Introduction to Algorithms

CS 430

Lecture L18 - 20



Outlines

- Greedy Algorithm
 - Activity selection problem
 - 0-1 Knapsack problem
 - Huffman code

Why Greedy?

When we are seeking for optimal solution

Dynamic programming

Decompose the problem to subproblems and take either top-down or bottom-up to solve the subproblems. The optimal solution depends on the combination of all solutions to subproblems.

- Problem exhibits optimal substructure.
- How about just to consider the local best choice and assume that all these local choices will lead to a global optimal solution?

Does this strategy work for all circumstances?

- Zero-Sum Game Theory
 - A zero-sum game is a mathematical representation of a situation in which each participant's gain or loss of utility is exactly balanced by the losses or gains of the utility of the other participants.

Example for local utility and global utility



Greedy Algorithm

- local optimal choice leads to global optimal solution
- a special case of dynamic programming
- dynamic programming: traversing the solutions of subproblems is equivalent to traversing subproblems; for bottom-up, a root has to consider the solutions of its children before solve itself;
- greedy: solutions of children do not affect the current choice; bottom-up is invalid.

Greedy algorithm

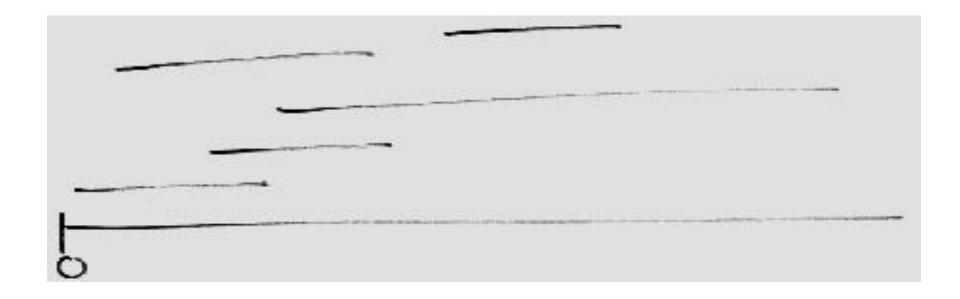
- definition:
 - A Greedy Algorithm always makes the choice that looks best at the moment. It makes a locally optimal choice in the hope that this choice will lead to globally optimal solution.
 - Greedy algorithms do not always yield optimal solutions, but for many problems, they do.

Applications of Greedy Algorithms

- Activity Selection Problem
- 0-1 Knapsack
- Huffman Codes

Activity Selection Problem*

- Set S of n activities each with start si, finish fi
- Maximum set of Compatible Activities (non overlapping)



Activity Selection Problem*

- An activity selection problem: we have a set S={a₁,a₂,...,a_n} of n proposed activities that wish to use a resource, which can serve only one activity at a time. Each activity a_i has a start time s_i and a finish time f_i. Where
 0 ≤ s_i ≤ f_i ≤ ∞.
- Activities a_i and a_j are compatible if the intervals [s_i, f_i) and [s_j, f_j) do not overlap. a_i and a_j are compatible if s_i,>=f_j or s_j,>= f_i.
- In the activity selection problem, we wish to select a maximum-size subset of mutually compatible activities.

Activity Selection – Brute Force*

Try ALL compatible meeting combinations, pick the one with the max # of activities.

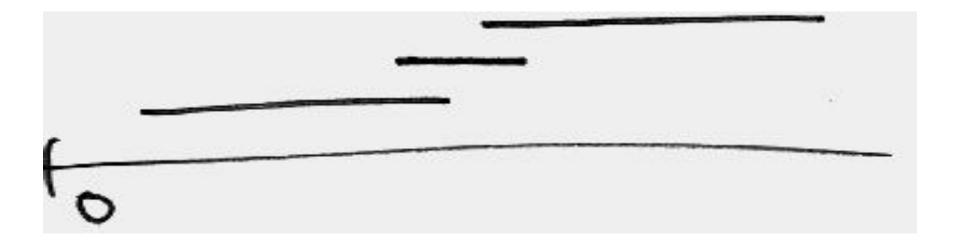
- "n" activities;
- total number of different combinations
 = 2ⁿ;
- subset of those are compatible

Activity Selection— Possible Greedy Approaches

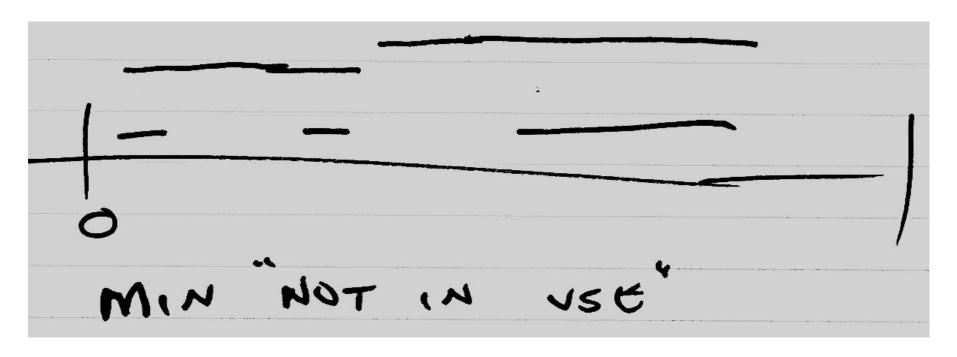
- Pick shortest meeting first
- Minimize "not in use" time between meetings
- Pick meetings with least number of conflicts
- Pick earliest start meeting

Divide problem into smaller sub-problems

Pick Shortest Meeting - Counter Example



Minimize "Not in use"- Counter Example



Pick Least Conflict - Counter Example

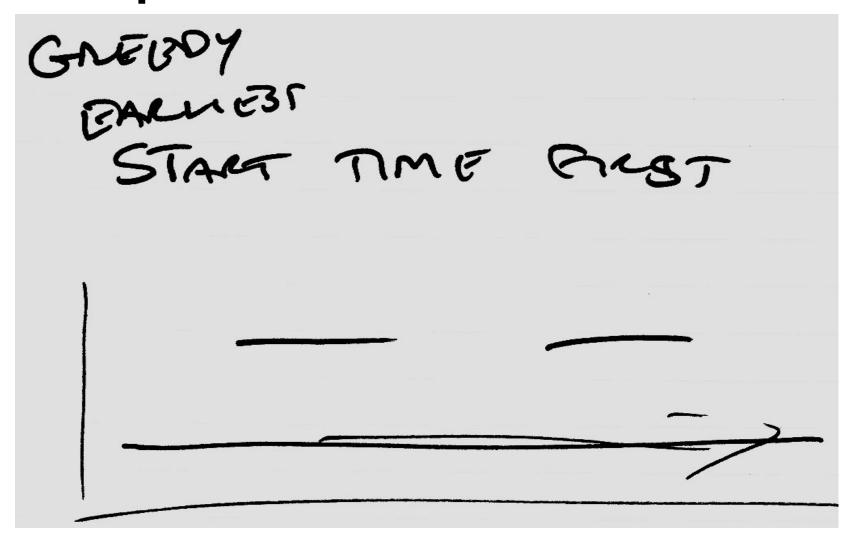
A=(4,6) B=(0,1) C=(1,5) D=(5,9) E=(9,10) F=(0,4) G=(6,10) H=(0,3) I=(7,10) J=(0,2) K=(8,10) L=(0,1) M=(9,10)

