Introduction to Algorithms

CS 430

Lecture L18 & 19

ILLINOIS INSTITUTE OF TECHNOLOGY

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Prelecture

Why DP?

- Optimization problem
- Overall problem can be decomposed into optimal sub-structures
- Overall optimization solution depends on the combination of subproblems' solutions

How to improve the complexity?

- With extra memoization
- Two ways
  - Top-down
  - Bottom-up

- Rod Cutting Decomposition

length i	1	2	3	4	5	6	7	8	9	10
price p_i	1	5	8	9	10	17	17	20	24	30

$r_1=1$

$r_2=5$

$r_3=8$

$r_4=p_2+p_2=10$

... ...

$r_7=p_1+p_6=p_2+p_2+p_3=18$

... ...

$r_5=13$ (p_2+p_3), $r_6=17$ (p_6)

$r_n=\max\{p_n, r_1+r_{n-1}, r_2+r_{n-2}, \ldots, r_{n-1}+r_1\} \Leftrightarrow r_n = \max_{1\le i\le n}(p_i + r_{n-i})$

- Top-down
  - We write the procedure recursively in a natural manner, but modified to save the results of sub problem.

```
MEMOIZED-CUT-ROD(p, n)
1  let r[0..n] be a new array
2  for i = 0 to n
3      r[i] = -∞
4  return MEMOIZED-CUT-ROD-AUX(p, n, r)
```

```
MEMOIZED-CUT-ROD-AUX(p, n, r)
1  if r[n] ≥ 0
2      return r[n]
3  if n == 0
4      q = 0
5  else q = -∞
6      for i = 1 to n
7          q = max(q, p[i] + MEMOIZED-CUT-ROD-AUX(p, n - i, r))
8  r[n] = q
9  return q
```

- Bottom-up
  - When solving a particular sub problem, we have already solved all of the smaller sub problems its solutions depend upon, and we have saved their solutions.

```
BOTTOM-UP-CUT-ROD(p, n)
1  let r[0..n] be a new array
2  r[0] = 0
3  for j = 1 to n
4      q = -∞
5      for i = 1 to j
6          q = max(q, p[i] + r[j - i])
7      r[j] = q
8  return r[n]
```

Outlines

- Greedy Algorithm
  - Activity selection problem
  - 0-1 Knapsack problem
  - Huffman code

Why Greedy?

When we are seeking for optimal solution

- Dynamic programming

  Decompose the problem to subproblems and take either top-down or bottom-up to solve the subproblems. The optimal solution depends on the combination of all solutions to subproblems.

- Problem exhibits optimal substructure.
- How about just to consider the local best choice and assume that all these local choices will lead to a global optimal solution?

Does this strategy work for all circumstances?

- Zero-Sum Game Theory
  - A zero-sum game is a mathematical representation of a situation in which each participant's gain or loss of utility is exactly balanced by the losses or gains of the utility of the other participants.

- Example for local utility and global utility

- Greedy Algorithm
  - local optimal choice leads to global optimal solution
  - a special case of dynamic programming
  - dynamic programming: traversing the solutions of subproblems is equivalent to traversing subproblems; for bottom-up, a root has to consider the solutions of its children before solve itself;
  - greedy: solutions of children do not affect the current choice; bottom-up is invalid.

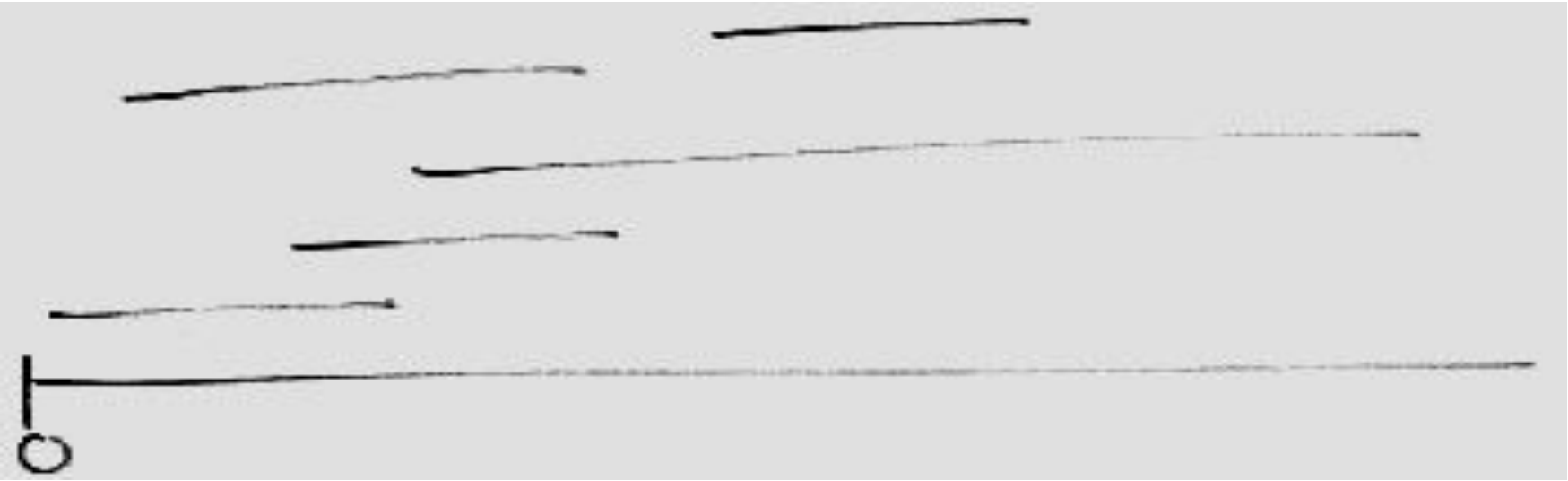
- Greedy algorithm
  - definition:
    - A Greedy Algorithm always makes the choice that looks best at the moment. It makes a locally optimal choice in the hope that this choice will lead to globally optimal solution.
    - Greedy algorithms do not always yield optimal solutions, but for many problems, they do.

Applications of Greedy Algorithms

- Activity Selection Problem
- 0-1 Knapsack
- Huffman Codes

Activity Selection Problem*

- Set S of n activities each with start si, finish fi
- Maximum set of Compatible Activities (non overlapping)



Activity Selection Problem*

- An activity selection problem: we have a set $S=\{a_1,a_2,\ldots,a_n\}$ of n proposed activities that wish to use a resource, which can serve only one activity at a time. Each activity a_i has a start time s_i and a finish time f_i. Where

  $0 \le s_i \le f_i \le \infty$.

- Activities a_i and a_j are compatible if the intervals $[s_i, f_i)$ and $[s_j, f_j)$ do not overlap. a_i and a_j are compatible if $s_i >= f_j$ or $s_j >= f_i$.
- In the activity selection problem, we wish to select a maximum-size subset of mutually compatible activities.

Activity Selection – Brute Force*

Try ALL compatible meeting combinations, pick the one with the max # of activities.

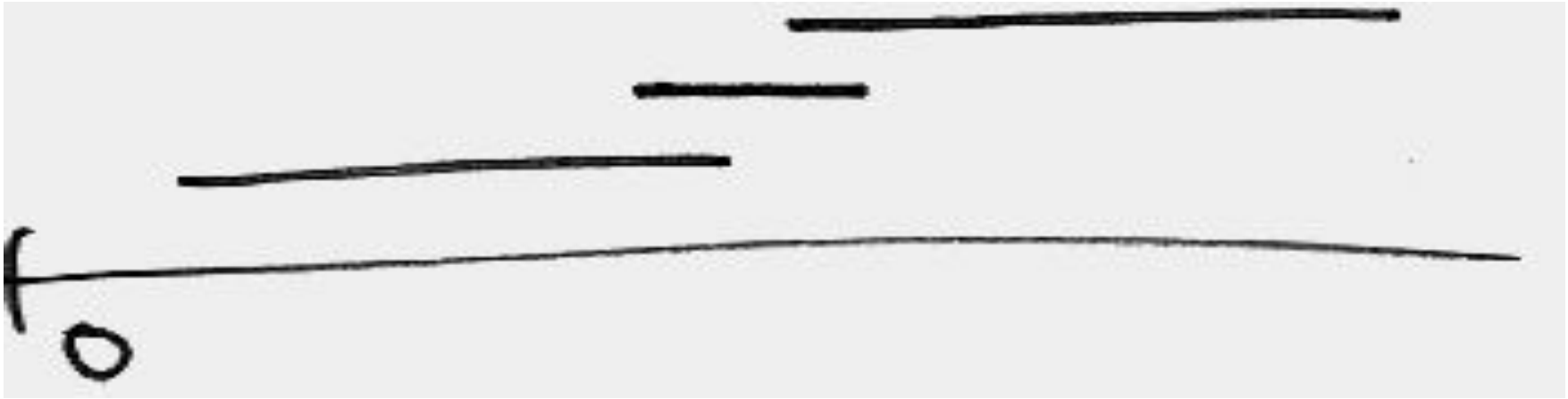
- “n” activities;
- total number of different combinations = 2^n;
- subset of those are compatible

