

# Example of B+ tree

# B+ Tree Example

- A balanced tree
- Each node can have **at most  $m$**  key fields and  **$m+1$**  pointer fields
- Half-full must be satisfied (except root node):
- $m$  is even and  $m=2d$ 
  - Leaf node half full: at least  $d$  entries
  - Non-leaf node half full: at least  $d$  entries
- $m$  is odd and  $m = 2d+1$ 
  - Leaf node half full: at least  $d+1$  entries
  - Non-leaf node half full: at least  $d$  entries (i.e.,  $d+1$  pointers)

# Show the tree after insertions

- Suppose each B+-tree node can hold up to 4 pointers and 3 keys.
- $m=3$  (odd),  $d=1$
- Half-full (for odd  $m$  value)
  - Leaf node, at least 2 ( $d+1$ ) entries
  - Non-leaf nodes, at least 2 ( $d+1$ ) pointers (1 entry)
- Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

# Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

- Insert 1

<b>1</b>			
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# Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

<b>1</b>			
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- Insert 3, 5

# Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

<b>1</b>		
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- Insert 3, 5

<b>1</b>	<b>3</b>	<b>5</b>	
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# Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

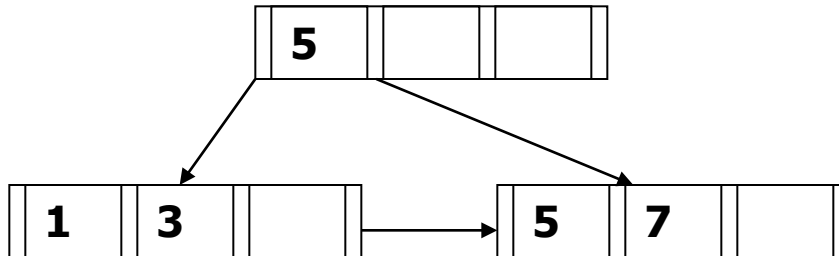
1	3	5	
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- Insert 7

# Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

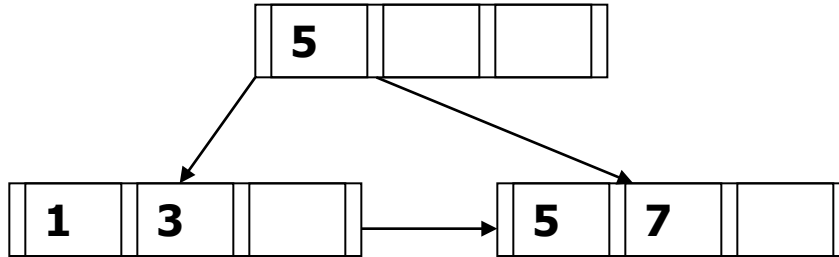


- Insert 7



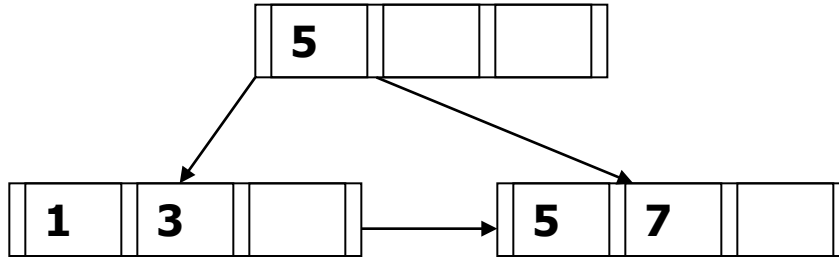


# Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

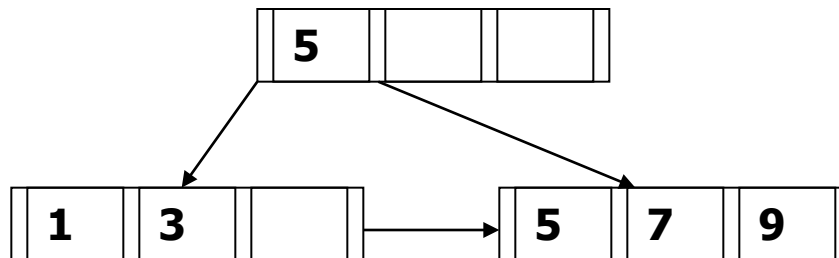


- Insert 9

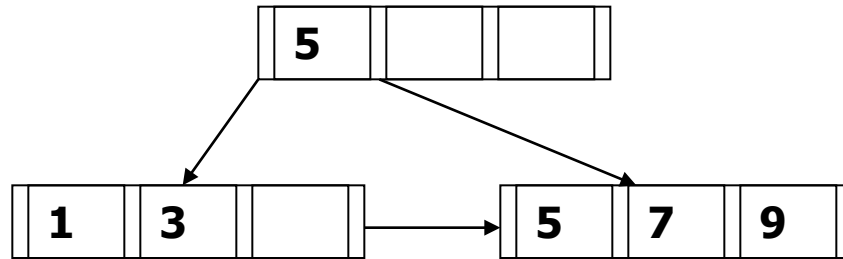
# Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10



