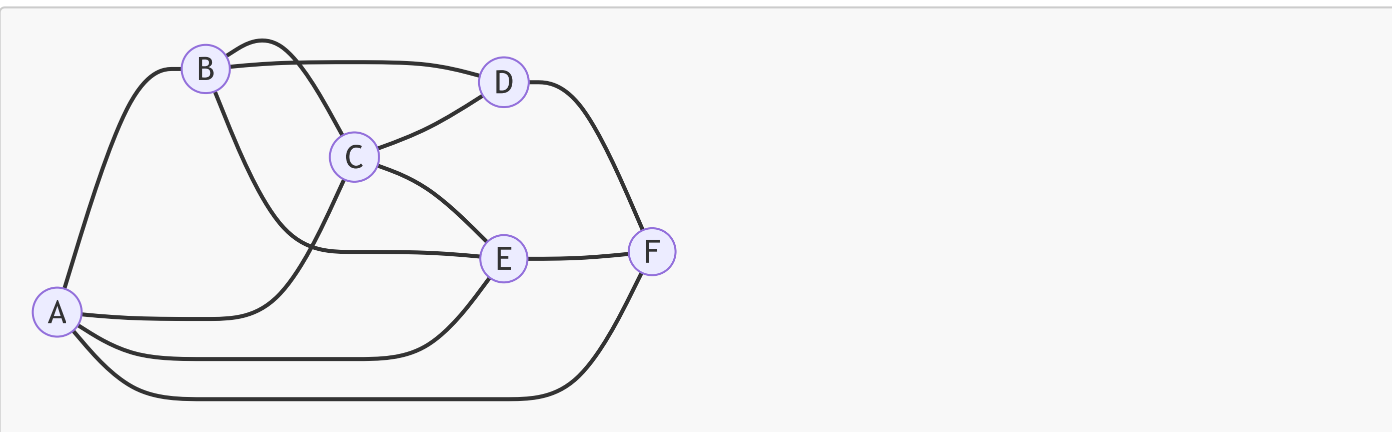
Homework 5

Problem 1

> Given the following adjacency matrix:
>
> $$\begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

The matrix size is 6×6, so there are 6 nodes in the graph.

$$\begin{array}{c|c:c:c:c:c:c} & A & B & C & D & E & F \\ \hline A & \infty & 1 & 1 & \infty & 1 & 1 \\ B & 1 & \infty & 1 & 1 & 1 & \infty \\ C & 1 & 1 & \infty & 1 & 1 & \infty \\ D & \infty & 1 & 1 & \infty & \infty & 1 \\ E & 1 & 1 & 1 & \infty & \infty & 1 \\ F & 1 & \infty & \infty & 1 & 1 & \infty \end{array}$$

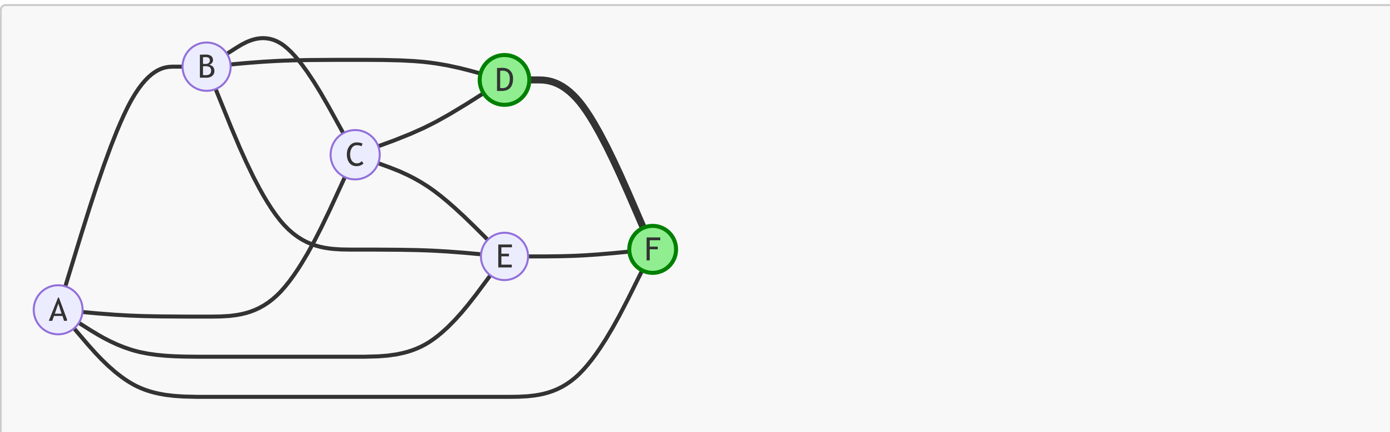


> 1. Provide the maximal 2-cliques for this graph.

Cliques refer to connected vertices where no additional vertices can be added to create a larger clique. With maximal 2-cliques, we consider two vertices that will not form a triangle with any other vertex.

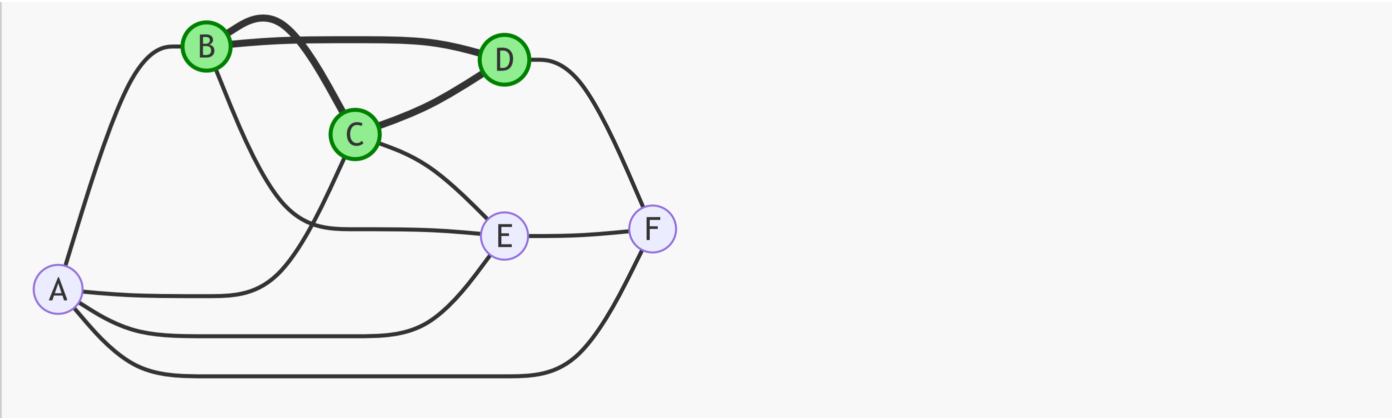
Clique ($k = 2$)	Shared neighbors	Can expand
A, B	C, E	✓
A, C	B, E	✓
A, E	B, C, F	✓

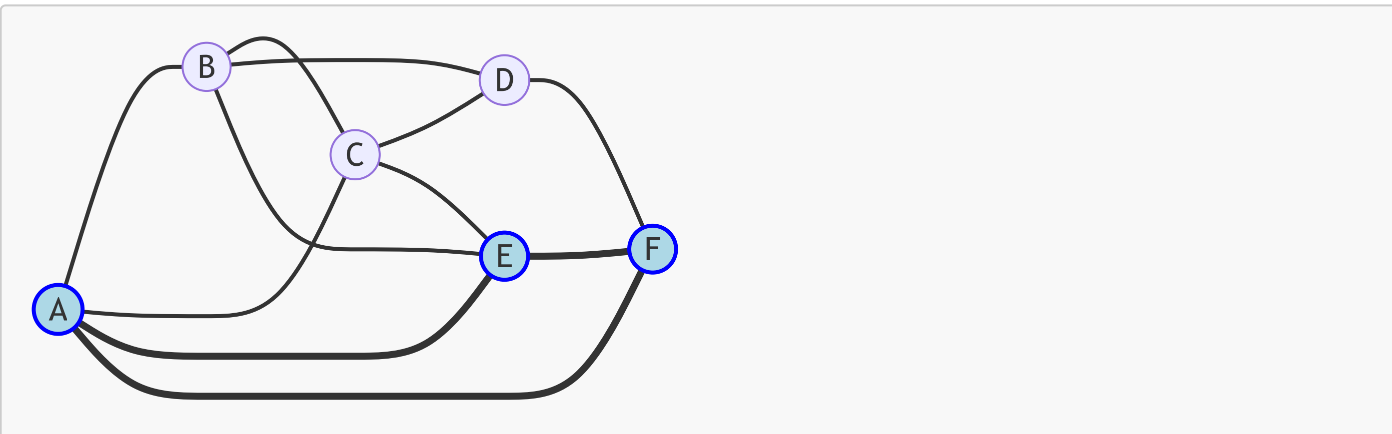
Clique ($k = 2$)	**Shared neighbors**	**Can expand**
A, F	E	✓
B, C	A, D, E	✓
B, D	C	✓
B, E	A, C	✓
C, D	B	✓
C, E	A, B	✓
D, F		✗
E, F	A	✓