Activity Selection– Possible Greedy Approaches

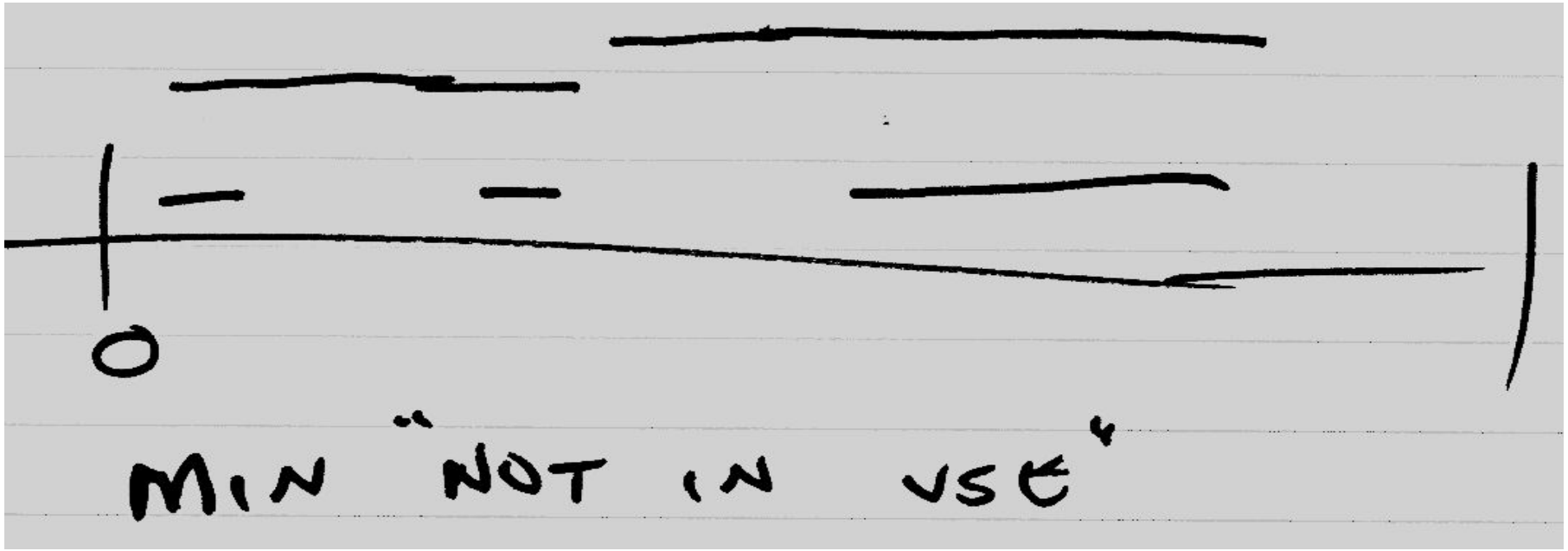
- Pick shortest meeting first
- Minimize “not in use” time between meetings
- Pick meetings with least number of conflicts
- Pick earliest start meeting

- Divide problem into smaller sub-problems

Pick Shortest Meeting - Counter Example

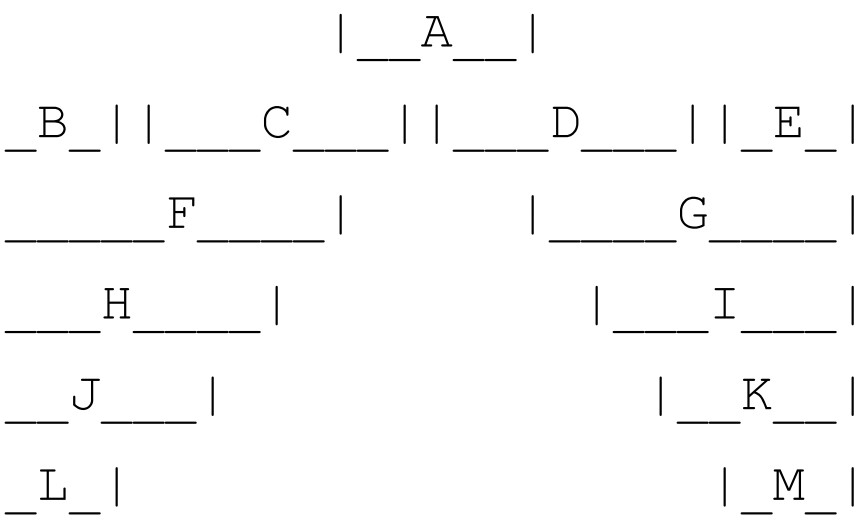


Minimize “Not in use”- Counter Example



Pick Least Conflict - Counter Example

A=(4,6) B=(0,1) C=(1,5) D=(5,9) E=(9,10) F=(0,4) G=(6,10) H=(0,3) I=(7,10) J=(0,2) K=(8,10) L=(0,1) M=(9,10)

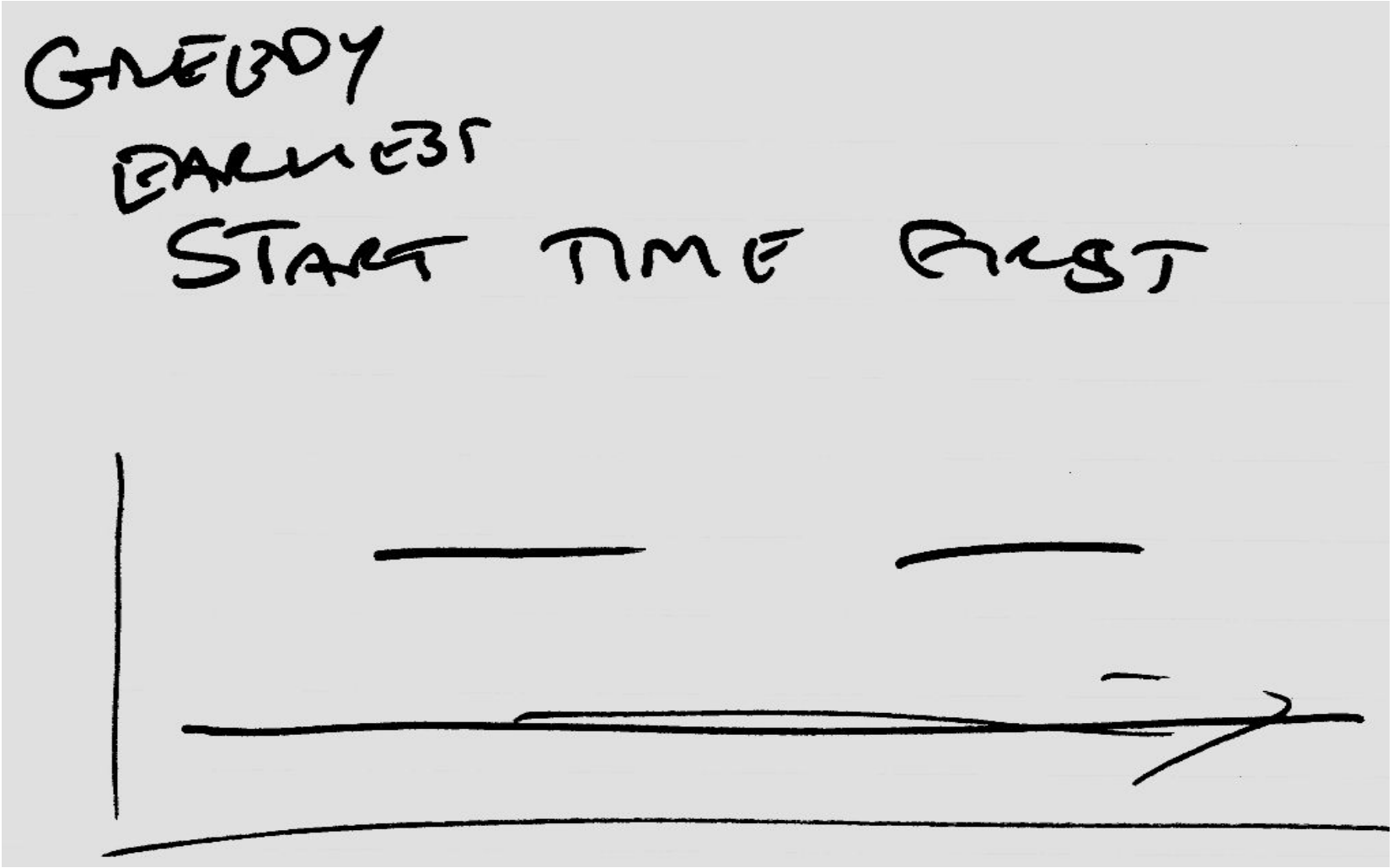


A overlaps 2 meetings, while everything else overlaps at least 3 mtgs

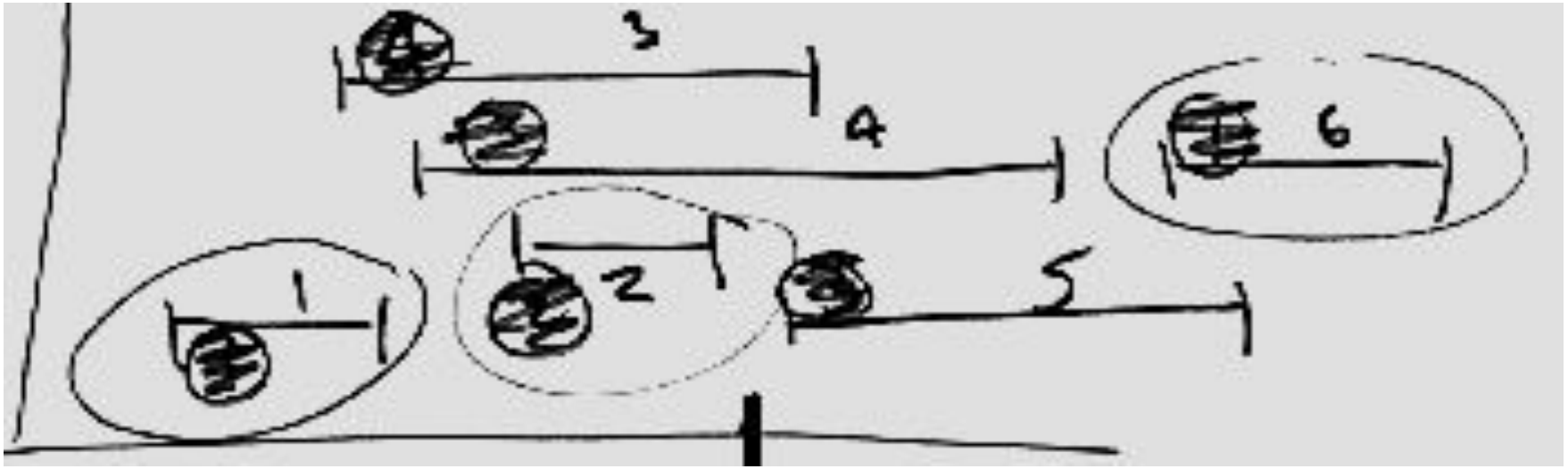
If we choose A, we exclude C and D, and the max we can get is 3 mtgs (A, and one from each side of A)

The optimal answer is 4 mtgs B, C, D, E (other similar answers length 4)

Earliest Start Time First - Counter Example



Earliest Finish Time first - Divide problem into smaller sub-problems -



1 2 6
1 2 5
1 4 6

a_i	1	2	3	4	5	6	7	8	9	10
s_i	1	3	1	9	6	7	1	5	3	7
f_i	4	4	2	11	7	8	2	7	6	8

Optimal Subs

Optimal solution to main problem contains optimal solutions to sub problems of main.

