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THE PROBLEM OF LONG-TERM STORAGE IN RESERVOIRS

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Le Groupe N° 3 recommande que ce Groupe de Travail se réunisse aussitôt que possible pour compléter le glossaire assez rapidement pour que la proposition complète puisse être mise en circulation au sous-comité avant la prochaine réunion.

Sections du glossaire

A. Termes généraux

1. Généralités
2. Mesures des débits des liquides en canaux ouverts
   (*a*) Hauteurs
   (*b*) Distances horizontales
   (*c*) Surfaces
   (*d*) Pentes

B. Méthodes basées sur la détermination des vitesses locales

1. Pitot
2. Moulinets
3. Flotteurs

C. Méthodes donnant immédiatement le débit total

1. Déversoirs
2. Canaux
3. Venturi et dérivés
4. Écrans
5. Méthodes de dilution
6. Orifices noyés

THE PROBLEM OF LONG-TERM STORAGE IN RESERVOIRS

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I. INTRODUCTION

The use of the water of a river for irrigation so as to extract the maximum benefit requires that the flow of the river shall be regulated by means of reservoirs. The ideal would be that the river should be completely controlled so as to send down a constant annual discharge distributed throughout the year according to the seasonal requirements of the crops.

In the following discussion, which is based on two earlier papers by the author,[1, 2] seasonal variation of flow and crop requirements within the year are not considered, since only annual totals are used. The effects of these must be considered separately and added to those dealt with here.

Given a series of annual discharges recorded for a past period, it is easy to calculate what storage would have been enough to equalize the flow and send down the mean discharge every year throughout the period. Much more than this is needed, however, before a policy can be laid down for the future, since the past is never exactly repeated.

II. CALCULATION OF STORAGE FROM PAST RECORDS OF PHENOMENA WHEN ALL THE SUPPLY IS USED

Suppose we have a record of the annual discharge of a stream over a number of years, and we assume that it flows into a reservoir of indefinite capacity, from which there is a constant outflow equal to the mean annual discharge. The storage which would have been required to make the annual discharge flow every year is obtained by computing the continued sums of the annual departures from the mean. Then the range R from maximum to minimum of these continued sums is either (*a*) the maximum accumulated storage when there is never a deficit, (*b*) the maximum accumulated deficit when

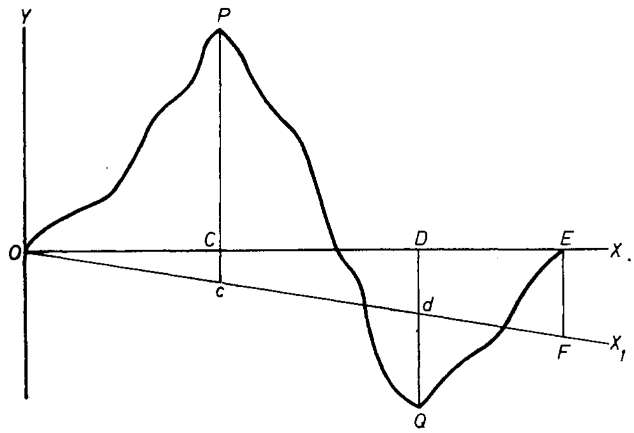


Figure 1.

there is never any storage, or (*c*) their sum when there is both storage and deficit.

This is shown in *Figure 1* where OX is the time axis and OY the axis of accumulated departures from the mean. The curve representing these is $OPQE$, and OE represents N years. During the period OC departures are positive and the initial storage is increased by PC. From C onwards to D departures are negative and the storage is decreased by $PC+DQ$, after which it increases up to the end of the record at E, when the total of departures is zero, and the storage is the same as at the commencement. $PC+DQ = R$, and this amount of storage would have enabled the mean discharge for the period to have been maintained throughout.

It is commonly assumed that the frequency distribution for river flows approximates to the Normal or Gaussian Curve, and this is usually true, but it is only part of the description of the phenomenon, since there is also a tendency for high or low years to be grouped together. A theoretical investigation shows that, if the individual years were entirely independent of each other and the discharges were randomly distributed, the most likely value of R/σ would be given by

$$R/\sigma = \sqrt{(\tfrac{1}{2}N\pi)} = 1{\cdot}25\sqrt{N}, \qquad (1)$$

where N is the number of years and σ is the standard deviation of the discharges for the period considered. Experiments with random events such as tossing coins agreed with this equation.

