CS 535 Design and Analysis of Algorithms	**Fall Semester, 2025**
Homework 5	
Assigned: November 4	*Due: November 18*

All students: please use Canvas to submit your solutions.
Notes for pseudocode usage:

1. C/Java instructions are fine. But do not write object-oriented additions. Do not declare or use any class. Declare only procedures (if necessary) and explain in words what each procedure does, and what is the use of each parameter. Feel free to use as procedures algorithms from the textbook; indicate page and edition. Please avoid Python as the running time can be obscured.
2. One instruction per line
3. Match the brackets with a horizontal line
4. Number your lines
5. Write down if your array is indexed $0 \dots n-1$ or $1 \dots n$.

Problem 1 A jogger wants to follow the least undesirable cycle of roads starting at her home. Each road has an "index of undesirability" (a positive integer) and can be traversed in either direction; the jogger must follow a nonempty cycle of roads and no road can be used twice. Formulated as a graph problem, the jogger has an undirected weighted multigraph $G = (V, E)$, and must determine the nonempty cycle of minimum weight starting (and ending) at vertex s, where the weight of a cycle is the sum of the weights of its edges.

1. Show how to use multiple applications of Dijkstra's shortest path algorithm to obtain the optimum jogger's route in time $O(|V|^2 \log |V| + |E||V|)$.

   Be precise: each time you want to use Dijkstra's explain which graph is the input of the algorithm. Justify the overall running time.

2. Prove the following. Let T be the shortest path tree constructed by Dijkstra's shortest path algorithm for starting vertex s in G. Then there exists an optimum jogger's route that has all but one of its edges in T, and furthermore, s is the only common ancestor in T of the endpoints of the edge not in T.

3. Use the result above and Dijkstra's shortest path algorithm and your own algorithm (which you need to describe) to find an optimum jogger's route in time $O(|V| \log |V| + |E|)$. Discuss the running time and correctness.

Problem 2 Let $G = (V, E, c, s, t)$ be a flow network with integer capacities, and let f be an integral maximum flow in G. Let $G' = (V, E, c', s, t)$ differ from G on one single edge e: $c'(e) = c(e) - 1$. Give a $O(|V| + |E|)$-time algorithm to obtain a maximum flow f' in G'.

Problem 3 Dining problem. Several families go out to dinner together. To increase their social interaction, they would like to sit at tables so that no two members of the same family are at the same table. Show how to formulate finding a seating arrangement that meets this objective as a maximum flow problem. Assume that q tables are available and that the j^{th} table has seating capacity of $b(j)$. Also assume there are p families and that the i^{th} family has $a(i)$ members.