If A is solution to S

Optimal Subset of MTGS

{2,4,k,8,10} No order {1,2,3,….n} No Order

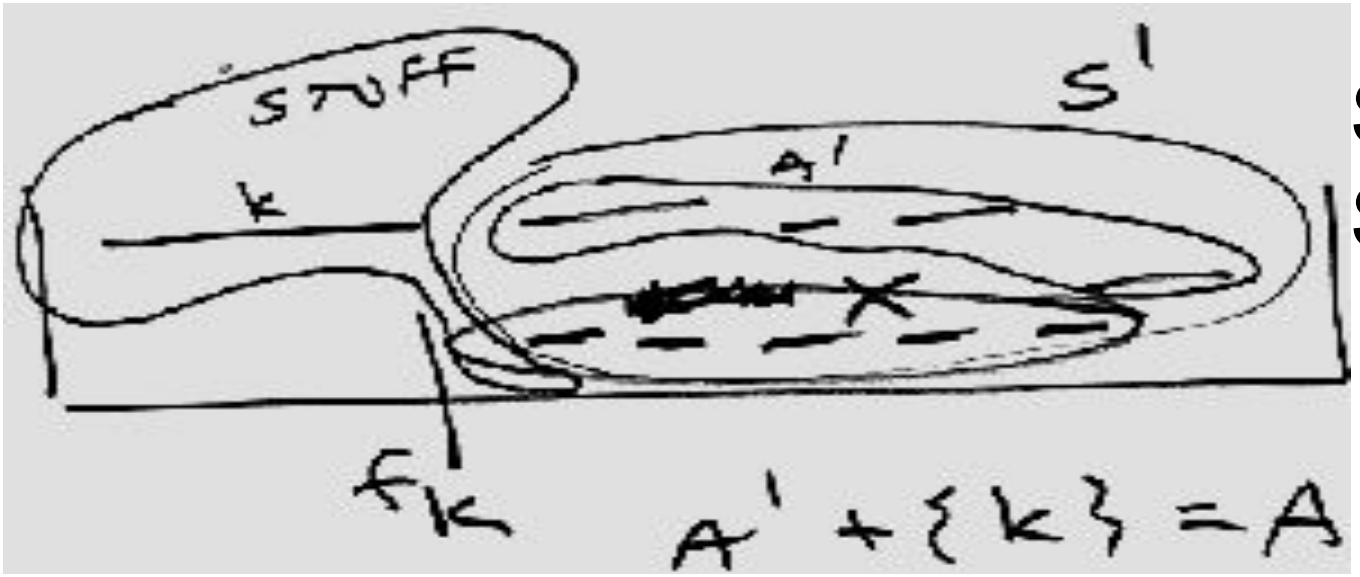
Remove earliest ending activity from A

(Happens to be MTG k) $f_k < f_i$

A’ = {2,4,8,10} must be solution to subproblem

S’={S:s_i>f_k} =All MTGS starting after k finishes

A’ must be solution to S’, if it isn’t then original solution to A was not optimal for S



S’ +
Stuff = S

Suppose that A’ is not the optimal solution to S’, there is an optimal solution X to S’:

$|X| + \{k\} > A$

Contradiction

A is Optimal

Announcements

Nov. 4

- HW #4 is due tonight.
- Unit test #2 is scheduled to the next Thursday (Nov. 11), covering R-B BST, DP, GA and Amortised Analysis. Online test will be available. The link to the google meet will be posted later.
- First section (8:30-9:20 am) of Recitation is cancelled
- Highly encourage you to attend the recitation afterwards.

- Greedy choice
  - We should choose an activity that leaves the resource available for as many other activities as possible.
  - Choose the activity in S with the earliest finish time since that would leave the resource available for as many activities the follow it as possible.
  - Let $S_k=\{a_i \in S: s_i >= f_k\}$ be the set of activities that start after a_k finishes.
  - If a_1 is in the optimal solution, then an optimal solution to the original problem consists of activity a_1 and all activities in an optimal solution to the subproblem S_1.

Prove greedy choice property for "Earliest Finish time first"

i	1	2	3	4	5	6	7	8	9	10	11
s_i	1	3	0	5	3	5	6	8	8	2	12
f_i	4	5	6	7	9	9	10	11	12	14	16

Assume MTGS ordered earliest finish time first.
Assume optimal solution A for problem S

I
t
(

Therefore, there is always an answer that contains the greedy choice

choice

meeting k ends first in solution A

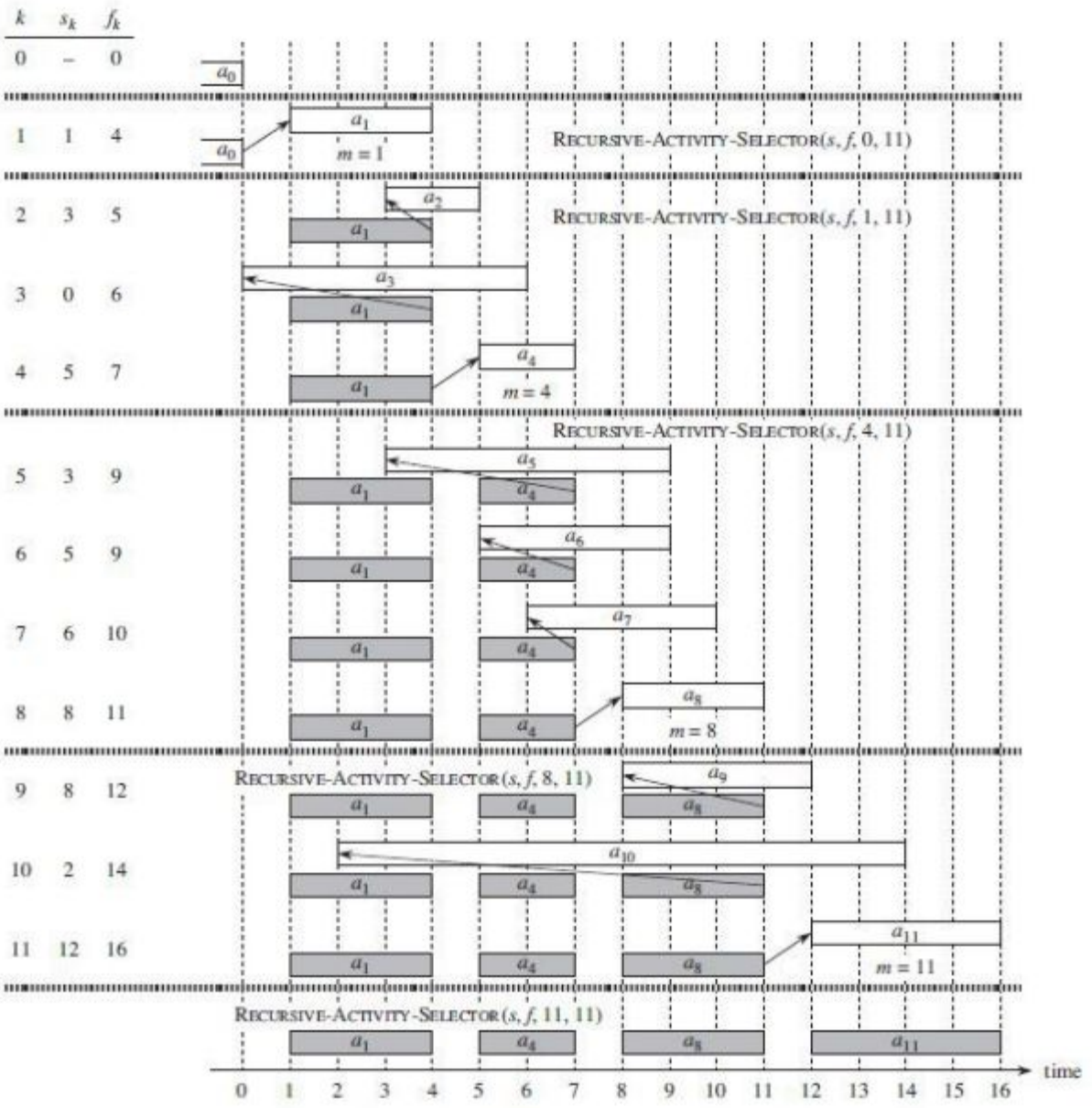
B = A - {K} +{1}

Replace non greedy first choice in A

with Greedy choice

This can be done $f_1 < f_k$

- Algorithm


k s_k f_k
0 – 0
1 1 4
2 3 5
3 0 6
4 5 7
5 3 9
6 5 9
7 6 10
8 8 11
9 8 12
10 2 14
11 12 16
RECURSIVE-ACTIVITY-SELECTOR(s, f, 0, 11)
RECURSIVE-ACTIVITY-SELECTOR(s, f, 1, 11)
RECURSIVE-ACTIVITY-SELECTOR(s, f, 4, 11)
RECURSIVE-ACTIVITY-SELECTOR(s, f, 8, 11)
RECURSIVE-ACTIVITY-SELECTOR(s, f, 11, 11)
m = 1
m = 4
m = 8
m = 11
time
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

- Algorithm

```
RECURSIVE-ACTIVITY-SELECTOR(s, f, k, n)
1  m = k + 1
2  while m ≤ n and s[m] < f[k]        // find the first activity in S_k to finish
3      m = m + 1
4  if m ≤ n
5      return {a_m} ∪ RECURSIVE-ACTIVITY-SELECTOR(s, f, m, n)
6  else return ∅
```

```
GREEDY-ACTIVITY-SELECTOR(s, f)
1  n = s.length
2  A = {a_1}
3  k = 1
4  for m = 2 to n
5      if s[m] ≥ f[k]
6          A = A ∪ {a_m}
7          k = m
8  return A
```

- The optimal solution
  - S_{ij} is the set of activities that start after activity a_i and that finish before activity a_j starts.
  - We wish to find a maximum set of mutually compatible activities in S_{ij}. Assume that the maximum set is A_{ij} which includes some activity a_k.
  - $A_{ij} = A_{ik} \cup \{a_k\} \cup A_{kj}$
  - The number of elements in A_{ij} is $|A_{ij}| = |A_{ik}| + |A_{kj}| + 1$.

Dynamic programming approach

- $c[i,j]$ is the size of an optimal solution to the set S_{ij};

$$c[i,j] = c[i,k] + c[k,j] + 1.$$

$$c[i,j] = \begin{cases} 0, & \text{if } S_{ij} = \phi \\ \text{Max}\{c[i,k] + c[k,j] + 1\}, & \text{when } a_k \in S_{ij} \text{ and } S_{ij} \neq \phi \end{cases}$$

Comparison on Dynamic and Greedy

- Greedy algorithm obtains an optimal solution to a problem by making a sequence of choices. At each decision point, the algorithm makes choices that seems best at the moment.
- This strategy does not always produce an optimal solution, but sometimes it does.