Records of the discharges of a number of rivers were examined and R was calculated for as many as were available. Unfortunately, there was no record then of the discharge of a river which covered much more than 70 years, and so R was computed for a number of rainfall records of which several covered more than 150 years. To these were added some records of river levels, temperatures, and pressures. A common feature of all the records was that their frequency distributions, ignoring order of occurrence, were of the humped type approximating to the normal Gaussian curve. When R was plotted against N an elongated group of points was produced represented by the equation

$$R/\sigma = 1{\cdot}65\sqrt{N}.$$

In this group there was nothing to distinguish one type of phenomenon from another, but it was clear that R increased more rapidly than was the case with random events. This was attributed to the tendency of natural phenomena to have runs when values on the whole were high and others when they were low. Owing to the scatter of the points the length of available records was not great enough to decide whether R/σ for natural phenomena was best represented by the square root or by some other function of N.

In the attempt to settle this point more long-term records of rainfall, temperature, pressure, and lake levels were computed and a search was made for other long-term records of natural phenomena. As a result the analysis was extended to the records of the Roda (Cairo) Nilometer, which carried back with gaps to A.D. 640, the thickness of tree rings which gave records up to 900 years, and varves for which 4,000 years were available.

Altogether 75 different phenomena were used. In the case of tree rings results from 4 different localities were taken separately, and the figures used for each locality are the means of a group of about 10 trees. A record was divided into periods for each of which R/σ was computed. For example, with 120 years recorded computations might be made for 3 periods of 40, two overlapping periods of 80, and the full period of 120 years. In general R was not computed for periods of less than 30 years. Altogether 690 values of R/σ were computed. A preliminary examination of the long-term data showed that R/σ increased more rapidly than $\sqrt{N}$ and less rapidly than N. To find the form of the relation the statistics were divided into sets containing similar phenomena, and the sets again into groups, each group containing a small number of values of R/σ with approximately the same value of N. *Table 1* gives the means of R/σ and N for these groups and also their logarithms.

Figure 2 shows log R/σ plotted against log N where R/σ and N are group means for each set of phenomena.*

* *Tables 1* and *2* and *Figure 2* are reproduced from ref. 1 by courtesy of the American Society of Civil Engineers.

Table 1. Relation of R/σ and N for groups of phenomena

Phenomena	*No. of cases*	*N years*	*R/σ*	log *N*	log *R/σ*	*K*
(*a*) *Group of* 99 *Cases*						
River levels, discharges, and runoff	8	35	7·5	1·54	0·85	0·68
	8	45	8·9	1·65	0·94	0·70
	8	62	13·1	1·79	1·08	0·72
	9	108	16·4	2·02	1·19	0·69
Roda Nilometer	12	105	19·6	2·02	1·27	0·75
	10	208	36·5	2·32	1·54	0·77
	9	309	53·9	2·49	1·72	0·79
	8	420	60·3	2·56	1·77	0·78
	7	511	67·7	2·71	1·82	0·75
	6	613	81·3	2·79	1·89	0·77
	5	716	104	2·85	2·01	0·79
	4	820	122	2·91	2·08	0·80
	3	927	129	2·96	2·11	0·79
	2	1,040	130	3·02	2·12	0·78
Mean of 99 cases	—	—	—	2·28	1·48	0·75
(*b*) *Group of* 168 *Cases*						
Rainfall stations with one value* of *R*	7	71	14·0	1·82	1·12	0·74
Two groups of values† of *R*	20	48	8·7	1·70	0·92	0·66
	12	98	13·7	1·94	1·11	0·68
Rainfall stations with three groups of values of *R*	66	38	8·2	1·57	0·91	0·72
	38	78	14·4	1·87	1·13	0·72
	25	121	22·2	2·08	1·31	0·74
Mean of 168 cases	—	—	—	1·79	1·08	0·70
(*c*) *Group of* 109 *Cases*						
Temperature and pressure stations with 3 values of *R*	27	37	7·4	1·55	0·85	0·68
	18	73	12·0	1·86	1·06	0·68
	10	110	18·0	2·04	1·24	0·71
Temperature stations with 4 values of *R*	24	44	8·8	1·63	0·93	0·70
	12	88	15·1	1·94	1·17	0·71
	12	132	21·0	2·12	1·32	0·72
	6	175	27·1	2·24	1·43	0·74
Mean of 109 cases	—	—	—	1·81	1·05	0·70
(*d*) *Group of* 85 *Cases*						
Annual growth of tree rings	41	50	13·6	1·69	1·11	0·80
	21	100	26·6	2·00	1·43	0·84
	14	200	45·0	2·30	1·64	0·82
	4	300	79·9	2·48	1·90	0·87
	4	462	76·6	2·66	1·87	0·79
	1	900	187·0	2·95	2·27	0·86
Mean of 85 cases	—	—	—	1·97	1·36	0·80
(*e*) *Group of* 90 *Cases*						
Thickness of annual layers of mud; Tamiskaming, Ont., Canada, and Moen, in the Sogne District, Norway	44	50	10·9	1·70	1·02	0·73
	22	100	21·3	2·00	1·31	0·77
	11	200	42·7	2·30	1·58	0·79
	4	300	82·5	2·48	1·90	0·87
	3	400	126	2·60	2·09	0·91
	4	550	115	2·74	2·00	0·82
	2	1,100	181	3·04	2·19	0·80
Mean of 90 cases	—	—	—	1·98	1·30	0·77

