



Chapter 19 - Fibonacci Heaps

Introduction to Algorithms, Third Edition

by Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest and Clifford Stein

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19.3 Decreasing a Key and Deleting a Node

In this section, we show how to decrease the key of a node in a Fibonacci heap in $O(1)$ amortized time and how to delete any node from an n -node Fibonacci heap in $O(D(n))$ amortized time. In [Section 19.4](#), we will show that the maximum degree $D(n)$ is $O(\lg n)$, which will imply that FIB-HEAP-EXTRACT-MIN and FIB-HEAP-DELETE run in $O(\lg n)$ amortized time.

Decreasing a Key

In the following pseudocode for the operation FIB-HEAP-DECREASE-KEY, we assume as before that removing a node from a linked list does not change any of the structural attributes in the removed node.

FIB-HEAP-DECREASE-KEY(H, x, k)

```

1 if  $k > x.key$ 
2   error "new key is greater than current key"
3  $x.key = k$ 
4  $y = x.p$ 
5 if  $y \neq \text{NIL}$  and  $x.key < y.key$ 
6   CUT( $H, x, y$ )
7   CASCADING-CUT( $H, y$ )
8 if  $x.key < H.min.key$ 
9    $H.min = x$ 
```

CUT(H, x, y)

```

1.remove  $x$  from the child list of  $y$ , decrementing  $y.degree$ 
2. add  $x$  to the root list of  $H$ 
3.  $x.p = \text{NIL}$ 
4.  $x.mark = \text{FALSE}$ 
```

CASCADING-CUT(H, y)

```

1.  $z = y.p$ 
2. if  $z \neq \text{NIL}$ 
3.   if  $y.mark == \text{FALSE}$ 
4.      $y.mark = \text{TRUE}$ 
5.   else CUT( $H, y, z$ )
6.   CASCADING-CUT( $H, z$ )
```

The FIB-HEAP-DECREASE-KEY procedure works as follows. Lines 1–3 ensure that the new key is no greater than the current key of x and then assign the new key to x . If x is a root or if $x.key \geq y.key$, where y is x 's parent, then no structural changes need occur, since min-heap order has not been violated. Lines 4–5 test for this condition.

If min-heap order has been violated, many changes may occur. We start by **cutting** x in line 6. The CUT procedure "cuts" the link between x and its parent y , making x a root.