- Insert 9

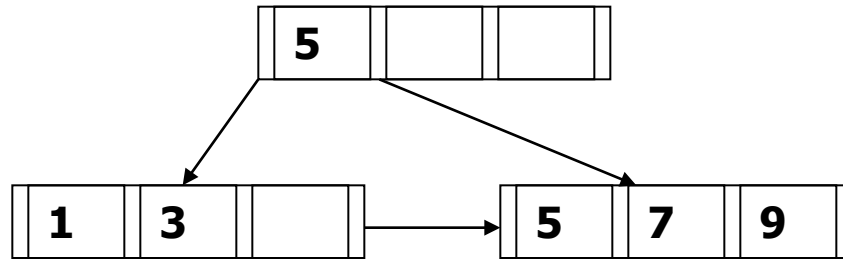


# Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

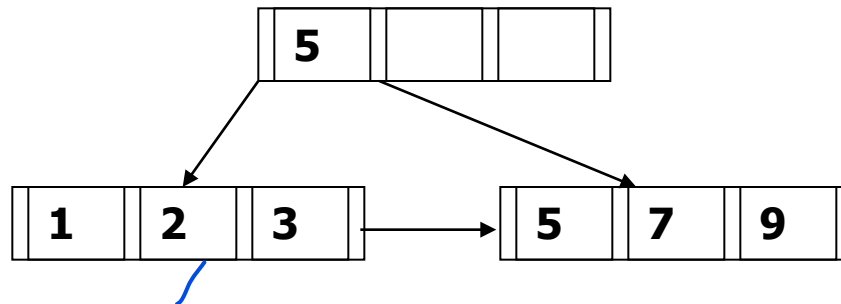


- Insert 2

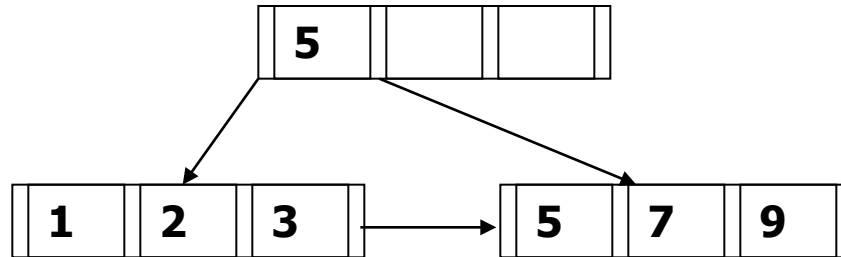
# Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10



- Insert 2

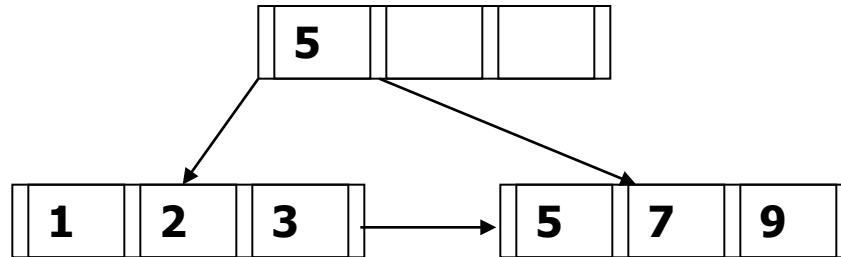


# Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

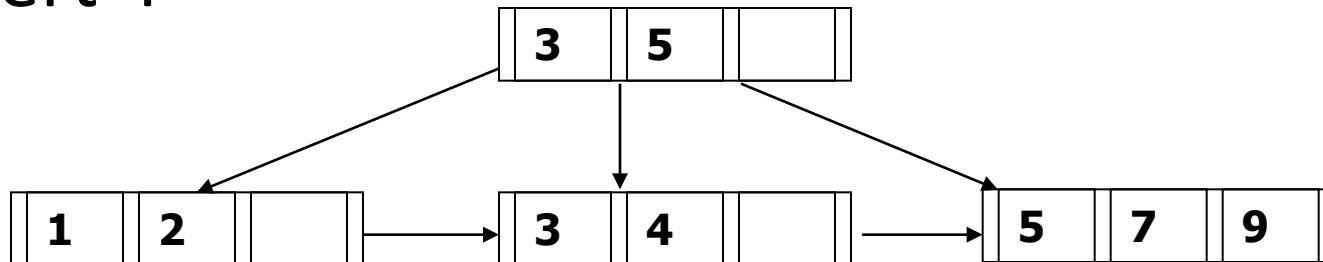


- Insert 4

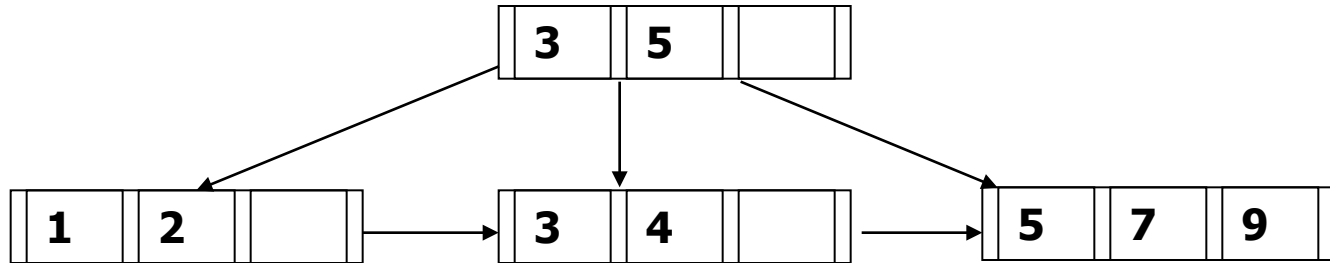
# Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10