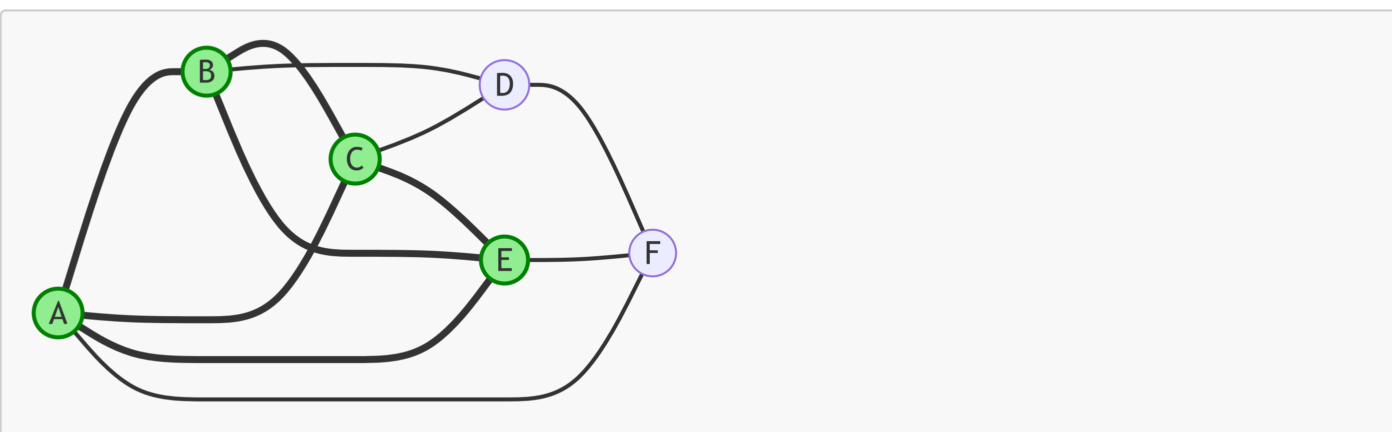
However, if maximal 2-cliques mean at least 2 vertices ($k \geq 2$), we need to consider larger sets of vertices:

Clique ($k = 3$)	**Shared neighbors**	**Can expand**
A, B, C	E	✓
A, B, E	C, F	✓
A, C, E	B, F	✓
B, C, D		✗
B, C, E	A	✓
A, E, F		✗





Clique ($k = 4$)	**Shared neighbors**	**Can expand**
A, B, C, E		✗

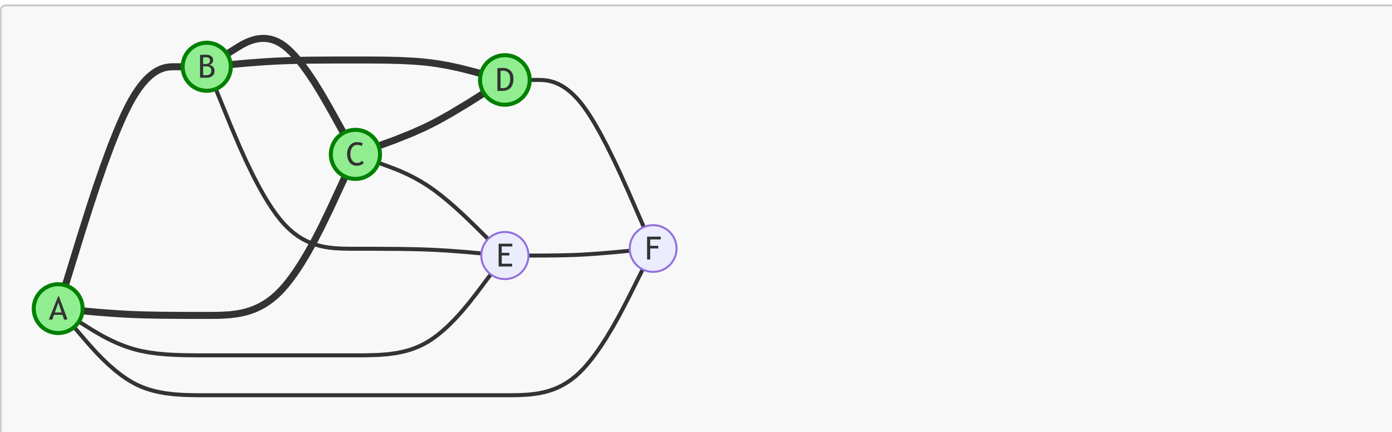


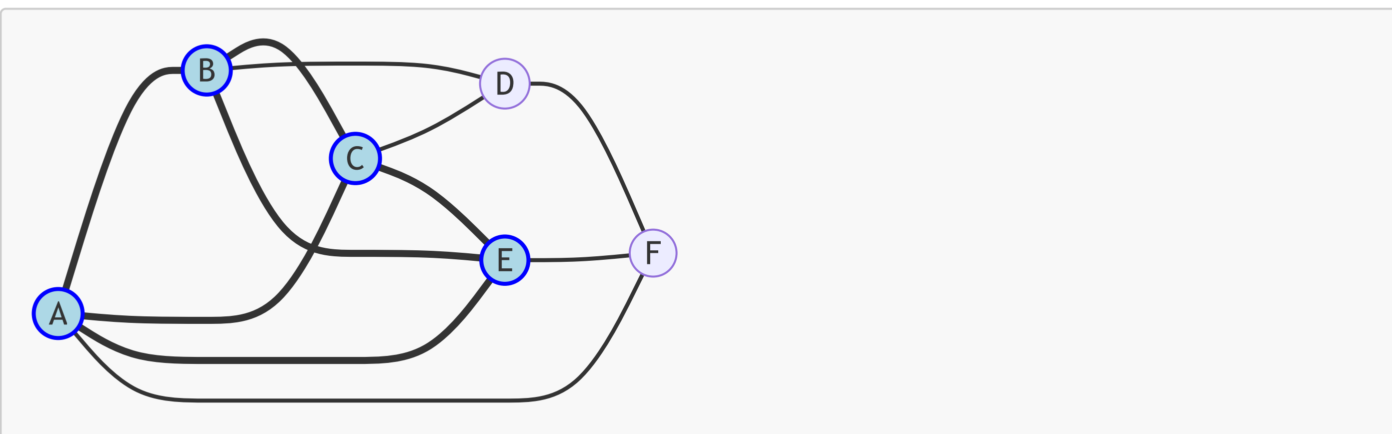
> 2. Provide the maximal 2-plexes for this graph.

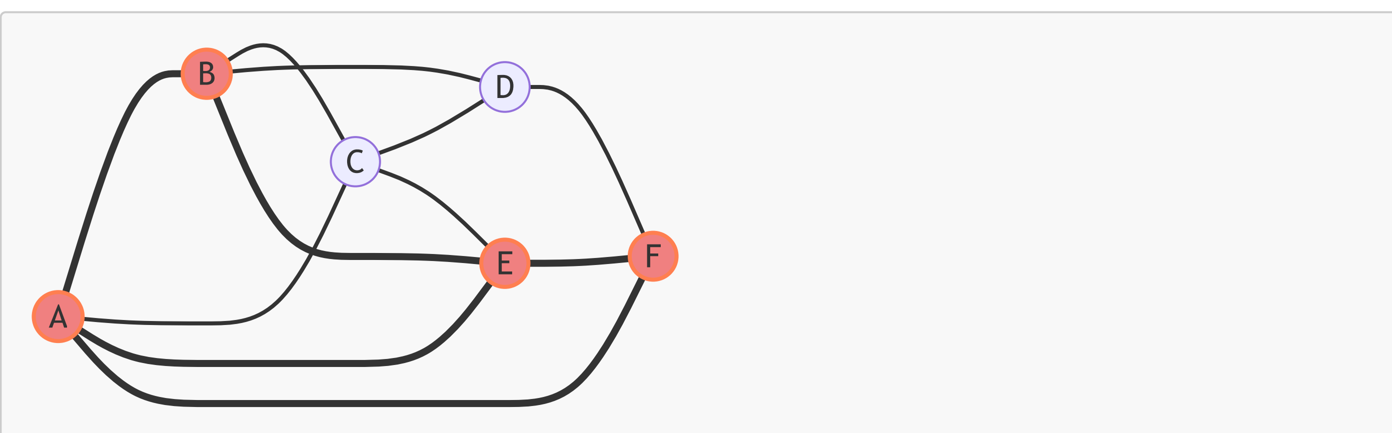
A set of vertices S is a 2-plex if every vertex $v \in S$ has a degree of at least $|S| - 2$ neighbors. For 6 vertices, the minimum required degree is 4. Note that a plex cannot contain a subsequent plex of a smaller size. In the table below, the set of vertices with size 5 is excluded because it contains sets of size 4 that are also 2-plexes.

