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Regarding: Point-by-point response to issues raised by referees (manuscript JMBBM-D-21-00364).

Dear Editors and Reviewers,

We thank you for your very valuable feedback. In response to your feedback, we have made the revisions listed in the section *List Of Changes* (*LoC*), which can be found on page 2 of this response letter. We provide a point-by-point response to the Reviewers' comments in the following pages. Our responses to Reviewer #1's and Reviewer #2's comments can be found on pages 3 and 8, respectively, of this response letter. Through these changes and our responses to your comments, we hope that we have addressed all of your concerns.

We thank you for your consideration of our revised manuscript.

Sincerely,

Wenqiang Fang, Sayaka Kochiyama, and Haneesh Kesari

List of Changes

Changes in response to Reviewer criticisms

Response to Reviewer #1's comments

1.1 **"The authors have presented a good piece of work, through which they are able to explain the sawtooth patterns observed in the force-displacement curves. This is done by allowing for slippage at the test supports and paying attention to the coefficient of friction."**

We thank the Reviewer for their consideration of our manuscript.

1.2 **" A few minor comments to help clarify and strengthen the work: a) In the SS experiments, how it is ascertained that the boundary conditions correspond to the simply supported conditions."**

We thank the reviewer for the comments on helping clarify and strengthen the work.

In structural mechanics, the essential features of a simply supported condition are given as follows (see Gere and Timoshenko ((1997))):

- *(SS1).* The material points of the beam that are in contact with the supports cannot move vertically.
- *(SS2).* The material points of the beam that are in contact with the supports cannot develop a moment reaction.

As for the setup in our SS experiments, we provided a schematic of the simply-supported setup in the submitted manuscript (see Fig. 1), in which the spicule is suspended over a trench. This suggests that SS1 is guaranteed. Photos of the deformed configuration of spicules just before failure can be found in Monn and Kesari ((2017)) (see Fig. 2). From the photos, we clearly see the spicules' segments beyond the trench edges (marked by red ellipses) are straight, which suggests that SS2 is valid.

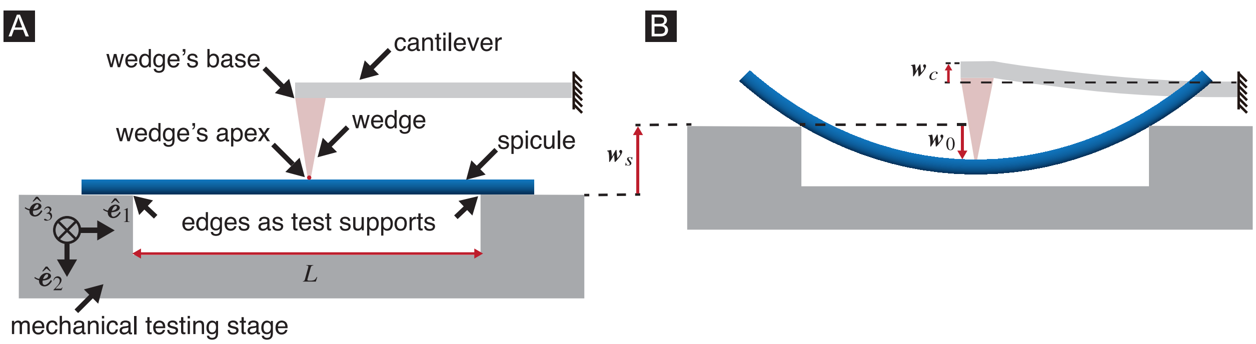

Figure 1. An illustration of the simply-supported setup (Figure 5 of the submitted manuscript).