Comparison on Dynamic and Greedy (cont'd)

- The greedy choice property: we can sample a globally optimal solution by making locally optimal (greedy) choices.
- When we are considering which choice to make, we make the choice that looks best to the current problem without considering effects from subproblems.
- In dynamic programming, we make a choice at each step. But the choice depends on the solution to subproblems. We solve dynamic programming problems processing from smaller subproblems to larger subproblems.

Does Greedy always work for optimization problems?

0-1 Knapsack Problem

- The thief robbing a store finds n items. The ith item is worth z_i dollars and the weight is w_i lbs where z_i and w_i are integers. The thief wants to take as valuable a load as possible, but he can carry at most W lbs in his knapsack. W is an integer. Which items should he take?
- The thief must either take or leave an item. He can not take a fractional amount of an item.

Fractional Knapsack Problem

- He can take a fractional amount of an item.

0-1 vs Fractional Knapsack*

n items

w_i weight

v_i values

Total weight limit = W

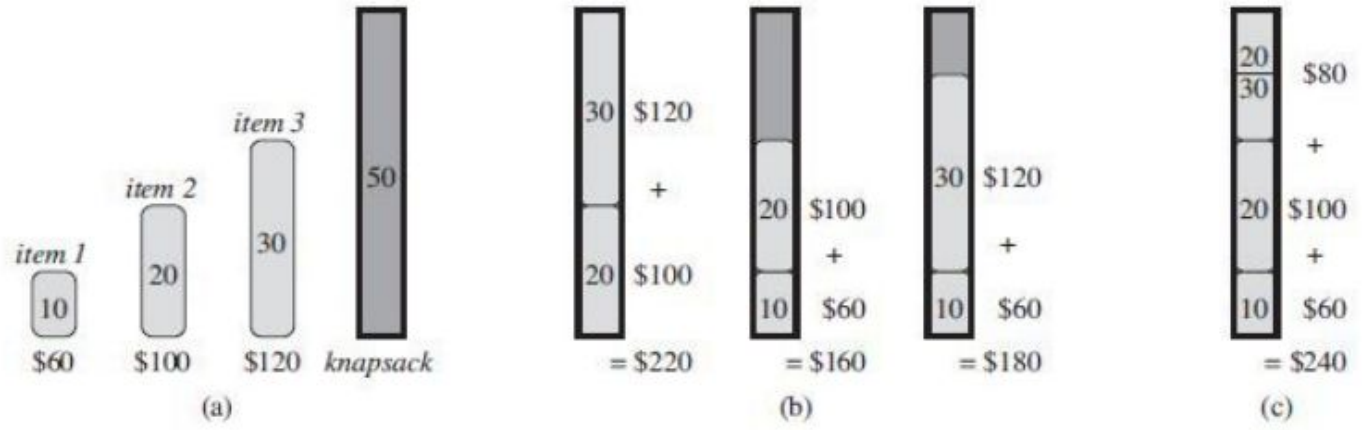
Maximize value

Total weight ≤ W

- Fractional amounts of items are not/ are allowed

Greedy Possibilities

- Max value first
- Min weight first
- Max value/weight first



10	20	30
$60	$100	$120

W=50

$$\frac{v}{w} = \frac{\$6}{lb} \quad \frac{\$5}{lb} \quad \frac{\$4}{lb}$$

All All

10lbs + 20lbs + 20lbs = 50

$60 $100 $80 = $240

10	20	30
$40	$100	$150

$$\frac{v}{w} = \frac{\$4}{lb} \quad \frac{\$5}{lb} \quad \frac{\$5}{lb}$$

$250

Fractional Knapsack Optimal Substructure

$$S = \left\{ \begin{matrix} w_1 \ldots\ldots\ w_i \ldots\ldots\ldots w_n \\ v_1 \ldots\ldots\ldots v_i \ldots\ldots\ldots\ v_n \end{matrix} \right\} \quad W$$

Given optimal solution A to fractional Knapsack problem ($w'_i <= w_i$ for all i)
A = {0,0,w'_3,0,w'_i,w'_5 …. w'_n}

Remove item i\\ (weight w'_i) from A

A' is same as A without w'_i

A' = {0,0,$w'_3$0, w'_5 …. w'_n}

A' is be optimal solution for sub-problem, S'

Total weight=W-w_i

original n-1 items (excluding item i)

Fractional Knapsack Optimal Substructure (continued)

B is solution to S'
If B was better than A'
(value B > value A')

then item i plus B would be better than A.

Contradiction

Fractional Knapsack Greedy Choice

$$S = \begin{Bmatrix} w_1 \ldots\ldots\ w_i \ldots\ldots\ldots w_n \\ v_1 \ldots\ldots\ldots v_i \ldots\ldots\ldots\ v_n \end{Bmatrix} \qquad W$$

(Take biggest unit value first)

Prove global optimal solution can be found from a local greedy choice

Assume items are in Greedy Choice order and we have optimal answer A (not necessarily created from greedy choice)

$$\frac{v_1}{w_1} \geq \frac{v_2}{w_2} \geq \frac{v_3}{w_3} \geq \ldots\ldots\ldots\ldots \geq \frac{v_n}{w_n}$$

$$A = \{w'_1, w'_{2,\ldots\ldots\ldots\ldots\ldots\ldots} w'_n\}$$

$0 <= w'_i <= w_i$

Fractional Knapsack Greedy Choice (continued)

If item 1 has the largest unit value, either A has the greedy choice (w'_1 != 0) or A doesn't have Greedy Choice ($w'_1 = 0$)

Show that there is an equally optimal solution B that does have greedy choice (removing some weight of item k and adding same weight of item 1)

B = A - {item k} + {item 1}
$W_A = W_B = W$

$$V_B \geq V_A$$

Fractional Knapsack Greedy Choice (continued)

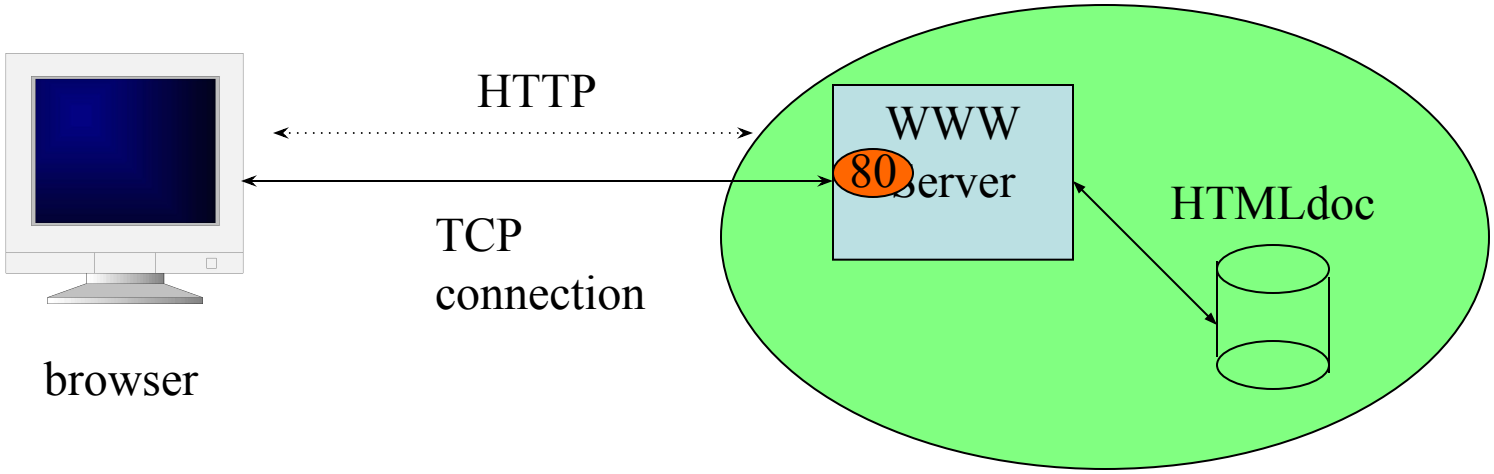
Replacing
min {weight of k or weight of 1} Lbs
item k with
min {weight of k or weight of 1} Lbs item 1
Replace the rest of k (if exist) with item 2
(continuing greedy choice)

$\frac{v_1}{w_1} >= \frac{v_k}{w_k}$ (items in greedy order)

