

Centrality and Community

CS 579 Online Social Network Analysis

Dr. Cindy Hood
9/25/25

Homework Assignments

- ▶ HW #3 - Network Metrics
 - ▶ Assigned yesterday - due by midnight 10/3, no submissions accepted after 5pm 10/6
 - ▶ Good prep for Exam 1
- ▶ HW #4 - Chicago Community Areas + Census Data
 - ▶ You may work in groups up to 4 students (no exceptions) on this hw
 - ▶ Assigned later this week
- ▶ Please contact TAs with questions

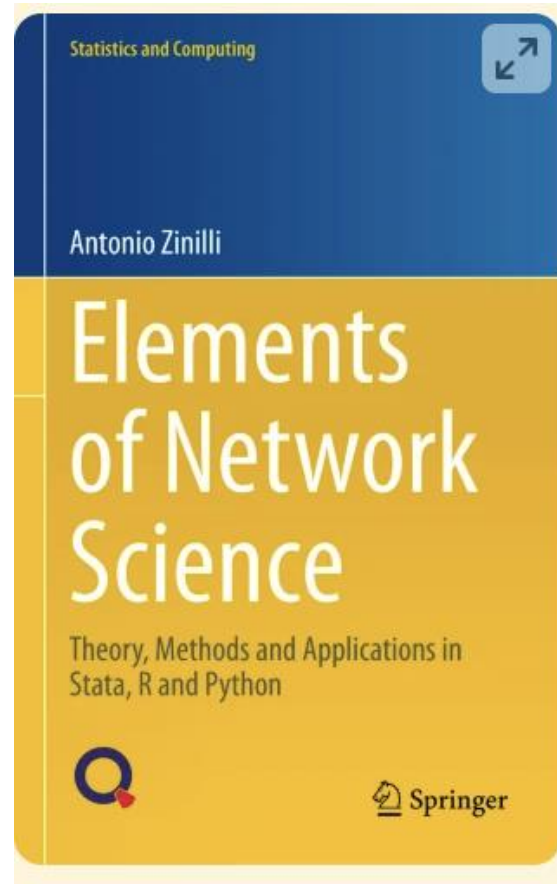
Exams and Final Project Poster Presentation

- ▶ Exam 1 - Oct 9 in class
- ▶ Exam 2 - Dec 2 in class
- ▶ Final Project Poster Session - Dec 4 in class
- ▶ Online students (sections 2 and 3) will have remote options

Teaching Assistants

- ▶ Siva Krishna Golla
 - ▶ sgolla2@hawk.illinoistech.edu
 - ▶ Mondays 2-3pm on zoom
- ▶ Khush Dhiren Patel
 - ▶ kpatel210@hawk.illinoistech.edu
 - ▶ Wednesdays 11-12 online
- ▶ Aswith Sama
 - ▶ asama@hawk.illinoistech.edu
 - ▶ Thursdays 3-4pm on zoom

Reference



<https://link.springer.com/book/10.1007/978-3-031-84712-7>

Recall Node's Neighborhood

- ▶ A node's neighborhood includes the group of nodes that it is connected to

$$N_i(G) = \{j : g_{ij} = 1\}$$

- ▶ Studying the neighborhood aids in identifying nodes or communities of nodes that share specific characteristics
- ▶ When individuals are motivated to emulate the behavior of their neighbors, cascading effects can occur
 - ▶ Can be observed when a new behavior initiates with a small group of early adopters and then spreads radially outward through the network

Neighborhood Degree Sequence

The neighborhood degree sequence for the node i , s_i , is derived from:

$$s_i = \{k_1^i, k_2^i, k_3^i, \dots, k_n^i\} \quad (3.19)$$

with k_i equal to the degree of the nodes to which i is connected. Being a sequence of degrees:

$$k_1^i \leq k_2^i \leq k_3^i \leq \dots \leq k_n^i \quad (3.20)$$

Neighborhood Degree Sequence

- ▶ Analysis
 - ▶ May help in examining variance of neighborhood degree sequences
 - ▶ Context of hierarchical complexity
 - ▶ May analyze neighbors of same degree to determine how similar they are