- Insert 4

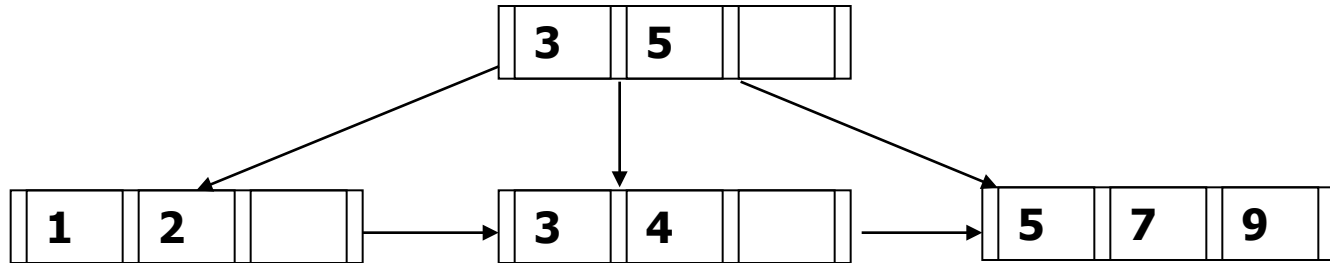


Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

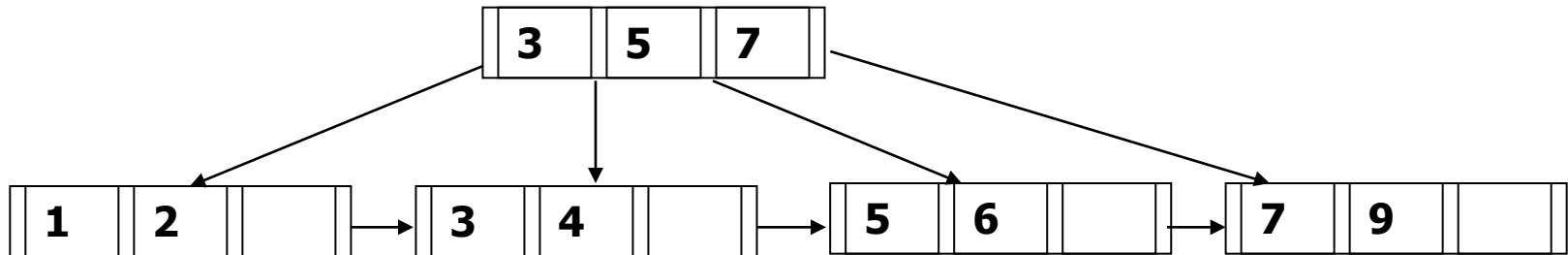


- Insert 6

# Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

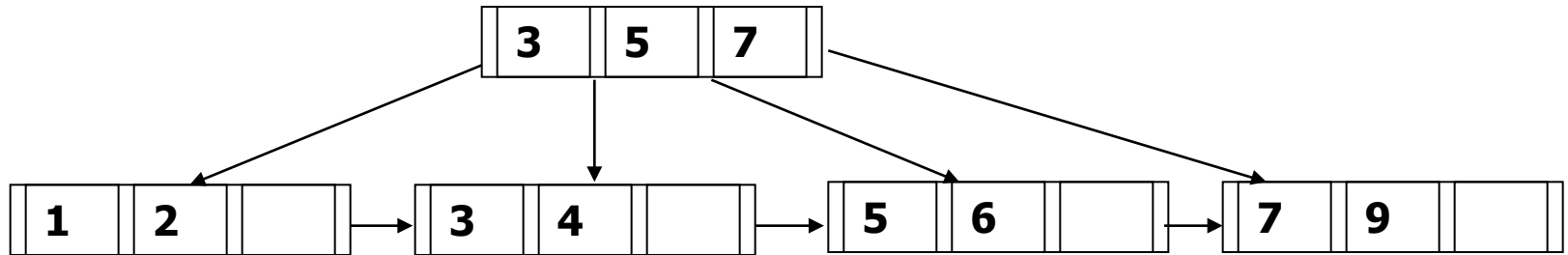


- Insert 6



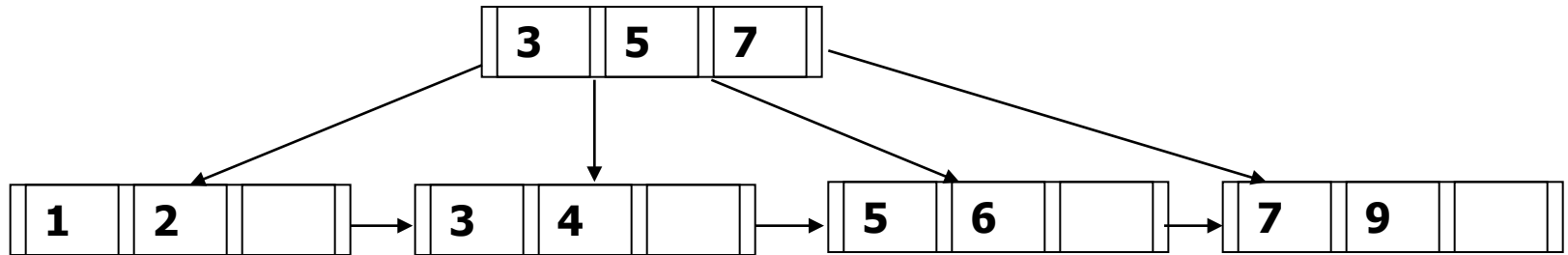


# Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

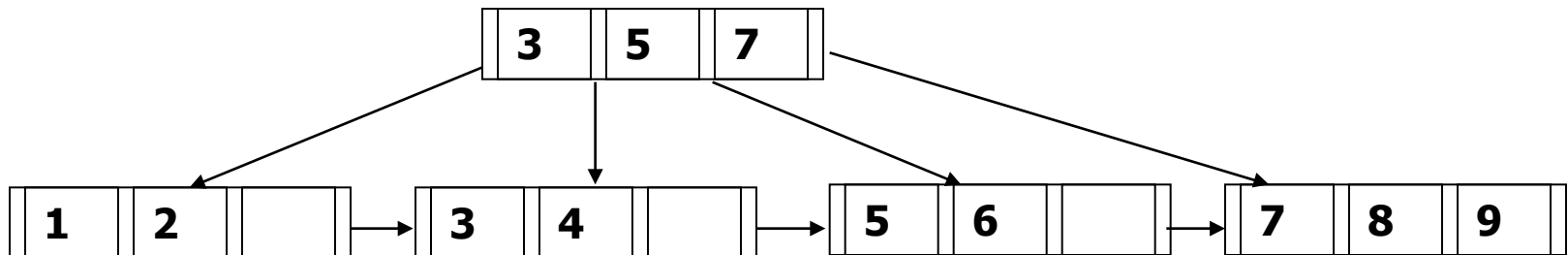


- Insert 8

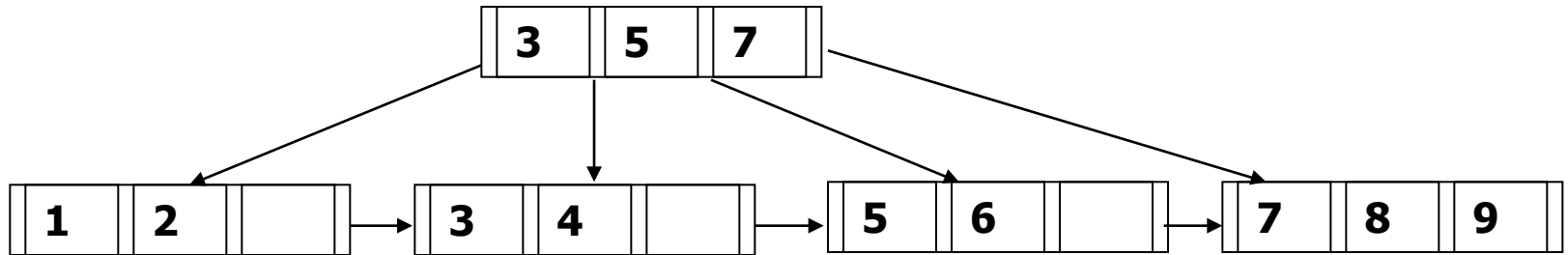
# Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10