A overlaps 2 meetings, while everything else overlaps at least 3 mtgs

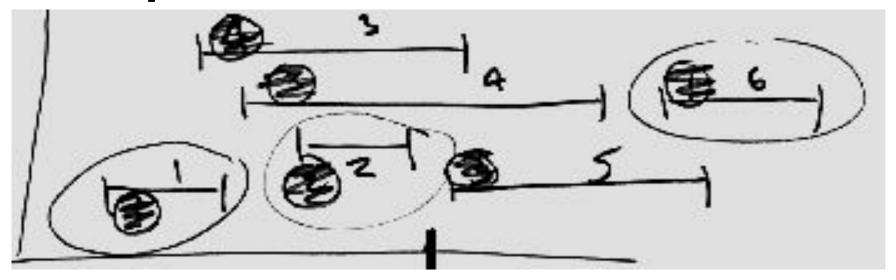
If we choose A, we exclude C and D, and the max we can get is 3 mtgs (A, and one from each side of A)

The optimal answer is 4 mtgs B, C, D, E (other similar answers length 4)

Earliest Start Time First - Counter Example



Earliest Finish Time first - Divide problem into smaller sub-problems -



- 1 2 6
- 1 2 5
- 1 4 6

Optimal Subs s_i 1 3 1 9 6 7 1 5 3 7 Optimal solution to main problem contains

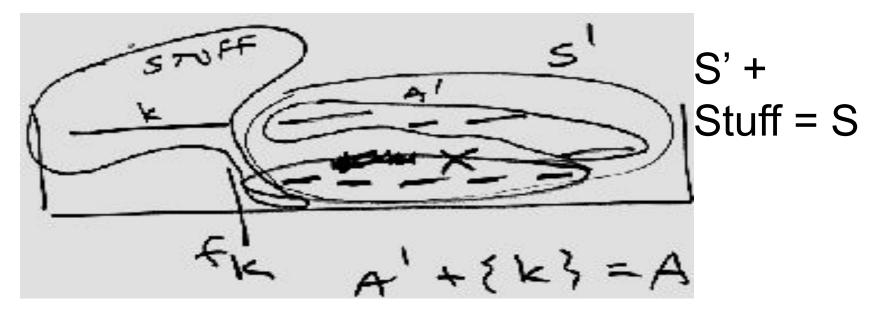
Optimal solution to main problem contains optimal solutions to sub problems of main. If A is solution to S

Optimal Subset of MTGS {2,4,k,8,10} No order {1,2,3,....n} No Order

Remove earliest ending activity from A (Happens to be MTG k) $f_k < f_i$

A' = $\{2,4,8,10\}$ must be solution to subproblem S'= $\{S:s_i>f_k\}$ =All MTGS starting after k finishes

A' must be solution to S', if it isn't then original solution to A was not optimal for S



Suppose that A' is not the optimal solution to S', there is an optimal solution X to S':

$$|X| + \{k\} > A$$

Contradiction

A is Optimal

- Greedy choice
 - We should choose an activity that leaves the resource available for as many other activities as possible.
 - Choose the activity in S with the earliest finish time since that would leave the resource available for as many activities the follow it as possible.
 - Let $S_k = \{a_i \in S: s_i > = f_k\}$ be the set of activities that start after a_k finishes.
 - If a₁ is in the optimal solution, then an optimal solution to the original problem consists of activity a₁ and all activities in an optimal solution to the subproblem S₁.

Prove greedy choice property for "Earliest Finish time first"

Assume MTGS ordered earliest finish time first. Assume optimal solution A for problem S

Therefore, there is always an answer that contains the greedy choice

choice

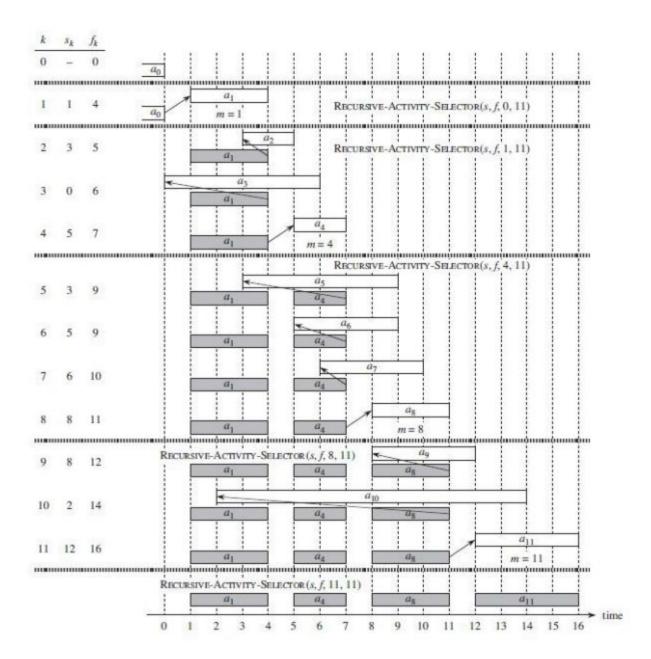
meeting k ends first in solution A

$$B = A - \{K\} + \{1\}$$

Replace non greedy first choice in A

with Greedy choice This can be done f₁ < fょ

Algorithm



Algorithm

return A

```
RECURSIVE-ACTIVITY-SELECTOR (s, f, k, n)
  m = k + 1
  while m \le n and s[m] < f[k] // find the first activity in S_k to finish
3 	 m = m + 1
4 if m \leq n
        return \{a_m\} \cup \text{RECURSIVE-ACTIVITY-SELECTOR}(s, f, m, n)
  else return Ø
 GREEDY-ACTIVITY-SELECTOR (s, f)
 1 \quad n = s.length
 A = \{a_1\}
 3 k = 1
 4 for m = 2 to n
5 if s[m] \ge f[k]
6 A = A \cup \{a_m\}
7 k = m
```

- The optimal solution
 - Sij is the set of activities that start after activity ai and that finish before activity aj starts.
 - We wish to find a maximum set of mutually compatible activities in S_{ij}. Assume that the maximum set is A_{ij} which includes some activity a_k.
 - ♠ A_{ij} = A_{ik} U {a_k} U A_{kj}
 - ◆ The number of elements in A_{ij} is |A_{ij}| = |A_{ik}| + |A_{kj}| +1.