Plex ($k = 2$)	**Missing edges**	**Required degree**	**Minimum degree**	**Has clique**
A, B, C, D	A $\rightarrow$ D	2	2	✗

Plex ($k = 2$)	Missing edges	Required degree	Minimum degree	Has clique
A, B, C, E		2	3	✗
A, B, E, F	B → F	2	2	✗
A, B, C, D, E	B → F	3	3	✓
A, B, C, E, F	D → B	3	3	✓
B, C, D, E, F		3	3	✓



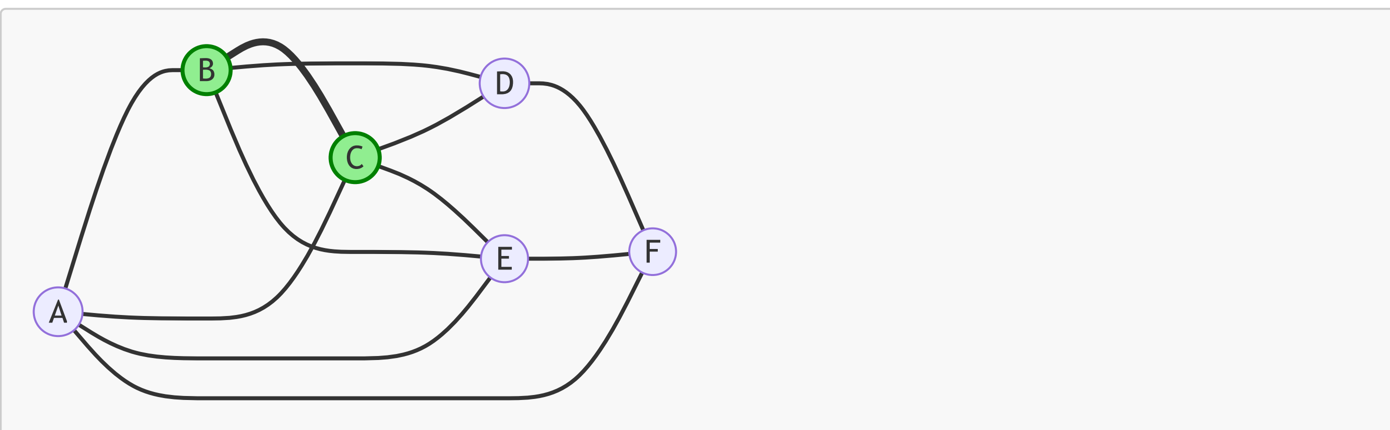




> 3. Are there any structually equivalent nodes in this graph? If yes, list them.

Structurally equivalent vertices have identical edges to all other vertices in the graph.

Path	Degrees	A	B	C	D	E	F	Equivalent
A → B	4/4	A: ↩ B: ✓	A: ✓ B: ↩	A: ✓ B: ✓	A: ✗ B: ✓	A: ✓ B: ✓	A: ✓ B: ✗	✗
A → C	4/4	A: ↩ C: ✓	A: ✓ C: ✓	A: ✓ C: ↩	A: ✗ C: ✓	A: ✓ C: ✓	A: ✓ C: ✗	✗
A → E	4/4	A: ↩ E: ✓	A: ✓ E: ✓	A: ✓ E: ✓	A: ✗ E: ✗	A: ✓ E: ↩	A: ✓ E: ✓	✗
B → C	4/4	B: ✓ C: ✓	B: ↩ C: ✓	B: ✓ C: ↩	B: ✓ C: ✓	B: ✓ C: ✓	B: ✗ C: ✗	✓
B → E	4/4	B: ✓ E: ✓	B: ↩ E: ✓	B: ✓ E: ✓	B: ✓ E: ✗	B: ✓ E: ↩	B: ✗ E: ✓	✗
C → E	4/4	C: ✓ E: ✓	C: ✓ E: ✓	C: ↩ E: ✓	C: ✓ E: ✗	C: ✓ E: ↩	C: ✗ E: ✓	✗
D → F	3/3	D: ✗ F: ✓	D: ✓ F: ✗	D: ✓ F: ✗	D: ↩ F: ✓	D: ✗ F: ✓	D: ✓ F: ↩	✗



Problem 2

> Using the graph from problem 1,
>
> 1. Find a minimum cut that creates partition P_1 and P_2 where $|P_1| = 4$ and $|P_2| = 2$.

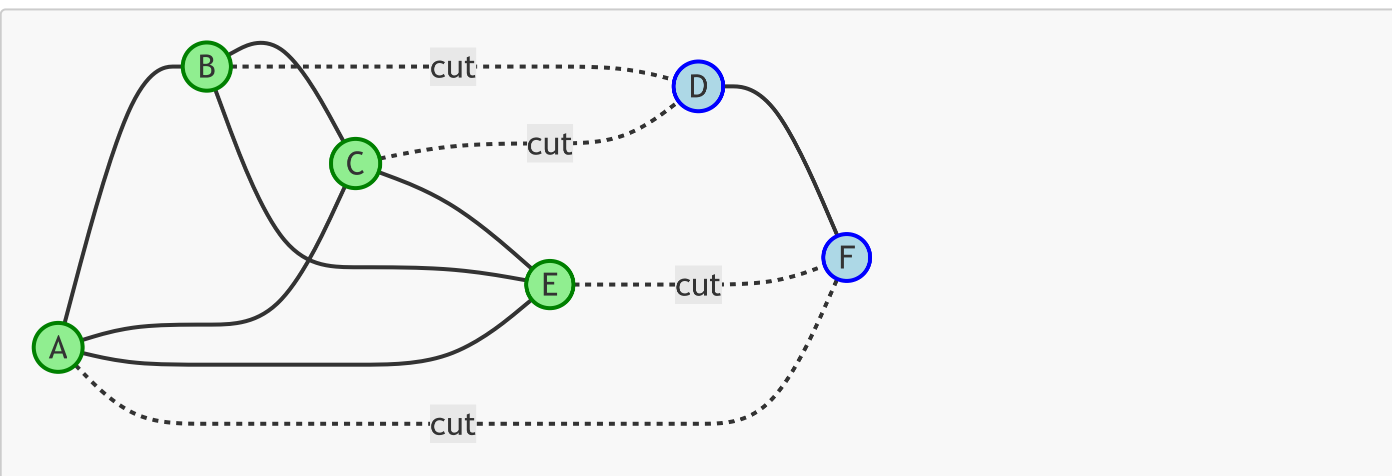
Iterate all permutations of partitions P_1 and P_2 to find the cut sizes. The minimum cut size is 4, found in $P_1 = \{A, B, C, E\}$ and $P_2 = \{D, F\}$.