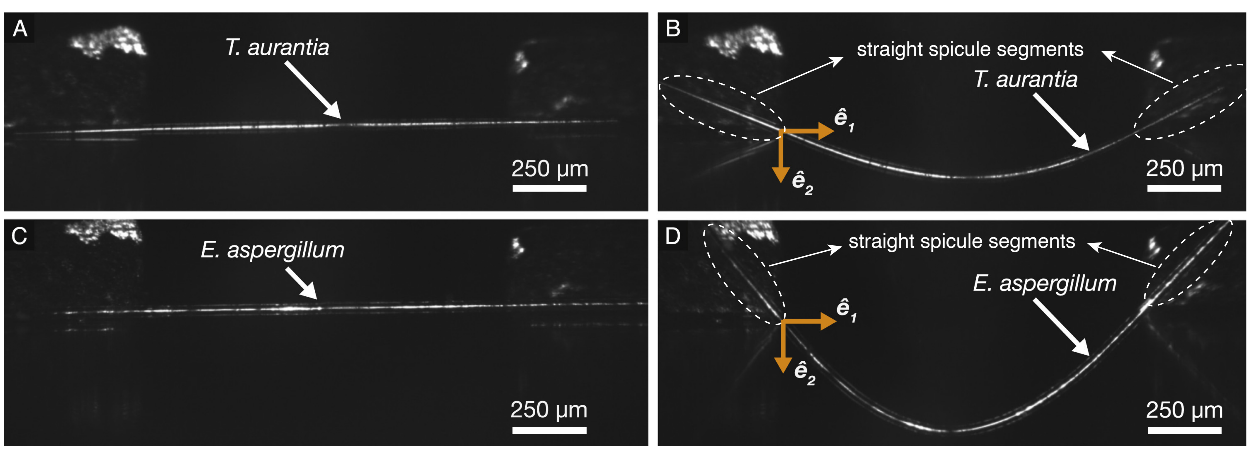

Figure 2. (A) and (B) (resp. (C) and (D)) show the undeformed configuration and deformed configuration just before failure of a representative *Tethya aurantia* (resp. *Euplectella aspergillum*) spicule (adapted from Figure 3 of Monn and Kesari ((2017))). The spicules' segments beyond the trench edges, marked by red ellipses, are straight.

Furthermore, we applied Euler-Bernoulli (EB) theory with simply-supported boundary condition to the SS experiments of spicules. From the experimentally measured force-displacement curves, we computed the Young's modulus of the tested spicules. For given applied forces, we were able to predict the deformed shapes of the spicules using EB theory with the calculated Young's modulus. When comparing the predictions with the optical micrographs of the spicules' deformed shapes, we found they match each other extremely well (for example, see Fig. 3). Therefore, we are certain that the boundary conditions in the SS experiments correspond to the simply supported conditions.

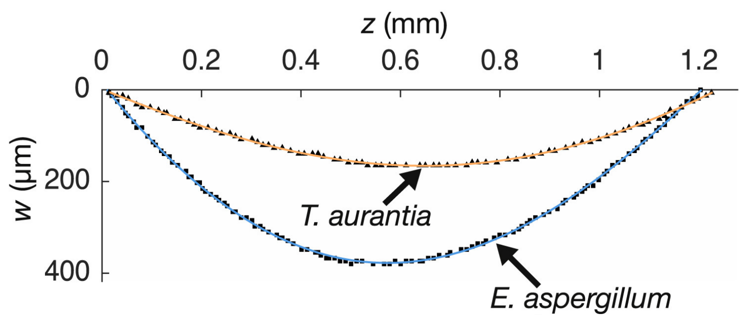

Figure 3. Comparison of spicule shapes between the predictions from Euler-Bernoulli (EB) theory and experimental measurements. The black triangles (resp. squares) correspond to the positions of material points for the representative *Tethya aurantia* (resp. *Euplectella aspergillum*) spicule shown in Fig. 2 (B) (resp. (D)). The orange (resp. blue) curves correspond to the deformed shapes predicted by EB theory. (Figure 4(D) of Monn and Kesari ((2017))).

1.3 **“b) The experiments all appear to be conducted for a static loading and this needs to be pointed out explicitly, in particular, when using friction in the model. The time scales over which the loading could be critical if dynamic friction is involved. Can any relevant details be provided in the paper? ”**

We thank the reviewer for pointing out this lapse on our part. In response to this suggestion, we have added the following details involving stage control and the associated time scales to the third paragraph of §3 in the manuscript:

The mechanical testing stage was driven by a DC servo motor, whose motion was controlled through PID algorithm. The stage was moved in a 2 μm increment with a minimum velocity of 50 μm/sec, maximum velocity of 200 μm/sec, and acceleration of 250 μm/sec^2, so that the stage was in motion for 10–40 ms at a time. After each increment, the stage was held still so that there was a 2000 ms interval between the initiation of each 2 μm step.

1.4 **“ c) With regard to the elastica theory, an extension would be the rod theory, which would be needed for a three-dimensional setting; see Antman, S. S., Problems in Nonlinear Elasticity, Springer.”**

We thank the reviewer for bringing up the rod theory by Antman Antman ((2005)). We will further improve our model by incorporating the rod theory in the future.

In fact, to account for the effects of the multi-layer architecture on the large deflection of *E. aspergillum* spicules, we have proposed a homogenized finite shear deformable beam theory Fang et al., in which we developed a shear deformable beam theory in a systematic way by following the general three-dimensional continuum theory and employing the Hellinger-Reissner variational principle. Neither the shear deformable beam theory or the rod theory by Antman comes with analytical solutions. When dealing with complicated beam deformation, such as buckling and looping formation, numerical instabilities are inevitable. Therefore, in order to get neat analytical solutions, we modeled the spicules' deformation using Euler's elastica theory in the submitted manuscript. We thank the reviewer again for providing such great insights and we will explore further in that direction in the future.

Response to Reviewer #2's comments

2.1 **"The authors report on the three-point bending tests of marine sponge fibers and the possible mechanism for the sawtooth-pattern during the bending tests. They argued that the sawtooth-pattern is originated from the stick-slip motion on the supporting substance and eliminated the Cook-Gordon mechanism. While Cook-Gordon mechanism is valid during the fracture of the materials, I am curious whether the experiment here caused the fracture of the spicule? If not, then the bending of spicule may remain in the "elastic deformation" and the stick-slip should be caused by the friction on the supporting layer, as stated by the authors. Then the initial argument is meaningless. If yes, how is the fractured surface looks like? Will the propagation of cracks have any correlation to the sawtooth-pattern? "**

All experiments that display the sawtooth patterns have caused the fracture of the spicules. The fractured surfaces are shown in Fig. 4.