- Insert 8

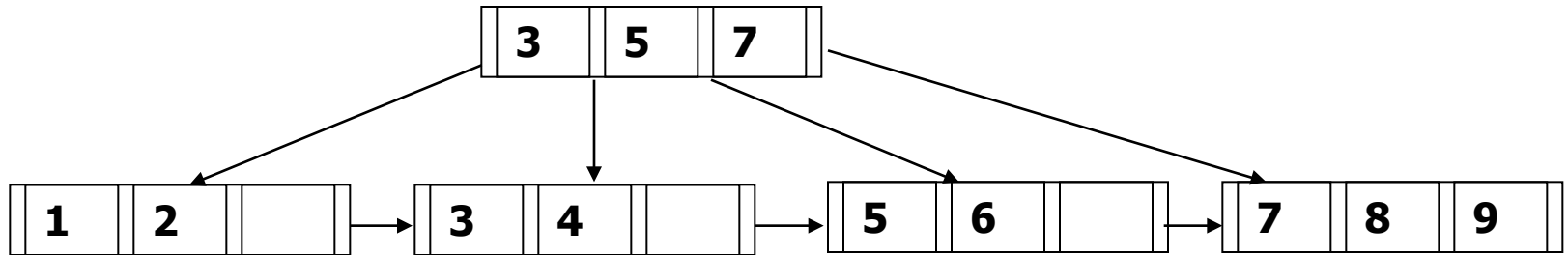


# Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

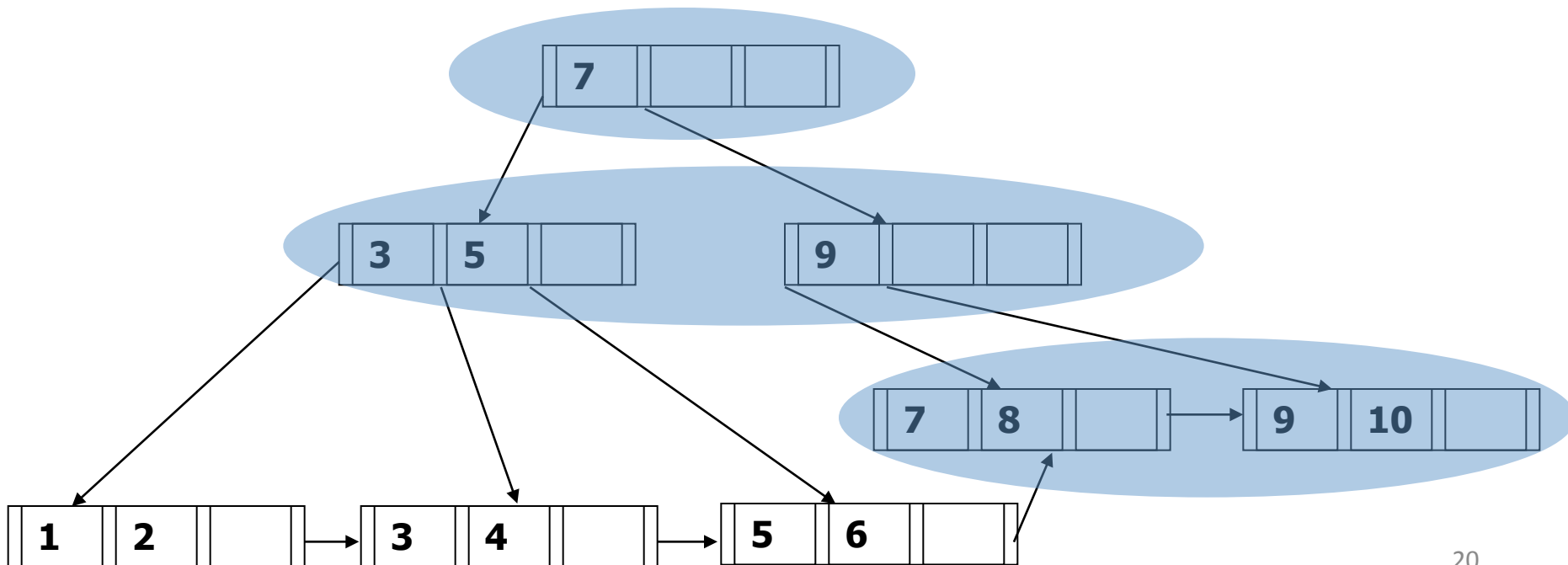


- Insert 10