Parition P_1	Parition P_2	Crossing edges	Cut size

Parition P_1	**Parition P_2**	**Crossing edges**	**Cut size**
A, B, C, D	E, F	A $\rightarrow$ E A $\rightarrow$ F B $\rightarrow$ E C $\rightarrow$ E E $\rightarrow$ F	5
A, B, C, E	D, F	A $\rightarrow$ F B $\rightarrow$ D C $\rightarrow$ D E $\rightarrow$ F	4
A, B, C, F	D, E	A $\rightarrow$ E B $\rightarrow$ D B $\rightarrow$ E C $\rightarrow$ D C $\rightarrow$ E	5
A, B, D, E	C, F	A $\rightarrow$ C A $\rightarrow$ F B $\rightarrow$ C C $\rightarrow$ D C $\rightarrow$ E D $\rightarrow$ F E $\rightarrow$ F	7
A, B, D, F	C, E	A $\rightarrow$ C A $\rightarrow$ E B $\rightarrow$ C B $\rightarrow$ E C $\rightarrow$ D C $\rightarrow$ E	6
A, B, E, F	C, D	A $\rightarrow$ C B $\rightarrow$ C B $\rightarrow$ D C $\rightarrow$ D C $\rightarrow$ E	5

Parition P_1	Parition P_2	Crossing edges	Cut size
A, C, D, E	B, F	A → B A → F B → C B → D B → E D → F E → F	7
A, C, D, F	B, E	A → B A → E B → C B → D B → E C → E	6
A, C, E, F	B, D	A → B B → C B → D B → E C → D	5
A, D, E, F	B, C	A → B A → C B → C B → D B → E C → D C → E	7
B, C, D, E	A, F	A → B A → C A → E A → F D → F	5
B, C, D, F	A, E	A → B A → C A → E A → F B → E C → E E → F	7

Parition P_1	Parition P_2	Crossing edges	Cut size
B, C, E, F	A, D	A → B A → C A → E A → F B → D C → D	6
B, D, E, F	A, C	A → B A → C A → E A → F B → C C → D C → E	7
C, D, E, F	A, B	A → B A → C A → E A → F B → C B → D B → E	7



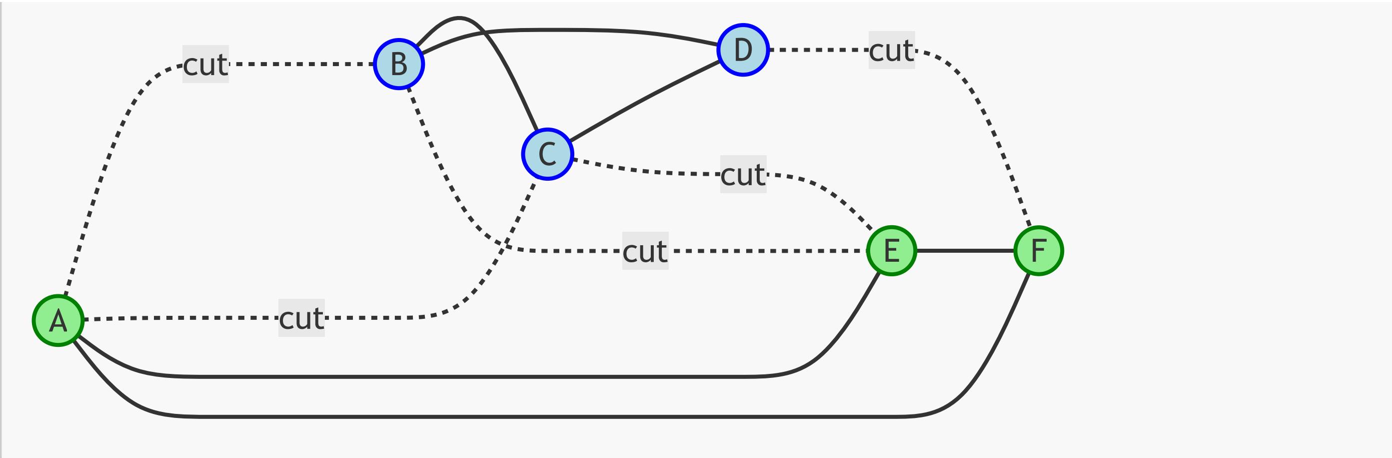
> 2. Find a minimum cut that creates partition P_1 and P_2 where $|P_1| = |P_2| = 3$.

For both partitions of size 3, the minimum cut size is 5 as seen in $P_1 = \{A, E, F\}$ and $P_2 = \{B, C, D\}$.

Partition P_1	Parition P_2	Crossing edges	Cut size

Partition P_1	Parition P_2	Crossing edges	Cut size
A, E, F	B, C, D	A → B A → C E → B E → C F → D	5
A, B, C	D, E, F	A → E A → F B → D B → E C → D C → E	6
A, B, E	C, D, F	A → C A → F B → C B → D E → C E → F	6
A, C, E	B, D, F	A → B A → F C → B C → D E → B E → F	6
A, D, F	B, C, E	A → B A → C A → E D → B D → C F → E	6
A, B, D	C, E, F	A → C A → E A → F B → C B → E D → C D → F	7

Partition P_1	**Parition P_2**	**Crossing edges**	**Cut size**
A, B, F	C, D, E	A $\rightarrow$ C A $\rightarrow$ E B $\rightarrow$ C B $\rightarrow$ D B $\rightarrow$ E F $\rightarrow$ D F $\rightarrow$ E	7
A, C, D	B, E, F	A $\rightarrow$ B A $\rightarrow$ E A $\rightarrow$ F C $\rightarrow$ B C $\rightarrow$ E D $\rightarrow$ B D $\rightarrow$ F	7
A, C, F	B, D, E	A $\rightarrow$ B A $\rightarrow$ E C $\rightarrow$ B C $\rightarrow$ D C $\rightarrow$ E F $\rightarrow$ D F $\rightarrow$ E	7
A, D, E	B, C, F	A $\rightarrow$ B A $\rightarrow$ C A $\rightarrow$ F D $\rightarrow$ B D $\rightarrow$ C D $\rightarrow$ F E $\rightarrow$ B E $\rightarrow$ C E $\rightarrow$ F	9



> 3. For each of the cuts in (1) and (2) above, calculate
>   1. Ratio cut (P)
>   2. Normalized cut (P)

The first problem is partitioned by 4-2.

$$
\begin{aligned}
P_1 &= \{A, B, C, E\} \\
P_2 &= \{D, F\} \\
\texttt{RATIO-CUT}(P_1, P_2) &= \frac{\texttt{CUT}(P_1, P_2)}{|P_1|} + \frac{\texttt{CUT}(P_1, P_2)}{|P_2|} \\
&= \frac{4}{4} + \frac{4}{2} \\
&= 1 + 2 &= \mathbf{3} \\
\texttt{NORMALIZED-CUT}(P_1, P_2) &= \frac{\texttt{CUT}(P_1, P_2)}{\sum_{v \in P_1} \deg(v)} + \frac{\texttt{CUT}(P_1, P_2)}{\sum_{v \in P_2} \deg(v)} \\
&= \frac{4}{16} + \frac{4}{6} \\
&= \frac{12}{48} + \frac{32}{48} \\
&= \frac{44}{48} &= \mathbf{\frac{11}{12}}
\end{aligned}
$$

Both partitions in the second problem are of size 3 (3-3).

$$
\begin{aligned}
P_1 &= \{A, E, F\} \\
P_2 &= \{B, C, D\} \\
\texttt{RATIO-CUT}(P_1, P_2) &= \frac{5}{3} + \frac{5}{3} &= \mathbf{\frac{10}{3}} \\
\texttt{NORMALIZED-CUT}(P_1, P_2) &= \frac{5}{11} + \frac{5}{11} &= \mathbf{\frac{10}{11}}
\end{aligned}
$$

> 4. Which cut is preferable based on the above metrics?

Looking at the cut size, the 4-2 partition is preferable because it is smaller. The 4-2 partition is also preferable in the ratio cut. However, the 3-3 partition is slightly better in normalized cut.

Metric	Partitions $\|P_1\| = 4$ and $\|P_2\| = 2$	Partitions $\|P_1\| = 3$ and $\|P_2\| = 3$
Cut size	4	5
Ratio cut	3	10/3
Normalized cut	11/12	10/11

Problem 3

Social Media Mining (SMM) *Ch. 6, problem 7:*

Community Detection

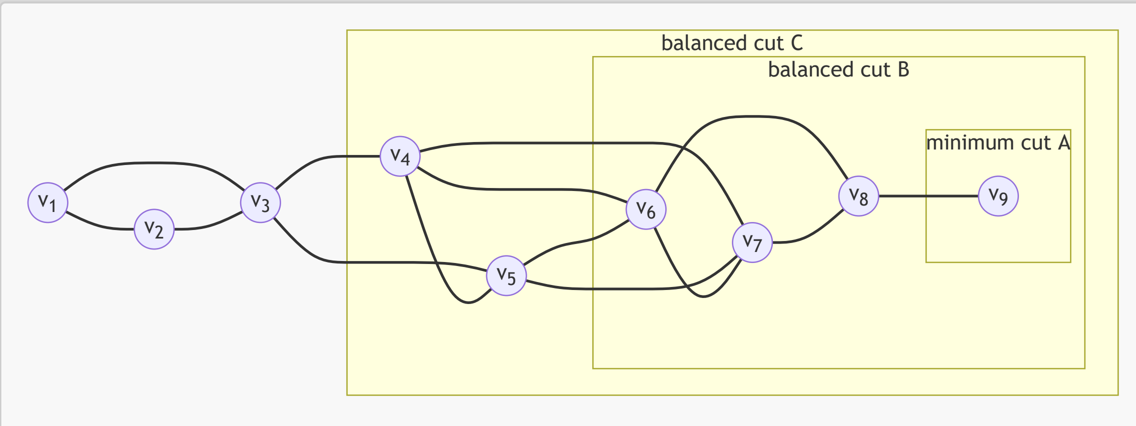


Figure 6.8: Minimum Cut (A) and Two More Balanced Cuts (B and C) in a Graph.