Dynamic programming approach

c[i,j] is the size of an optimal solution to the set Sij;
 c[i,j]=c[i,k]+c[k,j]+1.

$$c[i,j] = \begin{cases} 0, & \text{if } S_{ij} = \varphi \\ \\ \text{Max}\{c[i,k] + c[k,j] + 1\}, & \text{when } a_k \in S_{ij} \text{ and } S_{ij \neq} \varphi \end{cases}$$

Comparison on Dynamic and Greedy

- Greedy algorithm obtains an optimal solution to a problem by making a sequence of choices. At each decision point, the algorithm makes choices that seems best at the moment.
- This strategy does not always produce an optimal solution, but sometimes it does.

Comparison on Dynamic and Greedy (cont'd)

- The greedy choice property: we can sample a globally optimal solution by making locally optimal (greedy) choices.
- When we are considering which choice to make, we make the choice that looks best to the current problem without considering effects from subproblems.
- In dynamic programming, we make a choice at each step. But the choice depends on the solution to subproblems. We solve dynamic programming problems processing from smaller subproblems to larger subproblems.

Does Greedy always work for optimization problems?

0-1 Knapsack Problem

- The thief robbing a store finds n items. The ith item is worth z_i dollars and the weight is w_i lbs where z_i and w_i are integers. The thief wants to take as valuable a load as possible, but he can carry at most W lbs in his knapsack. W is an integer. Which items should he take?
- The thief must either take or leave an item.
 He can not take a fractional amount of an item.

Fractional Knapsack Problem

He can take a fractional amount of an item.

0-1 vs Fractional Knapsack*

n items

w_i weight

v_i values

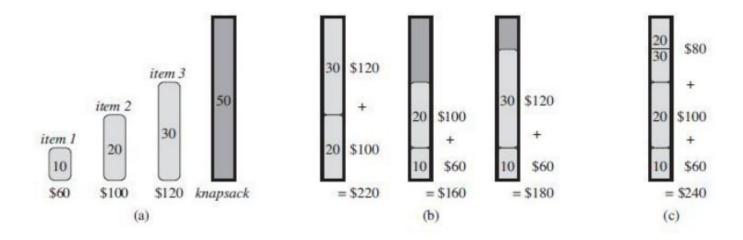
Total weight limit = W

Maximize value Total weight ≤ W

 Fractional amounts of items are not/ are allowed

Greedy Possibilities

- •Max value first
- •Min weight first
- Max value/weight first



W=50

\$250

Fractional Knapsack Optimal Substructure

$$S = \begin{cases} \mathbf{W}_1 \dots \mathbf{W}_i \dots \mathbf{W}_n \\ \mathbf{V}_1 \dots \mathbf{V}_i \dots \mathbf{V}_n \end{cases} \qquad \mathbf{W}$$

Given optimal solution A to fractional Knapsack problem $(w'_{i} \le w_{i} \text{ for all } i)$ $A = \{0,0,w'_{3},0,w'_{i},w'_{5},...,w'_{n}\}$

$$A = \{0,0,w'_{3},0,w'_{i},w'_{5},\dots,w'_{n}\}$$

Remove item i\\ (weight w',) from A

A' is same as A without w',

$$A' = \{0,0,w'_{3}0, w'_{5} \dots w'_{n}\}$$

A' is be optimal solution for sub-problem, S'

Total weight=W-w_i

original n-1 items (excluding item i)

Fractional Knapsack Optimal Substructure (continued)

B is solution to S'
If B was better than A'
(value B > value A')

then item i plus B would be better than A.

Contradiction

Fractional Knapsack Greedy Choice

$$S = \begin{cases} w_1 \dots w_i \dots w_n \\ v_1 \dots v_i \dots v_n \end{cases} \qquad W$$

(Take biggest unit value first)

Prove global optimal solution can be found from a local greedy choice

Assume items are in Greedy Choice order and we have optimal answer A (not necessarily created from greedy choice)

$$\frac{v_1}{w_1} \ge \frac{v_2}{w_2} \ge \frac{v_3}{w_3} \ge \dots \ge \frac{v_n}{w_n}$$

$$A = \{w'_1, w'_{2, \dots}, w'_{n}\}$$

$$0 <= w'_i <= w_i$$

Fractional Knapsack Greedy Choice (continued)

If item 1 has the largest unit value, either A has the greedy choice $(w'_1 != 0)$ or A doesn't have Greedy Choice $(w'_1 = 0)$

Show that there is an equally optimal solution B that does have greedy choice (removing some weight of item k and adding same weight of item 1)

$$B = A - \{\text{item k}\} + \{\text{item 1}\}$$

$$W_A = W_B = W$$

$$V_B \ge V_A$$

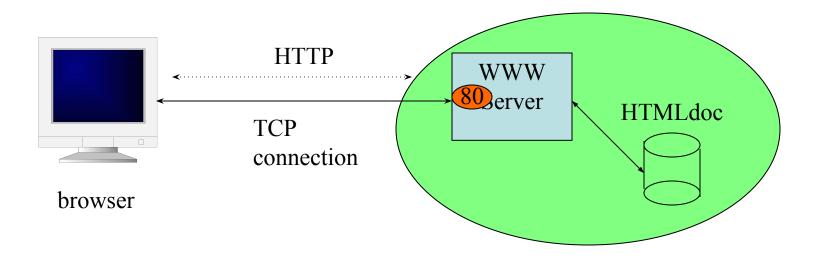
Fractional Knapsack Greedy Choice (continued)

Replacing
min {weight of k or weight of 1} Lbs
item k with
min {weight of k or weight of 1} Lbs item 1
Replace the rest of k (if exist) with item 2
(continuing greedy choice)

$$\frac{v_1}{w_1} > = \frac{v_k}{w_k}$$
 (items in greedy order)
$$B_{VALUE} > = A_{VALUE}$$

