Syllabus

Probability Theory

References: **Probability: Theory and Examples (https://uc.instructure.com/courses/1849416/files/202420191?wrap=1)** *by Rick Durrett*.

Measure Theory, Probability, and Stochastic Processes (https://uc.instructure.com/courses/1849416/files/202842113?wrap=1) *by Jean-Francois LeGall (Free e-copy can be found on the UC library website)*.

Lecture: TuTh 9:30--10:50 AM at 60WCHARL 245

Lecturer: Xiaoqin Guo (guoxq at ucmail dot uc dot edu)

Office hours: Tuesdays 1:20--2:10 PM or by appointment (at French Hall 4314)

Credits: 3

Course description:

This course provides a rigorous introduction to modern probability theory with an emphasis on measure-theoretic foundations and limit theorems. Topics include σ-fields, measures, and the Lebesgue integral; modes of convergence of random variables; expectations and distributions; independence and classical inequalities; laws of large numbers and central limit theorems; weak convergence and characteristic functions; and advanced tools such as Kolmogorov's inequalities and series theorems, central limit theorem for triangular arrays, and martingale theory. The course develops both conceptual understanding and technical skills, preparing students for further study in probability, statistics, stochastic processes, and related areas such as analysis and mathematical finance.

Prerequisites: Students will be expected to have a strong background in theoretical mathematics or statistics. A good knowledge of multivariable calculus and an introduction to analysis is a must. Advanced Calculus (MATH 6001), Mathematical Statistics (STAT 6021/6022), or equivalent is recommended. Enrollment Requirements: To take this course you must: Have taken the following Courses MATH5101 min grade B-, or MATH6001 min grade B-, or STAT5122 min grade B-, or STAT6022 min grade B-.

Course Grade: The final total score=10%Discussions+35%Homework+20%Midterm+35%Final.

The final grades will be determined according to the following scale:

A: [100,92],A-: (92,90],B+:(90, 87],B: (87,83],B-: (83,80],C+: (80,77],C:(77,72],C-:(72,70],D+: (70,67],D: (67,62],D-(62,60].

There will be no curving in the class, but the instructor reserves the right to modify the final grade lines.

- *Homework*: There will be several Homework assignments.

  The homework is graded according to its correctness, completeness and presentation. **Answers alone carry no credit**. One should provide clear argument and steps that lead to your solution/conclusion.

  - Observe the **Student Code of Conduct (https://www.uc.edu/campus-life/conduct/student-code-of-conduct.html)** . Handing in plagiarized work, whether copied from a fellow student or off the web, is not acceptable. Plagiarism cases will lead to sanctions.
  - Working in groups on homework assignments is encouraged; however, everyone must write his/her own assignments.
  - Organize your work neatly. Use proper English. Write in complete English or mathematical sentences. Answers should be simplified as much as possible.
  - Do not hand in your rough draft or first attempt. Papers that are unreadable or disorganized cannot be graded. I strongly encourage you to type up your solutions using Latex.
- *Discussion*: The Discussion score evaluates a student's attendance and performance in the class (i.e. asking/answering questions or giving comments).

- *Exams*: There will be one midterm exam and a comprehensive final exam. The time of the exams are TBA on our "**Schedule (https://uc.instructure.com/courses/1849416/pages/daily-schedule-will-be-updated-as-we-go)** " page.

Tentative Schedule:

Here is a tentative weekly schedule which will not be updated after the first day of class and may be significantly adjusted as we go. For the ***real*** schedule, please refer to our "**Schedule (https://uc.instructure.com/courses/1849416/pages/daily-schedule-will-be-updated-as-we-go)** " page.

Week	Topics
1 Jan/12	σ-field, Measure, and Lebesgue integral
2 Jan/19	Properties of the Lebesgue integral. Bounded convergence theorem, Fatou's lemma. Monotone convergence theorem, Dominated convergence theorem.
3 Jan/26	Distributions and expectations of random variables. Review of important distributions. Independence, Borel-Cantelli lemmas

4 Feb/2	Markov's inequality,Jensen's inequality, Cauchy-Schwarz inequality, definitions of covariance and variance. Modes of Convergence
5 Feb/9	Law of Large Numbers, the converse theorem of the SLLN, convergence in distribution
6 Feb/16	different characterizations of the weak limits, its relation with other modes of convergence
7 Feb/23	characteristic functions and properties.
8 Mar/2	Central Limit Theorems
9 Mar/9	Equivalent conditions for a.-s. convergence of a sequence, definition of "a.s. Cauchy". Midterm Exam.
10 Mar/16	Spring Break
11 Mar/23	Kolmogorov's maximal inequality, Kolmogorov-Khinchin's Two Series Theorem, Kolmogorov's Three Series Theorem.
12 Mar/30	triangular array, Lindeberg's CLT, Lindeberg's condition.
13 Apr/6	More examples on Lindeberg CLT
14 Apr/13	Computations of conditional Expectations, properties of conditional expectation,martingales
15 Apr/20	Martingale convergence theorem and martingale CLT
16	Final Exam