# Insert 1, 3, 5, 7, 9, 2, 4, 6, 8, 10

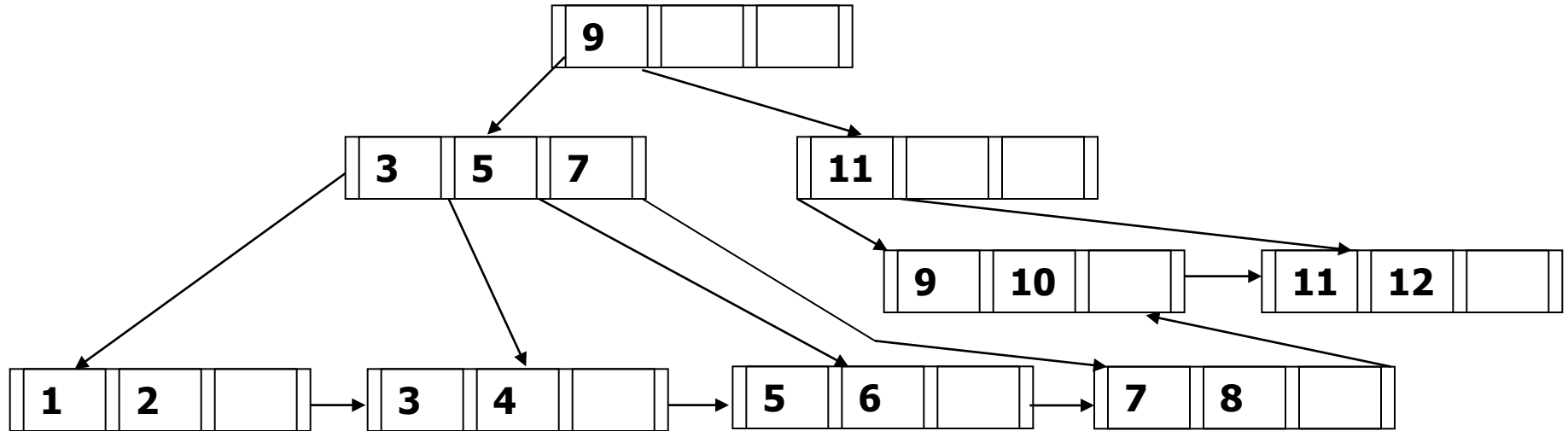


- Insert 10

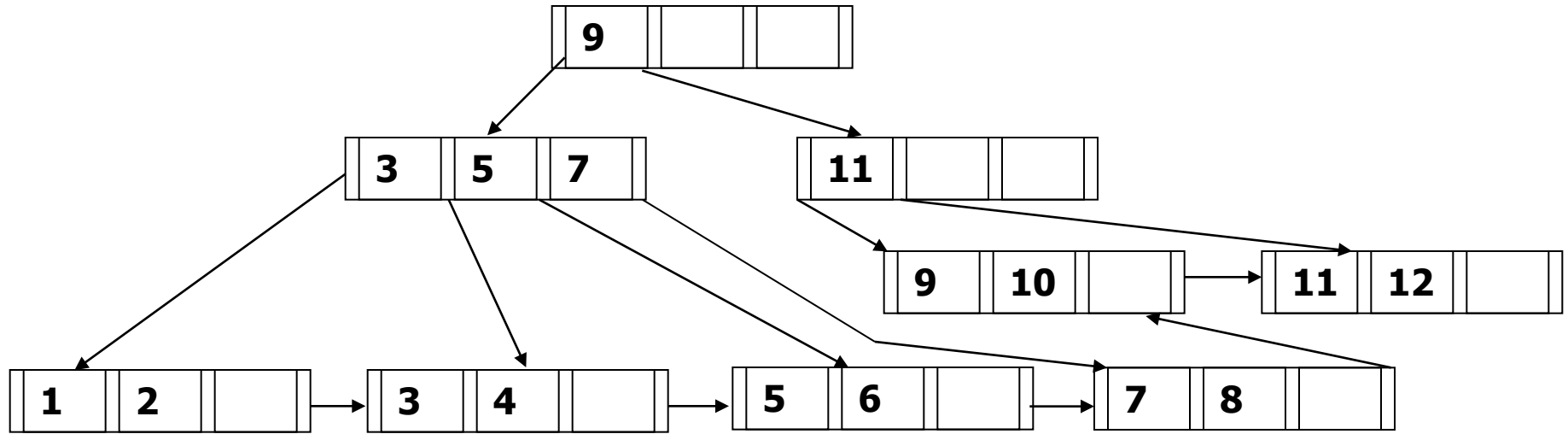


- Deletion

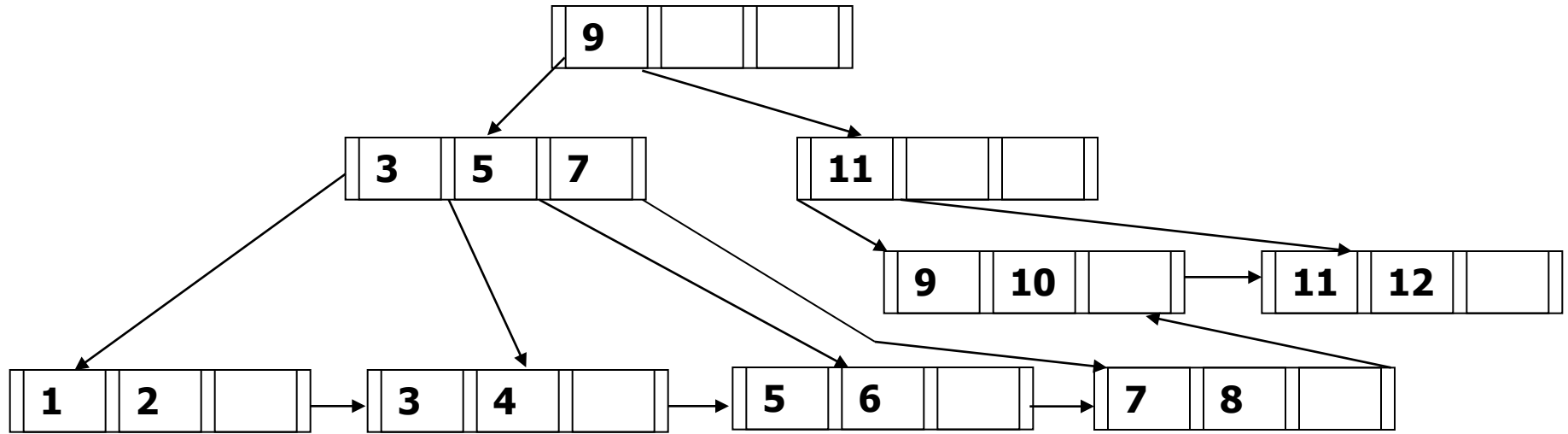