Coding problem

a webpage--HTTP



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(!DCTYPE html>(!— saved from url=(0020)https://www.ii.edu/ -->chtml lang="en" dir="ltr" prefix="content: http://purl.org/rss/1.0/modules/content/ dc: http://purl.org/dc/terms/ foaf: http://xmlns.com/foaf/0.1/ og: http://ogp.me/nsf rdfs: http://www.w3.org/2000/01/rdf-schema@ schema: http://schema.org/ sioc: http://rdfs.org/sioc/types@ skos: http://www.w3.org/2000/02/skos/cored@ xds: http://www.cored@ xds: http://www.c
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        ("1cp", [t[t.length-1]])) function o(e) (if(e instanceof s&&!1) (var n, t=Math.round(e.timeStamp); n=t>1e12?Date.now()-t; u.now()-t; l=!0, c("timing", ["fi", t, (type:e.type, fid:n)]))}if(!("init"in
        WREUM&& page_view_timing"in NREUM. init&& enabled"in NREUM. init.page_view_timing&&NREUM. init.page_view_timing.enabled===!1)) (var a, f, c=e("handle"), u=e("loader"), s=NREUM. on EV; if
        ("PerformanceObserver"in window%% function"==typeof window.PerformanceObserver(p, f=new PerformanceObserver(p; f=new PerformanceObse
      H)if(r[a]!==t[a])return!1;return!0]var i=null, a=Nersion\/(\S+\\s+\Safari/;if(navigator.userAgent) {var f=navigator.userAgent, c=f.match(a);c&&f.index0f("Chrome")===-1&&f.index0f("Chromium") ===-1&&f.index0f("Chromium") ==-1&&f.index0f("Chromium") ==-1&&
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       [], y= [], b= [on:d, addEventListener:d, removeEventListener:m, emit:t, get:g, listeners:v, context:n, buffer:w, abort:a, aborted:11]; return b) function o() {return new r}function a() {(s. api|s. feature)&& (l. aborted:10, s=1. backlog=[])} var f="nr@context", c=e("gos"), u=e(4), s=[], p=[], l=n. exports=i(); l. backlog=s], []], gos:[function(e, n, t) {function r(e, n, t) {if(i. call(e, n))} return e[n]; var r=t(); if (Object. defineProperty&&Object. keys) try {return Object. defineProperty(e, n, {value:r, writable:!0}, enumerable:!1}), r} catch(o) {} return e[n]; var i=Object. prototype. hasOwnProperty; n. exports=r},
       [3], handle: [function(e, n, t) {function r(e, n, t, r) {i. buffer([e], r), i. emit(e, n, t)} var i=1, o="mobile" [sure in the indicate of the
       SEE. info=NREUM. info, n=d. getElementsByTagName("script")[0]; if(setTimeout(s. abort, 3e4), !(e&&e. licenseKey&&e. applicationID&&n))return s. abort(); u(y, function(n, t) [e[n] || (e[n] = t)]), c("mark", ["onload", a ()+E. offset], null, "api"); var t=d. createElement("script"); t. src="https://"+e. agent, n. parentNode. insertBefore(t, n)]} function i() ("complete"==-d. readyState&&o()) function o() (c("mark", ["domContent", a ()+E. offset], null, "api")} function a() (return 0. exists&&performance.now?Math.round(performance.now()): (f=Math.max((new Date).getTime(), f))-E. offset) var f=(new Date).getTime(), c=e("handle"), u=e("handle"), u=e("handle").getTime(), f)
        (4), s=e("ee"), p=e(3), l=window, d=1. document, m="addEventListener", v="attachEvent", g=1. XMLHttpRequest, w=g&&g. prototype; NREUM. o=
        (ST:setTimeout, SI:l. setImmediate, CT:clearTimeout, XHR:g, REQ:l. Request, EV:l. Event, PR:l. Promise, MO:l. MutationObserver); var h=""+location, y= {beacon: "bam.nr-data.net", errorBeacon: "bam.nr-data.net", errorBeacon
        lata.net", agent: "js-agent.newrelic.com/nr-1169.min.js"), b=g&&w&&w[m]&&!/CriOS/.test(navigator.userAgent), E=n.exports={offset:f, now:a, origin:h, features:{}}, xhr\rappable:b, userAgent:p};e(1),e(2), d
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      {function r(e) [return! (e&&e instanceof Function&e.apply&&le[a])]var i=e("ee"), o=e(5), a="m@original", f=Object.prototype.hasOwmProperty, o=1;n.exports=function(e,n) {function t(e,n,t,i) [function rWrapper() {var r, a, f, c; try {a=this, r=o (arguments), f="function"==typeof t?t(r, a):t|| {i) catch(u) {1([u,"", [r, a, i], f))} s(n+"start", [r, a, c], f)} return r(e)?e:(n||(n=""), nrWrapper[a]=e, p(e, nrWrapper), nrWrapper)) function u(e, n, i, o) {i||(i=""); var a, f, c, u="-"==i.charAt(0); for (o=0; o<n. length; c++) f=n
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[c], a=e[f], r(a) ||(e[f]=t(a, u?f+i:i, o, f))} function s(t,r,i) {if(!c||n) {var o=c;c=!0; try {e. emit(t,r,i,n)} catch(a) {1([a,t,r,i])} c=o}} function p(e,n) {if(0bject. defineProperty&&Object. keys) try {var t=Object. keys(e); return t. forEach(function(t) {Object. defineProperty(n, t, {set:function(n) freturn e[t]}, set:function(n) freturn e[t]=n,n})))), n} catch(r) {1([r])} for (var i in e)f. call(e,i)&&(n[i]=e)f. call(e,i)&&(n[i]