$B_{VALUE} >= A_{VALUE}$

Coding problem

- a webpage--HTTP



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r(){}function i(e,n,t){return function(){return o(e,[u.now()].concat(f(arguments)),n?null:this,t),n?void 0:this}}var o=e("handle"),a=e(4),f=e(5),c=e("ee").get("tracer"),u=e("loader"),s=NREUM;"undefined"==typeof window.newrelic&&(newrelic=s);var p=["setPageViewName","setCustomAttribute","setErrorHandler","finished","addToTrace","inlineHit","addRelease"],l="api-",d=l+"ixn-";a(p,function(e,n){s[n]=i(l+n,!0,"api")}),s.addPageAction=i(l+"addPageAction",!0),s.setCurrentRouteName=i(l+"routeName",!0),n.exports=newrelic,s.interaction=function(){return(new r).get()};var m=r.prototype={createTracer:function(e,n){var t={},r=this,i="function"==typeof n;return o(d+"tracer",[u.now(),e,t],r),function(){if(c.emit((i?"":"no-")+"fn-start",[u.now(),r,i],t),i)try{return n.apply(this,arguments)}catch(e){throw c.emit("fn-err",[arguments,this,e],t),e}finally{c.emit("fn-end",[u.now()],t)}}};a("actionText,setName,setAttribute,save,ignore,onEnd,getContext,end,get".split(","),function(e,n){m[n]=i(d+n)}),newrelic.noticeError=function(e,n){"string"==typeof e&&(e=new Error(e)),o("err",[e,u.now(),!1,n])}},{}],2:[function(e,n,t){function r(e,n){var t=e.getEntries();t.forEach(function(e){"first-paint"===e.name?c("timing",["fp",Math.floor(e.startTime)]):"first-contentful-paint"===e.name&&c("timing",["fcp",Math.floor(e.startTime)])})}function i(e,n){var t=e.getEntries();t.length>0&&c("lcp",[t[t.length-1]])}function o(e){if(e instanceof s&&!l){var n,t=Math.round(e.timeStamp);n=t>1e12?Date.now()-t:u.now()-t,l=!0,c("timing",["fi",t,{type:e.type,fid:n}])}}if(!("init"in NREUM&&"page_view_timing"in NREUM.init&&"enabled"in NREUM.init.page_view_timing&&NREUM.init.page_view_timing.enabled===!1)){var a,f,c=e("handle"),u=e("loader"),s=NREUM.o.EV;if("PerformanceObserver"in window&&"function"==typeof window.PerformanceObserver){a=new PerformanceObserver(r),f=new PerformanceObserver(i);try{a.observe({entryTypes:["paint"]}),f.observe({entryTypes:["largest-contentful-paint"]})}catch(p){}}if("addEventListener"in document){var l=!1,d=["click","keydown","mousedown","pointerdown","touchstart"];d.forEach(function(e){document.addEventListener(e,o,!1)})}}},{}],3:[function(e,n,t){function r(e,n){if(!i)return!1;if(e!==i)return!1;if(!n)return!0;if(!o)return!1;for(var t=o.split("."),r=n.split("."),a=0;a<r.length;a++)if(r[a]!==t[a])return!1;return!0}var i=null,o=null,a=/Version\/(\S+)\s+Safari/;if(navigator.userAgent){var f=navigator.userAgent,c=f.match(a);c&&f.indexOf("Chrome")===-1&&f.indexOf("Chromium")===-1&&(i="Safari",o=c[1])}n.exports={agent:i,version:o,match:r}},{}],4:[function(e,n,t){function r(e,n){var t=[],r="",o=0;for(r in e)i.call(e,r)&&(t[o]=n(r,e[r]),o+=1);return t}var i=Object.prototype.hasOwnProperty;n.exports=r},{}],5:[function(e,n,t){function r(e,n,t){n||(n=0),"undefined"==typeof t&&(t=e?e.length:0);for(var r=-1,i=t-n||0,o=Array(i<0?0:i);++r<i;)o[r]=e[n+r];return o}n.exports=r},{}],6:[function(e,n,t){n.exports={exists:"undefined"!=typeof window.performance&&window.performance.timing&&"undefined"!=typeof window.performance.timing.navigationStart}},{}],ee:[function(e,n,t){function r(){}function i(e){function n(e){return e&&e instanceof r?e:e?c(e,f,o):o()}function t(t,r,i,o){if(!l.aborted||o){e&&e(t,r,i);for(var a=n(i),f=v(t),c=f.length,u=0;u<c;u++)f[u].apply(a,r);var p=s[y[t]];return p&&p.push([b,t,r,a]),a}}function d(e,n){h[e]=v(e).concat(n)}function m(e,n){var t=h[e];if(t)for(var r=0;r<t.length;r++)t[r]===n&&t.splice(r,1)}function v(e){return h[e]||[]}function g(e){return p[e]=p[e]||i(t)}function w(e,n){u(e,function(e,t){n=n||"feature",y[t]=n,n in s||(s[n]=[])})}var h={},y={},b={on:d,addEventListener:d,removeEventListener:m,emit:t,get:g,listeners:v,context:n,buffer:w,abort:a,aborted:!1};return b}function o(){return new r}function a(){(s.api||s.feature)&&(l.aborted=!0,s=l.backlog={})}var f="nr@context",c=e("gos"),u=e(4),s={},p={},l=n.exports=i();l.backlog=s},{}],gos:[function(e,n,t){function r(e,n,t){if(i.call(e,n))return e[n];var r=t();if(Object.defineProperty&&Object.keys)try{return Object.defineProperty(e,n,{value:r,writable:!0,enumerable:!1}),r}catch(o){}return e[n]=r,r}var i=Object.prototype.hasOwnProperty;n.exports=r},{}],handle:[function(e,n,t){function r(e,n,t,r){i.buffer([e],r),i.emit(e,n,t)}var i=e("ee").get("handle");n.exports=r,r.ee=i},{}],id:[function(e,n,t){function r(e){var n=typeof e;return!e||"object"!==n&&"function"!==n?-1:e===window?0:a(e,o,function(){return i++})}var i=1,o="nr@id",a=e("gos");n.exports=r},{}],loader:[function(e,n,t){function r(){if(!x++){var e=E.info=NREUM.info,n=d.getElementsByTagName("script")[0];if(setTimeout(s.abort,3e4),!(e&&e.licenseKey&&e.applicationID&&n))return s.abort();u(y,function(n,t){e[n]||(e[n]=t)}),c("mark",["onload",a()+E.offset],null,"api");var t=d.createElement("script");t.src="https://"+e.agent,n.parentNode.insertBefore(t,n)}function i(){"complete"===d.readyState&&o()}function o(){c("mark",["domContent",a()+E.offset],null,"api")}function a(){return 0.exists&&performance.now?Math.round(performance.now()):(f=Math.max((new Date).getTime(),f))-E.offset}var f=(new Date).getTime(),c=e("handle"),u=e(4),s=e("ee"),p=e(3),l=window,d=l.document,m="addEventListener",v="attachEvent",g=l.XMLHttpRequest,w=g&&g.prototype;NREUM.o={ST:setTimeout,SI:l.setImmediate,CT:clearTimeout,XHR:g,REQ:l.Request,EV:l.Event,PR:l.Promise,MO:l.MutationObserver};var h=""+location,y={beacon:"bam.nr-data.net",errorBeacon:"bam.nr-data.net",agent:"js-agent.newrelic.com/nr-1169.min.js"},b=g&&w&&w[m]&&!/CriOS/.test(navigator.userAgent),E=n.exports={offset:f,now:a,origin:h,features:{},xhrWrappable:b,userAgent:p};e(1),e(2),d[m]?(d[m]("DOMContentLoaded",o,!1),l[m]("load",r,!1)):(d[v]("onreadystatechange",i),l[v]("onload",r)),c("mark",["firstbyte",f],null,"api");var x=0,O=e(6)},{}],"wrap-function":[function(e,n,t){function r(e){return!(e&&e instanceof Function&&e.apply&&!e[a])}var i=e("ee"),o=e(5),a="nr@original",f=Object.prototype.hasOwnProperty,c=!1;n.exports=function(e,n){function t(e,n,t,i){function nrWrapper(){var r,a,f,c;try{a=this,r=o(arguments),f="function"==typeof t?t(r,a):t||{}}catch(u){l([u,"",[r,a,i],f])}s(n+"start",[r,a,i],f);try{return c=e.apply(a,r)}catch(p){throw s(n+"err",[r,a,p],f),p}finally{s(n+"end",[r,a,c],f)}}return r(e)?e:(n||(n=""),nrWrapper[a]=e,p(e,nrWrapper),nrWrapper)}function u(e,n,i,o){i||(i="");var a,f,c,u="-"===i.charAt(0);for(c=0;c<n.length;c++)f=n

button-open-text="Open the search panel" data-button-close-text="Close the search panel" data-button-enable-at="1024" data-button-open-class="search-panel-open">
Open the search panel
</button>
<form id="cludo-search-form" role="search" class="search-form search-block-form">
<label for="Search" class="visually-hidden">Search term or keyword...</label>
<input id="search" type="search" placeholder="Search term or keyword..." class="search-input" aria-label="Search" autocomplete="off">

<button type="submit" class="search-button" id="search-button">Search</button>
</form>

<nav id="utility-menu" class="utility-menu">
<ul class="menu">
<li class="menu-item--collapsed li--level-one">
Fall 2020

<li class="menu-item--collapsed li--level-one">
Directory

<li class="menu-item--collapsed li--level-one">
News

<li class="menu-item--collapsed li--level-one">
Events

<li class="li--level-one">
Giving

<li class="li--level-one">
myIIT

</nav><nav id="audience-menu" class="audience-menu accessible-menu" data-item-open-text="Open the %s menu" data-item-close-text="Close the %s menu">
<ul class="menu">
<li class="menu-item--expanded">
Resources for...
<button class="menu-item__button">Open the Resources for... menu</button><ul class="menu">
<li class="menu-item--collapsed li--level-one">
Admitted Students

<li class="li--level-one">
Parents and Families

<li class="li--level-one">
Current Students

<li class="li--level-one">
Alumni

<li class="li--level-one">
Faculty and Staff

<li class="li--level-one">
School Counselors

<li class="li--level-one">
Community

<li class="menu-item--collapsed li--level-one">
Coronavirus (COVID-19)

</nav>

<nav id="main-menu" class="main-menu accessible-menu" data-item-open-text="Open the %s menu" data-item-close-text="Close the %s menu">
<ul class="menu">
<li class="menu-item--expanded li--level-one">
Academics
<button class="menu-item__button">Open the Academics menu</button><ul class="menu">
<li class="li--level-two">
Academic Programs

<li class="menu-item--collapsed li--level-two">
Active Learning Initiatives

<li class="menu-item--collapsed li--level-two">
Undergraduate Programs

<li class="menu-item--collapsed li--level-two">
Graduate Programs

<li class="menu-item--collapsed li--level-two">
Pre-Health/Pre-Med

<li class="li--level-two">
Study Abroad

<li class="menu-item--collapsed li--level-two">
Pre-College Programs

<li class="menu-item--collapsed li--level-two">
Summer Courses

<li class="li--level-two">
Colleges and Institutes

<li class="li--level-two">
Libraries

<li class="li--level-two">
Academic Calendar

<li class="li--level-two">
Course Catalog

<li class="menu-item--expanded li--level-one">
Admission and Aid
<button class="menu-item__button">Open the Admission and Aid menu</button><ul class="menu">
<li class="li--level-two">
Apply

<li class="menu-item--collapsed li--level-two">
Undergraduate Admission

<li class="menu-item--collapsed li--level-two">
Graduate Admission

<li class="menu-item--collapsed li--level-two">
Tuition and Aid

<li class="menu-item--collapsed li--level-two">
Visit and Tour

<li class="li--level-two">
Request Information

<li class="menu-item--expanded li--level-one">
Student Experience
<button class="menu-item__button">Open the Student Experience menu</button><ul class="menu">
<li class="menu-item--collapsed li--level-two">
Our Campus and Housing