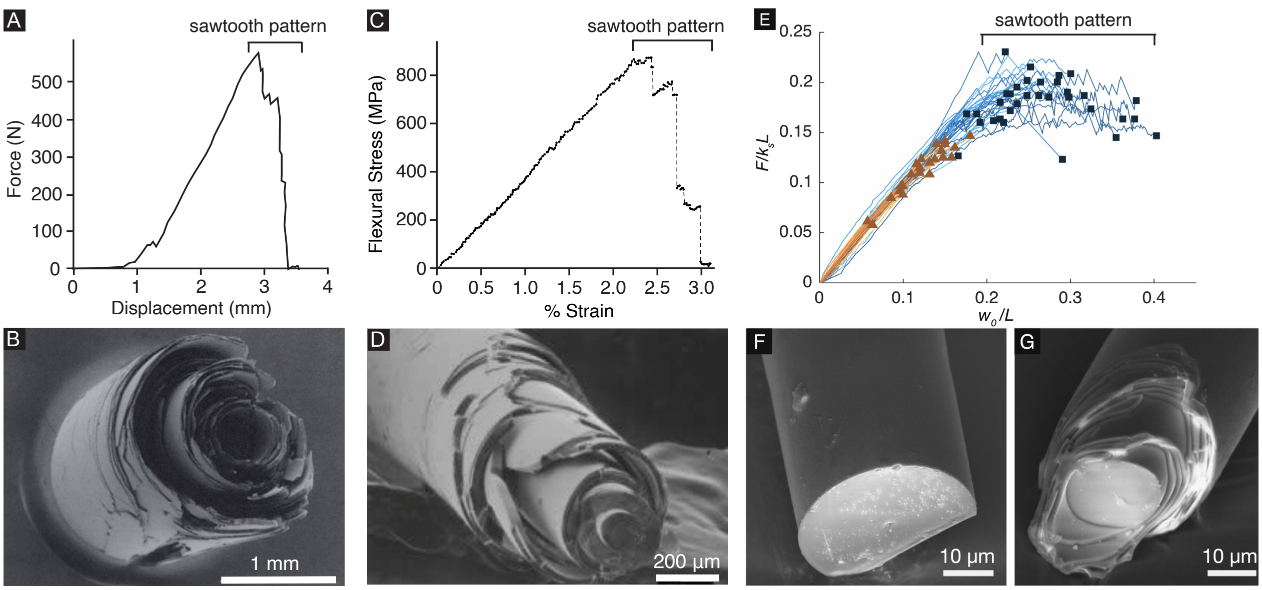

Figure 4. Force-displacement curves and optical images of the fractured surfaces for (A and B) *Monorhaphis chuni* anchor spicules, (C and D) *Rosella racovitzae* spicules (adpated from Figure 4 of Kochiyama et al. ((2021))), and (E and G) *Euplectella aspergillum* (*Ea.*) anchor spicules (adpated from Figure 3 of Monn and Kesari ((2017))).

Although we only tested *Ea.* spicules on our own, we brought up *M. chuni* and *R. racovitzae* in order to argue that our hypothesis and model can be applied to a broader range of structural biological materials.

In order to get direct evidence of whether the propagation of cracks have any correlation to the sawtooth patterns, it is necessary to monitor the crack propagation along with the measurement of force-displacement curves simultaneously during the three-point bending test. However, as the crack may grow inside the spicules, optical micrographs can not provide such evidence. Sophisticated experimental design involving electric circuit may be effective for the purpose of monitoring crack propagation. Since we have difficulty monitoring the crack propagation during our tests, we cannot conclude whether the propagation of cracks have any correlation to the sawtooth pattern solely from the final fractured surfaces of the spicules after the tests.

2.2 **“The labelling of figures is chaos! The language should be polished, it is quite spoken language.”**

In response to the reviewer’s above comments, we have made modifications to the labeling of Fig. 3 and revised the language of some of the sentences in the manuscripts.

Bibliography

S. S. Antman. Problems in nonlinear elasticity. *Nonlinear Problems of Elasticity*, pages 513–584, 2005.

W. Fang, J. C. Larsen, and H. Kesari. Finite shear deformable beam theory. *Under preparation*.

J. Gere and S. Timoshenko. Mechanics of materials, 1997. *PWS-KENT Publishing Company, ISBN 0*, 534(92174):4, 1997.

S. Kochiyama, W. Fang, M. A. Monn, and H. Kesari. Sawtooth patterns in flexural force curves of structural biological materials are not signatures of toughness enhancement: Part i. *Journal of the Mechanical Behavior of Biomedical Materials*, 119:104362, 2021.

M. A. Monn and H. Kesari. Enhanced bending failure strain in biological glass fibers due to internal lamellar architecture. *Journal of the mechanical behavior of biomedical materials*, 76:69–75, 2017.