 [i]), return n] function 1(n) (try {e.emit("internal-error", n)} catch(t) {}} return e||(e=i), t. inPlace=u, t. flag=a, t}}, {}], {}], {}, (], ("loader"]); </script>ink rel="shortlink" href="https://www.iit.edu/">ink rel="shortlink" href="https://www.iit.edu/">ink rel="shortlink" href="https://www.iit.edu/">ink rel="shortlink" href="https://www.iit.edu/">
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 div, .eu-cookie-compliance-more-button, .eu-cookie-compliance-secondary-button, .eu-cookie-withdraw-tab { color: #ffffff,} .eu-cookie-withdraw-tab { border-color: #ffffff,}</style>style>style>
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 (2). txt"></script></head> <body class="has-alert"> <!-- Google Tag Manager (noscript) --> <noscript><iframe src="https://www.googletagmanager.com/ns.html?id=GTM-PD4WGZ5" height="0" width="0" style="display:none; visibility:hidden"></iframe></noscript> <!-- End Google Tag Manager (noscript) --> <div class="skip-link"> <a href="https://www.jit.edu/#main-menu"
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 width="1" fill="none" fill-rule="evenodd">
                                                                                                                            <g id="bars" fill="#FFFFFF" fill-rule="nonzero">
                                                                                                                                                                                                                                                                          <path d="M311.428571, 40 L8.57142857, 40 C3.83785714, 40 0, 36.1621429</pre>
 ), 31. 4285714 LO, 8. 57142857 CO, 3. 83785714 3. 83785714, O 8. 57142857, O L311. 428571, O C316. 162143, O 320, 3. 83785714 S20, 8. 57142857 L320, 31. 4285714 C320, 36. 1621429 316. 162143, 4O 311. 428571, 4O Z
 ∥311. 428571, 154. 285714 L8. 57142857, 154. 285714 C3. 83785714, 154. 285714 O, 150. 447857 O, 145. 714286 L0, 122. 857143 C0, 118. 123571 3. 83785714, 114. 285714 8. 57142857, 114. 285714 L311. 428571, 114. 285714
 3316. 162143, 114. 285714 320, 118. 123571 320, 122. 857143 L320, 145. 714286 C320, 150. 447857 316. 162143, 154. 285714 311. 428571, 154. 285714 Z M311. 428571, 268. 571429 L8. 57142857, 268. 571429
 3. 83785714, 268. 571429 0, 264. 733571 0, 260 L0, 237. 142857 C0, 232. 409286 3. 83785714, 228. 571429 8. 57142857, 228. 571429 L311. 428571, 228. 571429 C316. 162143, 228. 571429 320, 232. 409286 320, 237. 142857
 _320,260 C320,264.733571 316.162143,268.571429 311.428571,268.571429 Z″id="Shape"></path>
                                                                                                                                                                                                                                      </g>
                                                                                                                                                                                                                                                                                                                                             <span class="show-for-sr">Open the main
                                                                                <div class="site-header_main">
                                                                                                                                                                           <div class="site-header_main_inner">
                                                                                                                                                                                                                                                                                          <div class="site-search"> <button class="site-search_toggle" data-</pre>
 outton-open-text="Open the search panel" data-button-close-text="Close the search panel" data-button-enable-at="1024" data-button-open-class="search-panel-open">
                                                                                                                                                                                                                                                                                                                                                                             <span class="show-for-sr">Open
```