Table 1—continued

Phenomena	*No. of cases*	*N years*	R/σ	log N	log R/σ	K
(*f*) *Group of* 114 *Cases*						
Thickness of annual layers of mud, Lake Saki in the Crimea	40	50	9·7	1·70	0·98	0·70
	40	100	15·3	2·00	1·17	0·69
	20	200	25·0	2·30	1·39	0·70
	8	500	47·9	2·70	1·66	0·69
	4	1,000	84·0	3·00	1·91	0·71
	2	2,000	179·0	3·30	2·24	0·75
Mean of 114 cases	—	—	—	2·06	1·22	0·69
(*g*) *Group of* 25 *Cases*						
Sunspot numbers and wheat prices	12	64	12·4	1·77	1·06	0·72
	6	124	22·1	2·09	1·34	0·75
	7	237	16·9	2·36	1·22	0·60
Mean of 25 cases	—	—	—	2·01	1·17	0·69
(*h*) *Group of* 259 *Cases*						
Weight mean of 259 cases‡ ..	—	97	17·8	—	—	—

* Rainfall stations with one value of R; includes temperature at one station.
† Rainfall stations with two groups of values of R; includes temperature and one pressure.
‡ N ranges from 81 to 120.

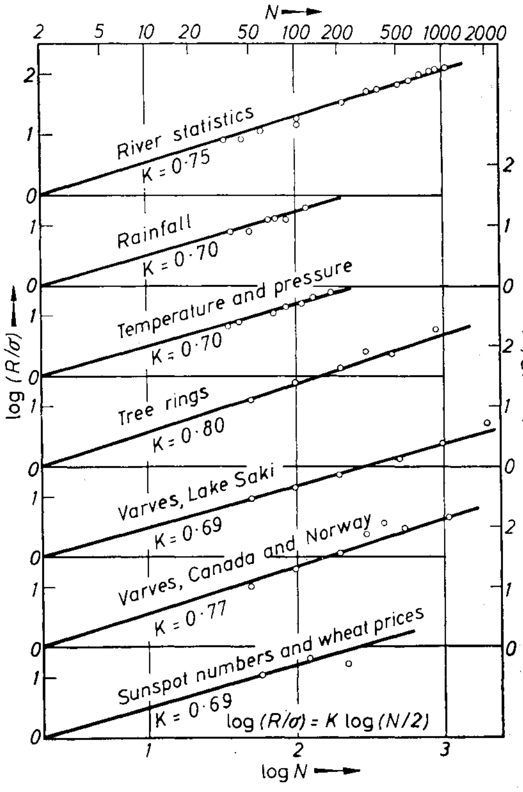


Figure 2. Relation between range of summation curve (R), *standard deviation* (σ), *and length of record* (N).

The figure shows that there is a linear relation between log R/σ and log N for each of the sets in which there are enough groups for it to show. R/σ can be found exactly for $N = 2$ and has the value 1. The lines in the figure are therefore drawn through the point (0·30,0) representing this, and through the centres of gravity of the groups of each set. The equations of these lines are

$$\log R/\sigma = K \log N/2 \qquad (2)$$

It is clear from the figure that for every set of phenomena a straight line is a good fit, and the remarkable fact appears that K, the slope of a line, varies very little from set to set. *Table 2* gives a summary of the mean values of K obtained from the 690 separate values which have been computed.