<li class="li--level-two">
Life in Chicago

<li class="li--level-two">
Career Services and Internships

<li class="menu-item--collapsed li--level-two">
Student Outcomes

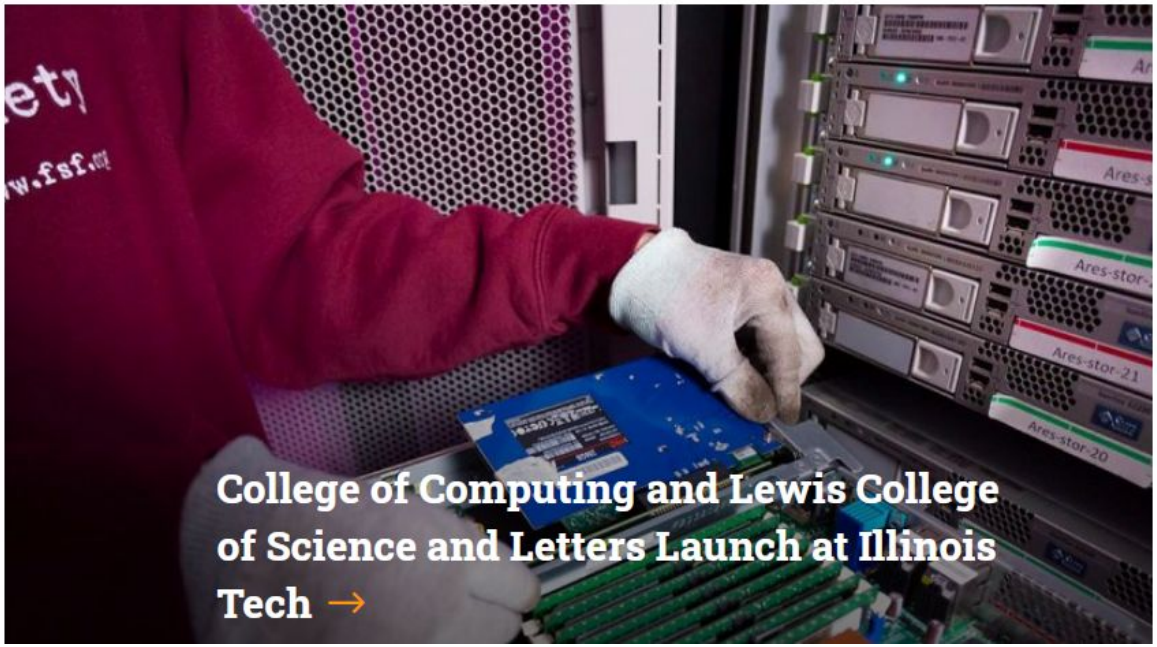
<li class="li--level-two">
Athletics

<li class="menu-item--collapsed li--level-two">
Student Organizations and Greek Life

<li

      </footer>   <script type="application/json" data-drupal-selector="drupal-settings-json">{"path":{"baseUrl":"\/","scriptPath":null,"pathPrefix":"","currentPath":"node\/1","currentPathIsAdmin":false,"isFront":true,"currentLanguage":"en"},"pluralDelimiter":"\u0003","suppressDeprecationErrors":true,"eu_cookie_compliance":{"popup_enabled":true,"popup_agreed_enabled":false,"popup_hide_agreed":false,"popup_clicking_confirmation":false,"popup_scrolling_confirmation":false,"popup_html_info":"\u003Carticle class=\u0022cookie-compliance-popup\u0022\u003E\n \u003Cdiv class=\u0022eu-cookie-compliance-banner eu-cookie-compliance-banner-info\u0022\u003E\n   \u003Cdiv class =\u0022popup-content info eu-cookie-compliance-content\u0022\u003E\n   \u003Cdiv id=\u0022popup-text\u0022 class=\u0022eu-cookie-compliance-message\u0022\u003E\n   \u003Cdiv class=\u0022grid-container\u0022\u003E\n   \u003Cdiv class=\u0022grid-x\u0022\u003E\n   \u003Cdiv class=\u0022cell small-12 medium-6\u0022\u003E\n   \u003Ch2\u003EWe use technologies, such as cookies, to customize content and advertising, to provide social media features, and to analyze traffic to the site. By using or registering on any portion of this site, you agree to our privacy and cookie statement.\u003C\/h2\u003E\n\n   \u003C\/div\u003E\n   \u003Cdiv class=\u0022cell small-12 medium-6\u0022\u003E\n   \u003Cdiv id=\u0022popup-buttons\u0022 class=\u0022eu-cookie-compliance-buttons\u0022\u003E\n   \u003Cbutton type=\u0022button\u0022 class=\u0022btn agree-button eu-cookie-compliance-default-button\u0022\u003EDismiss\u003C\/button\u003E\n   \u003Cbutton type=\u0022button\u0022 class=\u0022find-more-button eu-cookie-compliance-more-button\u0022\u003EPrivacy Statement\u003C\/button\u003E\n   \u003C\/div\u003E\n   \u003C\/div\u003E\n   \u003C\/div\u003E\n   \u003C\/div\u003E\n   \u003C\/div\u003E\n \u003C\/div\u003E\n\u003C\/article\u003E","use_mobile_message":false,"mobile_popup_html_info":"\u003Carticle class=\u0022cookie-compliance-popup\u0022\u003E\n \u003Cdiv class=\u0022eu-cookie-compliance-banner eu-cookie-compliance-banner-info\u0022\u003E\n   \u003Cdiv class =\u0022popup-content info eu-cookie-compliance-content\u0022\u003E\n   \u003Cdiv id=\u0022popup-text\u0022 class=\u0022eu-cookie-compliance-message\u0022\u003E\n   \u003Cdiv class=\u0022grid-container\u0022\u003E\n   \u003Cdiv class=\u0022grid-x\u0022\u003E\n   \u003Cdiv class=\u0022cell small-12 medium-6\u0022\u003E\n   \n   \u003C\/div\u003E\n   \u003Cdiv class=\u0022cell small-12 medium-6\u0022\u003E\n   \u003Cdiv id=\u0022popup-buttons\u0022 class=\u0022eu-cookie-compliance-buttons\u0022\u003E\n   \u003Cbutton type=\u0022button\u0022 class=\u0022btn agree-button eu-cookie-compliance-default-button\u0022\u003EDismiss\u003C\/button\u003E\n   \u003Cbutton type=\u0022button\u0022 class=\u0022find-more-button eu-cookie-compliance-more-button\u0022\u003EPrivacy Statement\u003C\/button\u003E\n   \u003C\/div\u003E\n   \u003C\/div\u003E\n   \u003C\/div\u003E\n   \u003C\/div\u003E\n   \u003C\/div\u003E\n   \u003C\/div\u003E\n \u003C\/div\u003E\n\u003C\/article\u003E","mobile_breakpoint":768,"popup_html_agreed":false,"popup_use_bare_css":false,"popup_height":"auto","popup_width":"100%","popup_delay":1000,"popup_link":"\/privacy","popup_link_new_window":true,"popup_position":false,"fixed_top_position":true,"popup_language":"en","store_consent":false,"better_support_for_screen_readers":false,"cookie_name":"","reload_page":false,"domain":"","domain_all_sites":false,"popup_eu_only_js":false,"cookie_lifetime":100,"cookie_session":0,"disagree_do_not_show_popup":false,"method":"default","whitelisted_cookies":"","withdraw_markup":"\u003Cbutton type=\u0022button\u0022 class=\u0022eu-cookie-withdraw-tab\u0022\u003EPrivacy settings\u003C\/button\u003E\n\u003Cdiv class=\u0022eu-cookie-withdraw-banner\u0022\u003E\n \u003Cdiv class=\u0022popup-content info eu-cookie-compliance-content\u0022\u003E\n   \u003Cdiv id=\u0022popup-text\u0022 class=\u0022eu-cookie-compliance-message\u0022\u003E\n   \u003Ch2\u003EWe use cookies on this site to enhance your user experience\u003C\/h2\u003E\n\u003Cp\u003EYou have given your consent for us to set cookies.\u003C\/p\u003E\n\n   \u003C\/div\u003E\n   \u003Cdiv id=\u0022popup-buttons\u0022 class=\u0022eu-cookie-compliance-buttons\u0022\u003E\n   \u003Cbutton type=\u0022button\u0022 class=\u0022eu-cookie-withdraw-button\u0022\u003EWithdraw consent\u003C\/button\u003E\n   \u003C\/div\u003E\n \u003C\/div\u003E\n\u003C\/div\u003E","withdraw_enabled":false,"withdraw_button_on_info_popup":false,"cookie_categories":[],"enable_save_preferences_button":true,"fix_first_cookie_category":true,"select_all_categories_by_default":false},"oho_cludo_search":{"cludo_settings":{"customerId":"10000893","engineId":"10001366","searchUrl":"\/search","language":"en","searchInputs":["cludo-search-form","cludo-search-form-full"],"template":"InlineBasic","hideSearchFilters":true,"focusOnResultsAfterSearch":true,"type":"inline"}},"user":{"uid":0,"permissionsHash":"e3568b825fb5f73e53ae577f5e3337337db31799e448047a326bda8da61ae590"}}</script><!-- Start of global snippet: Please do not removePlace this snippet between the <head> and </head> tags on every page of your site.--><!-- Global site tag (gtag.js) - Google Marketing Platform --><script async="" src="./Illinois Institute of Technology _ Illinois Institute of Technology_files/js"></script><script src="./Illinois Institute of Technology _ Illinois Institute of Technology_files/js_gtZlvNcFXqX9phnUNQqFO9n354iiaIuyXxqL5zPqWo4.js.下载"></script><script src="./Illinois Institute of Technology _ Illinois Institute of Technology_files/search-script.min.js.下载"></script><!--[if lte IE 9]><script src="//api.cludo.com/scripts/xdomain.js" slave="//api.cludo.com/proxy.html"></script><![endif]--><script src="./Illinois Institute of Technology _ Illinois Institute of Technology_files/js_vRZbgcFLaxCz03RuuoPQ3cGVqfiK-VS3EZ07jaYZHn8.js.下载"></script> <script type="text/javascript">window.NREUM||(NREUM={});NREUM.info={"beacon":"bam.nr-data.net","licenseKey":"7fc0e5db91","applicationID":"467337880","transactionName":"YAFQMEtYXkBQUkRYV1lLcwdNUF9dHnVCREhWCG4KV11Vb3JeXkVKWAheAUtlflxVVGZYXUAnXQpNS19fXVRCHAZBDVcT","queueTime":0,"applicationTime":1593,"atts":"TEZTRgNCTU4=","errorBeacon":"bam.nr-data.net","agent":""}</script><script> window.dataLayer = window.dataLayer || []; function gtag(){dataLayer.push(arguments);} gtag('js', new Date()); gtag('config', 'DC-9222161');</script><script type="text/javascript" id="">!function(b,e,f,g,a,c,d){b.fbq||(a=b.fbq=function(){a.callMethod?a.callMethod.apply(a,arguments):a.queue.push(arguments)},b._fbq||(b._fbq=a),a.push=a,a.loaded=!0,a.version="2.0",a.queue=[],c=e.createElement(f),c.async=!0,c.src=g,d=e.getElementsByTagName(f)[0],d.parentNode.insertBefore(c,d))}(window,document,"script","https://connect.facebook.net/en_US/fbevents.js");fbq("init","266976930448610");fbq("track","PageView");</script><noscript><img height="1" width="1" style="display:none" src="https://www.facebook.com/tr?id=266976930448610&ev=PageView&noscript=1"></noscript><script type="text/javascript" id="">!function(b,e,f,g,a,c,d){b.fbq||(a=b.fbq=function(){a.callMethod?a.callMethod.apply(a,arguments):a.queue.push(arguments)},b._fbq||(b._fbq=a),a.push=a,a.loaded=!0,a.version="2.0",a.queue=[],c=e.createElement(f),c.async=!0,c.src=g,d=e.getElementsByTagName(f)[0],d.parentNode.insertBefore(c,d))}(window,document,"script","https://connect.facebook.net/en_US/fbevents.js");fbq("init","221692042287635");fbq("track","PageView");</script><noscript><img height="1" width="1" style="display:none" src="https://www.facebook.com/tr?id=221692042287635&ev=PageView&noscript=1"></noscript><!-- End of global snippet: Please do not remove --><script>var _stk = "a0e675b95780f1f744a5d8c244c8eb6b449f3004";(function(){ var a=document, b=a.createElement("script"); b.type="text/javascript"; b.async=!0; b.src=('https:'==document.location.protocol ? 'https://' : 'http://') + 'd31y97ze264gaa.cloudfront.net/assets/st/js/st.js'; a=a.getElementsByTagName("script")[0]; a.parentNode.insertBefore(b,a); })();</script><iframe src="./Illinois Institute of Technology _ Illinois Institute of Technology_files/pixel.html" style="display: none;"></iframe></body></html>