# Show the tree after deletions



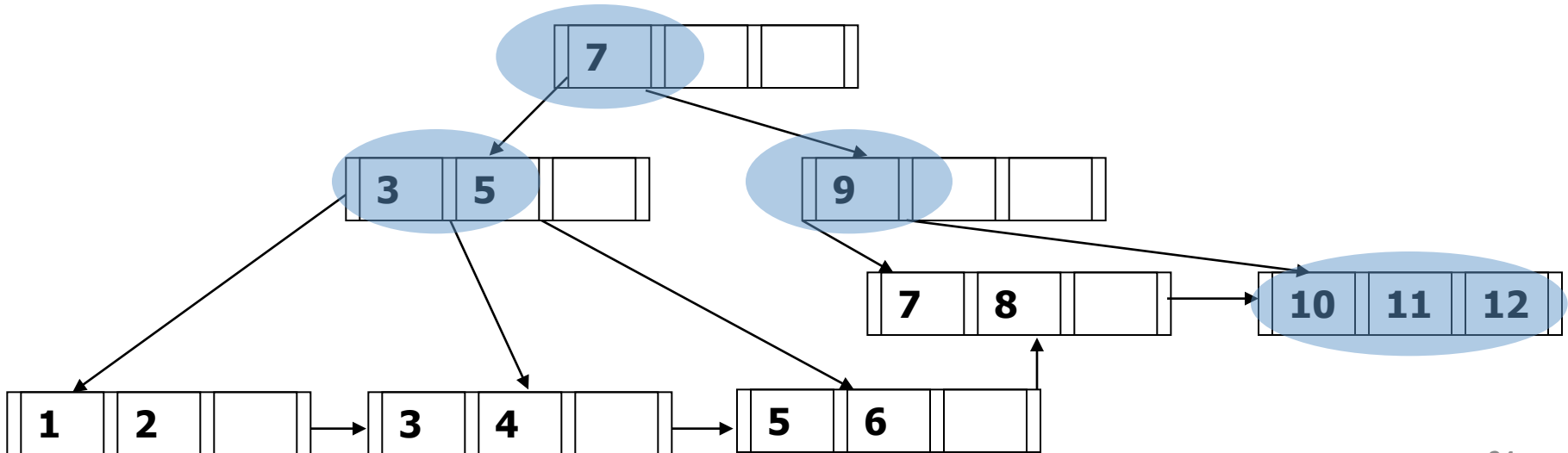
- Remove 9, 7, 8



- After removing 9

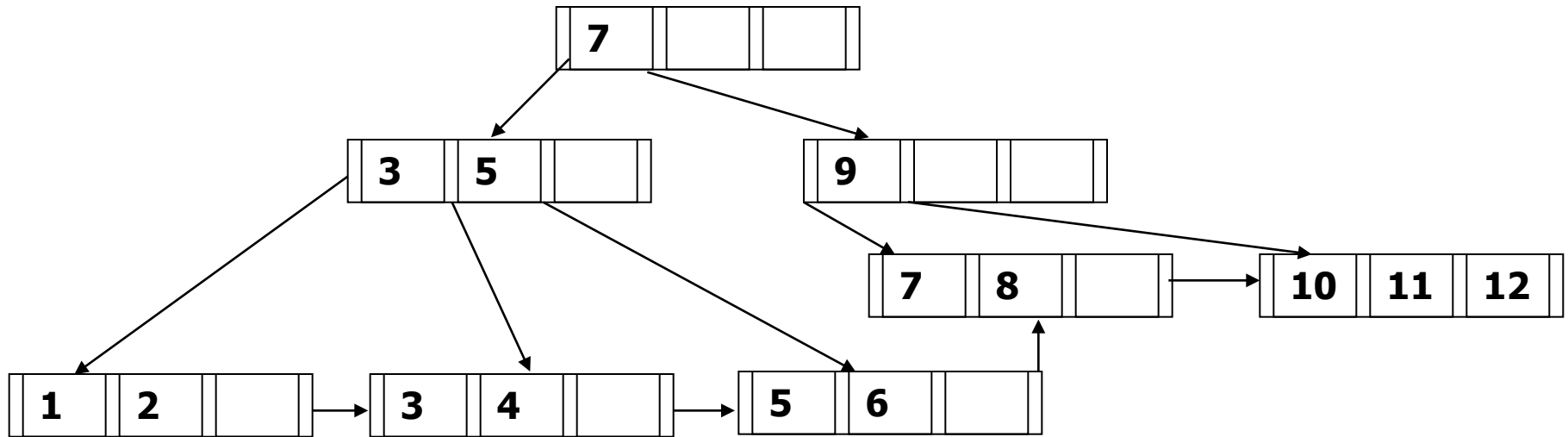