Table 2

Summary of values of K

Type of phenomenon	*No. of phenomena*	*No. of values of K*	*Mean*	*Std devn*
River levels, discharges, etc.	18	99	0·75	0·077
Rainfall	30	168	0·70	0·069
Temperature and pressure	19	115	0·70	0·085
Annual growth of tree rings	4	85	0·81	0·078
Varves (Lake Saki in Crimea)	1	114	0·69	0·064
Varves (Tamiskaming, Canada, and Moen, Norway)	2	90	0·77	0·094
Sunspot numbers and wheat prices (combined as miscellaneous)	2	25	0·69	0·086
Means and Totals	76	696	0·729	0·092

The remarkable facts which appear from the analysis of the figures are that a statistical relation of the same form exists for all the classes of phenomena which have been investigated, and that the parameter K in the relation varies very little from one class to another. This justifies the use of the other classes of data to supplement the data from discharge measurements on rivers. Individual values of K have a distribution which is approximately normal. The extreme values are 0·46 and 0·96. Considering the mean values of K for the different classes of phenomena the standard deviation of these means is about 4 times as great as it would be if the distribution of K was entirely random. This indicates that K does vary slightly with different classes of phenomena. Nevertheless it is a striking fact that the variation amongst the classes is so small. The largest mean value of K is 0·81 from tree rings derived from 4 sets of trees in different places whose means are 0·80, 0·81, 0·81, and 0·84. The standard deviation of a mean of 85 values of K on the assumption of a random distribution would be about 0·01, while the departure of the tree-ring mean K from the mean of all values is 0·08. The measurements of thickness of tree-rings, which were for American trees measured by Dr. Douglass, had been adjusted to eliminate as far as possible non-climatic factors, such as more rapid growth when the tree was young, spread of tree at the base, and more rapid growth of some trees owing to favourable environment. It is possible that this adjustment may be responsible for some of the difference in K.

If this difference is ignored the maximum difference of a group from the mean is still 0·04, which is too large for a random distribution. The conclusion is that on the present data the best value of K to adopt for calculations of reservoir capacity is 0·72, which gives a little more weight to the short-term and more precisely measured quantities—discharge, rainfall, temperature and pressure, of which it is the mean—than to varves and tree rings. We therefore arrive at the equation

$$R/\sigma = (N/2)^{0\cdot72} = 0\cdot61\, N^{0\cdot72}$$

as giving the most probable value of R.

It is clear from the above that in regard to the parameter R the natural phenomena so far considered have a similarity amongst themselves but differ from purely chance phenomena. One way in which this difference is shown is that if, for example, we have a record covering several hundred years and this is divided into sets of fifty years, for each of which we calculate the mean and standard deviation, these means and standard deviations are more variable than they would be for a normal Gaussian frequency distribution. Thus if we have a record for 40 or 50 years of the flow of a stream the extreme values which may occur in the future are likely to be greater than would be forecast on the ordinary theory of probability. The idea therefore that 30 or 40 years of observations on a stream give a full representation of what it can do is incorrect.

It is also fallacious to extend a short record of, say, 50 years by constructing a normal frequency curve and using some device of drawing a series of random values based on this curve, such as the author's probability pack of cards,[1] or the similar device described by F. B. BARNES;[3] these will give the distribution of R/σ represented by equation (1) instead of that of equation (2).

III. STORAGE FROM PAST RECORDS WHEN ONLY PART OF THE SUPPLY IS USED

We shall now consider the case where the annual water requirement is less than the mean flow of the river, but not less than the minimum recorded annual flow. In this case we have a draft B equal to the requirements and less than the mean M, and we need to know the greatest accumulated deficit S which has to be made up from storage. This can be found for any particular record from the curve of accumulated departures.

Referring to *Figure 1*, OX is the axis of departures from the mean (M). If now we give a draft B less than the mean the storage will each year be increased by $M-B$ over what it was when the draft was M. This can be found from the original curve of accumulated departures by drawing an axis OX_1, passing through the point F whose ordinate is $-N(M-B)$. The ordinates of the curve referred to axis OX_1 show the changes of storage with the draft B. The storage falls from P to Q and the fall is $Pc+dQ$.