Illinois Tech News



College of Computing and Lewis College of Science and Letters Launch at Illinois Tech →

Kenneth T. Christensen Named Armour College of Engineering Dean →

› Engineering Innovation

- Suppose that you sent 100,000 characters to a destination, and any of them may be one of {a,b,c,d,e,f}
- 2^n>=8 then n=3 bits
- the total traffic is: 3bits*100,000=300,000 bits
- how to compress this traffic?
  - Variable-length codeword is introduced to save space.
  - Strategy: the more frequently the character is used, the shorter code is designated for it.

Huffman Codes

Data Compression using a greedy algorithm to construct

Data is a sequence of characters

Use frequency of occurrence of characters to build an optimal way to represent each character as a binary string

	a	b	c	d	e	f
Frequency (in thousands)	45	13	12	16	9	5
Fixed-length codeword	000	001	010	011	100	101
Variable-length codeword	0	101	100	111	1101	1100

Figure 16.3 A character-coding problem. A data file of 100,000 characters contains only the characters a–f, with the frequencies indicated. If each character is assigned a 3-bit codeword, the file can be encoded in 300,000 bits. Using the variable-length code shown, the file can be encoded in 224,000 bits.

fixed-length: 3bits*100,000=300,000 bits

variable-length:

45,000*1+13,000*3+12,000*3+16,000*3+9,000*4+5,000*4

=224,000 bits

(300,000-224,000)/300,000>25%---big deal!

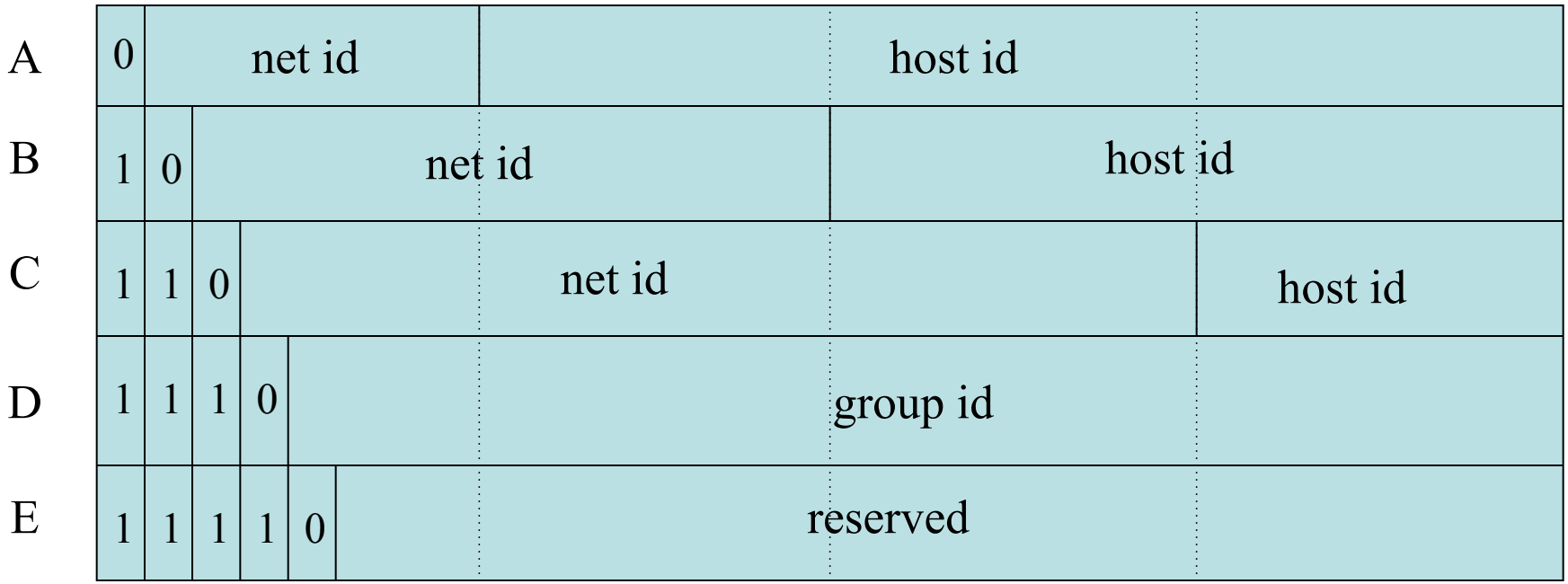
Fixed-Length vs. Variable Length Codes

Fixed Length – Each character is represented by a unique binary string. Easy to encode, concatenate the codes together. Easy to decode, break off 3-bit codewords and decode each one.

Variable Length – Give frequent characters shorter codewords, infrequent characters get long codewords. However, how do we decode if the length of the codewords are variable?

- Solution--Prefix Codes
  - We consider only codes in which no codeword is also a prefix of some other codeword. Such codes are called prefix codes.
  - Since no codeword is a prefix of any other, decoding is unambiguous.
    - classed IP address

net id+host



- Solution--Prefix Codes

  – Binary tree provides a convenient representation for the prefix code.
  – Binary tree method – 0 means go to left child, 1 means go to right child (not a binary search tree.

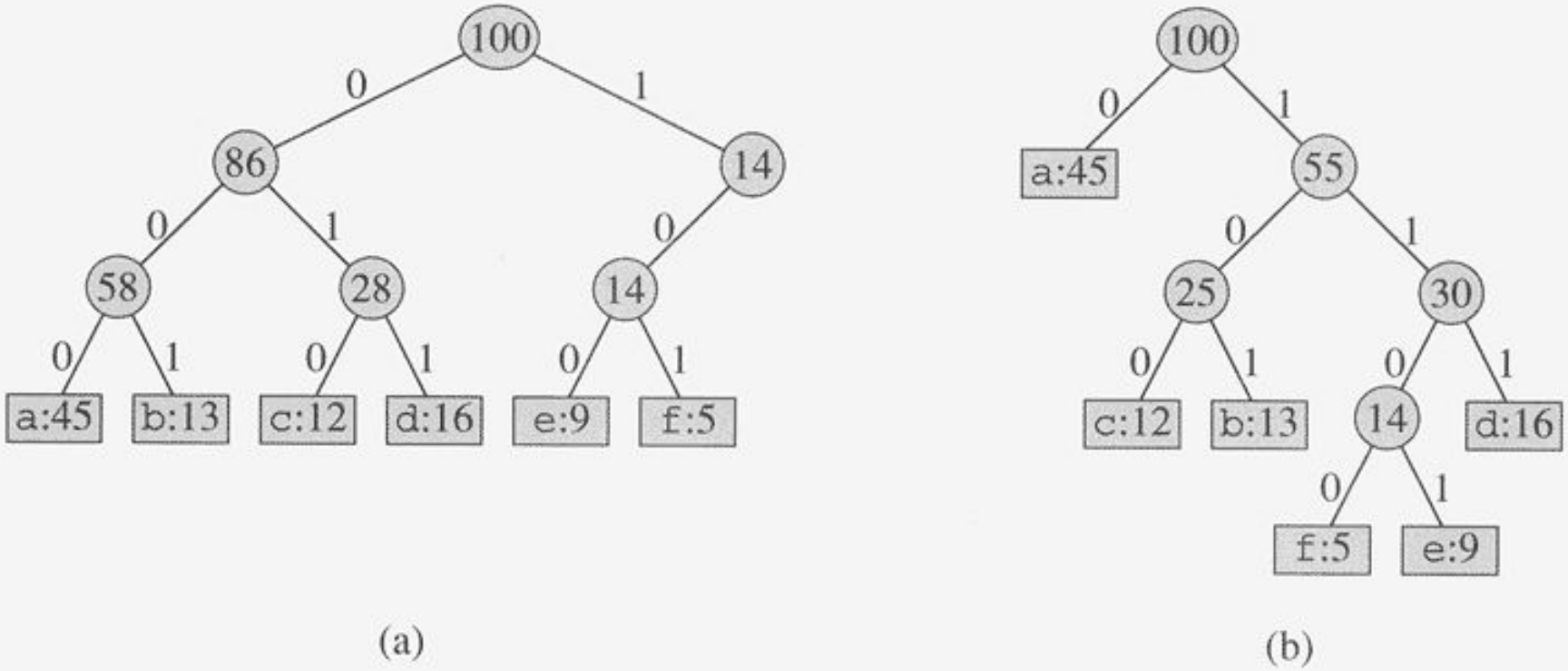


Figure 16.4 Trees corresponding to the coding schemes in Figure 16.3. Each leaf is labeled with a character and its frequency of occurrence. Each internal node is labeled with the sum of the frequencies of the leaves in its subtree. **(a)** The tree corresponding to the fixed-length code a = 000, ..., f = 101. **(b)** The tree corresponding to the optimal prefix code a = 0, b = 101, ..., f = 1100.

