



Inverse Kinematics

$$\begin{aligned}
 \text{WheelVelocities} = \begin{bmatrix} \dot{\phi}_l \\ \dot{\phi}_r \end{bmatrix} &= \frac{1}{r} \begin{bmatrix} -D & 1 & 0 \\ D & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ v_x \\ v_y \end{bmatrix} \\
 &= \frac{1}{r} \begin{bmatrix} -D\dot{\theta} + v_x \\ D\dot{\theta} + v_x \end{bmatrix} = \begin{bmatrix} (-D\dot{\theta} + v_x)/r \\ (D\dot{\theta} + v_x)/r \end{bmatrix} \begin{matrix} \textcircled{1} \\ \textcircled{2} \end{matrix}
 \end{aligned}$$

Forward Kinematics

Given WheelAngles $\begin{bmatrix} \phi_l \\ \phi_r \end{bmatrix}$, we can get the WheelVelocities $\begin{bmatrix} \dot{\phi}_l \\ \dot{\phi}_r \end{bmatrix}$

$$\text{now, } v_b = \begin{bmatrix} \dot{\theta} \\ v_x \\ v_y \end{bmatrix} = \begin{bmatrix} \frac{r}{2D}(\dot{\phi}_r - \dot{\phi}_l) \\ \frac{r}{2}(\dot{\phi}_l + \dot{\phi}_r) \\ 0 \end{bmatrix}$$

$$\begin{aligned}
 \textcircled{1} + \textcircled{2} &\Rightarrow \frac{2v_x}{r} = \dot{\phi}_l + \dot{\phi}_r \\
 v_x &= \frac{r}{2}(\dot{\phi}_l + \dot{\phi}_r)
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} - \textcircled{1} &\Rightarrow \dot{\phi}_r - \dot{\phi}_l = \frac{2D\dot{\theta}}{r} \\
 \dot{\theta} &= \frac{r}{2D}(\dot{\phi}_r - \dot{\phi}_l)
 \end{aligned}$$

by integrating v_b , we get $T_{bb'}$

finally, $T_{wb'} = T_{wb} T_{bb'}$