For Figure 6.8:

- Compute Jaccard and Cosine similarity between nodes v_4 and v_8, assuming that the neighborhood of a node excludes the node itself.

Let A, B be the neighborhood of v_4 and v_8, respectively, excluding the vertices themselves. Calculate the intersection and union of the two sets. Then, using the values, compute Jaccard and Cosine similarity.

$$
\begin{aligned}
A &= \{v_3, v_5, v_6, v_7\} \\
B &= \{v_6, v_7, v_9\} \\
A \cap B &= \{v_6, v_7\} \\
A \cup B &= \{v_3, v_5, v_6, v_7, v_9\} \\
\texttt{JACCARD}(A, B) &= \frac{|A \cap B|}{|A \cup B|} & = \mathbf{\frac{2}{5}} \\
\texttt{COSINE}(A, B) &= \frac{|A \cap B|}{\sqrt{|A| \cdot |B|}} \\
&= \frac{2}{\sqrt{4 \cdot 3}} \\
&= \frac{2}{2\sqrt{3}} & = \mathbf{\frac{1}{\sqrt{3}}}
\end{aligned}
$$

> - Compute Jaccard and Cosine similarity when the node is included in the neighborhood.

When the vertices themselves are included, both A and B will have an additional vertex.

$$\begin{aligned}
A &= \{v_3, v_4, v_5, v_6, v_7\} \\
B &= \{v_6, v_7, v_8, v_9\} \\
A \cap B &= \{v_6, v_7\} \\
A \cup B &= \{v_3, v_4, v_5, v_6, v_7, v_8, v_9\} \\
\texttt{JACCARD}(A, B) &= \frac{|A \cap B|}{|A \cup B|} &= \mathbf{\frac{2}{7}} \\
\texttt{COSINE}(A, B) &= \frac{|A \cap B|}{\sqrt{|A| \cdot |B|}} \\
&= \frac{2}{\sqrt{5 \cdot 4}} \\
&= \frac{2}{2\sqrt{5}} &= \mathbf{\frac{1}{\sqrt{5}}}
\end{aligned}$$

Problem 4

> SMM *Ch. 6, problem 9:*
>
> > **Community Evolution**
> >
> > Normalized mutual information (NMI) is used to evaluate community detection results when the actual communities (labels) are known beforehand.
>
> > - What are the maximum and minimum values for the NMI? Provide details.

According to the textbook, NMI starts from 0 and ends at 1. A value of 0 occurs when the detected communities are completely independent of the ground truth communities, that is, there is no mutual information shared between them. Conversely, 1 indicates that there is a perfect agreement between partitions, meaning that every vertex belongs to the correct community.

> - Explain how NMI works (describe the intuition behind it).

Mutual information (MI) is a metric to evaluate how much information regarding the ground truth communities is obtained from the detected communities. However, the value can be manipulated, resulting in higher purity values in some scenarios. Normalized mutual information (NMI) is designed to be a more accurate measurement because it accounts for the size and number of communities in both the detected and ground truth partitions.

Problem 5

> SMM *Ch. 6, problem 10:*
>
> > **Community Evaluation**

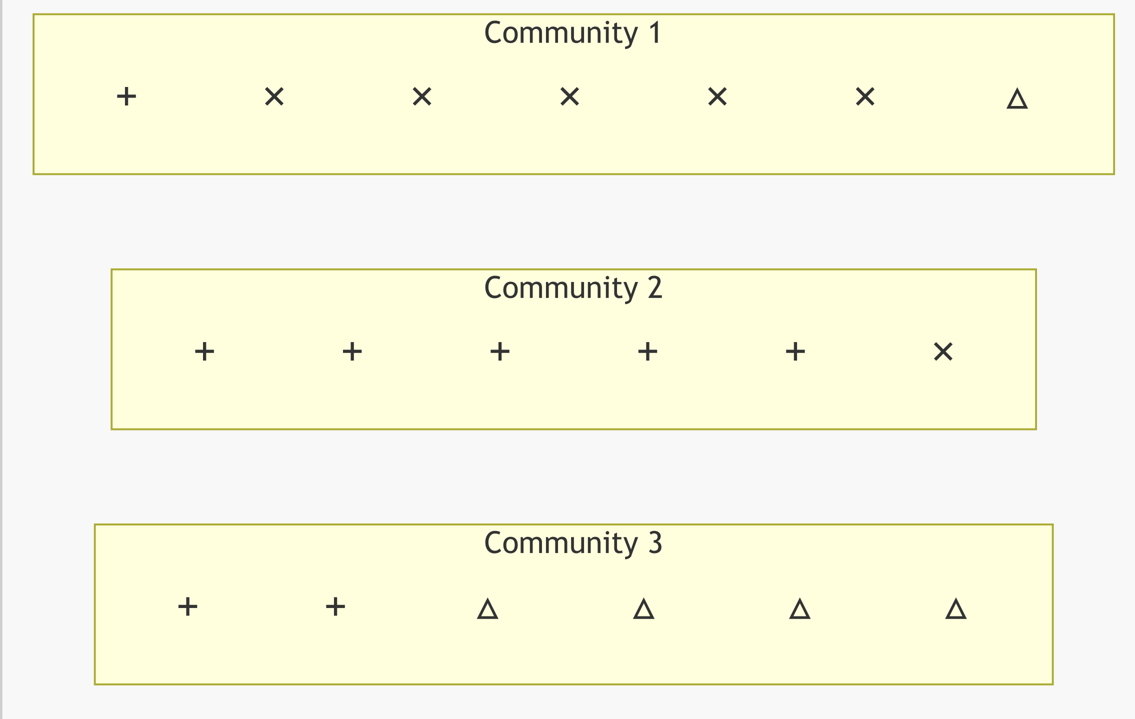


Figure 6.15: Community Evaluation Example. Circles represent communities, and items inside the circles represent members. Each item is represented using a symbol, **+, ×,** or **△,** that denotes the item's true label.

Compute NMI for Figure 6.15.

First, count the number of vertices in each community and label. In this example, the true labels are represented by symbols.

$$\begin{aligned}
|C_1| &= 7 \\
|C_2| &= 6 \\
|C_3| &= 6 \\
|L_1| &= \{+, \ldots\} & &= 8 \\
|L_2| &= \{\times, \ldots\} & &= 6 \\
|L_3| &= \{\triangle, \ldots\} & &= 5 \\
N &= 8 + 6 + 5 & &= \mathbf{19}
\end{aligned}$$

Community C	**Probability** $P(C)$	**Label** L	**Probability** $P(L)$
C_1	7/19	L_1	8/19
C_2	6/19	L_2	6/19
C_3	6/19	L_3	5/19

Then, calculate mutual information by summing over all non-zero $P_{i,j}$ community-label pairs.

