above the notches was loaded in compression. The specimen was observed to fail through the formation and propagation of two distinct crack systems. The first system (marked in red dashed lines) consists of a two cracks, each of which emanate from a notch and grow a short distance into the bottom portion of the specimen. The second system (marked in yellow dash-dot lines) consists of a single crack that nucleates in the top portion of the speciemen and propagates symmetrically towards the left and right. (b) shows the predictions of the RVF theory. As can seen, the RVF theory captures both system 1 and 2 of the cracks formed. Its predicted shape and length of the system 1 cracks is also quite consistent with what is experimentally observed. Its predicted shape and length of the system 2 crack is slightly different from what is observed in experiments. We believe that this disparity is due to the incomplete knowledge we have about the specimen's materials properties and the experiment's loading program. (c) shows the predictions from the XFEM (as implemented in Abaqus). The XFEM method is able to capture the system 1 cracks. However, it completely misses the system 2 cracks. This is expected, since as we discuss in the text, XFEM on its own cannot readily capture toplogy changes, of which crack nucleation is a special case. Also, the XFEM predicted length of the system 1 cracks is substantially different from what is observed experimentally. {Fig:Concre

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