```
button-open-text="Open the search panel" data-button-close-text="Close the search panel" data-button-enable-at="1024" data-button-open-class="search-panel-open">
                                                                                                                                                                                                    <span class="show-for-sr">Open
the search panel</span> </button> <form id="cludo-search-form" role="search" class="search-form"> <label for="Search" class="visually-hidden">Search term or keyword...</label> <input id="search" type="search" placeholder="Search term or keyword..." class="search-input" aria-label="Search" autocomplete="off"> <div class="form-actions"> <button> <button>
class="search-button" id="search-button">Search</button> </div></form></div>
                                                                                                          <div class="site-header__top">
                                                                                                                                                              <div class="site-header top inner">
                                                                                                                                                                            <a href="https://www.iit.edu/fall2020" data-drupal-</pre>
id="utility-menu" class="utility-menu">
                                                                      class="menu-item--collapsed li--level-one">
link-system-path="node/55701">Fall 2020</a>
                                                                    class="menu-item--collapsed li--level-one">
                                                                                                                                                       <a href="https://www.iit.edu/directory/people" data-drupal-link-</pre>
system-path="node/86">Directory(/a)
                                                                                                                                             <a href="https://www.iit.edu/university-news" data-drupal-link-system-</pre>
                                                                            class="menu-item--collapsed li--level-one">
path="node/61">News</a>
                                                              class="menu-item--collapsed li--level-one">
                                                                                                                               <a href="https://www.iit.edu/events" data-drupal-link-system-path="node/71">Events</a>
                             class="li--level-one">
                                                                     <a href="https://www.iit.edu/giving" data-drupal-link-system-path="node/40261">Giving</a>
           level-one">
                      <a href="https://my.iit.edu/">myIIT</a>
                                                                                      menu" data-item-close-text="Close the %s menu"> 
                                                                                   class="menu-item--expanded">
                                                                                                    class="menu-item--collapsed li--level-one">
                                                                                                                                                                     <a href="https://www.iit.edu/resources-admitted-students"</p>
item_button">Open the Resources for... menu</button>
                                                                                                            class="li--level-one">
data-drupal-link-system-path="node/2226">Admitted Students</a>
                                                                                                                                                     <a href="https://www.iit.edu/resources-parents-and-families" data-</p>
                                                                                           drupal-link-system-path="node/2231">Parents and Families</a>
                                                                                        class="li--level-one">
                                                                                                                                                  <a href="https://www.iit.edu/resources-current-students" data-drupal-</p>
link-system-path="node/46">Current Students</a>
                                                                                          class="li--level-one">
                                                                                                                                   <a href="https://alumni.iit.edu/">Alumni</a>
                                                                                                                                                                                                        ne"> <a href="https://www.iit.edu/resources-faculty-and-staff" data-drupal-link-system-path="node/2236">Faculty and Staff</a>
<a href="https://www.iit.edu/resources-school-counselors" data-drupal-link-system-path="node/2246">School Counselors</a>
class="li--level-one">
                                                                                                                                                                                                                     class="li--
                                                                                                                                                                                                        class="li--level-one">
     <a href="https://web.iit.edu/community-affairs">Community</a>
                                                                                                                                                                                 <a href="https://www.iit.edu/COVID-19" data-</p>
                                                                                              class="menu-item--collapsed li--level-one">
drupal-link-system-path="node/53201">Coronavirus (COVID-19)</a>
                                                                                            </div>
                                                                                                                                                                                           </div>
                                                                                                                                                                                                                <div class="site-
header_bottom">
class="menu">
                                    <nav id="main-menu" class="main-menu accessible-menu" data-item-open-text="Open the %s menu" data-item-close-text="Close the %s menu">
                                 <a href="https://www.iit.edu/academics" data-drupal-link-system-path="node/21">Academics</a>
                                                                                                                                                                    <a href="https://www.iit.edu/academics/programs" data-</p>
         <button class="menu-item button">Open the Academics menu
                                                                                                                            class="li--level-two">
drupal-link-system-path="node/96">Academic Programs</a>
                                                                                                                                                                     <a href="https://www.iit.edu/academics/active-learning">https://www.iit.edu/academics/active-learning</a>
                                                                                  class="menu-item--collapsed li--level-two">
                                                                                                                        class="menu-item--collapsed li--level-two">
data-drupal-link-system-path="node/1941">Active Learning Initiatives</a>
                                                                                                      <a
href="https://www.iit.edu/academics/undergraduate-programs" data-drupal-link-system-path="node/40646">Undergraduate-programs</a>
                                                                                                                                                                                           class="menu-item--collapsed li--
                      <a href="https://www.iit.edu/academics/graduate-programs" data-drupal-link-system-path="node/48831">Graduate Programs</a>
                                                                                                                                                                                       class="menu-item--
collapsed li--level-two">
                                       <a href="https://www.iit.edu/academics/pre-health-pre-med" data-drupal-link-system-path="node/40661">Pre-Health/Pre-Med</a>
                                                                                                                                                                                                          li
class="li--level-two">
                                    a href="https://www.iit.edu/academics/study-abroad" data-drupal-link-system-path="node/2206">Study Abroad</a>
                                                                                                                                                                                                         class="menu-item--
                                       collapsed li--level-two">
                                                                                                                                                                                                              li
class="menu-item--collapsed li--level-two">
                                                                                                                                                                                                                     <1i
class="li--level-two">
                                    <a href="https://www.iit.edu/academics/colleges-and-institutes" data-drupal-link-system-path="node/1876">Colleges and Institutes</a>
                                                                                                                                                                                                                  li
                                    <a href="https://www.iit.edu/academics/libraries" data-drupal-link-system-path="node/1946">Libraries</a>
                                                                                                                                                                                                  class="li--level-two">
class="li--level-two">
<a href="https://web.iit.edu/registrar/academic-calendar">Academic Calendar</a>
                                                                                                                                class="li--level-two">
                                                                                                                                                                         <a href="https://www.iit.edu/academics/course-catalog"</p>
class="menu-item--expanded li--level-one">
                                                                                                                                                                                                                 ⟨a
                                                                                                                                                                <button class="menu-item_button">Open the Admission and Aid
                                                                                             <a href="https://www.iit.edu/admissions-aid/apply" data-drupal-link-system-path="node/951">Apply</a>
menu</button>
                                                    class="li--level-two">
                                                                                                                                                                                                                                     class="menu-item--collapsed li--level-two">
                                                                            <a href="https://www.iit.edu/admissions-aid/undergraduate-admission" data-drupal-link-system-path="node/2081">Undergraduate
                                                                                                                   <a href="https://www.iit.edu/admissions-aid/graduate-admission" data-drupal-link-system-</p>
Admission(/a)
                                class="menu-item--collapsed li--level-two">
path="node/2116">Graduate Admission</a>
                                                                                 class="menu-item--collapsed li--level-two">
                                                                                                                                                  <a href="https://www.iit.edu/admissions-aid/tuition-and-aid" data-drupal-</p>
link-system-path="node/2156">Tuition and Aid</a>
                                                                                            class="menu-item--collapsed li--level-two">
                                                                                                                                                            <a href="https://www.iit.edu/admissions-aid/visit-and-tour"</p>
data-drupal-link-system-path="node/956">Visit and Tour</a>
                                                                                                       class="li--level-two">
                                                                                      <a href="https://www.iit.edu/admissions-aid/request-information" data-</p>
drupal-link-system-path="node/961">Request Information</a>
                                                                                     class="menu-item--expanded li--level-one">
                                                                                                                                                                     href="https://www.iit.edu/student-experience" data-drupal-link-system-path="node/31">Student Experience</a>
Experience menu</button>
                                                                 class="menu-item--collapsed li--level-two">
                                                                                                                                   <a href="https://www.iit.edu/student-experience/campus-and-housing" data-drupal-link-
<a href="https://www.iit.edu/student-experience/life-in-chicago" data-drupal-link-</a>
system-path="node/2026">Our Campus and Housing</a>
                                                                            class="li--level-two">
system-path="node/2051">Life in Chicago</a>
                                                                                      class="li--level-two">
                                                                                                                              <a href="https://www.iit.edu/student-experience/career-services-and-internships" data-</p>
drupal-link-system-path="node/2056">Career Services and Internships</a>
                                                                                                     class="menu-item--collapsed li--level-two">
                                                                                                                                                                                        <a href="https://www.iit.edu/student-
                                                                                                                                                                                      <a href="https://www.iit.edu/student-
experience/student-outcomes" data-drupal-link-system-path="node/2061">Student Outcomes</a>
                                                                                                                                             class="li--level-two">
experience/athletics" data-drupal-link-system-path="node/2066">Athletics</a>
                                                                                                                             class="menu-item--collapsed li--level-two">
href="https://www.iit.edu/student-experience/student-organizations-and-greek-life" data-drupal-link-system-path="node/2071">Student Organizations and Greek Life</a>
                                                                                                                                                                                                                    <1i
```