$$\{C_1, +\} = \frac{1}{19} \cdot \lg\left(\frac{\frac{1}{19}}{\frac{8}{19} \cdot \frac{7}{19}}\right)$$

$$\begin{aligned}
&= \frac{1}{19}\cdot\lg\left(\frac{\frac{1}{19}}{\frac{56}{361}}\right) && = \mathbf{\frac{1}{19}\cdot\lg\left(\frac{19}{56}\right)}\\
\{C_2,+\} &= \frac{5}{19}\cdot\lg\left(\frac{\frac{5}{19}}{\frac{8}{19}\cdot\frac{6}{19}}\right)\\
&= \frac{5}{19}\cdot\lg\left(\frac{\frac{5}{19}}{\frac{48}{361}}\right) && = \mathbf{\frac{5}{19}\cdot\lg\left(\frac{95}{48}\right)}\\
\{C_3,+\} &= \frac{2}{19}\cdot\lg\left(\frac{\frac{2}{19}}{\frac{8}{19}\cdot\frac{6}{19}}\right)\\
&= \frac{2}{19}\cdot\lg\left(\frac{\frac{2}{19}}{\frac{48}{361}}\right) && = \mathbf{\frac{2}{19}\cdot\lg\left(\frac{19}{24}\right)}\\
\{C_1,\times\} &= \frac{5}{19}\cdot\lg\left(\frac{\frac{5}{19}}{\frac{6}{19}\cdot\frac{7}{19}}\right)\\
&= \frac{5}{19}\cdot\lg\left(\frac{\frac{5}{19}}{\frac{42}{361}}\right) && = \mathbf{\frac{5}{19}\cdot\lg\left(\frac{95}{42}\right)}\\
\{C_2,\times\} &= \frac{1}{19}\cdot\lg\left(\frac{\frac{1}{19}}{\frac{6}{19}\cdot\frac{6}{19}}\right)\\
&= \frac{1}{19}\cdot\lg\left(\frac{\frac{1}{19}}{\frac{36}{361}}\right) && = \mathbf{\frac{1}{19}\cdot\lg\left(\frac{19}{36}\right)}\\
\{C_3,\times\} & && = \mathbf{0}\\
\{C_1,\triangle\} &= \frac{1}{19}\cdot\lg\left(\frac{\frac{1}{19}}{\frac{5}{19}\cdot\frac{7}{19}}\right)\\
&= \frac{1}{19}\cdot\lg\left(\frac{\frac{1}{19}}{\frac{35}{361}}\right) && = \mathbf{\frac{1}{19}\cdot\lg\left(\frac{19}{35}\right)}\\
\{C_2,\triangle\} & && = \mathbf{0}\\
\{C_3,\triangle\} &= \frac{4}{19}\cdot\lg\left(\frac{\frac{4}{19}}{\frac{5}{19}\cdot\frac{6}{19}}\right)\\
&= \frac{4}{19}\cdot\lg\left(\frac{\frac{4}{19}}{\frac{30}{361}}\right) && = \mathbf{\frac{4}{19}\cdot\lg\left(\frac{38}{15}\right)}\\
I(X,Y) &= \sum_{i,j} P_{i,j}\cdot\lg\left(\frac{P_{i,j}}{P(C_i)\cdot P(L_j)}\right)\\
&= \frac{1}{19}\cdot\lg\left(\frac{19}{56}\right)+\frac{5}{19}\cdot\lg\left(\frac{95}{48}\right)+\frac{2}{19}\cdot\lg\left(\frac{19}{24}\right)+\\
&\quad \frac{5}{19}\cdot\lg\left(\frac{95}{42}\right)+\frac{1}{19}\cdot\lg\left(\frac{19}{36}\right)+\frac{1}{19}\cdot\lg\left(\frac{19}{35}\right)+\\
&\quad \frac{4}{19}\cdot\lg\left(\frac{38}{15}\right)
\end{aligned}$$

$$= \frac{1,229}{1,903} \qquad = \mathbf{0.64}$$

Finally, get the entropy of true labels $H(Y)$ and entropy of detected communities $H(X)$. These values are used to compute NMI. The NMI is 0.41, which is closer to 0 (independent).

$$\begin{aligned}
H(X) &= -\sum_i P_i \cdot \lg(P_i) \\
&= -\frac{8}{19} \cdot \lg\left(\frac{8}{19}\right) - \frac{6}{19} \cdot \lg\left(\frac{6}{19}\right) - \frac{5}{19} \cdot \lg\left(\frac{5}{19}\right) \\
&= \frac{2,963}{1,903} && = \mathbf{1.55} \\
H(Y) &= -\sum_j P_j \cdot \lg(P_j) \\
&= -\frac{7}{19} \cdot \lg\left(\frac{7}{19}\right) - \frac{6}{19} \cdot \lg\left(\frac{6}{19}\right) - \frac{6}{19} \cdot \lg\left(\frac{6}{19}\right) \\
&= -\frac{7}{19} \cdot \lg\left(\frac{7}{19}\right) - 2 \cdot \frac{6}{19} \cdot \lg\left(\frac{6}{19}\right) \\
&= \frac{3,012}{1,903} && = \mathbf{1.58} \\
\texttt{NMI}(X,Y) &= \frac{2 \cdot I(X,Y)}{H(X) + H(Y)} \\
&= \frac{2 \cdot \frac{1{,}229}{1{,}903}}{\frac{2{,}963}{1{,}903} + \frac{3{,}012}{1{,}903}} \\
&= \frac{\frac{2{,}458}{1{,}903}}{\frac{5{,}975}{1{,}903}} \\
&= \frac{2,458}{5,975} && = \mathbf{0.41}
\end{aligned}$$

Problem 6

> SMM *Ch. 6, problem 11:*
>
> > **Community Evaluation**
> >
> > Why is high precision not enough? Provide an example to show that both precision and recall are important.

In the context of community detection, high precision itself is not considered sufficient since it only measures the accuracy of the detected communities without considering the degree of completeness. This is why the recall metric is also important.

For example, suppose there are 100 true community edges and the algorithm predicts 5 edge pairs. All 5 predicted edges produce 100% precision. However, the recall is only 0.05, missing 95% of the true structure compared to the precision metric.

$$
\begin{aligned}
TP &= 5 \\
FP &= 0 \\
FN &= 95 \\
\texttt{PRECISION} &= \frac{TP}{TP + FP} \\
&= \frac{5}{5 + 0} &= \mathbf{1.0} \\
\texttt{RECALL} &= \frac{TP}{TP + FN} \\
&= \frac{5}{5 + 95} &= \mathbf{0.05}
\end{aligned}
$$

Problem 7

> SMM *Ch. 6, problem 12:*
>
> > **Community Evaluation**
> >
> > Discuss situations where purity does not make sense.

Purity is a metric that evaluates the true labels within detected communities, that is, the extent to which a cluster contains a single class. For each community C_k, count nodes from the most frequent true label L_j.

$$
\texttt{purity} = \frac{1}{N} \sum_k \max_j |C_k \cap L_j|
$$

However, purity is not suitable when the clusters are fragmented into many tiny clusters, such as single-node clusters. Because the communities are split into distinct parts, the purity will not account for completeness (recall), achieving a perfect 1.0 for each cluster. An unusually high purity score may also occur when clusters are imbalanced in size, where one giant cluster dominates the rest.

Problem 8

> SMM *Ch. 6, problem 13:*
>
> > **Community Evaluation**
> >
> > Compute the following for Figure 6.17:

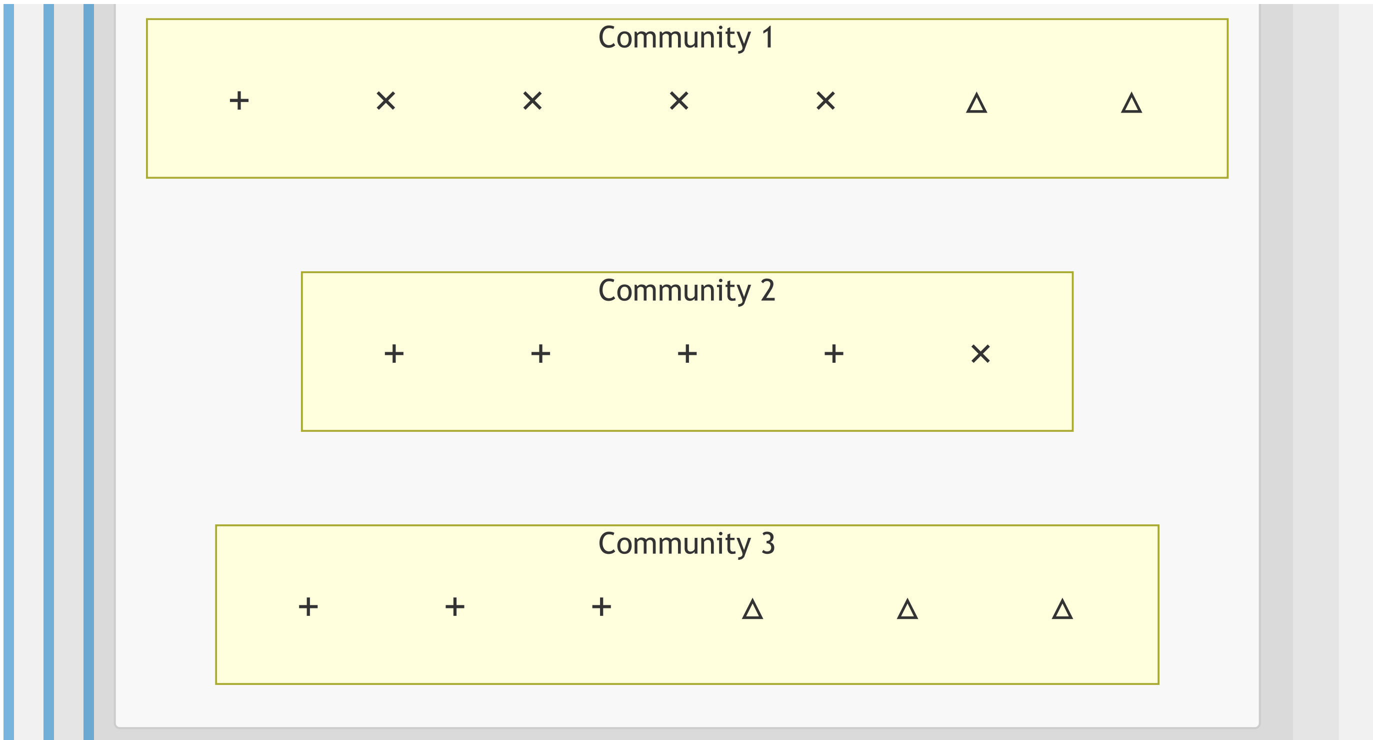


Figure 6.17: Community Evaluation Example.