- After removing 9



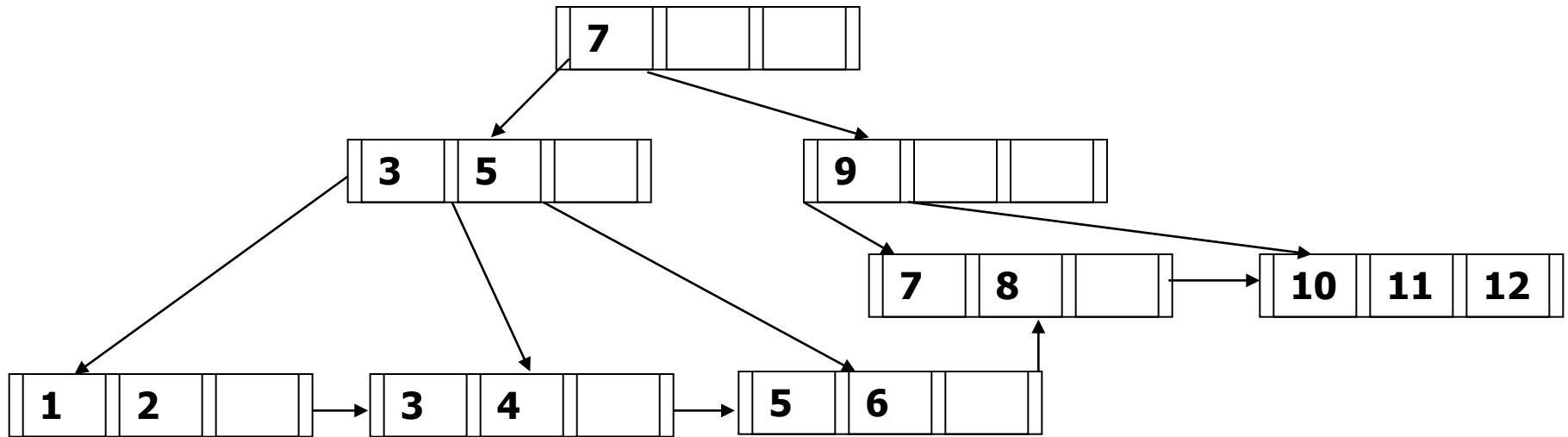


# Remove 9, 7, 8

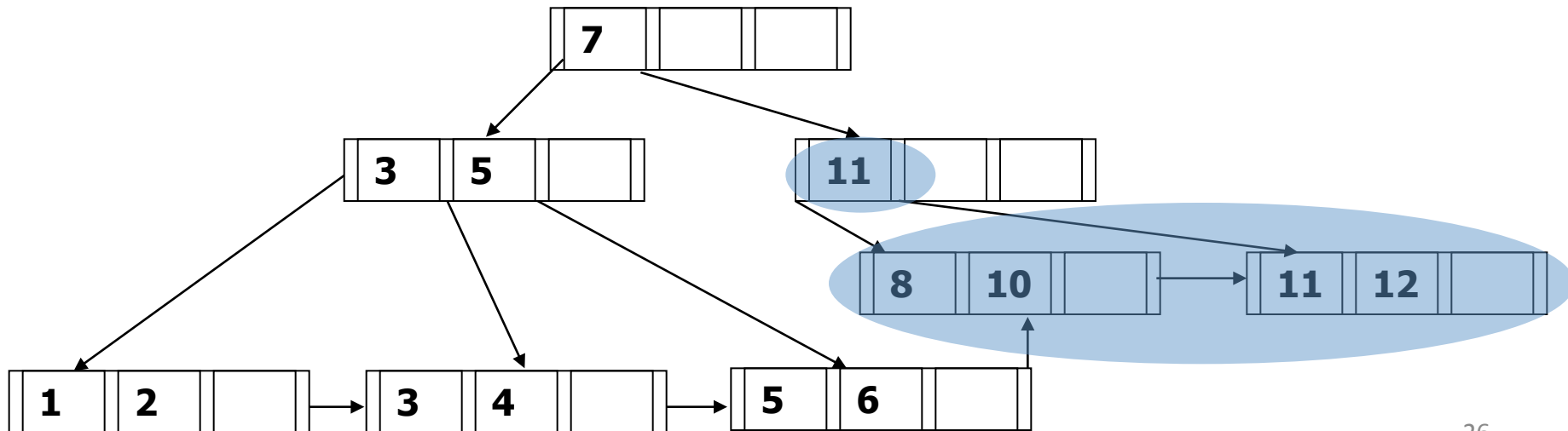


- After removing 7

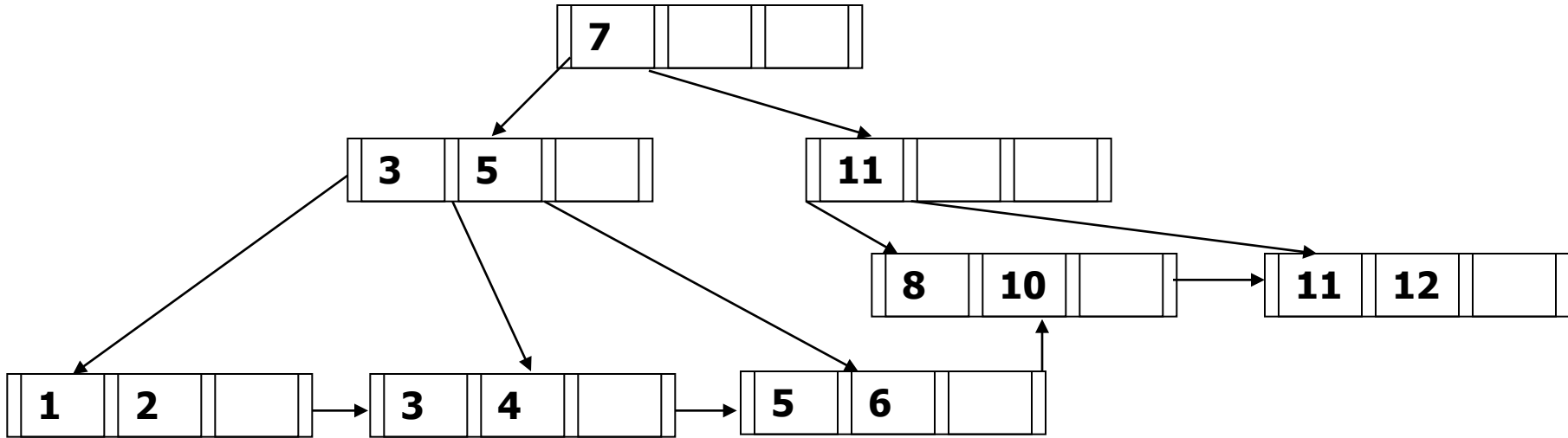