Jaccard Coefficient (JC) aka Jaccard Index

- ▶ Used to calculate how similar the neighbors of two vertices are

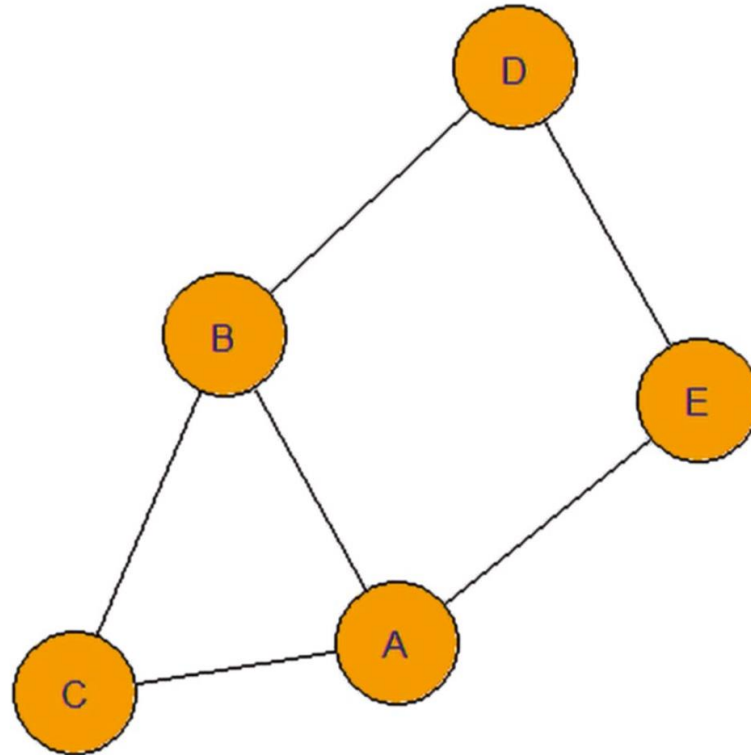
$$JC_{ij} = \frac{|N_i \cap N_j|}{|N_i \cup N_j|}$$

N_i = Neighborhood of i

- ▶ $0 \leq JC \leq 1$
 - ▶ 0 is maximum dissimilarity
 - ▶ 1 is maximum similarity

Example

Fig. 3.4 Graph example



		Degree	Degree Sequence
A	$N(A) = \{B, C, E\}$	3	$\{2, 2, 3\}$
B	$N(B) = \{A, C, D\}$	3	$\{2, 2, 3\}$
C	$N(C) = \{A, B\}$	2	$\{3, 3\}$
D	$N(D) = \{B, E\}$	2	$\{2, 3\}$
E	$N(E) = \{A, D\}$	2	$\{2, 3\}$

↑
Neighborhood

sequence

3

3

$$JC_{AB} = \frac{|N(A) \cap N(B)|}{|N(A) \cup N(B)|} = \frac{C}{ABCDE} = \frac{1}{5}$$

$$JC_{AC} = \frac{B}{ABCE} = \frac{1}{4}$$

$$JC_{AD} = \frac{B, E}{B, C, E} = \frac{2}{3}$$

$$JC_{AE} = 0$$

$$JC_{BC}$$

Jaccard Distance (JD)

- ▶ Measure of dissimilarity
- ▶ Complement of Jaccard Coefficient
- ▶ $JD = 1 - JC$

Community Detection

- ▶ Involves finding groups of nodes that are, in some way, more similar to each other than to other nodes

The screenshot shows the Merriam-Webster website with the word 'community' searched. The left sidebar contains navigation links: Dictionary, Definition (highlighted), Synonyms, Example Sentences, Word History, Phrases Containing, Rhymes, Entries Near, Related Articles, and Show More. A 'Save Word' button is at the bottom of the sidebar. The main content area displays the word 'community' as a noun with its phonetic transcription 'com·mu·ni·ty' and 'kə-ˈmyū-nə-tē'. It lists the plural 'communities' and notes it is 'often attributive'. A link for 'Synonyms of community' is provided. The definitions are listed with numbered and lettered bullets: 1: a unified body of individuals; such as a: the people with common interests living in a particular area; broadly: the area itself; the problems of a large community; b: a group of people with a common characteristic or interest living together within a larger society; a community of retired persons; a monastic community; c: a body of persons of common and especially professional interests scattered through a larger society; the academic community; the scientific community; d: a body of persons or nations having a common history or common social, economic, and political interests; the international community; e: a group linked by a common policy; f: an interacting population of various kinds of individuals (such as species) in a common location; g: STATE, COMMONWEALTH.

