Low-shot Learning via Covariance-Preserving Adversarial Augmentation Network

Hang Gao[1], Zheng Shou[1], Alireza Zareian[1], Hanwang Zhang[2], Shih-Fu Chang[1]
[1]Columbia University, [2]Nanyang Technological University
{hg2469, zs2262, az2407, sc250}@columbia.edu
hanwangzhang@ntu.edu.sg

1 Details about Neighborhood Batch Sampling

In this section, we cover more details with regarding to Neighborhood Batch Sampling (NBS). We have considered two instantiations of the translation mapping R and similarity scores α, based on hard k-nearest neighbor search and soft selection, respectively. Given a novel class y_n, we want to select the base classes $\{y_b\}$ that are semantically similar to the y_n query.

Hard assignments (NBS-H) This sampling method retrieves k uniformly weighted nearest base classes. NBS-H can be formulated as follows,

$$R(y_n) = \underset{\mathcal{Y}'_b \subset \mathcal{Y}_b, |\mathcal{Y}'_b| = k}{\arg\min} \sum_{y_b \in \mathcal{Y}'_b} \|\mathbf{l}_{y_b} - \mathbf{l}_{y_n}\|_2^2, \quad \alpha(y_b, y_n) = \frac{1}{k} \, \forall y_b \in R(y_n) \tag{1}$$

Similar heuristics are used in previous works [1, 2] as well by introducing a new hyper-parameter k. Though NBS-H may save computational resources, in practice, we find it too sensitive to the selection of k. In addition to that, it treats all selected base classes as equally related to the target novel class y_n, which slows the convergence and hurts the performance.

Soft assignments (NBS-S) In this case, all base classes are considered, and weighted by the softmax score over the learned metrics,

$$R(y_n) = \mathcal{Y}_b, \quad \alpha(y_b, y_n) = \frac{\exp\left(-\|\mathbf{l}_{y_b} - \mathbf{l}_{y_n}\|_2^2\right)}{\sum_{y'_b \in \mathcal{Y}_b} \exp\left(-\|\mathbf{l}_{y'_b} - \mathbf{l}_{y_n}\|_2^2\right)} \tag{2}$$

Through the ablation study, we showed that this batch sampling technique is more effective than NBS-H given enough computational resources.

2 Details about Intermediate GAN Objectives

In this section, we formulate our full objectives for intermediate variants derived for the imbalanced set-to-set translation.

`c-GAN`   Its full objective could be defined as a basic minimax game,

$$G_n^* = \arg\min_{G_n} \max_{D_n} \mathcal{L}_{\text{adv}}(G_n, D_n, \mathcal{B}, \mathcal{N}). \tag{3}$$

`cCyc-GAN` Accordingly, its full objective can be directly derived from cycle-consistency,

$$G_n^* = \arg \min_{G_n, G_b} \max_{D_n, D_b} \mathcal{L}_{\text{adv}}(G_n, D_n, \mathcal{B}, \mathcal{N}) + \mathcal{L}_{\text{adv}}(G_b, D_b, \mathcal{N}, \mathcal{B}) + \lambda_{\text{cyc}} \mathcal{L}_{\text{cyc}}(G_n, G_b). \quad (4)$$

3 Details about Computing Subgradient of Ky Fan m-norm

Theorem 1 *Given a matrix $\mathbf{X}$ and its Ky Fan m-norm $\|[\mathbf{X}]_m\|_* = \sum_i \sigma_i(\tilde{\mathbf{X}})$ where $\tilde{\mathbf{X}} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$ is the m-truncated SVD and $\sigma_i(\cdot)$ is the i-th largest singular value, we have,*

$$\frac{\mathrm{d}\|[\mathbf{X}]_m\|_*}{\mathrm{d}\mathbf{X}} = \mathbf{U}\mathbf{V}^T \quad (5)$$

Proof Rewrite Ky Fan m-norm by its sub-differential set,

$$\|[\mathbf{X}]\|_* = \mathrm{tr}(\mathbf{\Sigma}) = \mathrm{tr}(\mathbf{\Sigma}\mathbf{\Sigma}^{-1}\mathbf{\Sigma}) \quad (6)$$

Then,

$$\mathrm{d}\|[\mathbf{X}]_m\|_* = \mathrm{tr}(\mathbf{\Sigma}\mathbf{\Sigma}^{-1}\mathrm{d}\mathbf{\Sigma}) \quad (7)$$

Since we have,

$$\mathrm{d}\mathbf{X} = \mathrm{d}\mathbf{U}\mathbf{\Sigma}\mathbf{V}^T + \mathbf{U}\mathrm{d}\mathbf{\Sigma}\mathbf{V}^T + \mathbf{U}\mathbf{\Sigma}\mathrm{d}\mathbf{V}^T \quad (8)$$

Therefore,

$$\begin{aligned}
\mathbf{U}\mathrm{d}\mathbf{\Sigma}\mathbf{V}^T &= \mathrm{d}\mathbf{X} - \mathrm{d}\mathbf{U}\mathbf{\Sigma}\mathbf{V}^T - \mathbf{U}\mathbf{\Sigma}\mathrm{d}\mathbf{V}^T \\
\Rightarrow \mathrm{d}\mathbf{\Sigma} &= \mathbf{U}^T\mathrm{d}\mathbf{X}\mathbf{V} - \mathbf{U}^T\mathrm{d}\mathbf{U}\mathbf{\Sigma} - \mathbf{\Sigma}\mathrm{d}\mathbf{V}^T\mathbf{V}
\end{aligned} \quad (9)$$

By the diagonality of $\mathbf{\Sigma}$ and anti-symmetricity of $\mathbf{U}$, $\mathbf{V}$,

$$\begin{aligned}
\mathbf{U}^T\mathrm{d}\mathbf{U}\mathbf{\Sigma} + \mathbf{\Sigma}\mathrm{d}\mathbf{V}^T\mathbf{V} &= 0 \\
\Rightarrow \mathrm{d}\mathbf{\Sigma} &= \mathbf{U}^T\mathrm{d}\mathbf{X}\mathbf{V}
\end{aligned} \quad (10)$$

Substitute it into Equation 7,

$$\begin{aligned}
\mathrm{d}\|[\mathbf{X}]_m\|_* &= \mathrm{tr}(\mathbf{\Sigma}\mathbf{\Sigma}^{-1}\mathrm{d}\mathbf{\Sigma}) = \mathrm{tr}(\mathbf{U}^T\mathrm{d}\mathbf{X}\mathbf{V}) = \mathrm{tr}(\mathbf{U}^T\mathbf{V}\mathrm{d}\mathbf{X}) \\
\Rightarrow \frac{\mathrm{d}\|[\mathbf{X}]_m\|_*}{\mathrm{d}\mathbf{X}} &= \mathbf{U}\mathbf{V}^T
\end{aligned} \quad (11)$$

$\square$

References

[1] Bharath Hariharan and Ross Girshick. Low-shot visual recognition by shrinking and hallucinating features. *arXiv preprint arXiv:1606.02819*, 2016.

[2] Yu-Xiong Wang, Ross Girshick, Martial Hebert, and Bharath Hariharan. Low-shot learning from imaginary data. *arXiv preprint arXiv:1801.05401*, 2018.