What is Quasi-Brittle Fracture and How to Model its Fracture Behaviour

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There are many materials of great engineering significance, e.g. concrete, that are regarded as brittle which are to be used only under compression. Yet when attempts were made as far back as the 1950s to apply classical brittle fracture theories of Griffith and Irwin to concrete, these proved unsuccessful in the sense that the onset of fracture could not be quantified uniquely by the critical stress intensity factor or the critical energy release rate (i.e. fracture toughness). It was however observed that the behaviour of hardened cement paste with its fine microstructure was close to the predictions of linear elastic fracture mechanics (LEFM), but the behaviour deviated the more from the LEFM predictions the coarser, or the more heterogeneous, the microstructure of concrete became.

We now understand fully the reasons for the lack of success of LEFM as far as concrete is concerned (Karihaloo, 1995). These stem from the role of defects such as micro-cracks in the response of all cement-based materials that are traditionally regarded as being brittle, but in reality exhibit a far more sophisticated response. They are moderately strain hardening prior to the attainment of their ultimate tensile capacity, reminiscent of the response of high strength metallic materials. However, unlike the latter, they are characterised by an increase in deformation with decreasing tension carrying capacity past the ultimate strength. Such a response is called tension softening. The materials that exhibit moderate strain hardening prior to the attainment of ultimate tensile strength and tension softening thereafter may be called quasi-brittle. The softening is due to many fracture processes, such as localised micro-cracking, bridging by coarser aggregates, crack branching, etc. Included in the class of quasi-brittle materials are concrete, rocks, coarse-grained ceramics. However, we shall concentrate only on concrete in this talk.

The primary reason for the observed deviation of the behaviour of concrete from the LEFM prediction is the formation of an extensive fracture process zone (FPZ) ahead of a pre-existing notch/crack in which the material progressively softens due to the above-mentioned fracture processes. Therefore a fracture theory capable of describing the behaviour of concrete must include in it a description of the material softening taking place in the FPZ. Such a theory will necessarily be a non-linear one, but we must distinguish it from non-linear fracture theories applicable to ductile materials such as metals because in the latter the FPZ is very small and surrounded by a large non-linear plastic zone, whereas in a quasi-brittle material the FPZ practically occupies the entire large zone of non-linear deformation. In contrast, the non-linear zone is practically absent in brittle materials.

The first non-linear theory of fracture for concrete was proposed by Hillerborg et al. (1976). It includes the tension softening FPZ through a fictitious crack ahead of a pre-existing crack. The length of the fictitious crack is dictated by the microstructure of concrete; the coarser the microstructure the longer the fictitious crack. The term “fictitious” is used to underline the fact that this portion of the crack cannot be continuous with full separation of its faces. Indeed the latter are acted upon by certainclosing stresses such that the stress intensity factor vanishes at the tip of the extended crack. The closing stresses are not constant in the FPZ, as in Dugdale-BCS model of planar plastic zones ahead of a crack, but they increase from nothing at the tip of the pre-existing traction-free crack to the full uniaxial tensile strength of the material at the tip of the fictitious crack. The distribution of the closing stresses, $\sigma(w)$, along the fictitious crack depends on the opening of its faces, w. The fictitious crack model (FCM) for concrete also differs from Barenblatt's cohesive crack model, in that the size of the FPZ may not be small in comparison with the size of the traction-free pre-existing crack. Unlike LEFM, the description of the fracture of concrete requires two additional material parameters, namely the tension softening relationship $\sigma(w)$ in the FPZ and the area under the tension softening curve, namely the specific fracture energy G_F.

As the FPZ is not continuous (hence the notion of a fictitious crack) and as it does not necessarily develop in a narrow discrete region in line with the continuous traction-free crack, it has been argued by Bazant (1976) that the tension softening relation $\sigma(w)$ can equally well be approximated by a strain softening relation $\sigma(\varepsilon)$, i.e. a decreasing stress with increasing inelastic strain. This strain is of course related to the inelastic deformation w and the specific fracture energy G_F through a certain gauge length h. It was assumed that the FPZ is spread over a band of thickness h, hence the name of this model as the crack band model (CBM) or the smeared crack model to distinguish it from the discrete crack approach implied in the FCM.

The complete failure process of a concrete structure can therefore be modelled by the FCM or CBM once the two additional material parameters G_F and $\sigma(w)$ of the concrete mix are known. However, as the tension softening region (i.e. the fictitious crack) is generally discontinuous, any attempt at the precise determination of $\sigma(w)$ would seem to be doomed to failure from the outset. This has not prevented researchers from attempting to establish it directly from measurements using uniaxial tension tests. Such tests can be performed in a very limited number of laboratories in the world. By far the majority of the attempts have been directed at inferring the $\sigma(w)$ relationship from the measured specific fracture energy G_F, supplemented by other physical information. For example, it is observed that the part of the softening diagram immediately after the peak load is controlled by micro-cracking and is very steep, whereas the tail part of the diagram which is controlled by frictional processes such as bridging by coarser aggregates is shallow. Thus a bilinear approximation of what is essentially a continuous relation is quite adequate. We mention *en passant* that the tension softening relation can be established from rigorous micromechanical principles relating the microstructure of a concrete mix to its macroscopic response (Karihaloo, 1995).The determination of the specific fracture energy has been a subject of intense debate among researchers because it has been found to vary with the size and shape of the test specimen and with the test method. It has been confirmed recently by Abdalla & Karihaloo (2003) and Karihaloo et al. (2003) that the specific fracture energy of concrete measured on laboratory specimens is dependent on the shape and size of the specimen because the local energy in the fracture process zone decreases as the crack approaches the back face of the specimen, as suggested earlier by Hu & Wittmann (2000). It was also observed that size-independent specific fracture energy of concrete could be obtained by testing three point bend (TPB) or wedge splitting (WS) specimens of just one size. However, it is necessary that half the number of specimens contains a very shallow, while the other half contains a deep starter notch. This observation was based on limited numbers of TPB and WS specimens made from normal and high strength concretes. Specific fracture energy data of concretes published in the literature (26 available data sets) were then re-evaluated and confirmed this observation. Thus the determination of the true specific fracture energy of concrete has become a simple and straightforward task requiring few specimens of the same size and shape. Abdalla & Karihaloo (2004) also proposed a method based on the concept of a nonlinear hinge (Ulfkjaer et al. 1995) for constructing a bilinear approximation of the tension softening relation consistent with this true specific fracture energy. The parameters of this bilinear approximation are inferred in an inverse manner from the load-displacement diagrams registered in TPB or the load-crack mouth opening diagrams registered in WS tests.