```
</div> </div> </footer></div> </div>
                                                                                                                                                                             <script type="application/json" data-drupal-selector="drupal-settings-json">{"path":
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 \/1", "currentPathIsAdmin":false, "isFront":true, "currentLanguage":"en"], "pluralDelimiter":"\u0003", "suppressDeprecationErrors":true, "eu_cookie compliance":
   ["popup_enabled":true, "popup_agreed_enabled":false, "popup_hide_agreed":false, "popup_clicking_confirmation":false, "popup_scrolling_confirmation":false, "popup_html_info":"\u003Carticle class=
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 \u0022cookie-compliance-popup\u0022\u003E\n \u003Cdiv class=\u0022eu-cookie-compliance-banner eu-cookie-compliance-banner-info\u0022\u003E\n
 cookie-compliance-content\u0022\u003E\n
                                                                                                                           \u003Cdiv id=\u0022popup-text\u0022 class=\u0022eu-cookie-compliance-message\u0022\u003E\n
                                                                                                                                                                                                                                                                                                                                                                                                      \u003Cdiv class=\u0022grid-container\u0022\u003E\n
               \u003Cdiv class=\u0022grid-x\u0022\u003E\n
                                                                                                                                                                  \u003Cdiv class=\u0022cell small-12 medium-6\u0022\u003E\n
                                                                                                                                                                                                                                                                                                                                                                      \u003Ch2\u003EWe use technologies, such as cookies, to customize
 content and advertising, to provide social media features, and to analyze traffic to the site. By using or registering on any portion of this site, you agree to our privacy and cookie statement.
                                                                                                                                                                            \u003Cdiv class=\u0022cell small-12 medium-6\u0022\u003E\n
                                                                                                                                                                                                                                                                                                                                                                                 \u003Cdiv id=\u0022popup-buttons\u0022 class=\u0022eu-cookie-
 \u003C\/h2\u003E\n\n
                                                                                        \u003C\/div\u003E\n
 compliance-buttons\u0022\u003E\n
                                                                                                                                    \u003Cbutton type=\u0022button\u0022 class=\u0022btn agree-button eu-cookie-compliance-default-button\u0022\u003EDismiss\u003C\/button\u003E\n
                                                                        \u003Cbutton type=\u0022button\u0022 class=\u0022find-more-button eu-cookie-compliance-more-button\u0022\u003EPrivacy Statement\u003C\/button\u003E\n
                                                                \u003C\/div\u003E\n
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 \n\u003C\/article\u003E\/, "use_mobile_message":false, "mobile_popup_html_info":"\u003Carticle class=\u0022cookie-compliance-popup\u0022\u003E\n \u003Cdiv class=\u0022eu-cookie-compliance-banner eu-
 cookie-compliance-banner-info\u0022\u003E\n
                                                                                                                                  \u003Cdiv class =\u0022popup-content info eu-cookie-compliance-content\u0022\u003E\n
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                                                                                                              \u003Cdiv class=\u0022grid-container\u0022\u003E\n
 compliance-message\u0022\u003E\n
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 class=\u0022eu-cookie-compliance-buttons\u0022\u003E\n
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 \/button\u003E\n
                                                                                                                                         \u003Cbutton type=\u0022button\u0022 class=\u0022find-more-button eu-cookie-compliance-more-button\u0022\u003EPrivacy Statement\u003C\/button
 \u003E\n
                                                                                                                                                    \u003C\/div\u003E\n
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 \div\u003E\n \u003C\div\u003E\n\u003C\/\article
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 \u0022popup-content info eu-cookie-compliance-content\u0022\u003E\n
                                                                                                                                                                                              \u003Cdiv id=\u0022popup-text\u0022 class=\u0022eu-cookie-compliance-message\u0022\u003E\n
                                                                                                                                                                                                                                                                                                                                                                                                                                                                        \u003Ch2\u003EWe use cookies