<https://www.merriam-webster.com/dictionary/community>

Clustering Coefficient aka Transitivity

- ▶ Feature of real world networks
- ▶ Reflects the degree to which a node's neighbors are related to one another
- ▶ Frequently used to understand how nodes in a network tend to cluster together
- ▶ Also used to understand whether a graph has a small world, random or scale-free property

Local Clustering Coefficient

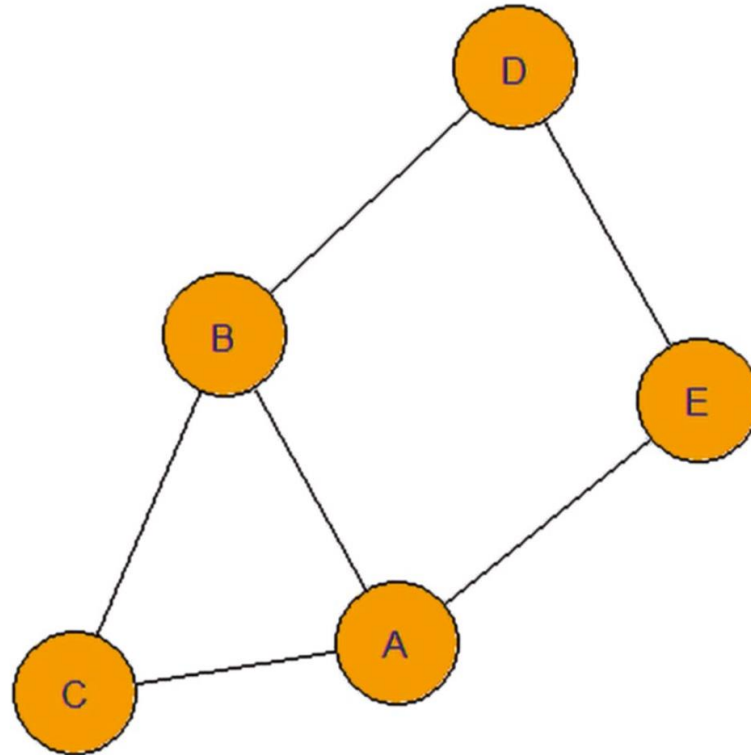
- ▶ Assume there is a node i of degree k
 - ▶ Let b_i be the number of edges that exist between the k neighbors of i
 - ▶ The local clustering coefficient $C_{local}(i)$ is the proportion between the actual number of edges amongst the neighbors, b_i and the maximum possible value of edges among the the neighbors.
 - ▶ Likelihood that two neighbors of node i are also connected
 - ▶ The maximum possible value can be calculated using classical binomial coefficient $\binom{k}{2}$
 - ▶ $\binom{k}{2} = \frac{k!}{2!(k-2)!} = \frac{k!}{2(k-2)!}$
- ▶ $C_{local}(i) = \frac{b_i}{\binom{k}{2}}$ with $0 \leq C_{local}(i) \leq 1$

Global Clustering Coefficient

- ▶ Average value of the local clustering coefficient $C_{local}(i)$ over all nodes in the network
- ▶ $C_{global} = \frac{1}{n} \sum_{i=1}^n C_{local}(i)$

Example

Fig. 3.4 Graph example



$$C_{\text{local}}(i) = \frac{b_i}{\binom{k_i}{2}} = \frac{b_i}{\frac{k_i!}{2!(k_i-2)!}}$$

$$b_A = 1 \quad K_A = 3 \quad C_{\text{local}}(A) = \frac{\frac{1}{3!}}{\frac{2!(3-2)!}} = \frac{\frac{1}{6}}{2} = \frac{1}{3}$$

$$b_B = 1 \quad K_B = 3 \quad C_{\text{local}}(B) = \frac{1}{3}$$

$$C_{\text{local}}(C) = 1$$

$$C_{\text{local}}(D) = 0$$

$$C_{\text{local}}(E) = 0$$

$$\begin{aligned} &= \frac{1}{3} \quad n=5 \\ C_{\text{global}} &= \frac{1}{5} \left(\frac{1}{3} + \frac{1}{3} + 1 + 0 + 0 \right) \\ &= \frac{1}{5} \left(\frac{5}{3} \right) = \frac{1}{3} \end{aligned}$$