# Remove 9, 7, 8



- After removing 7

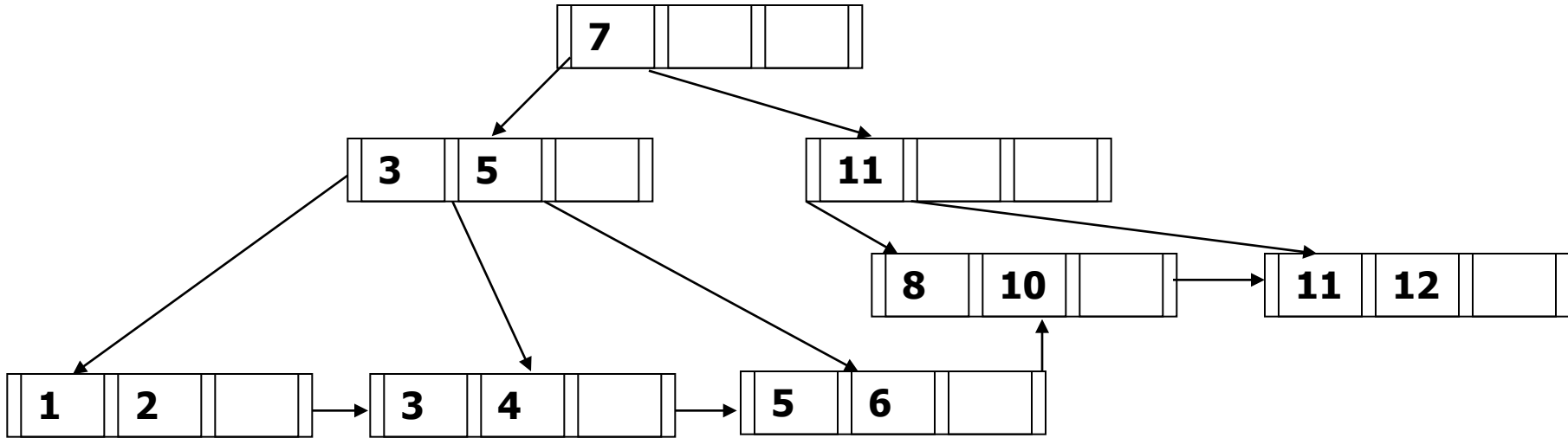


# Remove 9, 7, 8



- After removing 8

# Remove 9, 7, 8



- After removing 8