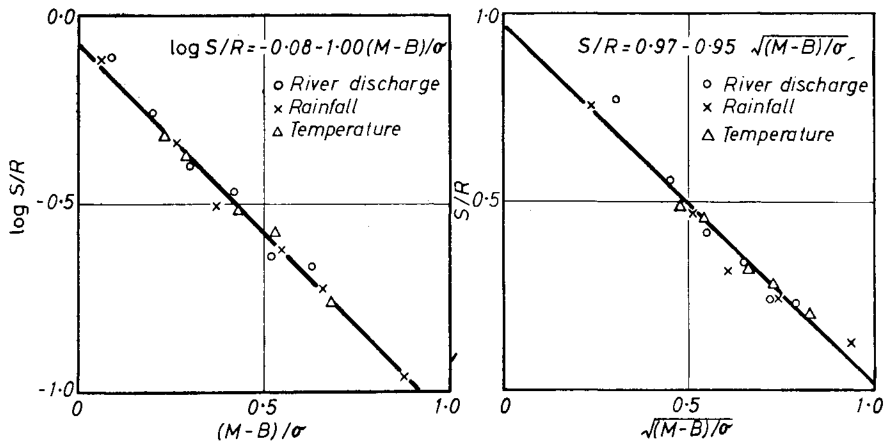
This is the amount of storage S which would have been needed to give the draft B.

S was found for several different drafts for each of 38 phenomena taken from the classes of river discharges, rainfall, and temperature. The results for each class were grouped according to the draft, and the means of these groups have been plotted. Two relations fit the results equally well. The first is

$$\log_{10}(S/R) = -0{\cdot}08 - 1{\cdot}00(M-B)/\sigma, \tag{3}$$

and this is shown in *Figure 3(a)*.
The second is

$$S/R = 0{\cdot}97 - 0{\cdot}95\sqrt{\{(M-B)/\sigma\}}, \tag{4}$$



(*a*) (*b*)

Figure 3. Relation between draft (B) and maximum deficit (S).

which is shown in *Figure 3(b)*. The average value of N from which these results are deduced is 96. It will be seen that as far as closeness of fit is concerned, over the range of observations, there is no significant difference between one type of relation and the other. At some future time it may perhaps be possible to decide that one of the types has some theoretical justification. If the figures are examined it will be noticed that the values from the three classes of phenomena are indistinguishable. This is a further example of the statistical similarity of river discharges, rainfall, and temperature shown by *Table 2*. Similar relations, but with slightly different constants, have been found for the phenomena with long-term records, Roda Gauge, varves, and tree rings (average N = 430).

A feature of the results is that a small reduction of draft below the mean makes a much greater reduction in the storage required to give the draft. For example a reduction of $0{\cdot}1\sigma$ in the draft reduces the storage by 35 per cent. This fact provides a very practical means of applying a factor of safety to compensate for the natural variations of R.

IV. REMARKS ON THE PARAMETER R

Equation (1) giving the mean value of R for a quantity Q whose variation is random has also been derived by Professor W. FELLER[4] of Princeton University by a more abstruse method. He also states that when N is large the mean value of R is independent of the frequency distribution of Q. It is probable that this applies also to natural phenomena since it is derived from a general theorem of statistical theory. An inference from this is that the difference between equation (2) for natural phenomena, and equation (1) for random events is not due to differences from the normal distribution, but to the occurrence of runs of high or low values leading to greater variation of the means and standard deviations for periods of years than would occur in random distributions. The fact that frequency distributions of many natural phenomena approximate to the normal Gaussian form is not therefore as relevant as the author originally supposed, and the small variability of K (*Table 2*) is to be taken as a property only remotely connected with the form of distributions.

V. METHODS OF USING LONG-TERM STORAGE. GENERAL REMARKS

The previous investigation dealt with past events, for which complete records exist, and from which conclusions were drawn as to the relations existing between R, σ, and N, and between S, R, and B. The important practical problems to which these relations will be applied are (*a*) the determination of the size of a long-term storage reservoir, and (*b*) the means to be adopted to regulate the storage to the best advantage to meet an unknown future.

It may be mentioned that although the approach of the writer to the problem was from the point of view of irrigation, power, and flood-protection, nevertheless there are other matters to which the investigation could be applied, as for example the accumulation of stocks of food, munitions, or other materials. Although a variety of phenomena have provided data it will be convenient to talk of all of it as if it referred to river discharges.