Given the communities C and true labels L, get the total number of vertices N and the purity score. Find the matching cluster with the highest true positive count for each label. The purity for this set of communities is 11/18.

$$\begin{aligned}
|C_1| &= 7 \\
|C_2| &= 5 \\
|C_3| &= 6 \\
|L_1| &= \{+, \ldots\} && = 8 \\
|L_2| &= \{\times, \ldots\} && = 5 \\
|L_3| &= \{\triangle, \ldots\} && = 5 \\
N &= 8 + 5 + 5 && = \mathbf{18} \\
\texttt{purity} &= \frac{1}{N} \sum_k \max_j |C_k \cap L_j| \\
&= \frac{\max(C_1) + \max(C_2) + \max(C_3)}{N} \\
&= \frac{\max(1, 4, 2) + \max(4, 1, 0) + \max(3, 0, 3)}{18} \\
&= \frac{4 + 4 + 3}{18} && = \mathbf{\frac{11}{18}}
\end{aligned}$$

Label	**Matching cluster**	**True positive TP**
$L_1 : \{+, \ldots\}$	C_2	4
$L_2 : \{\times, \ldots\}$	C_1	4
$L_3 : \{\triangle, \ldots\}$	C_3	3

Specific to each label, calculate precision, recall and F-measure.

$$
\begin{aligned}
\texttt{PRECISION}(L_1) &= \frac{TP}{TP+FP} \\
&= \frac{4}{4+1} && = \mathbf{\frac{4}{5}} \\
\texttt{RECALL}(L_1) &= \frac{TP}{TP+FN} \\
&= \frac{4}{4+4} \\
&= \frac{4}{8} && = \mathbf{\frac{1}{2}} \\
\texttt{F-MEASURE}(L_1) &= \frac{2 \cdot \texttt{PRECISION}(L_1) \cdot \texttt{RECALL}(L_1)}{\texttt{PRECISION}(L_1) + \texttt{RECALL}(L_1)} \\
&= \frac{2 \cdot \frac{4}{5} \cdot \frac{1}{2}}{\frac{4}{5} + \frac{1}{2}} \\
&= \frac{\frac{8}{10}}{\frac{8}{10} + \frac{5}{10}} \\
&= \frac{\frac{8}{10}}{\frac{13}{10}} && = \mathbf{\frac{8}{13}} \\
\texttt{PRECISION}(L_2) &= \frac{4}{4+3} && = \mathbf{\frac{4}{7}} \\
\texttt{RECALL}(L_2) &= \frac{4}{4+1} && = \mathbf{\frac{4}{5}} \\
\texttt{F-MEASURE}(L_2) &= \frac{2 \cdot \frac{4}{7} \cdot \frac{4}{5}}{\frac{4}{7} + \frac{4}{5}} \\
&= \frac{\frac{32}{35}}{\frac{20}{35} + \frac{28}{35}} \\
&= \frac{\frac{32}{35}}{\frac{48}{35}} && = \mathbf{\frac{2}{3}} \\
\texttt{PRECISION}(L_3) &= \frac{3}{3+3} \\
&= \frac{3}{6} && = \mathbf{\frac{1}{2}} \\
\texttt{RECALL}(L_3) &= \frac{3}{3+2} && = \mathbf{\frac{3}{5}} \\
\texttt{F-MEASURE}(L_3) &= \frac{2 \cdot \frac{1}{2} \cdot \frac{3}{5}}{\frac{1}{2} + \frac{3}{5}} \\
&= \frac{\frac{6}{10}}{\frac{5}{10} + \frac{6}{10}} \\
&= \frac{\frac{6}{10}}{\frac{11}{10}} && = \mathbf{\frac{6}{11}}
\end{aligned}
$$

Label	Precision	Recall	F-measure
$L_1 : \{+, \ldots\}$	4/5	1/2	8/13
$L_2 : \{\times, \ldots\}$	4/7	4/5	2/3
$L_3 : \{\triangle, \ldots\}$	1/2	3/5	6/11

Problem 9

> SMM *Ch. 7, problem 2:*
>
> > **Herd Effect**
> >
> > What are the minimum requirements for a herd behavior experiment? Design an experiment of your own.

Herd behavior experiments are designed to study when and why individuals in a group act collectively, conforming to the majority. Such an experiment requires two components: individual connections and the capability to observe others' actions. The study will analyze whether such behavior is transferable to other contexts.

My proposed experiment involves participants posting product reviews on e-commerce platforms. It is a variation of the classic urn problem by Anderson and Holt, where two urns represent verified and likely fake reviews. To be verified as a good review, some criteria, such as when the reviewer joined the platform and how many reviews they have written, must be met. It is important to note that negative reviews can still be considered genuine if they meet the verification criteria. Participants will post reviews sequentially to update the private signals reflecting the quality of the reviews.

When the probability is set to a good enough threshold, like 0.7, the reviews are more likely to be genuine. However, if the initial reviews are fake, subsequent reviewers may also post fake reviews, leading to an information cascade. Identifying the cascade point is crucial to determining the success of this experiment.

Problem 10

> SMM *Ch. 7, problem 3:*
>
> > **Diffusion of Innovation**
> >
> > Simulate internal-, external-, and mixed-influence models in a program. How are the saturation levels different for each model?

Internal, external and mixed in diffusion models refer to how the saturation levels differ based on the source of influence. With external influence, where only mass campaigns are considered, the growth is exponential until it reaches a saturation point. In contrast, internal influence's adoption happens through peer-to-peer interactions, resulting in logistic growth. Meanwhile, mixed influence combines both factors, leading to a bass diffusion model.

$$\begin{aligned}
\texttt{EXTERNAL-N}(t) &= p \cdot (M - N(t)) \\
\texttt{INTERNAL-N}(t) &= q \cdot \frac{N(t)}{M} \cdot (M - N(t)) \\
\texttt{MIXED-N}(t) &= p(M - N(t)) + q \cdot \frac{N(t)}{M} \cdot (M - N(t))
\end{aligned}$$

Suppose the total market size $M = 1,000$ and time steps $T = 50$. Let the external influence $p = 0.03$, representing a very low decimal value. The internal influence $q = 0.38$ indicates moderate peer influence. The external model is expected to show exponential decay towards saturation, never quite reaching the 100% mark. The internal model starts slowly, but accelerates before leveling off as it approaches saturation, visually resembling an S-curve. The mixed model is the fastest method to reach the highest saturation level. At time step $t = 50$, $N(t)$ values are approximately 80, 90 and 100 for external, internal and mixed models, respectively.

Problem 11

> SMM *Ch. 7, problem 4:*
>
> > **Diffusion of Innovation**
> >
> > Provide a simple example of diffusion of innovations and suggest a specific way of intervention to expedite the diffusion.