Optimal Variable Length Prefix Codes

An optimal (minimal) code is always represented by a full binary tree (every non-leaf has 2 children. WHY?

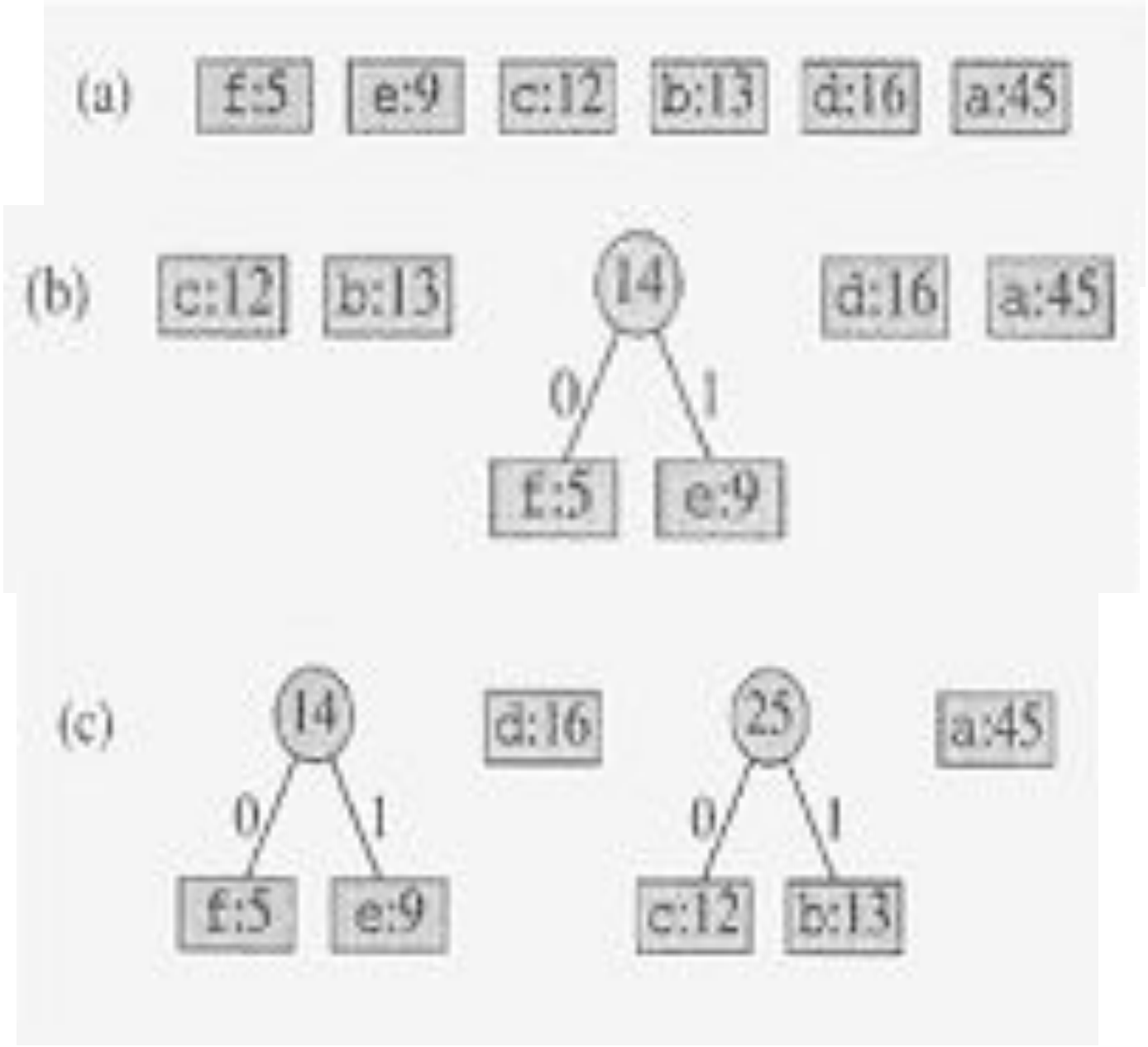
If “C” is the alphabet to be compressed (coded), you need |C| leaves and |C|-1 internal nodes (in a full binary tree).

Cost of Tree in bits

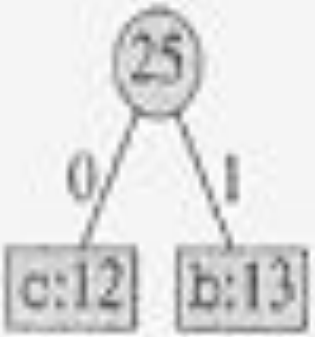
$$B(T) = \sum_{\text{for all } c \in C} \text{freq}(c) * \text{depth}(c)$$

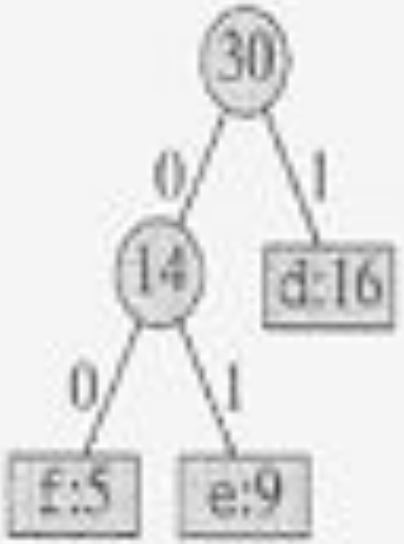
Greedy Algorithm to construct an Optimal Huffman Code

- Builds tree bottom up
- the character with the least frequency should be on the longest path
- Uses minimum priority queue to merge 2 least frequent objects (objects are leaf nodes or other subtrees) together into new subtree.


(a)
f:5
e:9
c:12
b:13
d:16
a:45
(b)
c:12
b:13
14
0
1
f:5
e:9
d:16
a:45
(c)
14
0
1
f:5
e:9
d:16
25
0
1
c:12
b:13
a:45

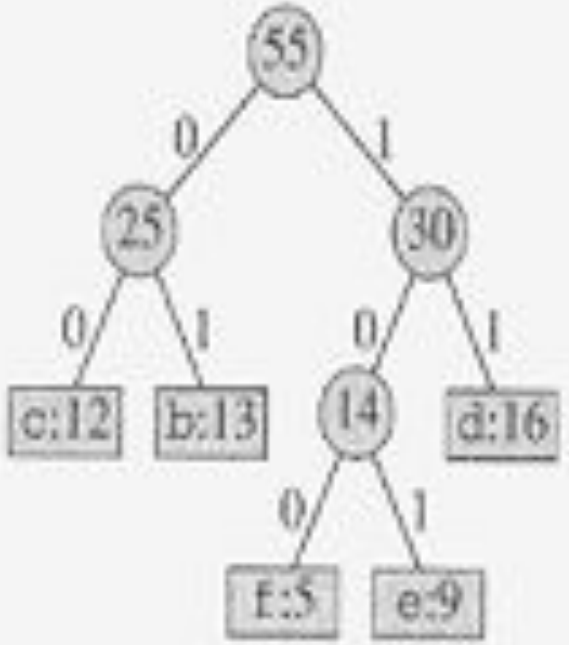
(d)


25
0
1
c:12
b:13

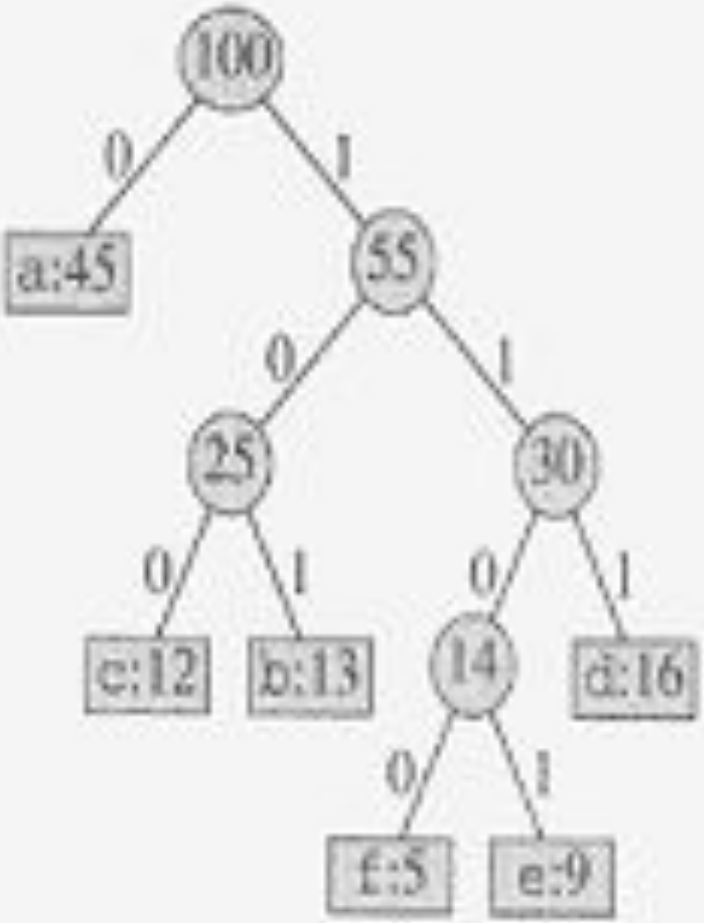

30
0
1
14
d:16
0
1
f:5
e:9

a:45

(e) a:45


55
0
1
25
30
0
1
0
1
c:12
b:13
14
d:16
0
1
f:5
e:9

(f)



```
HUFFMAN(C)
1  n ← |C|
2  Q ← C
3  for i ← 1 to n − 1
4       do allocate a new node z
5          left[z] ← x ← EXTRACT-MIN(Q)
6          right[z] ← y ← EXTRACT-MIN(Q)
7          f[z] ← f[x] + f[y]
8          INSERT(Q, z)
9  return EXTRACT-MIN(Q)          ▷ Return the root of the tree.
```