With regard to the size of a reservoir, besides the statistical conditions, there are many other determining factors such as available sites and their capacities, cost in relation to size, how the storage will be used and where the results will be employed, and possible political considerations; or conflicting interests in the use of the water, as for instance agricultural and industrial. The present discussion is confined to statistical considerations, and is based on trial regulations applied to some of the data which were used in the foregoing investigation.

In the work of the author and his colleagues on Nile Projects[5] the long period of 100 years was taken as the basis of calculations of storage. This was done because of the persistence of low or high values of discharges over periods of years as long as 50 or even more, and because 100 years is a long time to look forward in the life of a project, especially when one considers the present very rapid development of physical and engineering science. The parameter R_{100} is therefore the capacity which has been used as a basis in the schemes of regulation of the draft, and hence of the storage, which will now be described. It may again be pointed out that equation (2), from which R_{100} is calculated by substituting $N = 100$, gives only a mean value for R_1, and that generally, if the circumstances of a project permit, the larger the value assumed for the reservoir capacity the better.

The following is taken from ref. 2. The problem which has been investigated first is what system can be applied automatically to guarantee as large an annual draft as possible, but which will not empty the reservoir over a long period of years. Reservoir losses have not been considered, but in some cases in practice would need to be taken into account. A preliminary trial was made with 32 different phenomena by plotting accumulated departures for each record and then calculating $R_{100} = 16{\cdot}7\sigma$. Using R_{100} as the capacity of the reservoir and starting with the reservoir half-full, if the average discharge for the period was taken as the draft, it was found that the reservoir filled in 66 per cent of the cases and emptied in 59 per cent. Filling or emptying might be prevented either by increasing the value of R or by making the draft variable. If the reservoir was full at the start it emptied in 41 per cent of the cases.

In these cases regulation takes place after the event, using the actual mean and standard deviation of the period. Even so, it is clear that a factor of safety is needed, and still more will this be the case when a mean and standard deviation from the past have to be applied to the future.

VI. REGULATIONS WHICH WOULD BE POSSIBLE IN PRACTICE

To find regulations which would be possible the assumption was made that the first 30 years records were available, and these were used as initial data to form a basis for regulating the remainder of the record, taking account of the additional data as it appeared. The following possibilities were tried:

(*a*) The draft was kept constant.
(*b*) The draft was varied with the inflow.
(*c*) The draft was varied with the amount in the reservoir (content).
(*d*) The draft was varied both with inflow and with content.

It was found that with natural phenomena the standard deviation tended to increase with the length of the period, so in calculating R_{100} the standard deviation (σ') was taken as $1{\cdot}1\sigma_{30}$ (subscripts denote number of years in period). M denotes a mean.

Table 3 shows the results of trying regulations of types (*a*) and (*b*). It shows the advantage of the draft varying with the supply, over a draft fixed at M_{30}, and the effect of increasing the starting content, which however decreases as the content becomes larger. A draft M_{10} changing every year is better than one changing every 5 years, both from the point of view of running dry and of wasting water by spilling. In practice if flood protection has to be considered there may be a limit to the permissible draft and consequent amount of spilling and it will be better to start with the reservoir half full.

In the case of Regulation 7 a reduction of the draft of $0{\cdot}1\sigma$ would have

Table 3

Results of regulations based on initial data from 30 years
Capacity $R_{100} = 16{\cdot}7\sigma' = 18{\cdot}4\sigma$

Regulation No.	*No. of phenomena*	*Starting content*	*Draft*	*Percentage of cases*	
				Reservoir fills	*Reservoir empties*
3	51	$\frac{1}{2}R'_{100}$	M_{30}	44	38
4	51	$\frac{1}{2}R'_{100}$	M_{10} changing every 5 years	23	19
5	51	$\frac{3}{4}R'_{100}$	,, ,, ,, ,, ,,	56	5
6	51	R'_{100}	,, ,, ,, ,, ,,		2
7	51	$\frac{1}{2}R'_{100}$	M_{10} changing every year	12	15

prevented the reservoir emptying in all but 6 per cent. of the cases. A regulation of type (*c*) where the draft was varied on the basis of reservoir content only was tried in one case, but it was not thought a good enough scheme compared with others to be worth much labour to examine.