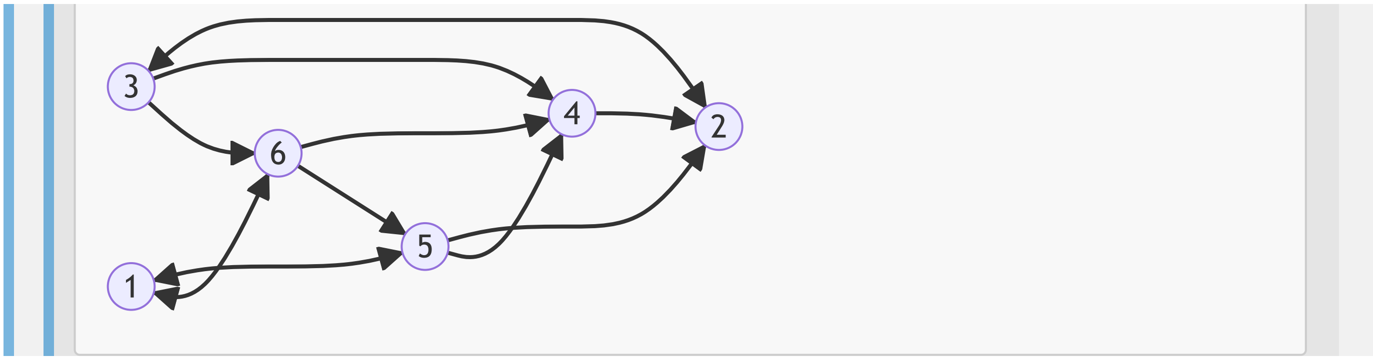
Diffusion of innovations is a process of understanding how new products, practices or technologies are adopted over time. One example is the adoption of radio-frequency identification (RFID) technology found in credit cards, transit passes and identification badges. When it was first patented in 1973, the equipment and infrastructure to support RFID were too expensive for commercial use. However, the adoption rate quickly surged when the cost plummeted. In 2011, a tag could be produced for less than 10 cents.

The diffusion model for RFID technology follows a mixed model where both internal and external influences play a role. Early adopters were limited to large enterprises and government agencies that could afford the high cost, which is the external influence. As the technology matured and prices dropped, internal influence such as businesses adopting the technology because their competitors did so too.

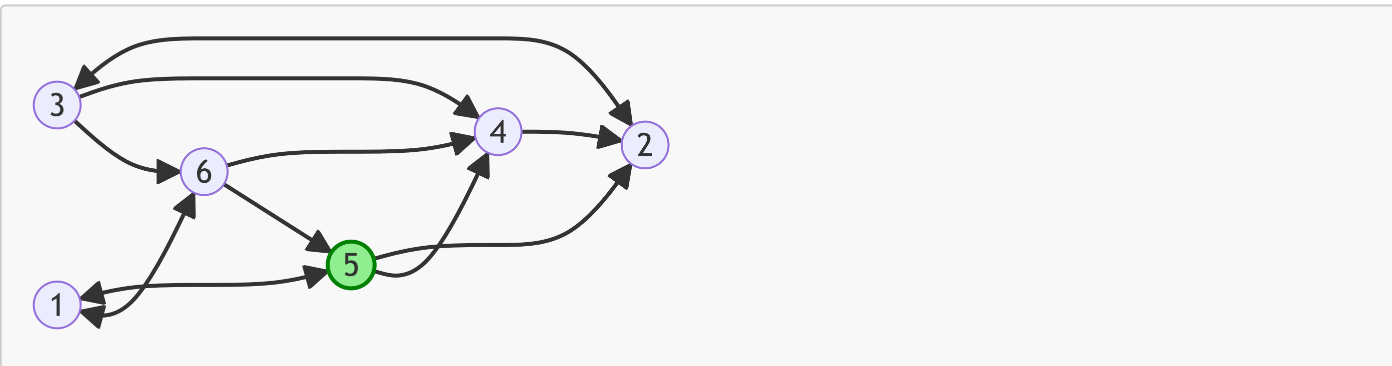
Expediting the RFID diffusion process can be achieved through funding initiatives to subsidize the initial costs for smaller businesses. Internally, create a set of agreed standards, such as frequency bands, to promote compatibility and security.

Problem 12

> SMM *Ch. 7, problem 7:*
>
> > **Information Cascades**
> >
> > Follow the ICM procedure until it converges for the following graph. Assume that node i activates node j when $i - j \equiv 1 \pmod 3$ and node 5 is activated at time 0.



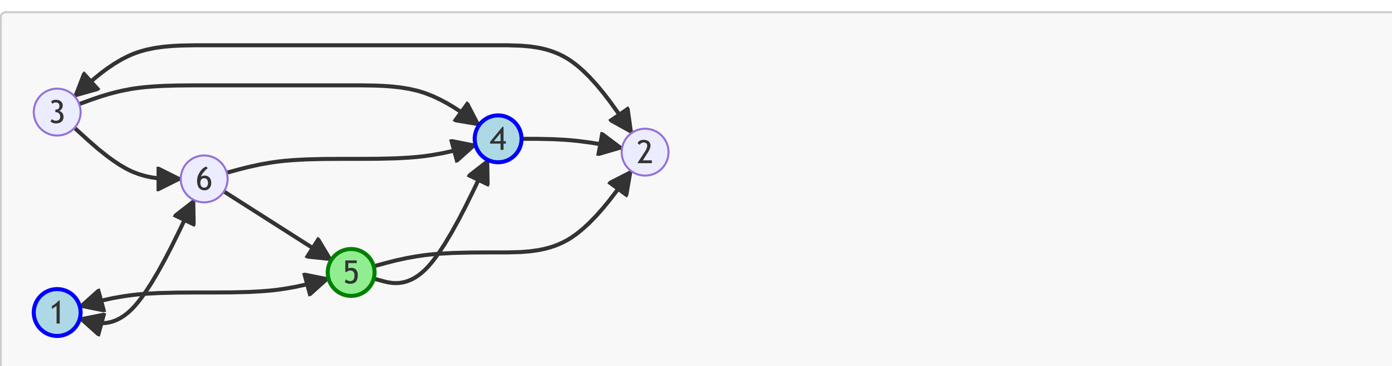
The independent cascade model (ICM) is a form of diffusion model where information spreads in a network through cascading effects. Initially, only node 5 is active.



Iteration 1

Node 5 can go to either 1, 2 or 4. However, the activation rule restricts it to only 1 and 4.

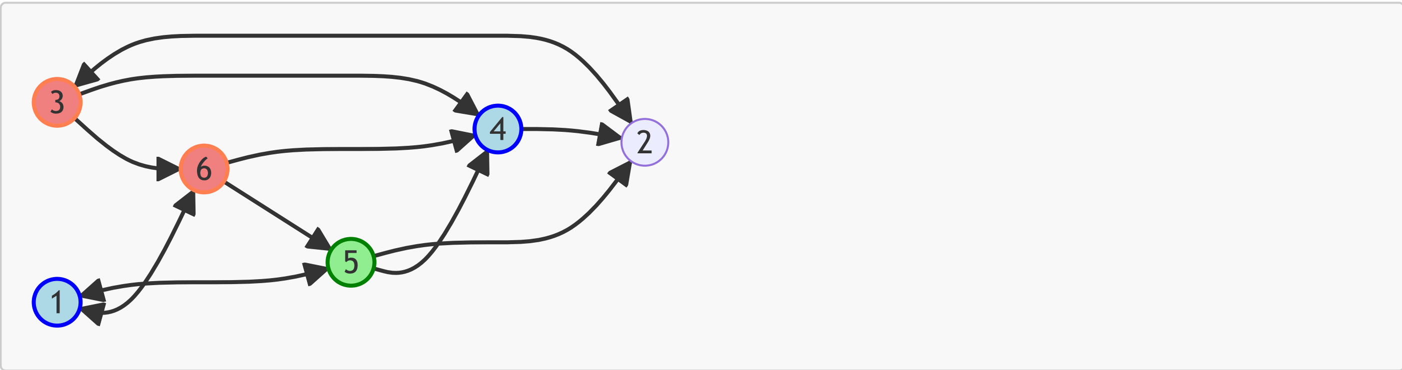
$$\begin{aligned} 1 &= (5-1) \bmod 3 \\ &= 4 \bmod 3 && = \mathbf{1} \\ 2 &= (5-2) \bmod 3 \\ &= 3 \bmod 3 && = \mathbf{0} \\ 4 &= (5-4) \bmod 3 \\ &= 1 \bmod 3 && = \mathbf{1} \end{aligned}$$



Iteration 2

With nodes 1 and 4 active, they can now activate their neighbors 3 and 6.

$$\begin{aligned} 2 &= (1-2) \bmod 3 \\ &= -1 \bmod 3 && = \mathbf{2} \\ 3 &= (1-3) \bmod 3 \\ &= -2 \bmod 3 && = \mathbf{1} \\ 6 &= (1-6) \bmod 3 \\ &= -5 \bmod 3 && = \mathbf{1} \end{aligned}$$



Iteration 3

Finally, node 3 can activate node 2. There are no other nodes to activate, ending the iterations.

$$\begin{aligned} 2 &= (3-2) \bmod 3 \\ &= 1 \bmod 3 && = \mathbf{1} \end{aligned}$$