Structures made of quasi-brittle materials exhibit size and scale effects. The apparent strength would appear to decrease as the size of the structure increases. Also, structures made of the same material exhibit a transition from ductile to brittle response as the size increases. Leicester (1973) seems to have been the first to identify two fundamental causes of size effect in structures made from quasi-brittle materials, such as concrete, namely the material heterogeneity (i.e. statistical size effect) and the occurrence of discontinuities in the flow of stress, such as at cracks and notches (i.e. deterministic fracture mechanical size effect). In quasi-brittle materials, any crack or notch tips are blunted by the formation of a process zone ahead of them. In this process zone the stresses are redistributed and energy dissipated which is thus not available for crack propagation. The size of this fracture process zone (FPZ) can be commensurate with that of most structural elements (W). Only in very large structures can this size be regarded as small in comparison with the characteristic dimensions. The redistribution of stresses and dissipation of energy in the FPZ was not accounted for by Leicester. That was done by Bazant (1984) who derived the following formula for geometrically similar structures

$$(\sigma_N)_u = \frac{A_2}{(1+W/B_2)^{\frac{1}{2}}}$$

where A_2 and B_2 are positive coefficients. The above formula reduces to the linear elastic fracture mechanics as W when the size of the FPZ is very small in comparison with W. In fact the formula can be established by Taylor's expansion from this asymptotic limit (Karihaloo, 1995). Since its appearance in the literature in 1984, it has been re-derived from energy considerations and asymptotic matching techniques (see, e.g. Bazant, 1997). The positive coefficients A_2 and B_2 are related to the specific fracture energy G_f and the FPZ size c_f measured on a very large specimen (W with a/W = fixed), as well as the non-dimensional geometry factor g() and its first derivative g(). The geometry factor g() depends on the notch to depth ratio $= a/W$ and is different for different test specimen shapes. However, it transpires that both G_f and c_f cannot be regarded as material properties because they vary with , W and the shape of the test specimen. Thus, it is not clear how much of the size effect in the strength of a quasi-brittle structure predicted by this formula is a result of the intrinsic size effect in the G_f itself? In other words, if the specific fracture energy of a quasi-brittle material that did not depend on the shape and size of the test specimen could be independently determined, would a structure made of such a material still exhibit a strong size effect in strength? This question was recently answered in the affirmative by Karihaloo et al. (2006) who showed that

$$\frac{(\sigma_N)_u}{f_t} = D_1(\alpha)\left(1+\frac{W/l_{ch}}{D_2(\alpha)}\right)^{-\frac{1}{2}} + \frac{D_1(\alpha)}{2D_2(\alpha)}\frac{W}{l_{ch}}\left(1+\frac{W/l_{ch}}{D_4(\alpha)}\right)^{-1}$$

Here, the coefficients $D_1(\alpha)$, $D_2(\alpha)$ and $D_4(\alpha)$ are obtained by nonlinear regression of the test results on notched specimens of any shape and D_3 is related to the other coefficients via

$$D_3 = \frac{D_1 D_4}{2D_2}$$

The concrete mix characteristic length l_{ch} is a derived material parameter related to its stiffness E, tensile strength f_t and specific fracture energy G_F via $l_{ch} = E\, G_F/(f_t)^2$.

The progressive failure process of a structure made of a quasi-brittle material or a particulate composite under loading can be simulated by lattice models (see, e.g. Burt & Dougill, 1977) used for solving classical problems of elasticity. Bazant et al. (1990) and Schlangen & van Mier (1992) extended lattice models to concrete; the former used truss elements, while the latter adopted Euler-Bernoulli beam elements. The lattice model at the mesolevel projects directly the material multi-phase structure on to the lattice. It is a relatively simple and powerful technique to identify micro-cracking, crack branching, crack tortuosity and bridging, thus allowing the fracture process to be followed until complete rupture. However, these models have produced unreasonably brittle post-peak response of plain and reinforced concrete beams. A recent improvement by Karihaloo et al. (2003) who allowed for the non-linear behaviour of the matrix (i.e. cement mortar) and/or the interface between the matrix and hard phase (i.e. coarse aggregate) produced the expected ductile response.

Lattice models are however useful only for small structures. For medium and large size structures one has to resort to finite element analysis. Both the FCM and the BCM have been implemented in almost all commercially available finite element codes. In particular, Karihaloo and his co-workers have recently shown how to analyse cracked concrete structures very accurately using rather coarse meshes by combining FCM with the extended finite element methodology (XFEM) (Xiao et al. 2007).

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