In the cases, summarized in *Table 3*, where there was difficulty in preventing the reservoir from emptying, the variable draft of Regulation 7 was reduced by the application of a sliding scale, depending on the amount of water in the reservoir (Regulation 9). In Regulation 9 the content of the reservoir was divided into 9 parts. When the content was in the middle ninth the draft was M_{10}, in the ninth below the draft was $M_{10} - g$ and so on down to $M_{10} - 4g$, where g is the step of the sliding scale. As a measure of safety against floods the step can also be applied to the upper half of the content. With Regulation 9, in the worst cases the steps g which would have prevented emptying were

$$0{\cdot}001\sigma',\ 0{\cdot}01\sigma',\ 0{\cdot}04\sigma',\ 0{\cdot}4\sigma',\ 0{\cdot}5\sigma',\ 0{\cdot}5\sigma' \qquad (5)$$

with one other case where an insignificant step would have prevented emptying. The result of Regulation 9 is that in 14 per cent. of cases a sliding scale would be needed to prevent emptying, and in only 6 per cent. would the reduction of draft have been more than trivial. Regulation 9 is probably the type which would be most generally useful when prevention of emptying is the main consideration. The type of regulation, however, depends upon the special circumstances of the case and probably no single type will meet all cases. The choice of a regulation depends on the risk of emptying, the size of the disaster if emptying occurs, and the cost of remedial measures.

The difficult cases have been analysed and in each case the main cause of difficulty was the pronounced reduction of the mean discharge after the regulation had started, which sometimes lasted for 50 years, and this was sometimes assisted by an increase of σ or K.

The variation of draft for 46 of the phenomena used in ref. 2 has been found from the curves of accumulated departures by comparing the lowest 10-year mean with the lowest 30-year mean. Thirty years is a long time in the life of a farm or a hydro-electric scheme, and covers more than a generation, so it gives a useful standard of comparison. Taking means at 5-year intervals, the results were

	Mean value
(Lowest 10-year mean)/(Lowest 30-year mean)	0·95
(Lowest 10-year mean − $0 \cdot 1\sigma$)/(Lowest 30-year mean)	0·93

VII. FLOOD PROTECTION

The previous sections deal only with safeguarding against drought. Flood protection requires that the discharge out of the reservoir must not exceed a certain limit. This limit depends on local circumstances, but as an example annual discharges which would be exceeded on the average 1 year in 10, 1 year in 20, and 1 year in 50 have been taken as criteria. Since on the average their frequency distributions are approximately normal the above frequencies roughly correspond to departures from the mean greater than 1·3, 1·7, and 2·1 times the standard deviation.

Regulations 4 and 5 have been carried out on all the phenomena considered in section VI, and the criteria have been applied to those cases in which the reservoir filled. As an example we take Charleston rainfall, which was the most extreme case, and the diagram for which is shown in *Figure 4*.

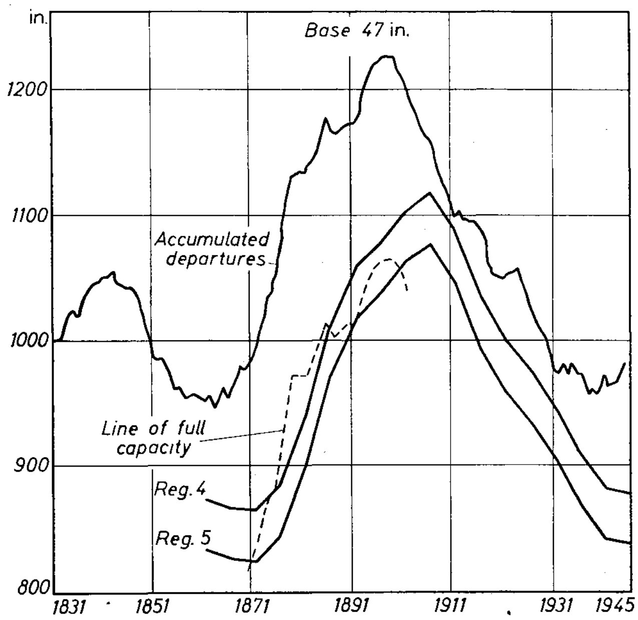


Figure 4.