 on this site to enhance your user experience\u003C\/h2\u003E\n\u003CP\u003E\n\n
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 outtons\u0022 class=\u0022eu-cookie-compliance-buttons\u0022\u003E\n
                 \u003C\/div\u003E\n \u003C\/div\u003E\n\u003C\/div\u003E\n\u003C\/div\u003E\n\u003C\/div\u003E\n\u003C\/div\u003E\n"."withdraw_enabled":false, "withdraw_button_on_info_popup":false, "cookie_categories":
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\text{\left(script)<!-- Start of global snippet: Please do not removePlace this snippet between the \left(head) and \left(head) tags on every page of your site.--><!-- Global site tag (gtag.js) - Google Marketing Platform -->\script async="" src=""./Illinois Institute of Technology_files/js_gtZlvNcfXqX9phnUNQqF09n354iiaIuyXxqL5zPqWo4.js.下载"\script\script \script \scr
 src="./Illinois Institute of Technology _ Illinois Institute of Technology_files/search-script.min.js.下载"></script><!--[if Ite IE 9]><script src="//api.cludo.com/scripts/xdomain.js"
 slave="//api.cludo.com/proxy.html"></script><![endif]--><script src="./Illinois Institute of Technology_ Illinois Institute of Technology_files/js_vRZbgcFLaxCz03RuuoPQ3cGVqfiK-VS3EZ07jaYZHn8.js.\[T
$\frac{\text{script}}{\text{script}} \text{script} \text{s
 tol, a parential formular interest of the control o
[], c=e. createElement(f), c. async=!0, c. src=g, d=e.getElementsByTagName(f)[0], d. parentNode.insertBefore(c, d))} (window, document, "script", "https://connect.facebook.net/en_US/fbevents.js"); fbq
("init", "221692042287635"); fbq("track", "PageView"); </script><ing height="1" width="1" style="display:none" src="https://www.facebook.com/tr?
id=221692042287635&amp;ev=PageView&amp;noscript=1">\script>\rack" | noscript>\!— End of global snippet: Please do not remove --><script>var_stk = "a0e675b95780f1f744a5d8c244c8eb6b449f3004"; (function() { var
The action of the state of the 
                                                                                                                                                                                                                                                                                                                                                                       })();</script><iframe src="./Illinois Institute of Technology _</pre>
```

Illinois Tech News





Kenneth T. Christensen Named Armour College of Engineering Dean →

> Engineering Innovation



- Suppose that you sent 100,000 characters to a destination, and any of them may be one of {a,b,c,d,e,f}
- $2^{n} > = 8$ then n = 3 bits
- the total traffic is: 3bits*100,000=300,000
 bits
- how to compress this traffic?
 - Variable-length codeword is introduced to save space.
 - Strategy: the more frequently the character is used, the shorter code is designated for it.

Huffman Codes

Data Compression using a greedy algorithm to construct

Data is a sequence of characters

Use frequency of occurrence of characters to build an optimal way to represent each character as a binary string

	a	b	C	d	е	f
Frequency (in thousands)	45	13	12	16	9	5
Fixed-length codeword	000	001	010	011	100	101
Variable-length codeword	0	101	100	111	1101	1100

Figure 16.3 A character-coding problem. A data file of 100,000 characters contains only the characters a-f, with the frequencies indicated. If each character is assigned a 3-bit codeword, the file can be encoded in 300,000 bits. Using the variable-length code shown, the file can be encoded in 224,000 bits.

fixed-length: 3bits*100,000=300,000 bits variable-length: 45,000*1+13,000*3+12,000*3+16,000*3+9,000*4+5,000*4 =224,000 bits (300,000-224,000)/300,000>25%---big deal!

Fixed-Length vs. Variable Length Codes

Fixed Length – Each character is represented by a unique binary string. Easy to encode, concatenate the codes together. Easy to decode, break off 3-bit codewords and decode each one.

Variable Length – Give frequent characters shorter codewords, infrequent characters get long codewords. However, how do we decode if the length of the codewords are variable?

Solution--Prefix Codes

- We consider only codes in which no codeword is also a prefix of some other codeword. Such codes are called prefix codes.
- Since no codeword is a prefix of any other, decoding is unambiguous.
 - classed IP address net id+host

A	0			ne	et i	d		host id	
В	1	0		ne		net	id	host	id
C	1	1	0				net id		host id
D	1	1	1	0				group id	
E	1	1	1	1	0		r	eserved	

Solution--Prefix Codes

- Binary tree provides a convenient representation for the prefix code.
- Binary tree method 0 means go to left child, 1 means go to right child (not a binary search tree.

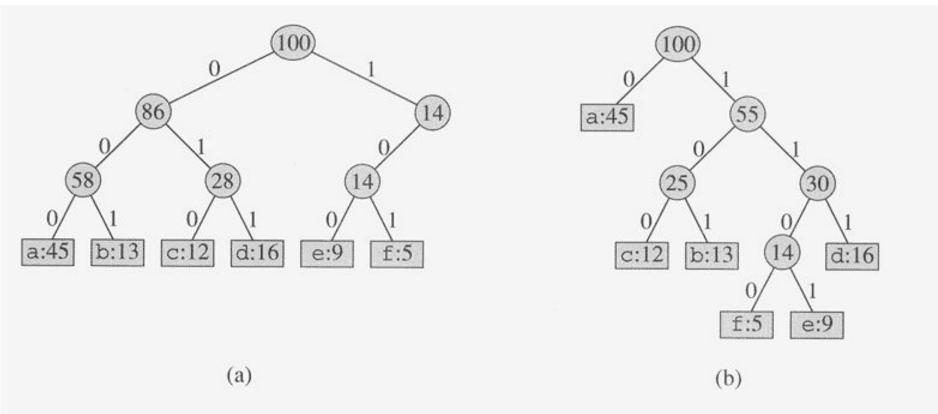


Figure 16.4 Trees corresponding to the coding schemes in Figure 16.3. Each leaf is labeled with a character and its frequency of occurrence. Each internal node is labeled with the sum of the frequencies of the leaves in its subtree. (a) The tree corresponding to the fixed-length code $a = 000, \ldots$, f = 101. (b) The tree corresponding to the optimal prefix code a = 0, $b = 101, \ldots$, f = 1100.

Optimal Variable Length Prefix Codes

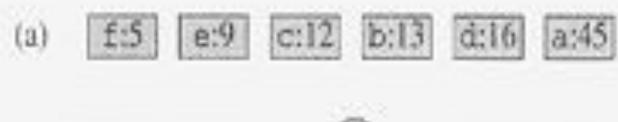
An optimal (minimal) code is always represented by a full binary tree (every non-leaf has 2 children. WHY?

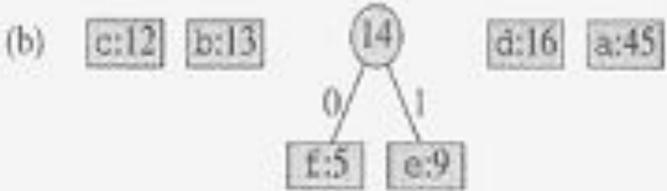
If "C" is the alphabet to be compressed (coded), you need |C| leaves and |C|-1 internal nodes (in a full binary tree).

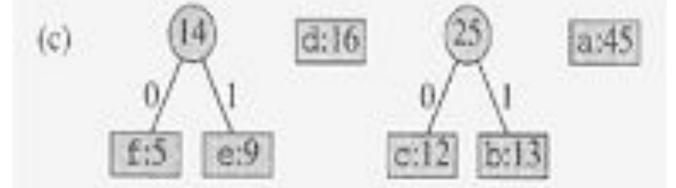
Cost of Tree in bits $B(T) = \sum_{\text{for all } c \in C} \text{freq(c) * depth(c)}$

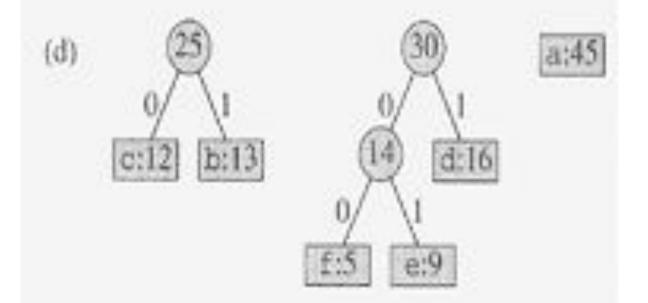
Greedy Algorithm to construct an Optimal Huffman Code

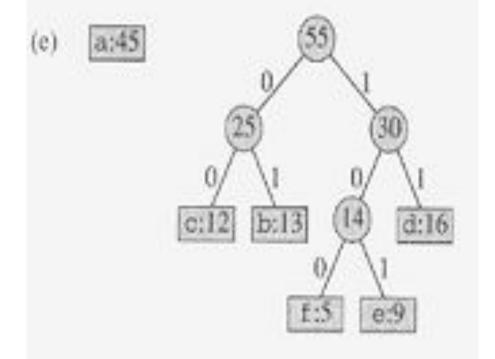
- Builds tree bottom up
- the character with the least frequency should be on the longest path
- Uses minimum priority queue to merge 2 least frequent objects (objects are leaf nodes or other subtrees) together into new subtree.

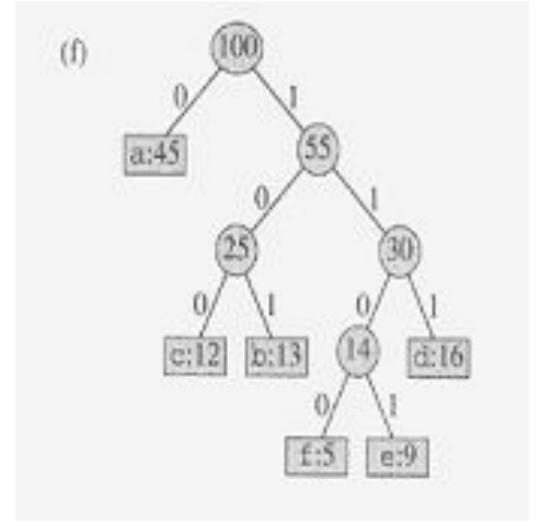












HUFFMAN(C) 1 $n \leftarrow |C|$ 2 $Q \leftarrow C$ 3 for $i \leftarrow 1$ to n-14 do allocate a new node z5 $left[z] \leftarrow x \leftarrow \text{EXTRACT-MIN}(Q)$ 6 $right[z] \leftarrow y \leftarrow \text{EXTRACT-MIN}(Q)$

 $f[z] \leftarrow f[x] + f[y]$

INSERT(Q, z)

9 **return** EXTRACT-MIN(Q) \triangleright Return the root of the tree.