As already explained the regulations start at the end of the first 30 years. Regulation 4 starts with the reservoir half-full and this content is represented by the initial distance between the regulation curve and the curve of accumulated departures. A dotted line to represent the content when the reservoir is full has been drawn at a distance R'_{100} below the accumulated departures curve. When this line is above the regulation curve the reservoir will be full and the difference between the two curves represents the content which will have to be discharged in excess of the draft, or alternatively stored, in whole or in part, in some extra capacity reserved for flood protection.

Figure 5 shows the time of excess on a larger scale. It also shows lines

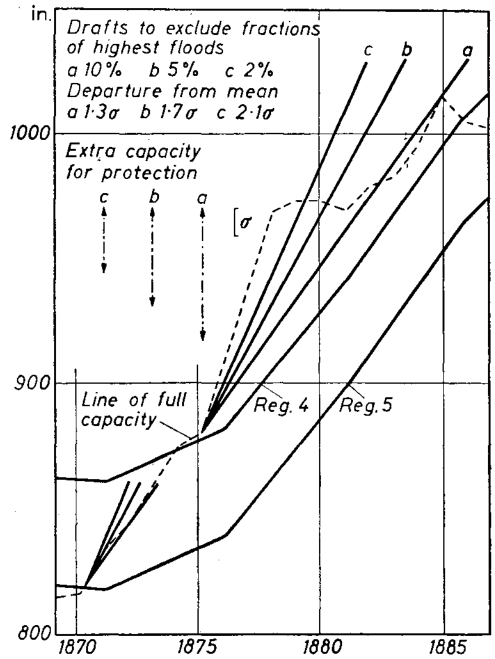


Figure 5.

of draft *a*, *b*, and *c* representing the different permissible limits applied to Regulation 5, but from 1875 the same lines also apply to Regulation 4. The maximum storages required in these cases are

$$(a)\ 4{\cdot}7\sigma,\ (b)\ 3{\cdot}5\sigma,\ \text{and}\ (c)\ 2{\cdot}3\sigma.$$

The preceding applies to Regulations 4 and 5, which, however, are not the most likely to be adopted. The results of applying a sliding scale to Regulation 7 are given below, where the step is just enough to prevent the reservoir spilling over.

Charleston rainfall was very unusual, as the three highest years occurred in succession and had departures from the mean of $2{\cdot}9\sigma$, $2{\cdot}9\sigma$, and $2{\cdot}8\sigma$. With a purely random distribution a departure of $2{\cdot}8\sigma$ or more would occur on the average about once in 400 years. In the cases examined Albany rainfall is the only other where departures greater than $2{\cdot}7\sigma$ occurred, and they did so in successive years.

It is interesting to compare the steps in (7) with those in *Table 4.*

Table 4. Regulation 9

Sliding scales, depending on content, applied to Regulation 7

Phenomenon	*Step*
Cape Town rainfall	$0{\cdot}0067\sigma$
Charleston rainfall	$0{\cdot}165\sigma$
Helsingfors temperature	$0{\cdot}067\sigma$
Roda gauge A.D. 1341–1440	$0{\cdot}040\sigma$
Trier rainfall	$0{\cdot}0025\sigma$

In the first case the draft is regulated so that the reservoir just empties and does not remain empty, in the second the draft is regulated so that the reservoir just fills and there is no necessity to spill excess water. In both tables the number of cases is nearly the same and the steps of the sliding scale are of the same order in size. It seems legitimate therefore to combine the tables and say that a sliding scale of $0{\cdot}2\sigma$, reducing the 10-year-mean draft at the smaller contents and increasing it at the larger would leave only 4 cases out of 51 in which the reservoir capacity would have been insufficient.

The above analysis deals only with annual average or total discharges, leaving variations within the year to be considered separately. It is probable that in regard to flood protection, in many cases, individual high floods may be more important than the cumulative effects discussed in the present paper.

VIII. CONCLUSION

An outline of the general theory underlying long-term storage has been given. Each problem, however, must be treated separately, having regard to the practical conditions by which it is governed. The records of the phenomenon which is to be regulated are not themselves sufficient to give a solution. This must also be based, as has been shown, on data relating to similar phenomena. Finally it is to be remembered that, as in most civil engineering problems, the data are not certainties for the future, but only probabilities of varying degrees. They must therefore be dealt with by the recognized methods of statistics, and the factors of safety deducible from these must be applied.

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